Design and Analysis of Tasks: An Onto-Semiotic Approach

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1.INTRODUCTION

- The **onto-semiotic approach (OSA)**

- Design and analysis of geometric tasks
(some examples of a broader research)

- Research the possibility of using other models of plane geometry

2. The onto-semiotic approach (OSA)

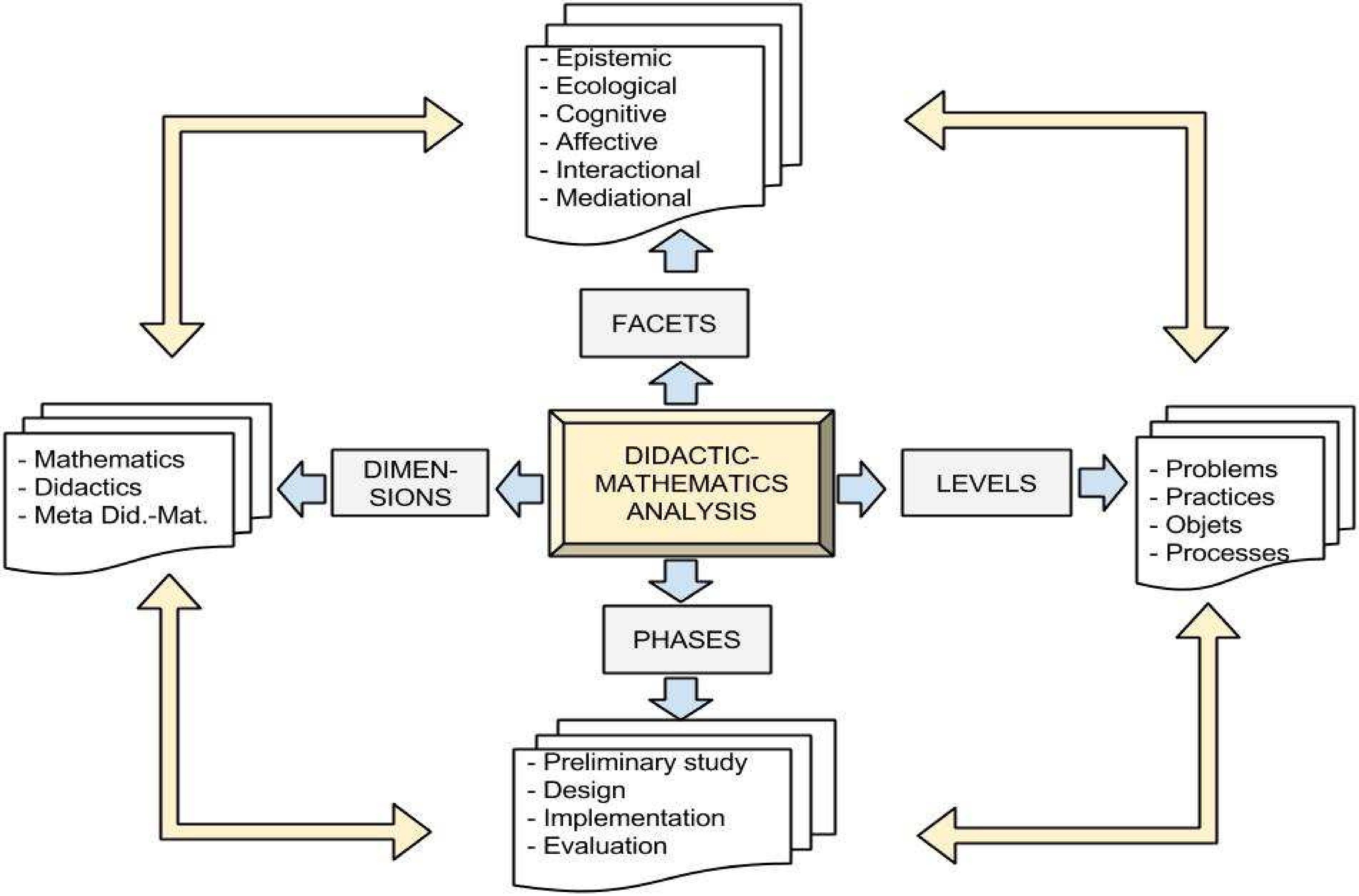
Godino and Batanero have developed a systemic and integrative approach to research in mathematical education, the Onto-Semiotic Approach (OSA).

