Book Prize – The Committee's report

The Prize Committee has been communicating solely via email. The Committee ascertained that two monographs have been presented for the Prize, namely:
1. "Almost global existence of solutions for capillarity-gravity water waves equations with periodic spatial boundary conditions" by Berti-Delort,
2. "Kähler immersions of Kähler manifolds into complex space forms" by Loi-Zedda.

The Committee approved the following reports on the two monographs:

1. Report on the monograph "Almost global existence of solutions for capillarity-gravity water waves equations with periodic spatial boundary conditions".

The study of the water wave equations is a classical problem that has received the attention of many mathematicians. Let us recall the classical results of Levi-Civita, the Hamiltian formulation by Zakharov, the seminal papers by Craig, Ioos, Plotnikov and Toland. Very recently several interesting results about existence of long time or even global solutions for the water waves problems have been obtained by several authors (Alazard, Delort, Deng, Germain, Ifrim, Ionescu, Masmoudi, Pausader, Pusateri, Shatah, Tataru, etc.)

This monograph gives a relevant contribution to the field, showing that *any solution of the Cauchy problem for the capillarity–gravity water waves equations, in one space dimension, with periodic, even in space, initial data of small size ϵ, is almost globally defined in time on Sobolev spaces, i.e., it exists on a time interval of length of magnitude $\epsilon^{?N}$ for any N, as soon as the initial data are smooth enough, and the gravity-capillarity parameters are taken outside an exceptional subset of zero measure.*

This is a new and important result, being the first on long time existence

for water waves equations on the torus. Similar results are known for the equation on the real line, based however on the dispersive properties of the linear flow.

To prove the result the Authors mix tools in dynamical systems and functional analysis. In particular they follow a (non standard) normal form procedure, which, to be applied in the present case, requires several technically highly nontrivial steps: a paralinearization of the equation, several paradifferential reductions of the equations, and finally the normal form procedure.

The strategy used here to tackle the water wave problem is highly innovative, with many potential applications to other relevant classes of nonlinear differential equations.

2. Report on the monograph "Kähler immersions of Kähler manifolds into complex space forms".

This monograph is devoted to the problem of understanding when a Kähler manifold admits an isometric holomorphic immersion in one of the "model space forms" with constant curvature (the complex Euclidean space, the complex projective space, the complex hyperbolic space), even with infinite dimension. If the variety to be immersed is compact and the space form has finite dimension, the only possibility is the projective space (hence the variety has to be algebraic).

The first remark is that the Hermitian curvature of a complex subvariety is always bounded from above by the curvature of the ambient space, which gives a necessary condition for having such an immersion. In his Ph.D. thesis in 1950, Eugenio Calabi went much further by obtaining necessary and sufficient conditions for the existence of such immersions in the case of Kähler manifolds with an analytically real metric. These conditions were formulated in terms of a new object defined on such varieties, that he defines the *diastatic function*.

The diastatic function is obtained from the holomorphic prolongation of any local Kähler potential to a neighborhood of the diagonal in the product of a local chart with itself, via a sort of "algebraic polarization". It enjoys some important properties, and often can be explicitely computed. This notwithstanding, the diastatic function remained somehow aside in the current Kählerian literature, in particular there is no book or monograph on this subject.

The present monograph is therefore the first book on this topic. In the first two chapters the authors give a clear exposition of the results in Calabi's thesis. In chapter 3 they report on results by Di Scala-Ishi-Loi in the case in which the variety to be immersed is homogeneous. Chapter 4 is devoted to a famouse open problem in thi. s field, i.e., prove that the only compact Kähler–Einstein varieties which can be isometrically immersed in a projective space are homogeneous. The authors present results by Hulin, Chen and Tsukada. Chapter 5 concerns the immersion of Cartan-Hartogs' domains. In chapter 6 they treat the concept of "relatives" (two Kähler varieties admitting a common Kähler subvariety). The last chapter contains further examples. Each chapter contains various exercises and several open problems are mentioned.

The Committee then started a broad and in–depth discussion about the value of the two monographs and the eventual attribution of the prize. It has been unanimously observed that the two monographs have a quite different character. Both being research monographs, the former is mainly directed to the exposition of a new, original, important result, whereas the latter is devoted to an original, systematic and concise exposition of deep and important results, which are worth being presented in the form of a book to the community of researchers, and whose value is related not only to the expounded theory, but also to the relevant open problems introduced and discussed in it. In any event, both monographs are highly recommended for publication on the UMI-Springer Lecture Notes in Mathematics series.

Eventually the Committee observes that, given the different character of the two monographs and the high quality of both, it is quite difficult, if not impossible, to decide which deserves the prize more. For this reason, the Committee unanimously decides to attribute the Book Prize ex aequo to the the two monographs, with the following motivations:

1. "Almost global existence of solutions for capillarity-gravity water waves equations with periodic spatial boundary conditions" is an excellent, original monograph presenting a new relevant, innovative contribution to the classical problem of the study of water wave equations. The authors use, and clearly expound, different techniques and tools from various field of analysis, like dynamical systems and functional analysis. The monograph is an essentially self contained presentation of the subject that will for sure be a fundamental reference book for everybody working in the field of Hamiltonian PDE's. Moreover, it will be a very useful text for all people interested in the tools

that in this book are presented and generalized, like paradifferential calculus, normal forms for PDEs, etc.

2. "Kähler immersions of Kähler manifolds into complex space forms" is and excellent monograph presenting in a clear and original way the latest developments in an active and important research area of complex analysis, to which the authors made important contributions in recent years. Besides its intrinsic quality, the value of the monograph is stressed by the fact that the important and difficult subject treated here has never been the object of previous expositions in books or monographs. This monograph is very well written, with interesting open problems presented and discussed. This book will certainly be of the greatest interest for researchers in the field.

The Committee definitely recommends the publication of both monographs in the UMI-Springer Lecture Notes in Mathematics series.

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