

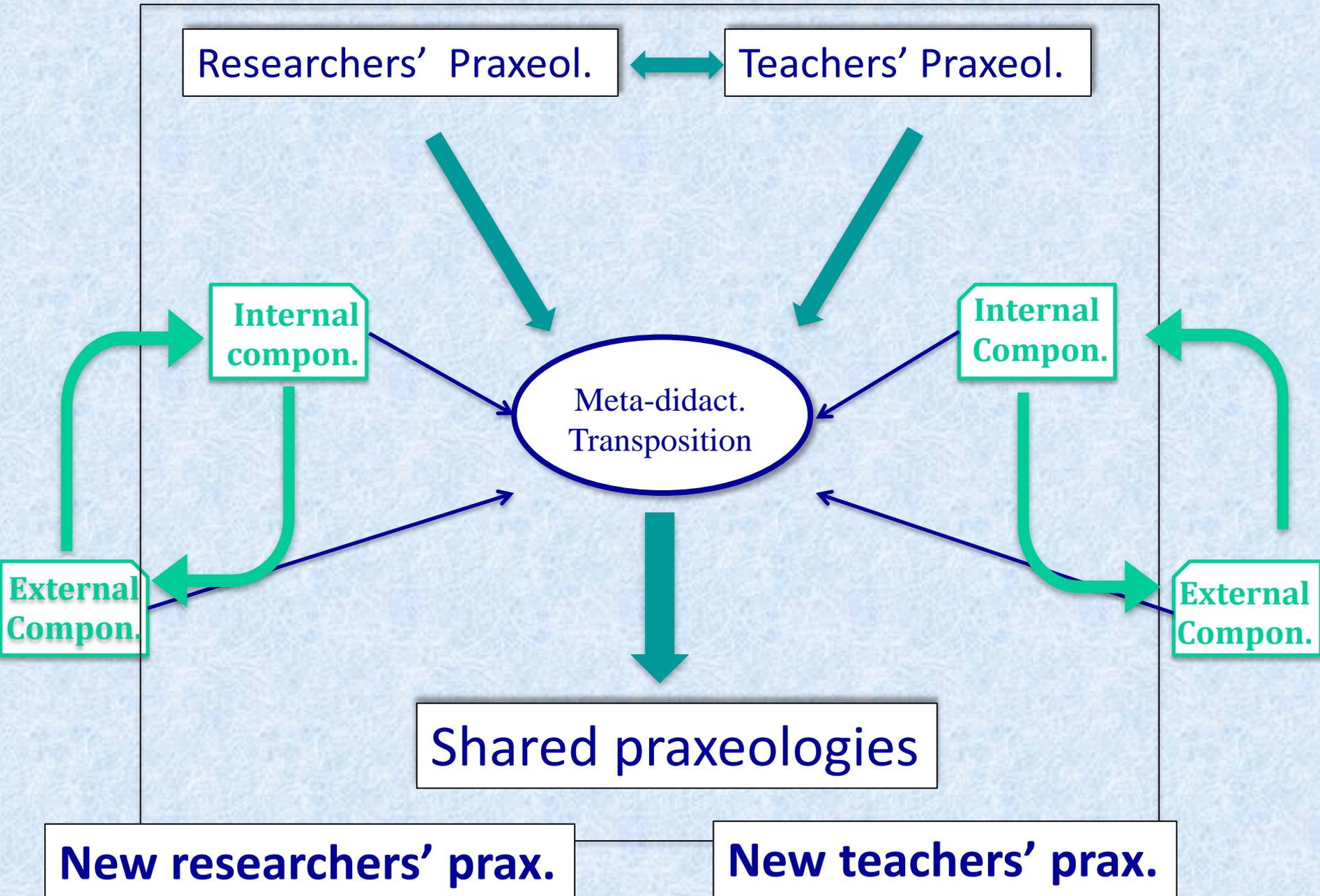
***The MDT Model as a tool
to highlight fundamental features
of teacher education processes***

Annalisa Cusi
Università di Modena
e Reggio Emilia

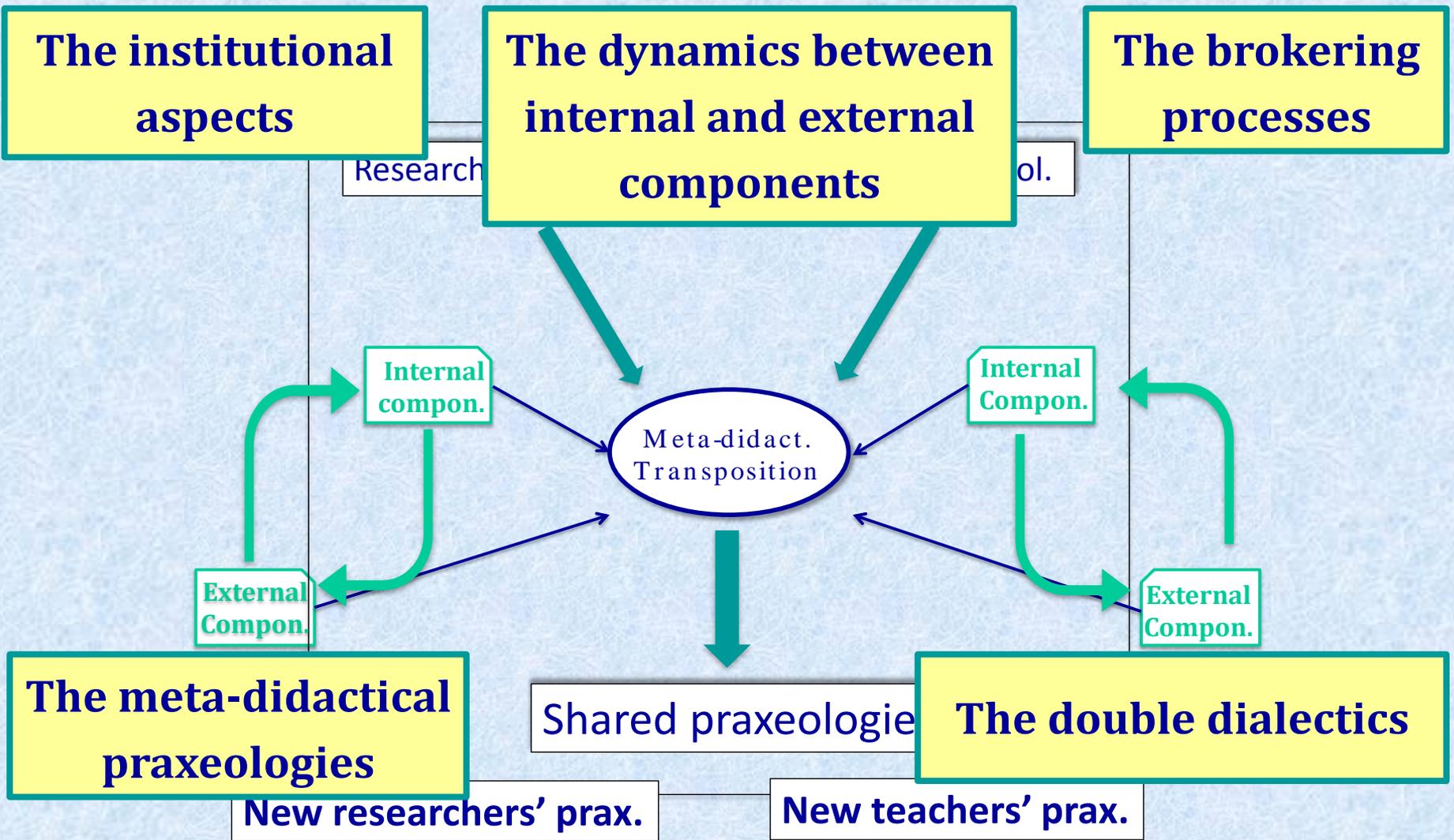
Francesca Martignone
Università del
Piemonte Orientale