Enfoque Ontosemiótico del Conocimiento y la Instrucción Matemática

Un marco teórico integrativo para la Didáctica de la Matemática

The starting point of the onto-semiotic approach is the formulation of an ontology of mathematical objects which takes into account three aspects of mathematics: as the **socially shared**, as a **symbolic language** and as **a logically organised conceptual system.**

2. The onto-semiotic approach (OSA)



2. The onto-semiotic approach (OSA)

Primary entities:
problem-situations;

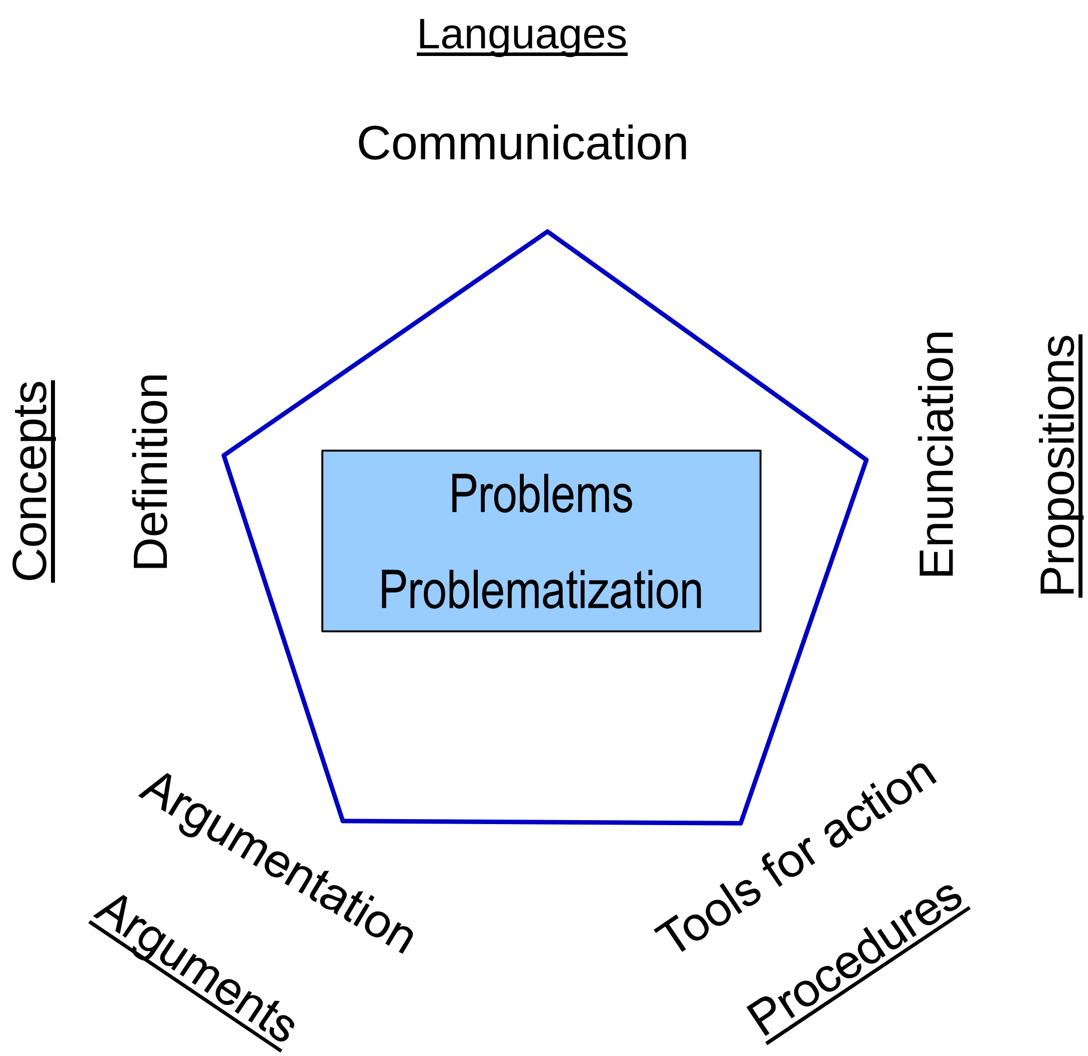
languages (e.g., terms, expressions, notations and graphs) in its various forms (e.g., written, oral, sign language);

concepts (approached through definitions or descriptions);

propositions (statements on concepts);
procedures (e.g., algorithms, operations, calculation techniques) and

arguments (statements used to validate or explain the propositions and the procedures, of deductive nature or any other type).

Objects Involved and Emerging



3. Teaching Experiment

The teaching experiment dealt with the **proof processes** of **Secondary School students**

Reasons

- Secondary School syllabus contemplates the development of various structuring forms of logic reasoning
- The mathematical concepts and their properties should be stimulated intuitively, until the students are able to work on them and reach precise mathematical formulations.

Sample

- A group of 20 students in their 1st grade of secondary school (age 15-16 years old).
- Two students (both girls 16 years old) selected for follow-up in case study according to abilities and attitudes (from high to average).

Objectives

Teaching and learning geometry, according to a diversified approach.

Poincaré half-plane
Spherical model for Riemann geometry

Intuition promotes the accomplishment of deductive reasoning.

