The use of technology to teach mathematically gifted students

Angel Gutiérrez
angel.gutierrez@uv.es
www.uv.es/angel.gutierrez

Adela Jaime
adela.jaime@uv.es
www.uv.es/ajaime

Departamento de Didáctica de la Matemática
Universidad de Valencia
Valencia (Spain)

MINISTERIO DE ECONOMÍA Y COMPETITIVIDAD
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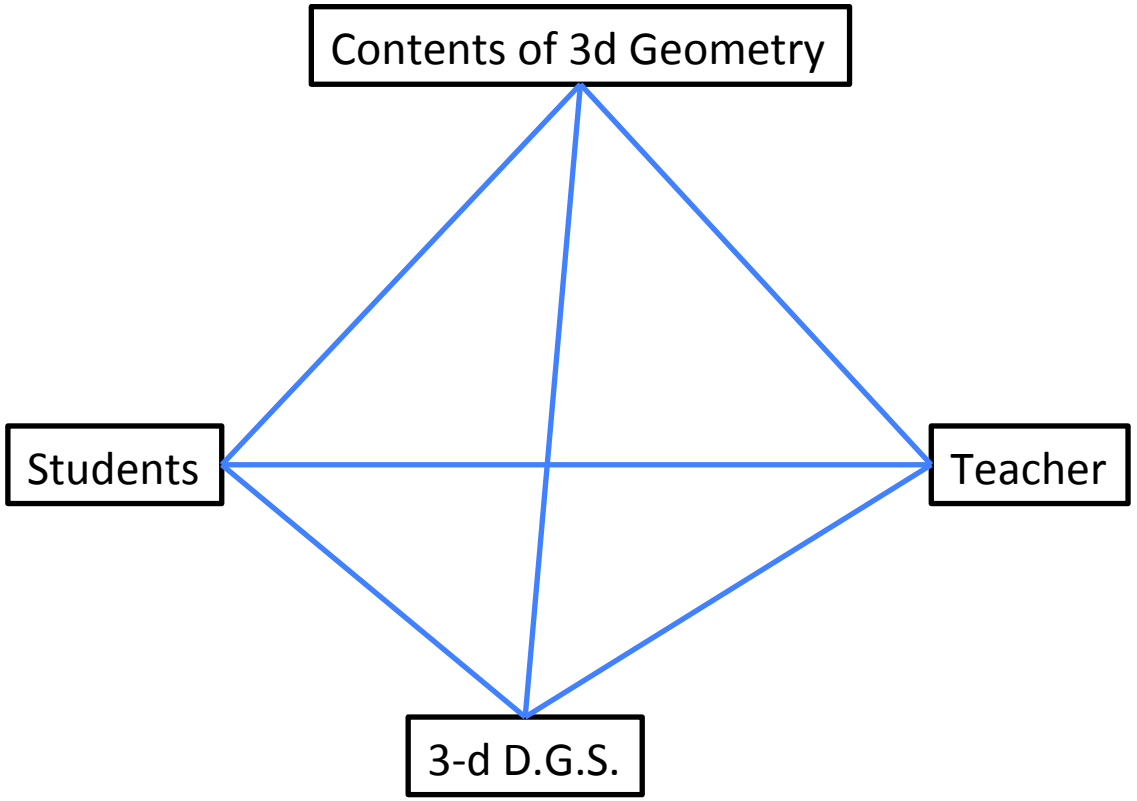
THE CONTEXT