Cristina Sabena
Università di Torino

THE META-DIDACTICAL TRANSPOSITION



FIVE INTERTWINED FEATURES



FIVE INTERTWINED FEATURES

The institutional aspects

The dynamics between internal and external components

The brokering processes

Research

pl.

Internal compon.

Internal Compon.

Meta-didact. Transposition

External Compon.

External Compon.

The meta-didactical praxeologies

Shared praxeologies

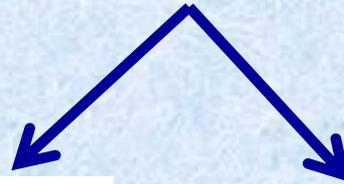
The double dialectics

New researchers' prax.

New teachers' prax.

CONTENTS OF THIS PRESENTATION:

The analysis of two examples to highlight



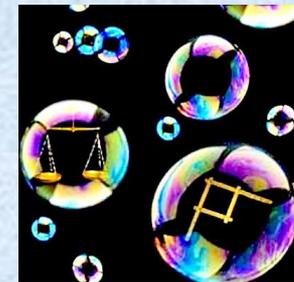
The Double Dialectic
and the consequent evolution
of the teachers' praxeologies

The praxis and logos levels of
researchers' praxeologies and
their evolution over time

The ArAI Project



The MMLab-ER Project



**FIRST
EXAMPLE**



Arithmetic

Algebra

ArAl

FIRST EXAMPLE



Arithmetic

Algebra

ArAl

- To promote a **linguistic and constructive approach to early algebra** starting from primary school or even kindergarten;
- To constitute an **integrated teacher education program.**

Malara & Navarra 2003
Cusi Malara & Navarra 2010

Our methodology of work with and for teachers

HYPOTHESIS

The **observation and the critical-reflective study of socio-constructive class processes** are necessary conditions to **foster teachers' development of awareness about**

- ✓ the **roles** he/she must play in the class,
- ✓ the **dynamics** which characterize the mathematical collective construction,
- ✓ the **variables** which intervene.

**FIRST
EXAMPLE**



Our methodology of work with and for teachers

**Community
of inquiry**

The teachers are organized in groups

Each group is coordinated by a researcher-mentor

**Frequent face-to-face and e-mail exchanges
between the teachers and the researcher-mentor**

Work-sessions conducted by the project leader

**Joint meetings with all the teachers and researchers
involved in the research**

Our methodology of work with and for teachers

Teachers are involved in a **complex activity of critical analysis of the transcripts of audio-recordings of classroom processes and associated reflections**

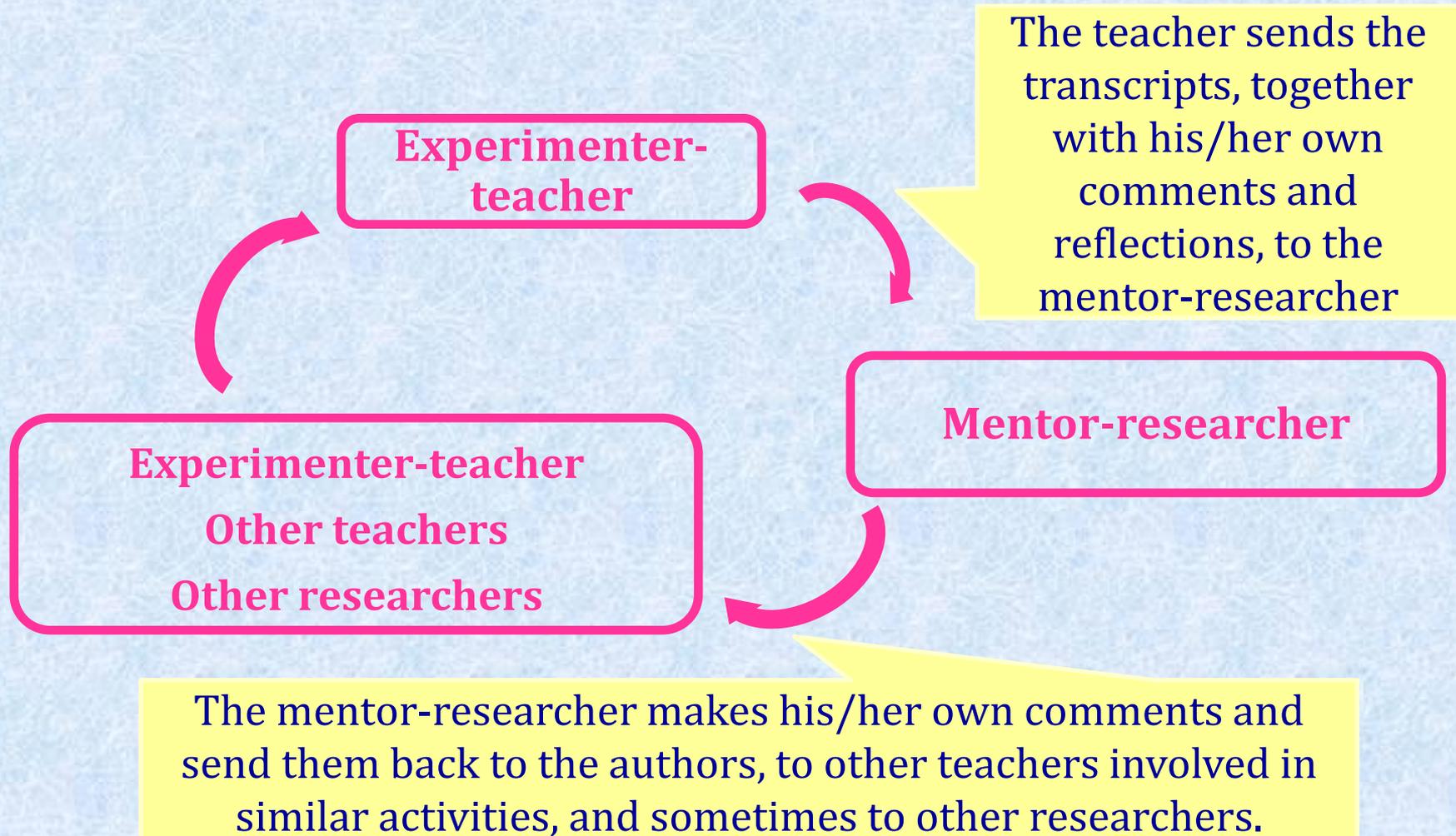
The Multicommented Transcripts (MT)