- PHASE 1

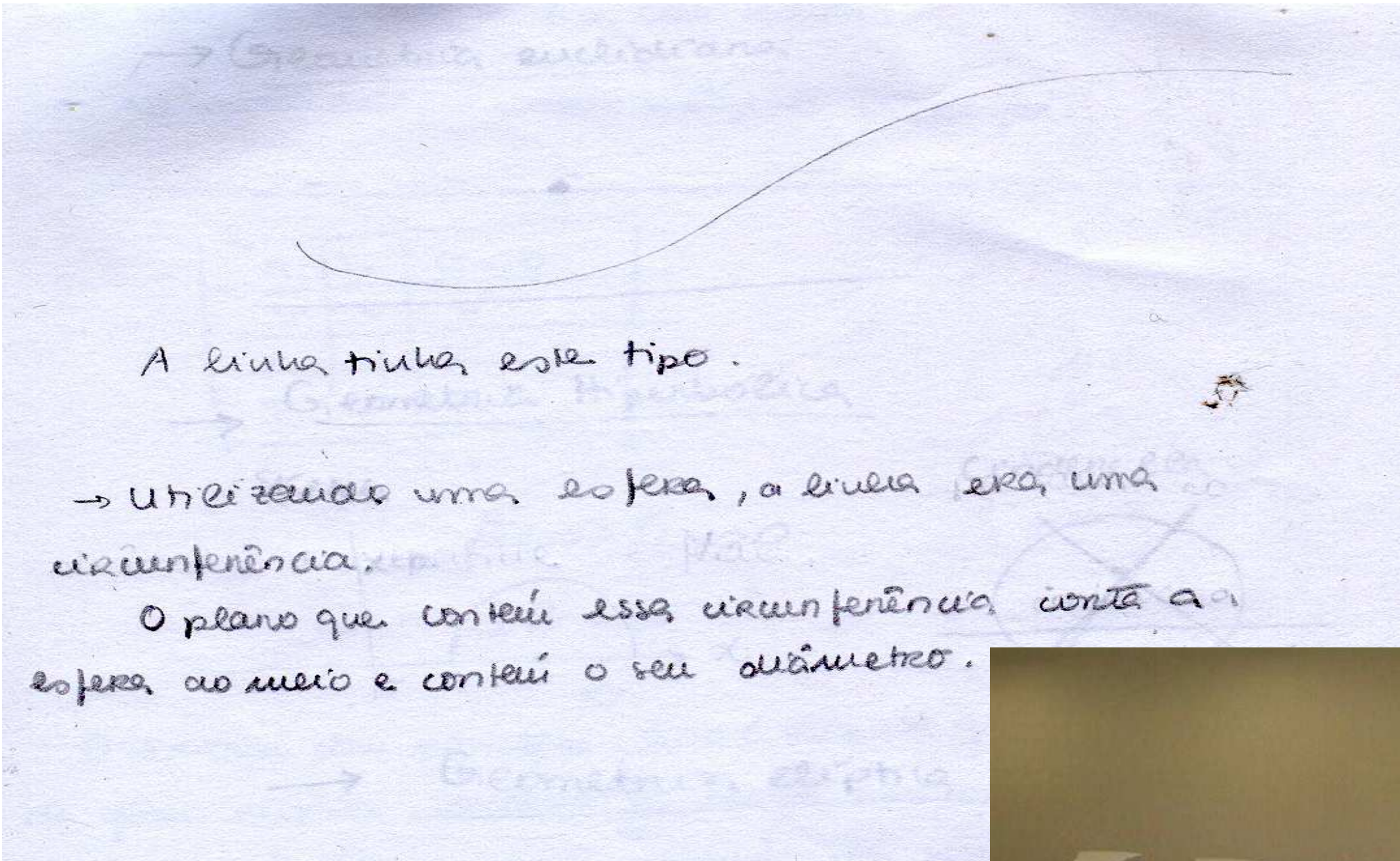
Introduction of distinct geometry models of the Euclidean model was done by resorting to objects (**percussion instrument**, **acrylic sphere**, rubber balloons, among others)

3. Teaching Experiment

- PHASE 1

Record made by one of the students regarding the shape of the line represented by the wire over part of the musical instrument and of the acrylic sphere. Note that, while working with the acrylic sphere, the students became aware that they were only able to “hold” the sphere if the wire completed a full circle.

3. Teaching Experiment



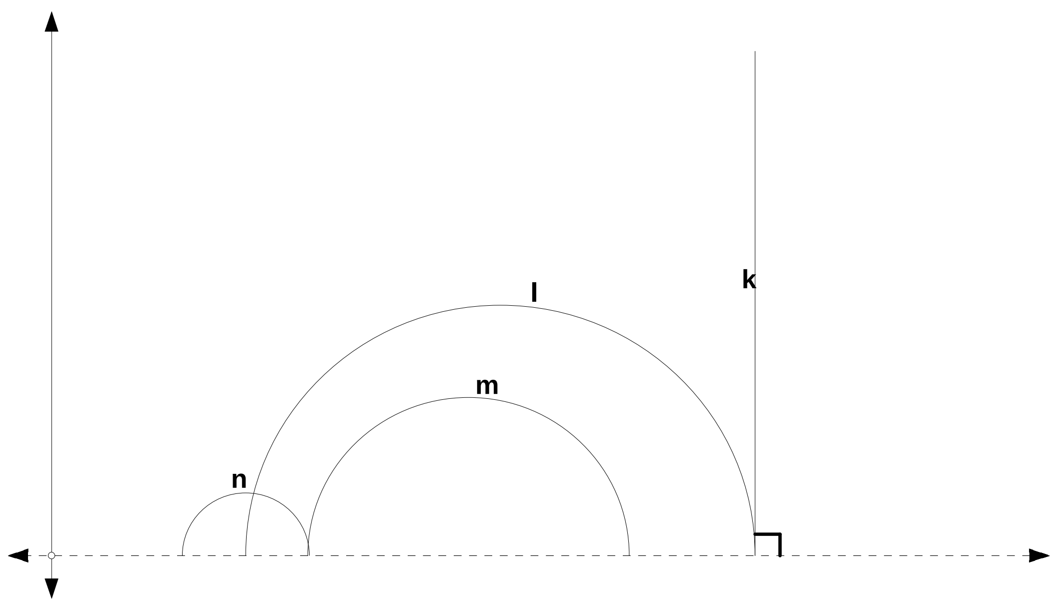
A linha tinha este tipo.