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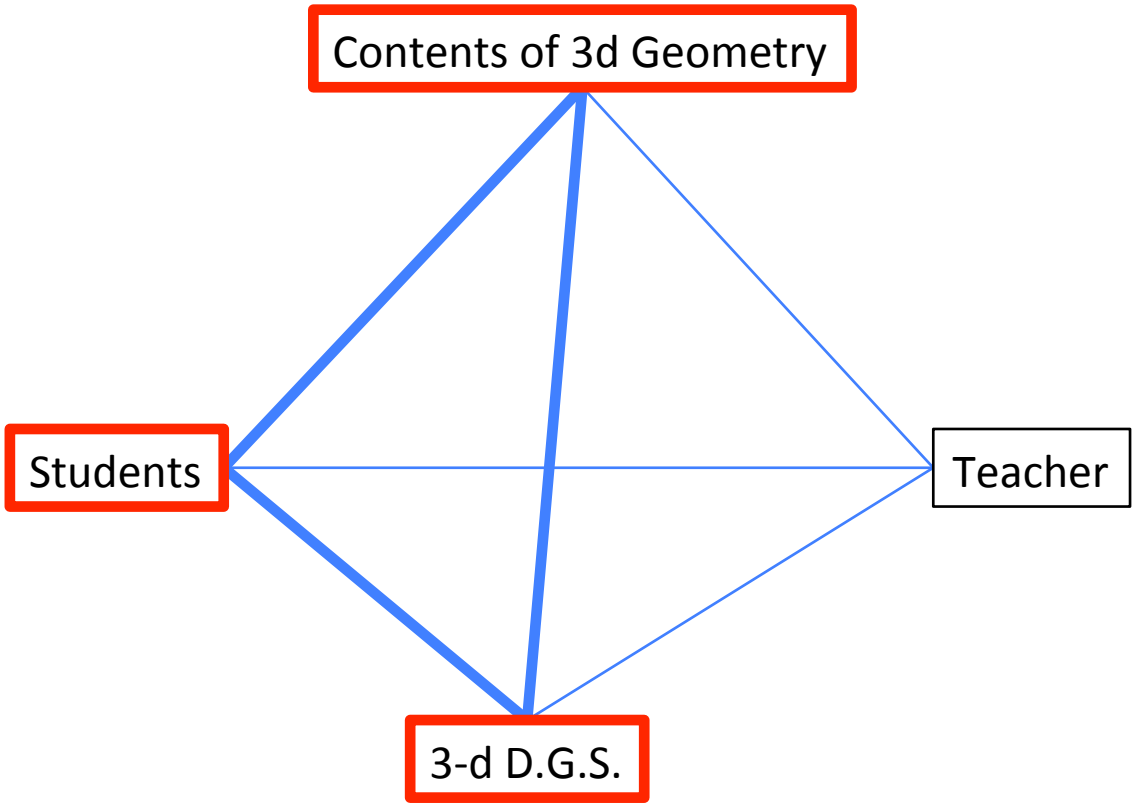
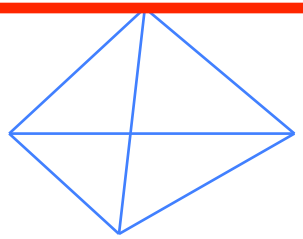
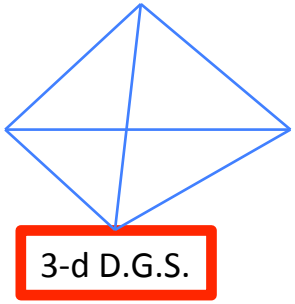


Contents of 3d Geometry



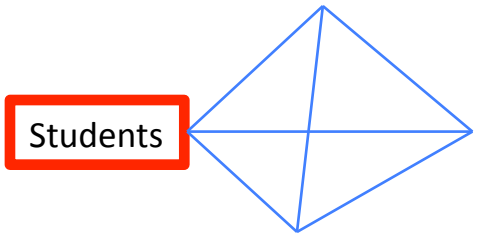
THE CONTEXT

- Properties of straight lines and planes in space.
  - Relative positions of straight lines and/or planes in space: parallels, perpendiculars, incident (cutting), crossing, pertaining.
    - *Parallelism in space.*



THE CONTEXT

- Cabri 3d.
  - Comands for construction of objects.
    - Points, straight lines and planes.
    - Parallel and perpendicular objects.
  - Comands for dragging objects and to change de point of view (glass ball).
  - Ways objects are represented in the screen.