TO MAKE TEACHERS

- **become increasingly able to interpret the complexity of class processes, reflecting on the effectiveness of their own role**
- **control their behavior and communicative styles.**

FIRST EXAMPLE

The Multicommented Transcripts (MT)

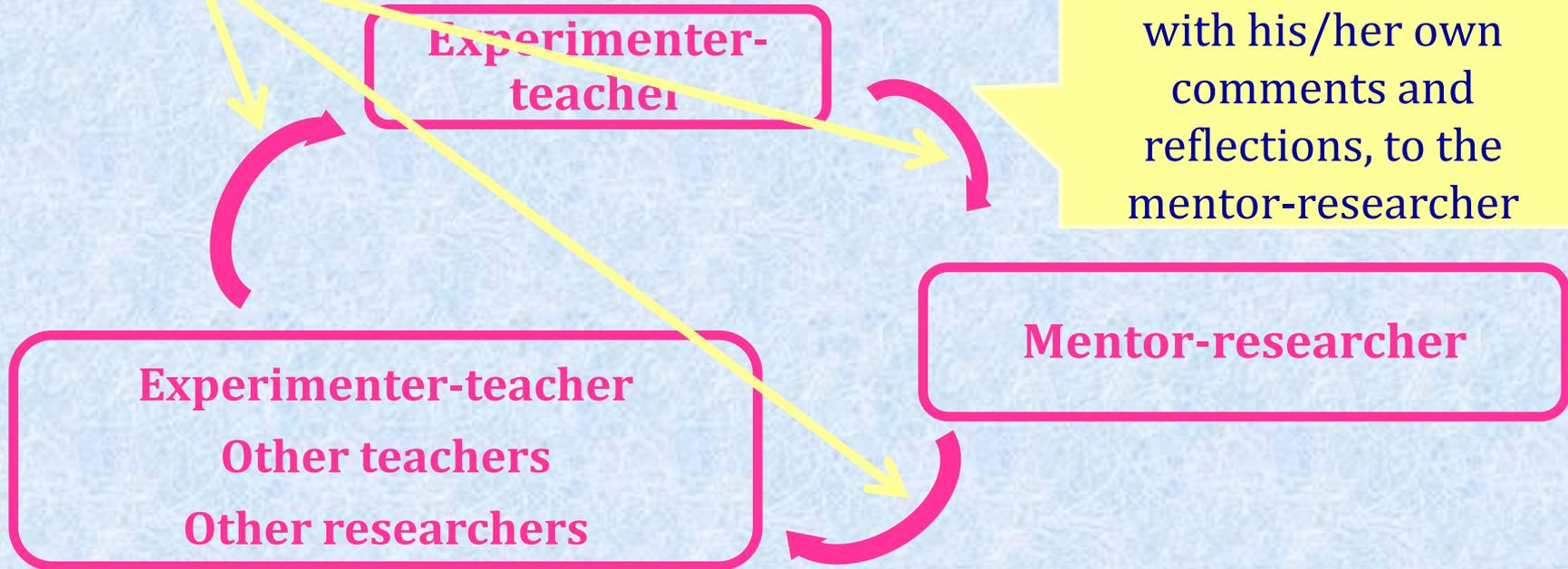


FIRST

Often, both teachers and researchers make further interventions in this cycle, commenting on comments or inserting new ones.

Mediated Transcripts (MT)

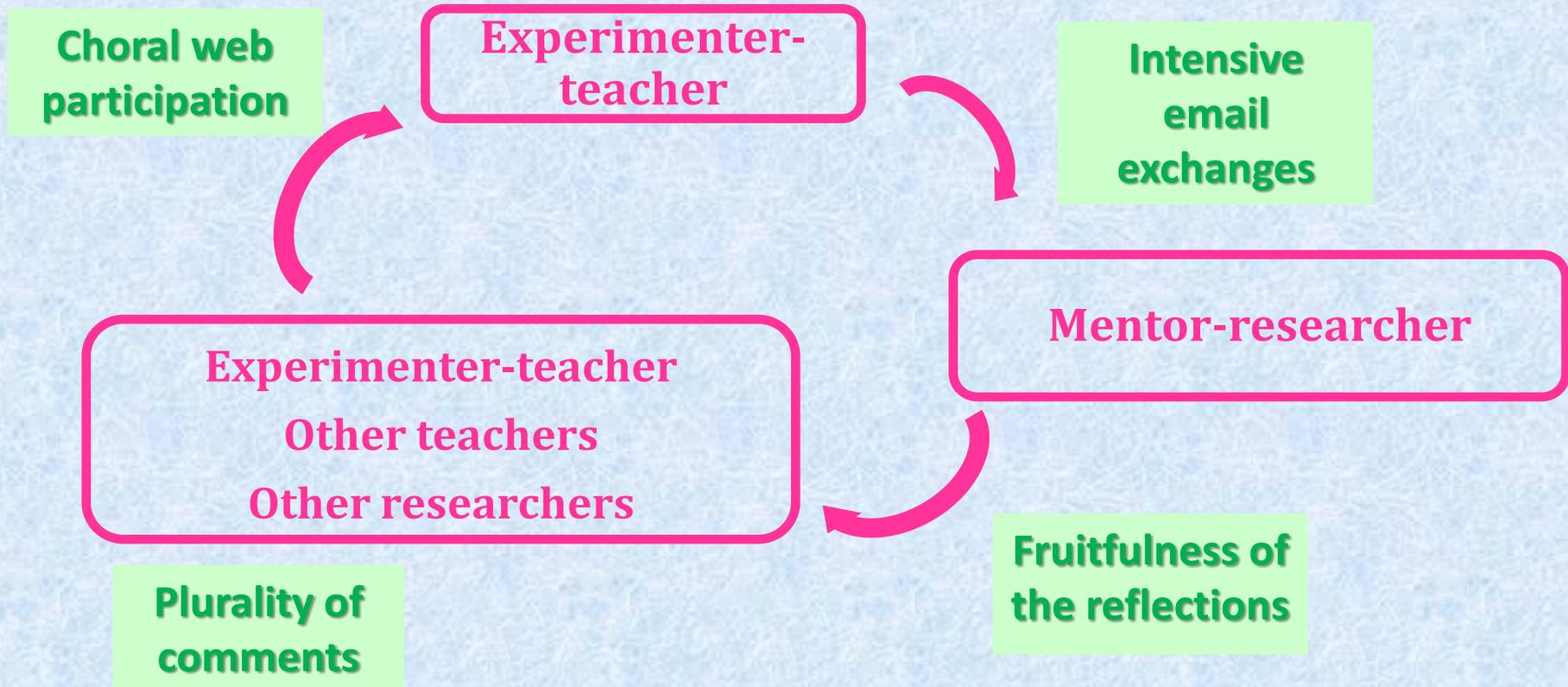
The teacher sends the transcripts, together with his/her own comments and reflections, to the mentor-researcher



The mentor-researcher makes his/her own comments and send them back to the authors, to other teachers involved in similar activities, and sometimes to other researchers.