→ utilizando uma esfera, a linha era uma circunferência.

O plano que contém essa circunferência corta a esfera ao meio e contém o seu diâmetro.

- PHASE 1

The students become familiar with Dynamic Geometry Software, GSP - Geometer's Sketchpad, (half-plane model: hy_line.gss; hy segment gss)



- PHASE 1

Problem situation

Given the propositions:

A – For a given straight line and a point in the plane outside of this straight line one can draw one and only one straight line parallel to the line given.

B – The sum of the measure of the interior angles of a triangle equals 180 degrees.

Prove that the above statements are **equivalent.**

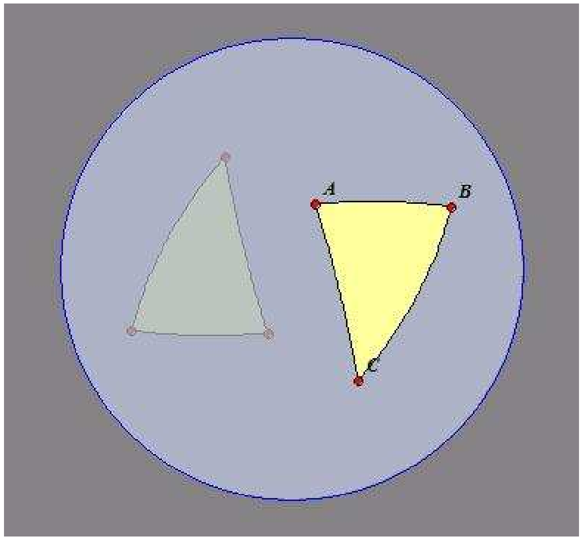
Problem situation

If the A is not verified one cannot verify the following statement: the sum of the measure of the interior angles of a triangle equals 180 degrees.

Then, the sum of the interior angles of a triangle **is less** or **greater** than 180 degrees?

And which geometry model is being used here?

After the students prepared an conjecture, they were asked to confirm or refute using GSP (half-plane model: hy_seg.gss; hy_angle.gss) and rubber balloons (Cinderella_ vistas), to visualize a triangle on a spherical surface.



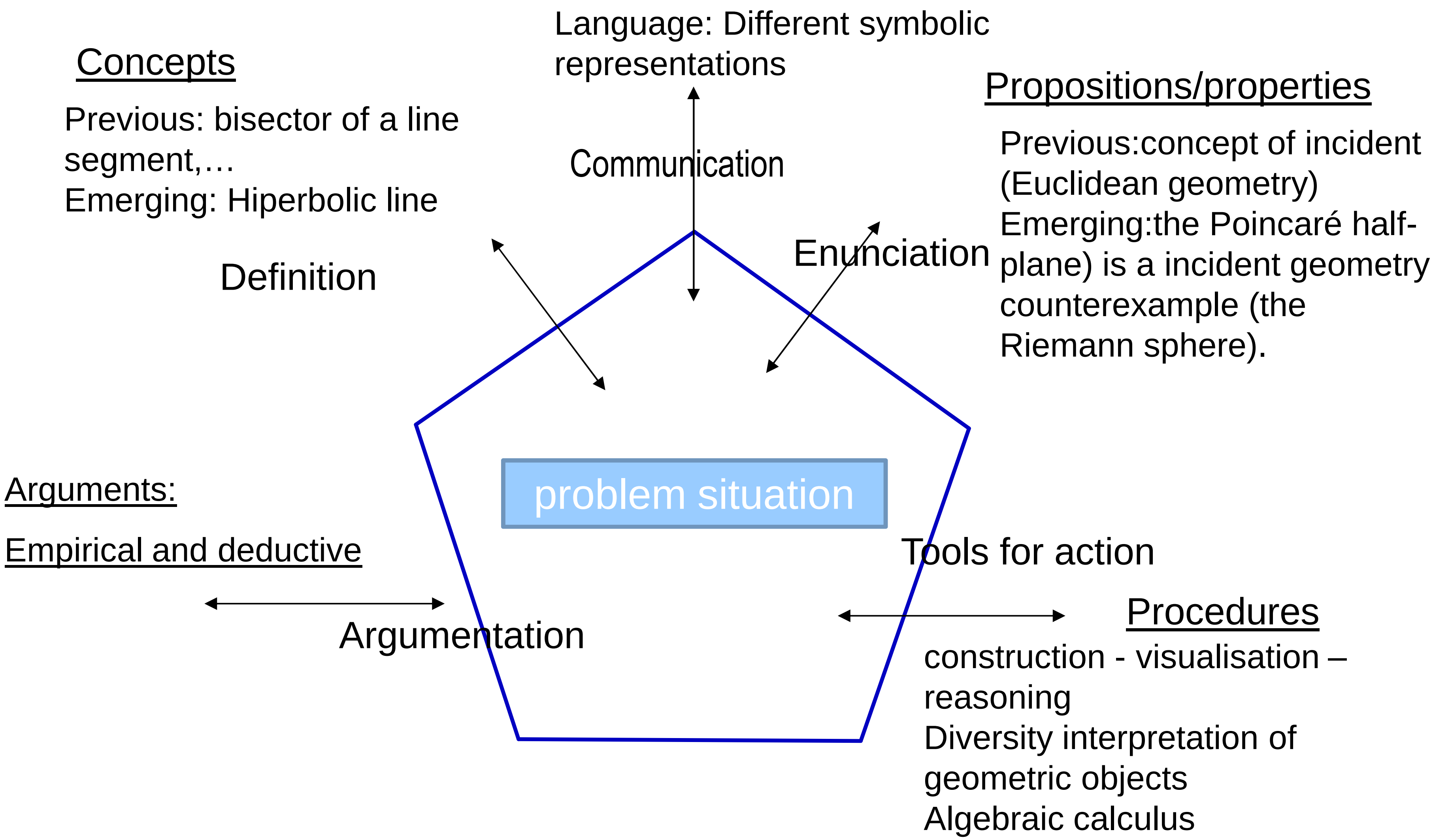
- PHASE 2 (to encourage their reasoning process)