THE CONTEXT

- A gifted student in 2nd grade of Secondary School (grade 8).
- He had studied at school:
  - Relative positions of straight lines in 2d.
    - Parallelism in 2d.
  - Standard solids (prism, pyramid, cilinder, cone, sphere).
- He had not studied straight lines nor planes in space.
- Previous use of D.G.S.:
    - Confident with geometry in Geogebra.
    - Basic knowledge of Cabri 3d (polyhedra).

RESEARCH QUESTIONS

- How are the processes of learning the concepts of parallelism in espace in a 3-d D.G.S. environment?
- Are environments based only on 3-d D.G.S. feasible to learn space geometry?
- Which benefits and deficiencies do these environments show for mathematically talented (m.t.) students?

✧ Which obstacles do m.t. students suffer when using 3-d D.G.S. environments?

✧ How do they overcome those obstacles?

✧ How are their instrumental genesis processes?

THEORETICAL FRAMEWORK

- Components of the theoretical framework:
  - Analysis of students' processes of learning space geometry.
  - Specific features of m.t. students.
  - Instrumental genesis.
  - Plane representations of space objects.

THEORETICAL FRAMEWORK

- Analysis of students' processes of learning space geometry.
  - Reasoning strategies (Van Hiele levels).
  - Creation of concept images (Vinner).

THEORETICAL FRAMEWORK

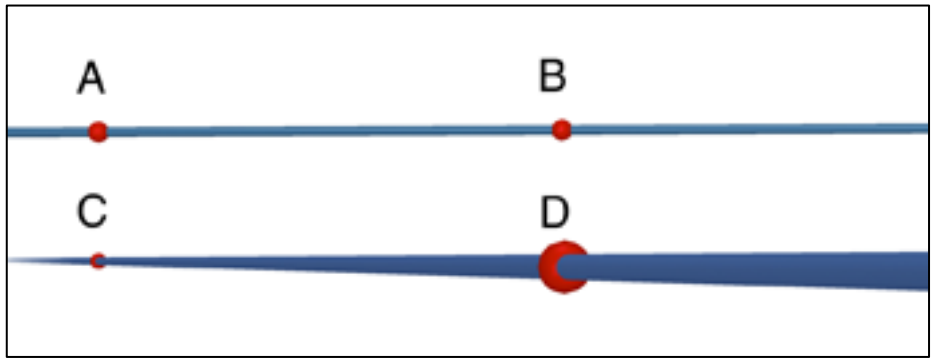
- Specific features of mathematically talented students.
  - High ability to reason analytically or spatially.
  - Ability or potential to get creative results.
  - Ability to quickly and accurately solve problems.
  - Capacity to see relationships between topics or concepts.
  - Capacity to identify the key to solve a problem.
  - Intuitive understanding of processes, allowing them to skip steps to find answers.
  - Ability to use analogic reasoning by generalizing, abstracting and transfering knowledge.
  - Competency to examine examples and counter-examples, to make conjectures or get similar structures.
  - Ability to shorten processes when solving similar problems.
  - Ability to verbalize, explain or reason.

THEORETICAL FRAMEWORK

- Instrumental genesis.
  - Relationship among *subject* (student) and the *artefact,* that has to become an *instrument* by means of *instrumentalization* and *instrumentation*.

THEORETICAL FRAMEWORK

- Plane representations of space objects.
  - Loss of information (Parzysz).
    - Use of codes characteristic of the 3-d D.G.S. to represent points, stright lines and planes.



  - Learning to draw and read plane representations (Jaime, Gutiérrez).

METHODOLOGY

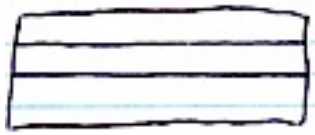
- Exploratory descriptive study of one case.

✧ Identification of student's previous knowledge (18 questions without computer).
✧ Teaching (24 activities with Cabri 3d).
✧ Identification of student's progress (same questionnaire).
✧ Teaching (10 activities with computer).

- Clinical interview (student and researcher-teacher).
- Recording of computer screen + audio. Written answers. Cabri 3d files.