**FIRST
EXAMPLE**

The Multicommented Transcripts (MT)



FIRST EXAMPLE

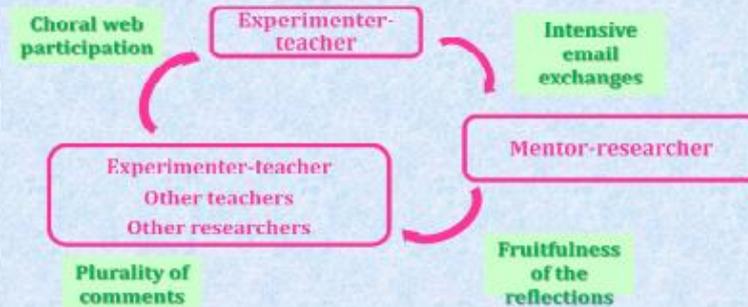


THE DOUBLE DIALECTIC within the ArAl Project

Thanks to the MT teachers have the possibility to become aware of:

The contrast/interaction between the personal sense their students attribute to class activities and the institutional meaning of both the activities and the mathematical concepts involved.

The possible different interpretations, given by teachers and researchers, of the dynamics activated during class activities.



(1) **First dialectic**
at the *didactical level*
in the classroom

THE
DOUBLE
DIALECTIC

(2) **Second dialectic**
at the *meta-didactical level*

FIRST EXAMPLE

(1) *First dialectic*
at the *didactical level*
in the classroom

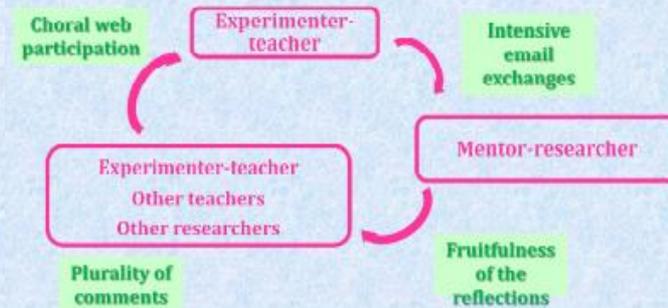


(2) *Second dialectic*
at the *meta-didactical level*

The double-level dialectic engendered thanks to the MT fosters the development of **new teachers' praxeologies related to both**

the roles
they should play
in their classrooms

the ways
of pursuing their
professional
development



FIRST EXAMPLE

(1) *First dialectic*
at the didactical level

“I believe that a teacher who plans his/her didactical activities constantly referring to these (theoretical) aspects could really educate students to think, instead of only training them to reproduce algorithms and meaningless contents”
(Reflection of C)

at the meta
level

the roles
they should play
in their classrooms

the ways
of pursuing their
professional
development

Theoretical elements for the analysis of class processes become internal components of the teachers' community.

Choral web participation

Experimenter-teacher

Intensive email exchanges

FIRST EXAMPLE

“My initial idea of myself as the only guide, unassailable from both the didactical and methodological point of view ... I realised , instead, the advantages of this kind of work: reflecting on some negative attitudes/interventions, highlighted during the a-posteriori analysis with the mentor-researcher and the other researchers, enabled me to become less impulsive and to activate, when necessary, suitable strategies aimed at motivating my students”

(Reflection by M)

(1) First
at the did
in the c

(2) Second dialectic
at the meta-didactical
level

the roles
they should play
in their classrooms

the ways
of pursuing their
professional
development



MMLab-ER

**“Mathematical Machines Laboratories
for the Emilia-Romagna region”**

MMLab-ER Project



Supported by Region Emilia Romagna and coordinated by the Mathematical Machine Laboratory.

<http://www.mmlab.unimore.it/site/home/progetto-regionale-emilia-romagna.html>

The MMLab-ER project, framed in the **Italian research for Innovation**, takes into account the studies on **teacher knowledge and teacher education** and responds to national and international standards about **Inquiry Based Science Education**.

It aims at the construction of a network of well prepared in-service teachers about **mathematical laboratory**

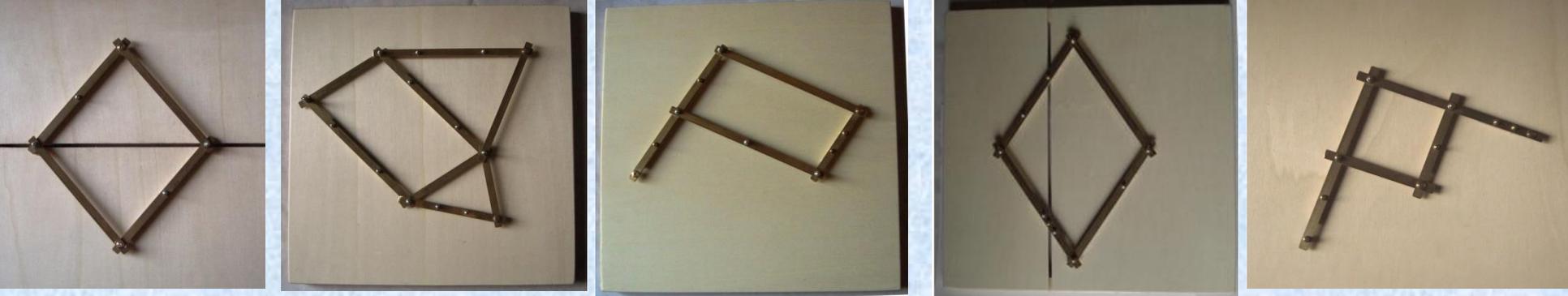
(Arzarello & Bartolini Bussi, 1998)

(Schulman, 1986)
(Wood (ed), 2008)
(Ball et al., 2008)

(Rochard et al., 2007)



www.umi.dm.unibo.it/downloads/icme10.pdf



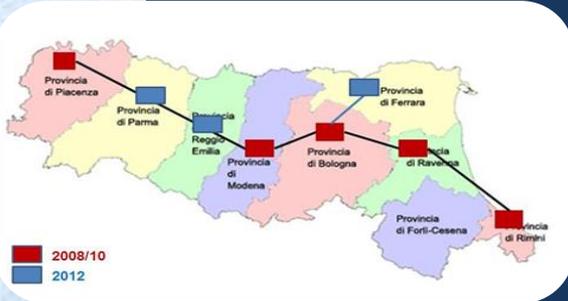
Teachers and then students worked on different **geometrical machines** (e.g. pantographs for geometric transformations and curve drawers) and **arithmetical machines** (e.g. the Pascaline)

The focus was on the analysis of **exploration and argumentation processes** and **of cultural aspects** involved.

(Boero, 2007, Martignone & Antonini, 2009; Antonini & Martignone, 2011; Martignone, 2011; Bartolini Bussi et. Al, 2011; Bartolini Bussi & Martignone, 2013)

<http://www.macchinematematiche.org/>

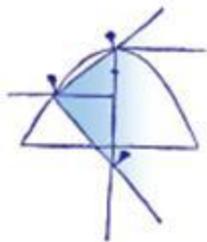
MMLab-ER Phases (2008/10-2012)



Set up of laboratories with many collections of reconstructions of cultural-historical artefacts: the mathematical machines

Teacher education program: teachers and researchers (as teacher educators) designed and developed different teaching experiments sharing and discussing the results.