Problem-situation

-**Part 1** – Define, in the Poincaré half-plane, the line passing through points A (1,1) and B (3,3). How many distinct lines pass through these two points? Support your answer. (The same question was formulated for Euclidean geometry).

-**Part 2** – Considering that l_1 and l_2 are lines in the Poincaré half-plane. If $l_1 \cap l_2$ has two or more points, then $l_1$coincides with l_2. Support your answer.

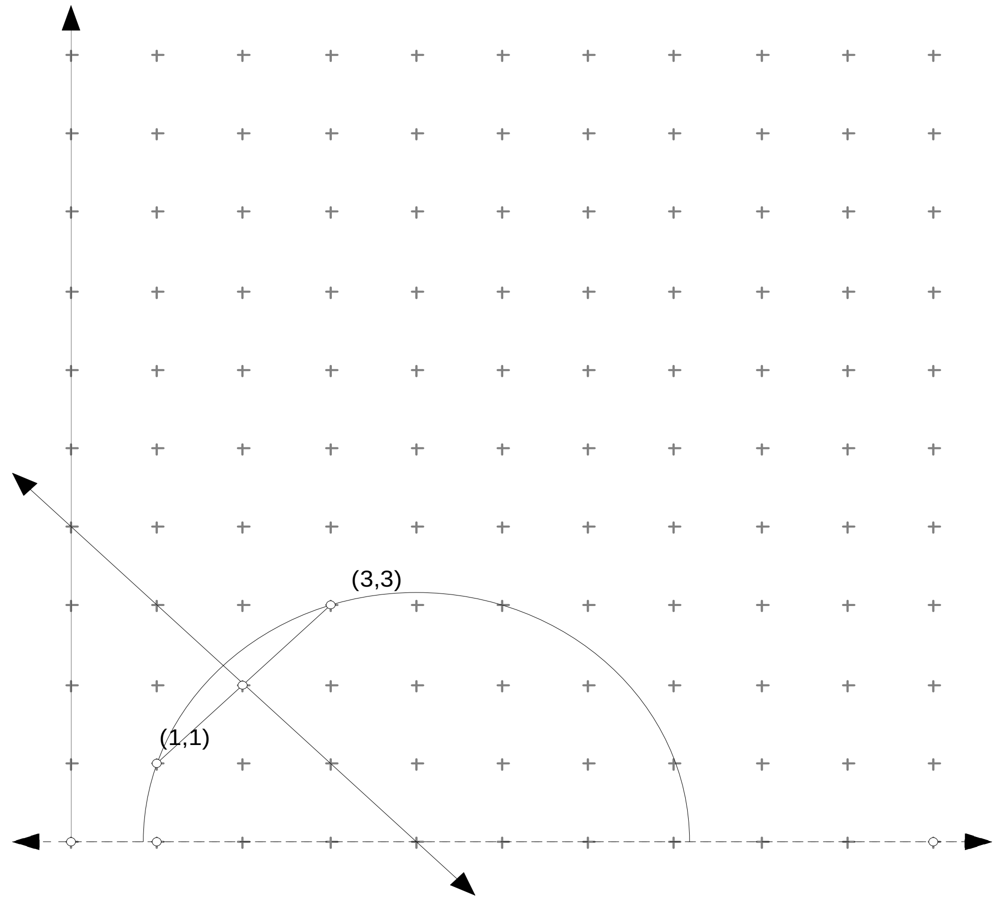
Primary relations



Problem-situation

Part 1

When **student X** tried to draw the centre of the semicircle, coordinate point (4,0), she was unable to do so as the point does not belong to the Poincaré half-plane.