DEVELOPMENT OF THE EXPERIMENT
(previous knowledge about parallelism)

Q1. Draw a plane and two parallel lines within the plane.



Prototypical figure

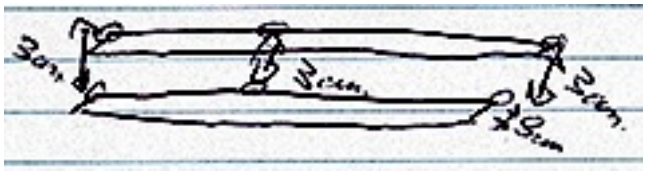
Q2. How many points have in common two parallel lines?

S. *They don't have any point in common, because parallel lines never cut each other.*

He combined definition, graphical representation and mathematical argument -> advanced level 2 reasoning.

DEVELOPMENT OF THE EXPERIMENT
(previous knowledge about parallelism)

Q6. Draw two parallel planes.



He extrapolated the definition of // lines.

Q7. How many points have in common two parallel planes?

S. *None (see questions 2 and 6).*

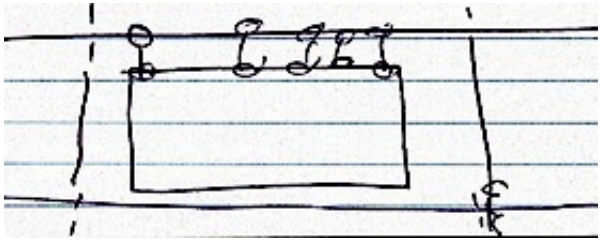
Capacities to • see relationships between topics or concepts and • generalize and transfer knowledge.

He justified refering to previous answers.

Ability to shorten processes when solving similar problems.

DEVELOPMENT OF THE EXPERIMENT
(previous knowledge about parallelism)

Q11. Draw a plane and a parallel line.



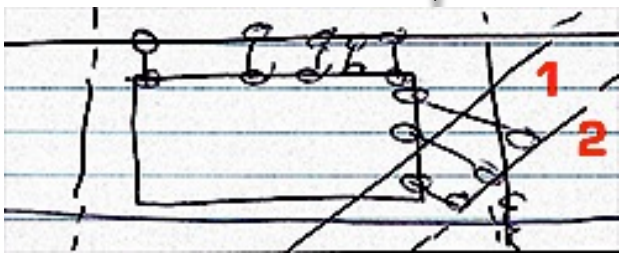
Wrong concept images of plane (bounded region) and parallelism in 3d.

S. *Because they* [the lines] *never touch it. ... And, furthermore, because the distance from here to here is the same as from here to here and from here to here, etc.*

R. *Can you also draw a non-parallel line?*

S. *Yes.* [line **1**]

R. *What does it happen if the line is outside, how would it be?* [line **2**]

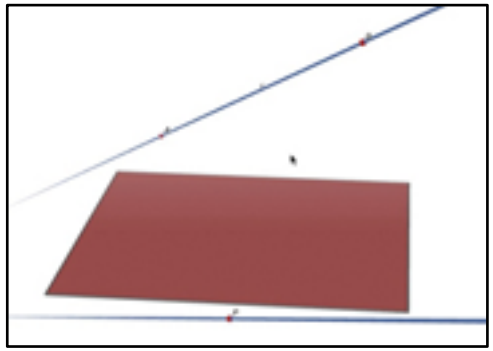


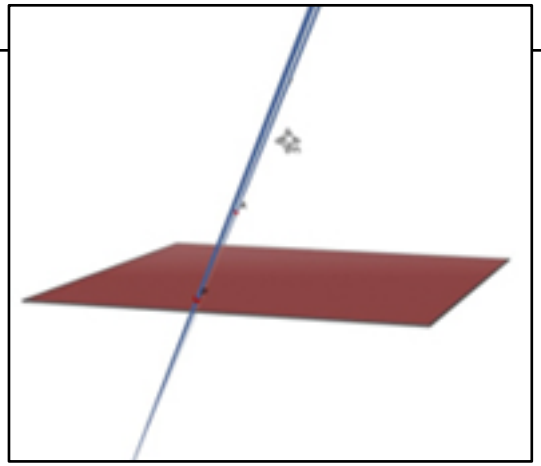
S. *Non-parallel, because the distance from here to here is not the same as from here to here nor from here to here, etc., etc.*

He used his knowledge of // lines in 2d.