Teaching experiments in primary and secondary schools are still going on

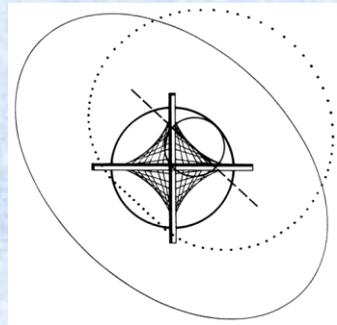


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delle Macchine
Matematiche

UNIVERSITÀ DEGLI STUDI
DI MODENA E REGGIO EMILIA
Anno Indotto nel 1175



www.mmlab.unimore.it
www.macchinematematiche.org



The MDT-Model gives us some interpretative tools to control the overall development of researchers'/teacher educators' **Meta-Didactical** praxeologies, identifying how these praxeologies changed at different levels: praxis level and logos level.

The term “meta-didactical” denotes that these praxeologies deal with the actions and reflections of researchers about the educational activities.

interpretative
development of

researchers’/teacher educators’ **Meta-Didactical** praxeologies, identifying how these praxeologies changed at different levels: praxis level and logos level.

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interpretative
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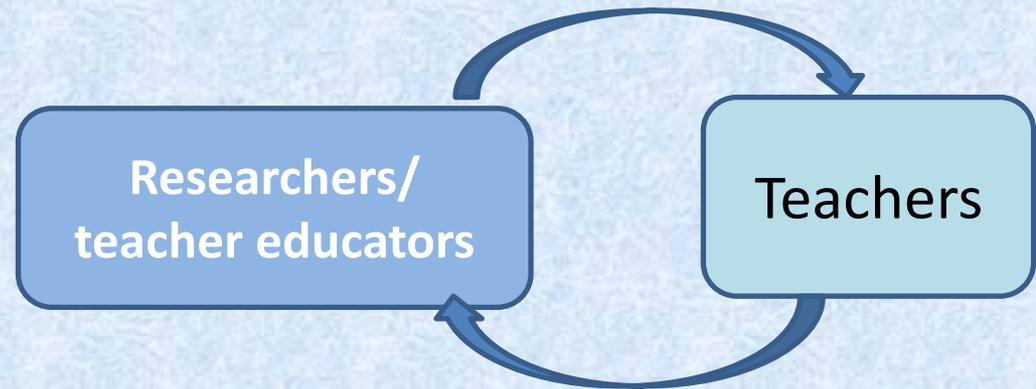
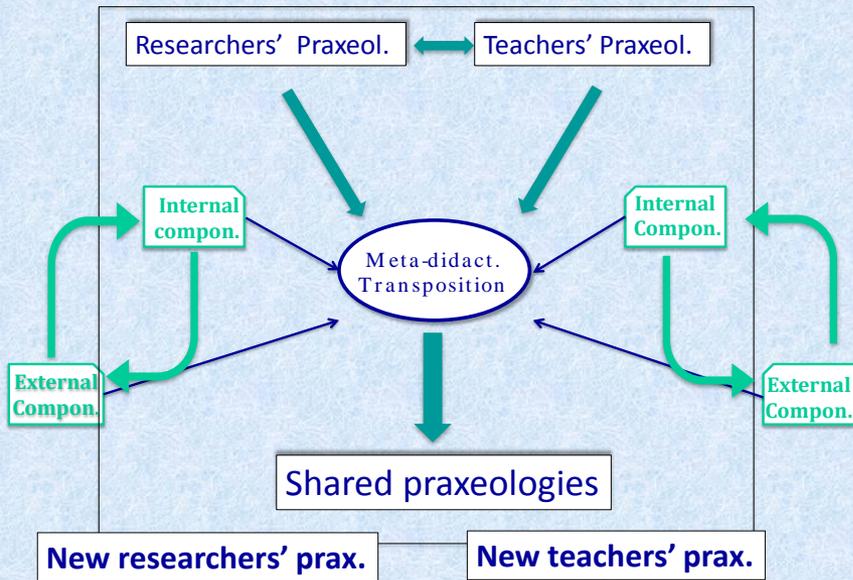
researchers’/teacher educators’ **Meta-Didactical** praxeologies, identifying how these praxeologies changed at different levels: praxis level and logos level.

The questions that we have answered are:

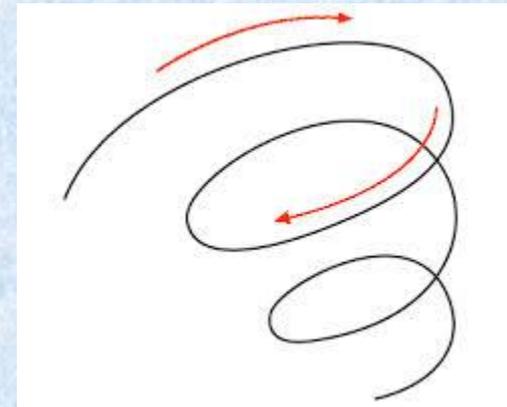
What has changed? In what way? At what level?

Why?

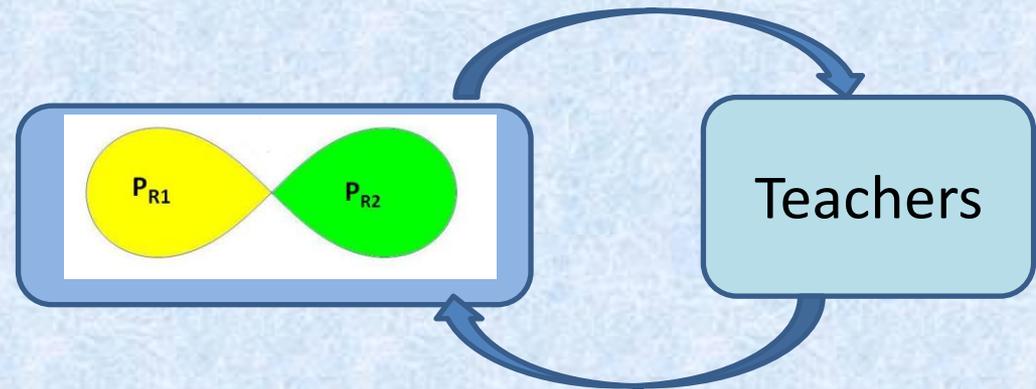
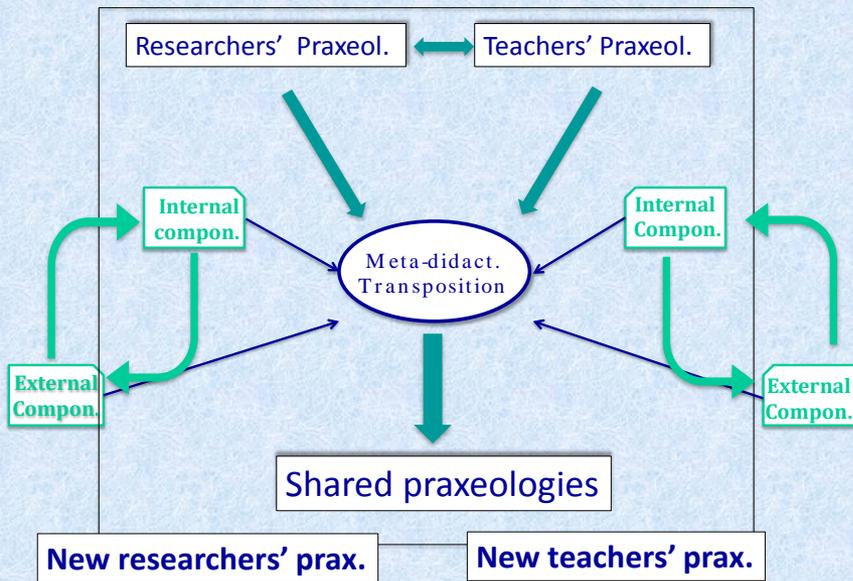
During the MMLab-ER Teacher Education Program



