Rotating Spirals in segregated reaction-diffusion systems

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We give a complete characterization of the boundary traces φ_i $(i = 1, \dots, K)$ supporting spiraling waves, rotating with a given angular speed ω, which appear as singular limits of competition-diffusion systems of the type

$$\begin{cases} \partial_t u_i - \Delta u_i = \mu u_i - \beta u_i \sum_{j \neq i} a_{ij} u_j & \text{in } \Omega \times \mathbb{R}^+ \\ u_i = \varphi_i & \text{on } \partial\Omega \times \mathbb{R}^+ \\ u_i(\mathbf{x}, 0) = u_{i,0}(\mathbf{x}) & \text{for } \mathbf{x} \in \Omega \end{cases}$$

as $\beta \to +\infty$. Here Ω is a rotationally invariant planar set and $a_{ij} > 0$ for every i and j. We tackle also the homogeneous Dirichlet and Neumann boundary conditions, as well as entire solutions in the plane. As a byproduct of our analysis we detect explicit families of eternal, entire solutions of the pure heat equation, parameterized by $\omega \in \mathbb{R}$, which reduce to homogeneous harmonic polynomials for $\omega = 0$.

It is a joint work with A. Salort, G. Verzini and A. Zilio

References

[1] A. Salort, S. Terracini, G. Verzini, and A. Zilio., *Rotating Spirals in segregated reaction-diffusion systems*, preprint, 2022.

[2] S. Terracini, G. Verzini, and A. Zilio. Spiraling asymptotic profiles of competition-diffusion systems. *Comm. Pure Appl. Math.*, 72(12):2578–2620, 2019.