DEVELOPMENT OF THE EXPERIMENT
(learning of parallelism)

A14. Make a straight line *r*. Make a line parallel to *r* through a point *P* exterior to *r*.





He used a (wrong) visual criterion to justify that the lines are parallel: It is possible to see them superposed in the screen.

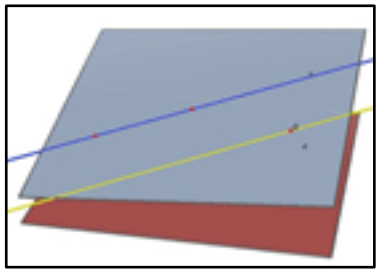
A15. How many points do two parallel lines have in common?

S. *None.*
R. *Why none?*
S. *Because they never touch each other.*
R. *Why do they never touch each other?*
S. *Because they are parallel.*
R. *For the definition of parallel lines.*

He had difficulty to express the justification by using the definition and/or a property. This drived him to a circular argument.

DEVELOPMENT OF THE EXPERIMENT
(learning of parallelism)

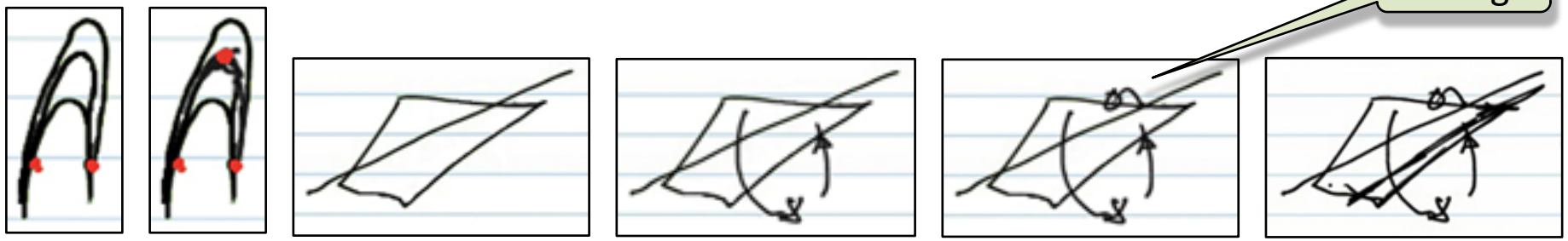
A17. Make planes passing through the two parallel lines *c* and *d*.
How many planes were you able to make?



S. [Used the command "Plane" and clicked 4 times on *c* and *d*]
R. *How many different planes have you created?*
S. *Different, none, because there is only one.*
R. *Then, ¿what did you make four times?*
S. *The same plane.*
R. *Why?*
. . . . . .

DEVELOPMENT OF THE EXPERIMENT
(learning of parallelism)

A17. Make planes passing through the two parallel lines *c* and *d*.
How many planes were you able to make?



Wrong.

R. *Why?*

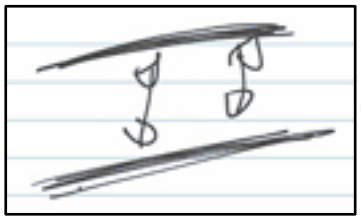
S. *Because ... For instance, if we have a parabola and we are given two points, it may be this one, this one, this one* [1]. *But now, we put another point and what we do is to get the one passing through here* [the new point, 2]. *Then, if we are given a straight line, the plane can be like this* [3]*, or rotated in this way* [4] *or also rotated this other way* [5]. *But now, when we are given other line* [6]*, we are told that the plane has to be rotated to a direction* [position]*, so we are told which one is the plane.*

Capacity to transfer knowledge by reasoning by analogy.