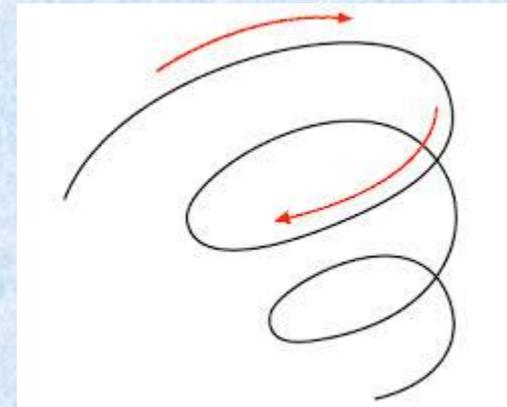
By means of the teacher education program, in which theory and practice are strictly intertwined, new ***shared praxeologies*** developed and also the previous teachers' and researchers' praxeologies improved.



During the MMLab-ER Teacher Education Program



By means of the teacher education program, in which theory and practice are strictly intertwined, new **shared praxeologies** developed and also the previous teachers' and researchers' praxeologies improved.



Researchers' Praxelologies P_{R1}

Meta-didactical because they reflect on educational activities

Developed when the researchers study theories, plan, observe and analyse classroom activities, construct theoretical lenses to describe and interpret students' difficulties and successes.

Praxis level

Tasks

To study the educational potential of the laboratory activities with mathematical machines



Bilbao, 2014

Techniques

Design and analysis of activities for primary and secondary school students.

Development and analysis of clinical interviews

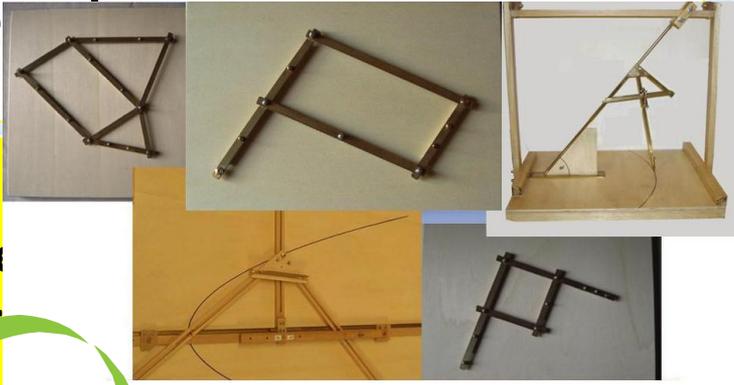


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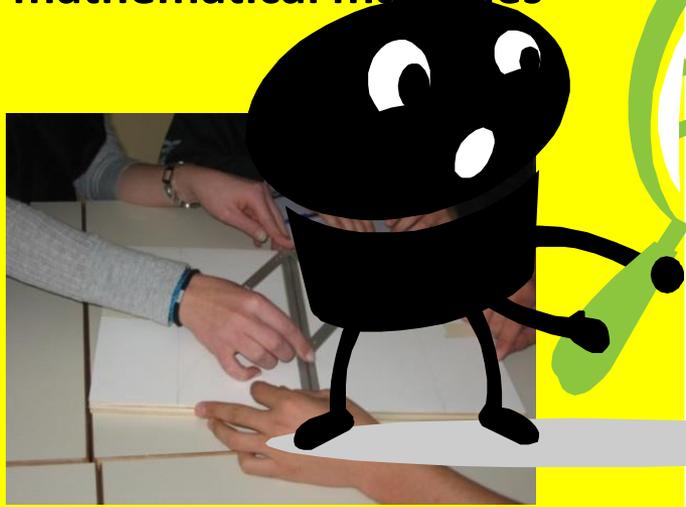
Praxis lenses



Tasks

To study the educational potential of the laboratory activities with mathematical machines

De
for



Descrizione della macchina

- Quali figure geometriche formano le aste del sistema articolato? Le aste del sistema articolato formano un poligono che ruota attorno ad un punto fisso detto il polo AB e il segmento della parte di B in movimento che Q sia il simmetrico di A rispetto al polo O .

$Q = \sigma_B(A)$

Quali possono essere i suoi movimenti?
 Il punto O è fisso, il punto Q si muove e descrive una circonferenza nel piano α . La parte trapezoidale è quella in cui il punto Q non può arrivare. In questa estremità il movimento è perché abbiamo i punti Q e P della macchina di sistema, mentre in questa estremità lo stesso è per lo spessore delle aste, il foglio di carta e il punto Q non riesce a raggiungere quella parte del piano.

• Quali sono le regioni piane messe in corrispondenza dalla macchina? Descrivere, anche attraverso disegni, tali regioni e giustificare la forma in relazione alle caratteristiche della macchina.

Quella non trapezoidale nel disegno precedente. Si forma una circonferenza perché dato che O è un punto fisso il punto Q è equidistante rispetto al polo (come una sfera di compasso), la seconda circonferenza si forma a causa dello spessore delle aste, quindi il punto Q non ci può arrivare.