Student Y Similarly to the previous group, both students used the hyp_line.gss script and drew a line passing through A (1,1) and B (3,3).
The following figure illustrates the solution proposed by the students.

Problem-situation

Part 1

Next, the students determined the value of the radius and provided an answer.

How many distinct lines pass through these two points? Support your answer.

At this point, the **student X** asked for help in the drafting of a support answer and were explained the method of proof by reductio ad absurdum.

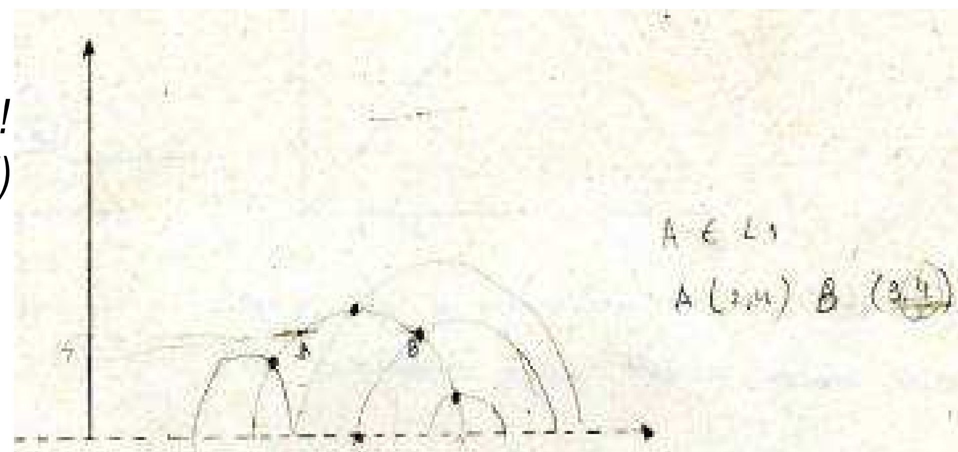
Student Y used GSP, the Hyp_line script and supported their answer geometrically stating there was only one hyperbolic line passing through the given points.

Problem-situation

Part 2

In the second part of the problem-situation, the **student Y** established connections with the work done in the 1st part.

Y.: Haven't we solved this problem already? (the question arose immediately after the student had read the question)
T.: If they intersect in two points, they are coincident!...
Y.: There is one, see?... (the student drew a diagram to explain)It is not possible to draw another line (passing through these two points).
T.: They intersect?
Y.: The two lines only intersect in one point. See?! (the student continues explaining using visual aid)



Student X not established connections with the work done in the 1st part.