He had learned the codes to draw 3d objects.

DEVELOPMENT OF THE EXPERIMENT (learning of parallelism)

A19. How many point have in common two parallel planes?



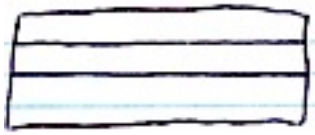
He had learned the codes to draw 3d objects.

S. [without making any action on the computer] *None, because we have a line and now we have another line* [representing planes drawn orthogonal to the sheet]*. They are parallel because the distance from here to here is the same as from here to here* [he drew the vertical arrows] *and, as they never cut each other, then they do not have any point in common*.

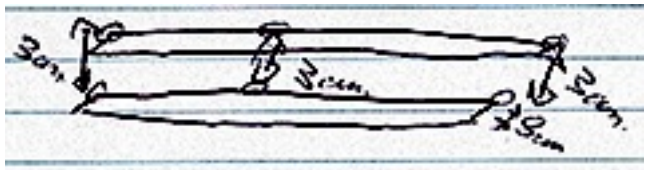
In A17 and A19, the drawings are not specific examples, but a help for his abstracts verbal justifications -> initial level 3 reasoning.

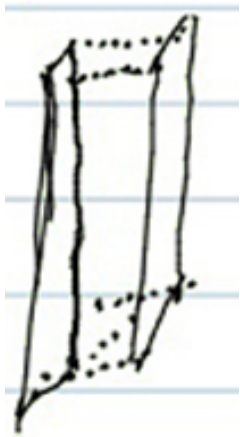
DEVELOPMENT OF THE EXPERIMENT (instrumental genesis - drawing of planes)

Draw a plane and two parallel lines within the plane



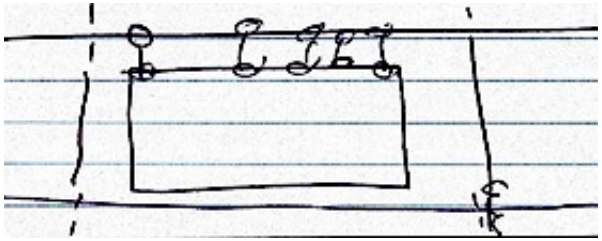
Draw two parallel planes.

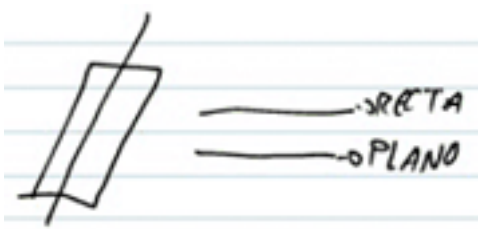




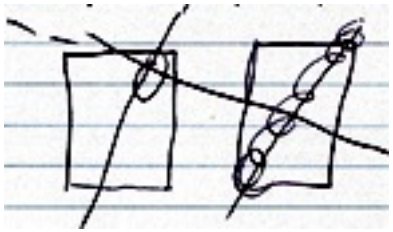
DEVELOPMENT OF THE EXPERIMENT (instrumental genesis - drawing of planes)

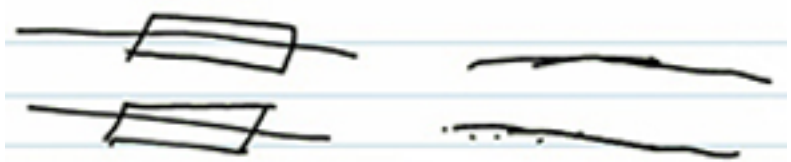
Draw a plane and a parallel line.



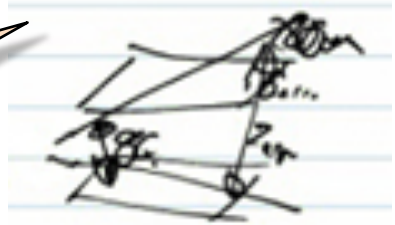


Draw two straight lines in parallel planes.





The way to draw planes has improved. Instrumental genesis developed.



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