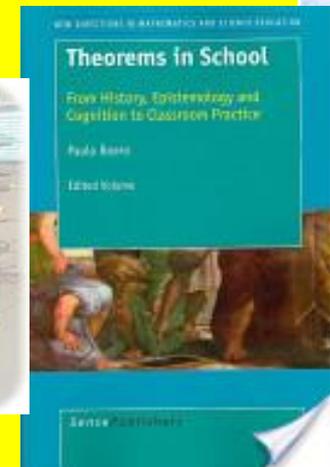
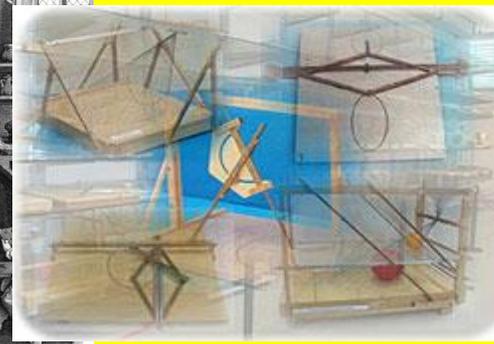
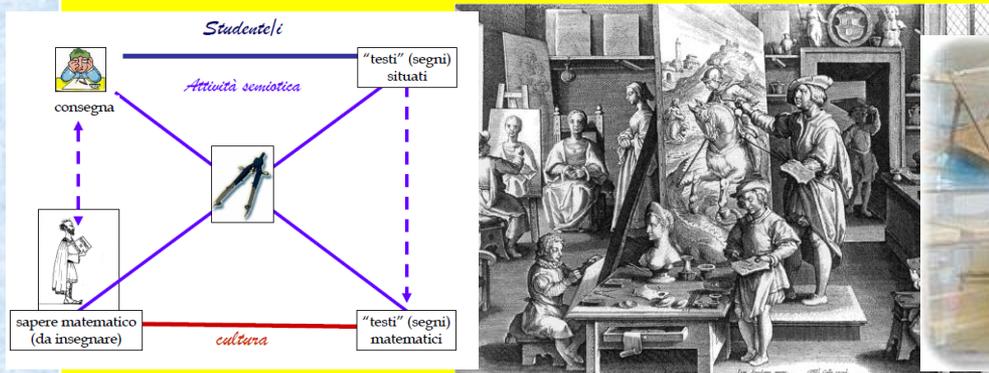
Logos level

Semiotic mediation framework (Bartolini Bussi & Mariotti, 2008)

Educational studies about proof (Garuti, Boero, Lemut & Mariotti, 1996; Garuti, 2003; Theorems in School (Boero ed., 2007)...)

Mathematics laboratory (Curricula and Standards)

Studies on mathematical machines utilization schemes (Martignone & Antonini, 2009)



Researchers' (as teacher educators) praxeologies P_{R2}

Praxeologies linked to their work as teacher educators and their research on teacher education processes (Boero & Guala (ACC), Shulman (PCK), Watson & Sullivan (*Task for teachers*))

Meta-didactical because they reflect on educational activities

Praxis level

Tasks

Design and analyse the activities for developing teachers' attention on the exploration processes, on the conjecture productions and on the proof constructions by means of laboratory sessions with mathematical machines.

Tecniques

The development of tasks for teachers that include, for example, the selection of suitable educational paths to be discussed with teachers and the analysis of different teaching experiments. The analysis of teachers' actions during the program and of teachers' reflections by means of logbooks and final reports

Researchers' (as teacher educators) praxeologies P_{R2}

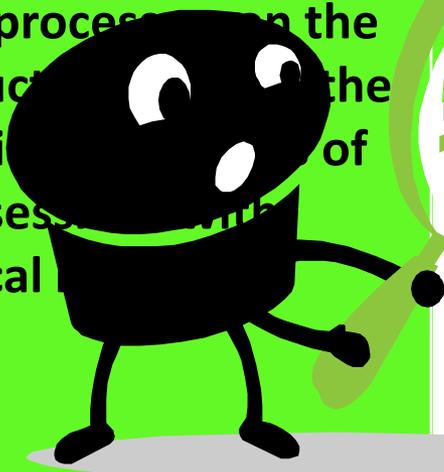
Praxeologies linked to their work as teacher educators and their research on teacher education processes (Boero & Guala (ACC), Shulman (PCK), Watson & Sullivan (*Task for teachers*))

Meta-didactical because they reflect on educational activities

Praxis le

Tasks

Design and analyse the activities for developing teachers' attention on the exploration process, on the conjecture production, on the proof construction, of laboratory sessions with mathematical



Diario di bordo

Luca: per formare un rombo servono 4 listelli della stessa misura e 4 fermacampioni. Mettere i quattro listelli obliqui e paralleli a 2 a 2. Forare le estremità dei 4 listelli e unirli con i 4 fermacampioni ed il rombo è fatto. (OBBLIQUI? non penso sia necessario scriverlo).

Pietro: prendere 4 listelli uguali, fare i buchi ai listelli all'estremità. Unire i listelli con i fermacampioni, in modo da angoli ottusi e due angoli acuti.

Salvatore: un rombo si costruisce prendendo 4 listelli uguali e 4 fermacampioni. Poi si bucano i listelli nella punta e si mettono i fermacampioni. Infine si assembla il tutto. La proprietà del rombo è quella di essere una figura articolata, infatti viene usato in molte macchine matematiche, tra cui il "sistema articolato".

Mirco: il rombo si costruisce con 4 listelli di cartoncino uguali. Poi, bisogna unire le estremità dei listelli formando 4 angoli di 90°. Il rombo è una figura articolabile. Esso è un particolare quadrato. Ha i lati tutti uguali e paralleli (PARALLELI A DUE A DUE). Ha due diagonali perpendicolari che formano nel punto di incidenza 4 angoli retti. (IL QUADRATO È UN PARTICOLARE ROMBO).

Diego: per prima cosa bisogna procurarsi 4 listelli di cartoncino e 4 fermacampioni. Poi unisci i listelli in modo che venga un quadrato, poi, mettilo in posizione come rappresentato nel disegno.

perpendicolari) o semplicemente vuole riprodurre percettivamente la sua immagine mentale di rombo con i lati "obliqui" rispetto alle direzioni verticali e orizzontali del foglio... si potrebbe domandare se il quadrato è un rombo. Le cose che mi colpiscono sono due: 1) Luca fa entrare in gioco la nozione di parallelismo (proprietà che invece esce "gratia" dalla costruzione del rombo usando quattro aste di uguale lunghezza e incastrate nelle estremità) 2) nel disegno che fa ci sono le diagonali di cui non parla. Mi chiedo: se è consapevole del punto 1 e se poi nel disegno usi la tecnica delle diagonali perpendicolari che si bisecano per disegnare il rombo

Commento [F33]: Anche qui si cerca di riprodurre l'immagine di "rombo generico" e quindi valgono alcune delle osservazioni fatte sopra

Commento [F34]: Qui nella discussione si potrebbero confrontare i vari modi di dire che sono usciti: alle estremità, nella punta... e poi chiedere se si capisce bene la tecnica di costruzione da queste descrizioni

Commento [F35]: Ok, qui è il contrario degli altri: sarebbe bella una discussione su questo!

Commento [F36]: Diego ha del disappunto: il rombo sembra essere un quadrato sono messo in una posizione diversa... quindi qui c'è da lavorare facendogli vedere e leggere il lavoro dei compagni.

educational paths to be discussed

REFLESSIONI CONCLUSIVE DELL'INSEGNANTE

Lo svolgimento della sperimentazione nell'ultimo scorcio dell'anno scolastico, l'aver proposto l'attività senza una specifica valutazione, forse un po' troppo "rilassata", ed una metodologia diversa dal solito, ha fatto sì che non tutti gli studenti abbiano lavorato al massimo delle loro potenzialità e che i momenti di discussione, di focalizzazione sui contenuti e di sintesi teorica non siano stati colti appieno nella loro importanza. La classe, però, ha confermato di aver assimilato in modo accettabile il metodo dimostrativo e le conoscenze già affrontate. Tutti hanno accolto bene l'attività e gli allievi più appassionati e brillanti ne hanno colto i punti nodali. Per completare il laboratorio sarebbe stata necessaria un'altra lezione di restituzione e discussione dell'ultima scheda, con qualche disegno dinamico con GeoGebra, nonché una qualche forma di valutazione. Forse si potrà fare a settembre, per consolidare anche questa modalità di lavoro. Ho un bel lavoro estivo di geometria sui luoghi geometrici e sicuramente e proporrò qualche esercitazione con GeoGebra all'inizio del nuovo anno scolastico.