Objects Involved and Emerging

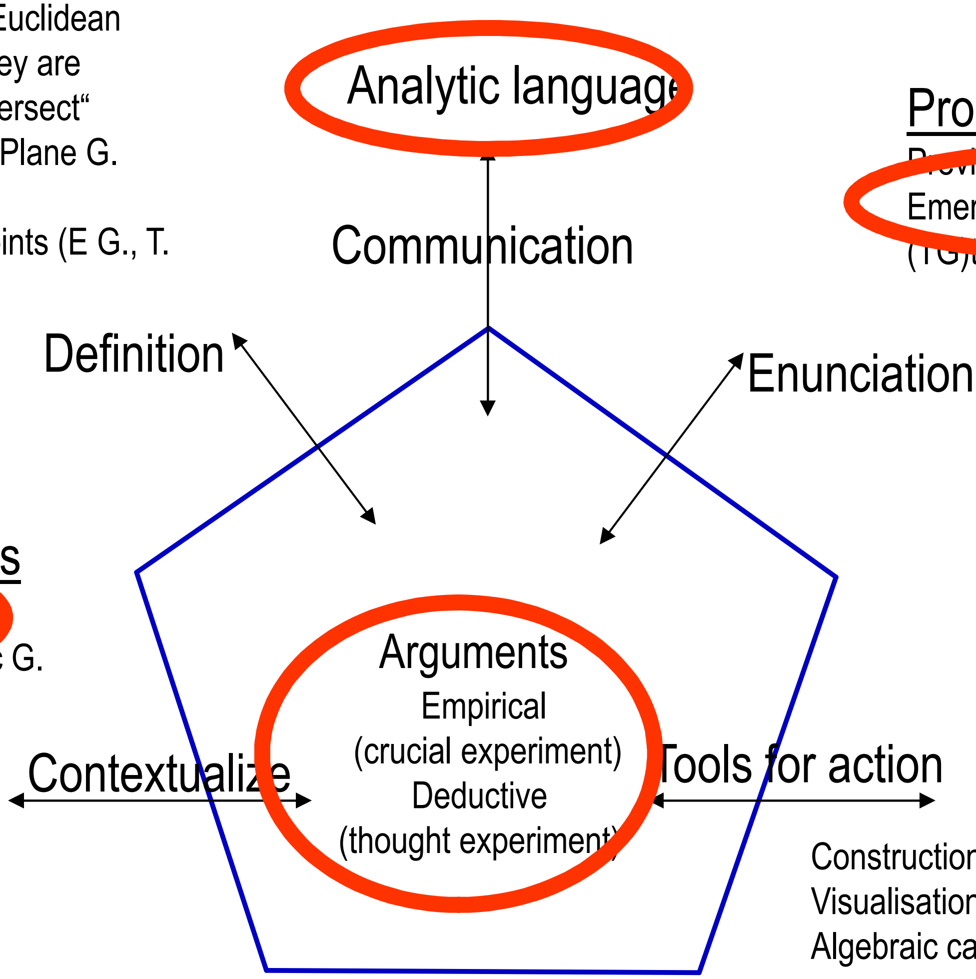
Student X

Concepts

Previous: Parallel lines (Euclidean G.)-“no matter how far they are prolonged, they never intersect“
Emerging: Parallel lines (Plane G. models)
Distance between two points (E G., T. G.)

Propositions/properties

Previous: Triangle inequality
Emerging:Incidence G.-Examples (TG)triangle inequality holds



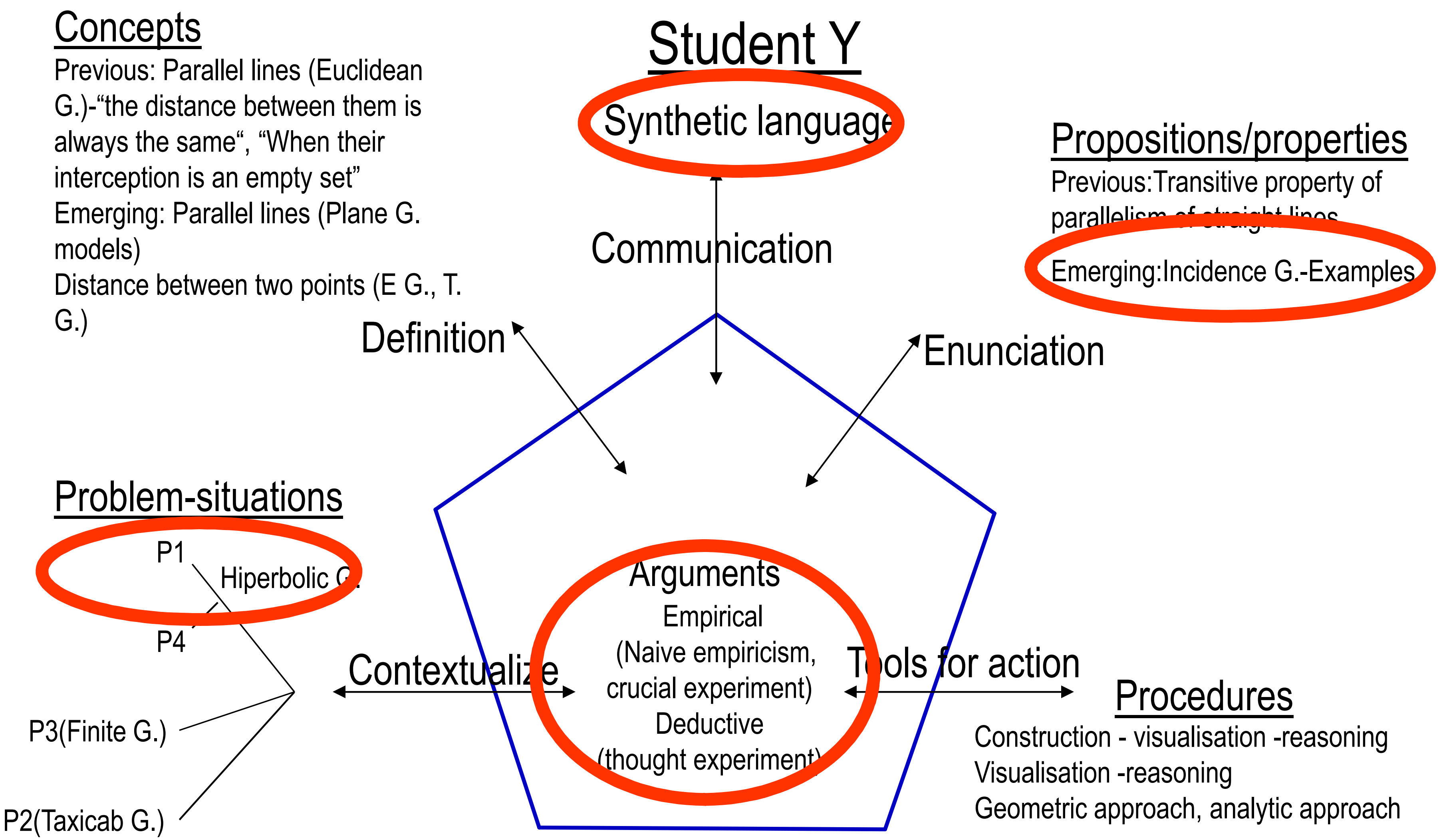
Problem situations

P1
Hiperbolic G.
P4
P3(Finite G.)
P2(Taxicab G.)