Researchers' (as teacher educators) praxeologies P_{R2}

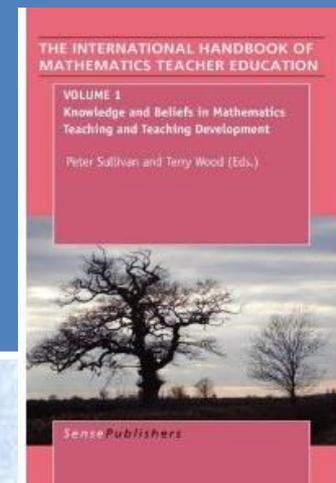


Logos level

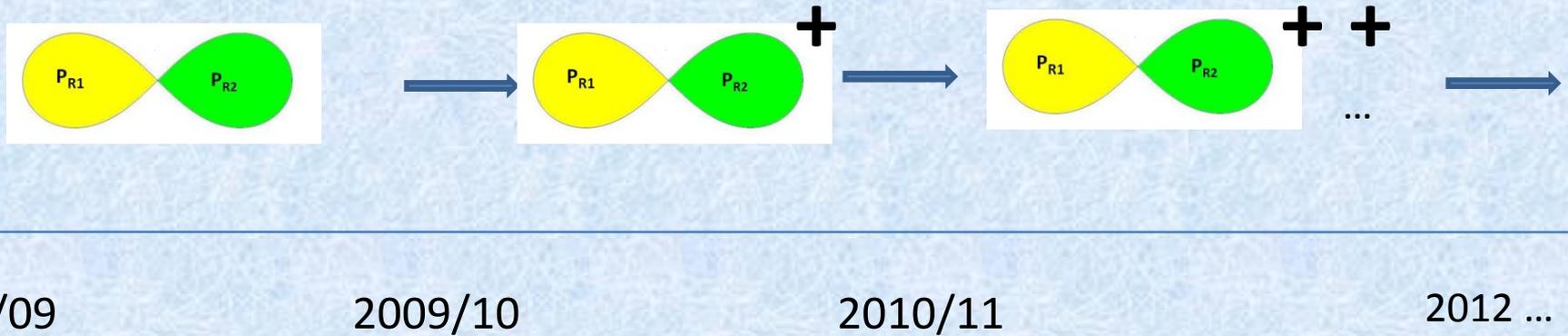
Cultural Analysis of Content (Boero & Guala, 2008)

Research in Mathematics Education and studies about argumentation processes (Gutierrez & Boero, 2006; Boero ed., 2007)

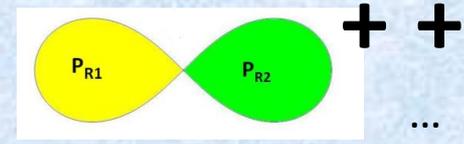
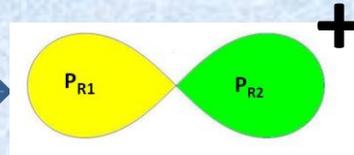
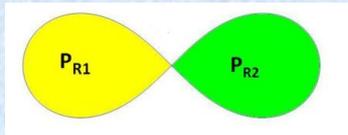
Research on teacher education (Schulman, 1986; Wood Ed., 2008; PME and Cerme studies...)



The MDT model is useful in describing and analyzing the evolution over time of the different components of the praxeologies. We can identify the aspects that do not change and what and how in the praxis and logos levels are modified.



What has changed? In what way? At what level? Why?



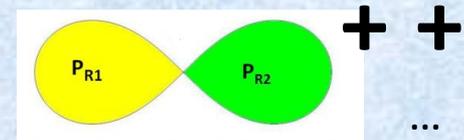
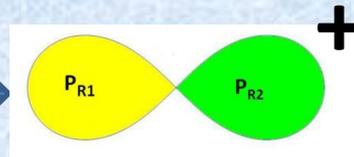
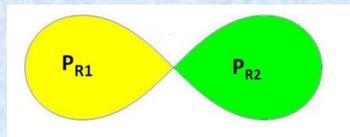
2008/09

2009/10

2010/11

2012 ...

The model is useful in describing and analyzing the evolution over time of the different components of the praxeologies. We can identify the aspects that do not change and what and how in the praxis and logos levels are modified.



2008/09

2009/10

2010/11

2012 ...

Praxis level

- **Improvement of techniques:** modifying some activities, removing others, introducing new tasks elaborated with the teachers and analysing previous logbooks to use them as tools for reflection

Logos Level

- **Improvement of technologies and theory:** developing new studies on cognitive, cultural and educational aspects involved

Cognitive studies

(Antonini & Martignone, 2011)

Reflections about the cultural aspects

(Bartolini Bussi & Martignone, 2013)

Studies about educational aspects

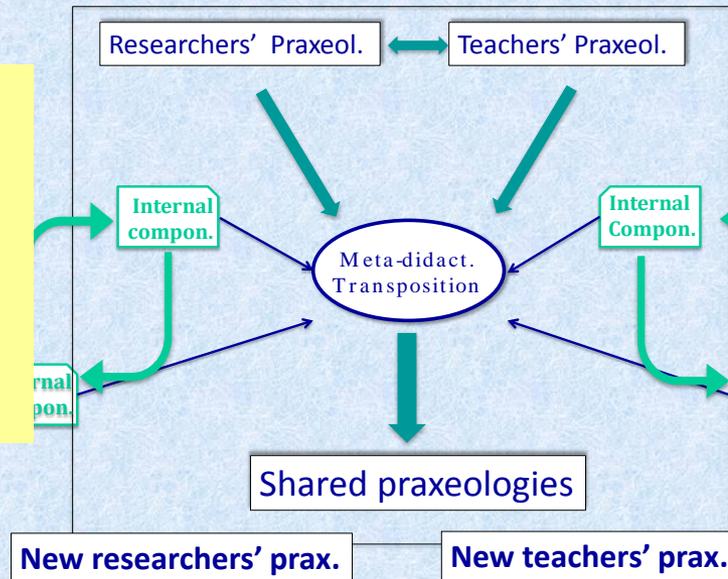
(Garuti & Martignone, 2010;
Martignone, 2012;
Banchelli & Martignone, 2012...)

The MDT Model

To describe

To analyse

To understand what changes over time and why



To highlight what kind of dynamics could be engendered at both the didactical and meta-didactical level

To compare different processes and different programs

It could enable a re-design and planning of future activities

www.ehu.es

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Meeting RSME-SCM-
SEMA-SIMAI-UMI

Thank you!