Procedures

Construction - visualisation -reasoning
Visualisation -reasoning
Algebraic calculus

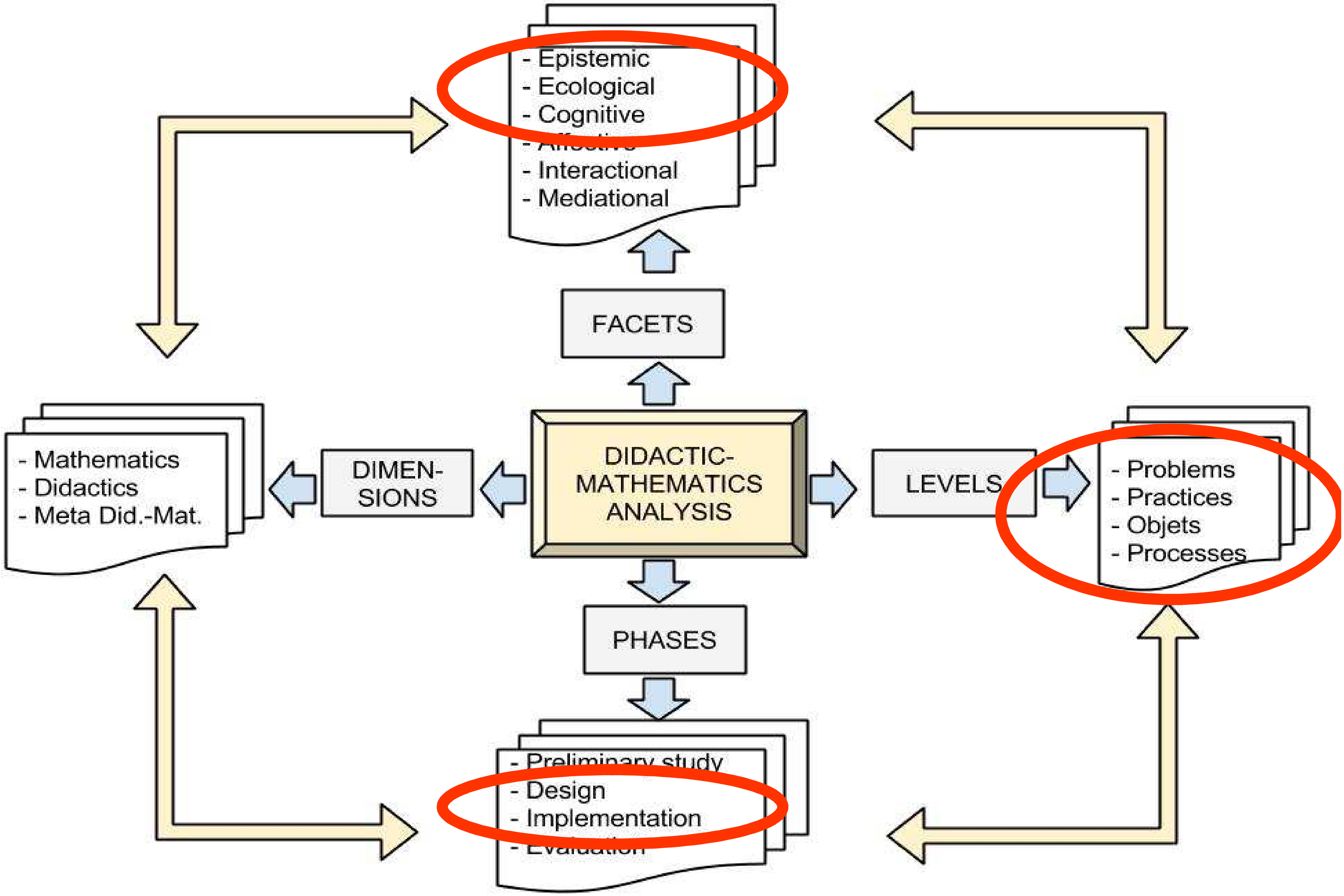
Objects Involved and Emerging



Types of Justification
(Marrades and Gutiérrez, 2000)

- Empirical
  - Failed
  - *Naive empiricism*
    - Perceptual
    - Inductive
  - Crucial experiment
  - Generic example
    - Example- based
    - Constructive
    - Analytical
    - Intellectual
- Deductive
  - Failed
  - Thought experiment
  - Formal
    - Transformative
    - Structural

4. Final considerations



4. Final considerations

The focus of attention of a didactic and mathematical analysis, presented in the theoretical framework, allowed for supported articulation of the design and implementation phases, focusing on the epistemic and cognitive facets and involving various levels (problem-situations, practices and processes).

4. Final considerations

Regarding the various levels, the design and implementation of this teaching experiment enabled, through an understanding of the primary entities of the ontology and epistemology of EOS (Godino, 2014), the emergence of objects, in particular the definition of incident geometry, the definition of the Poincaré half-plane, hyperbolic line and the analysis of propositions (Proposition: Euclidean geometry is an incident geometry; Proposition: Hyperbolic geometry is an incident geometry).

4. Final considerations

Onto-semiotic perspective on mathematics argumentation promoted insight about this process but more research is needed to systematically document these theoretical framework in this field.

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Thank you!

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