Realistic Mathematics Education

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Realistic Mathematics Education [RME]

- Freudenthal: anti-didactical inversion = endpoint of the work of mathematicians (e.g. set theory as organizing tool) is used as a starting point for instruction

- Alternative: mathematics as an activity
  - organizing subject matter from reality
  - organizing mathematical subject matter

Realistic Mathematics Education

mechanistic approach to learning

applications

target

constructivist approach to learning

applications

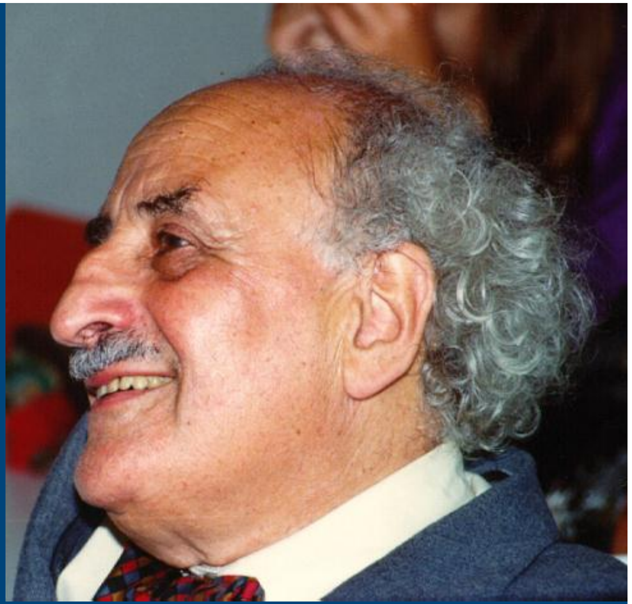
source

applications

target

Hans Freudenthal

(1905 – 1990)



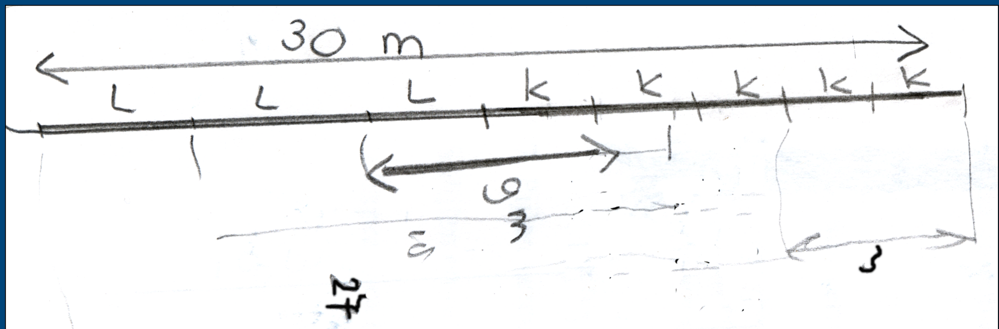
Rather than beginning with abstractions or definitions to be applied later, one must start with rich contexts that ask for mathematical organization; or, in other words, one must start with *contexts* that can be *mathematized*.

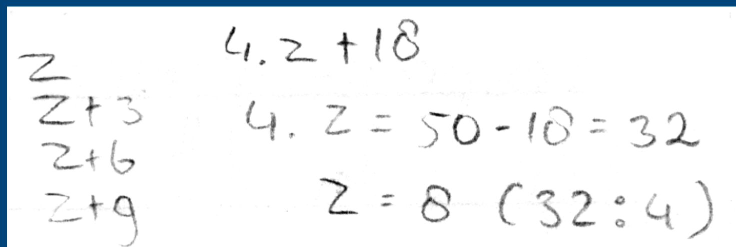
Content

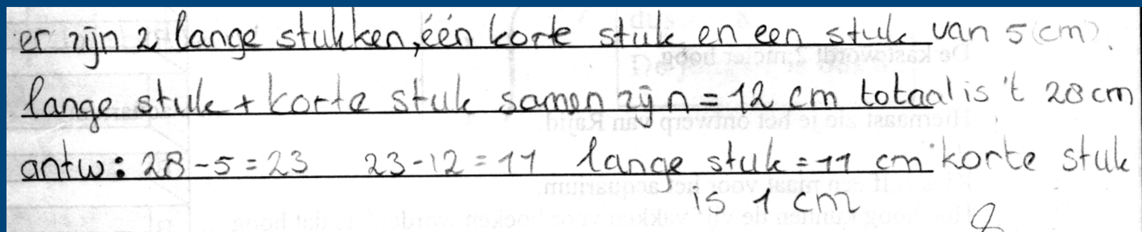
1. Two examples
2. RME: Mathematizing & Symbolizing
3. Emergent modeling
4. Use of contexts
5. Concluding remarks

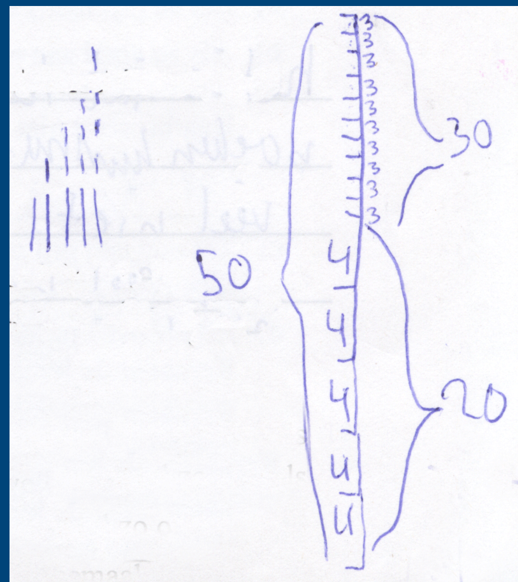
Example 1: How long?

- A rope of 30 meter is divided in 5 short and 3 long parts. A short and a long part together are 9 meter.
- How long is a short part?









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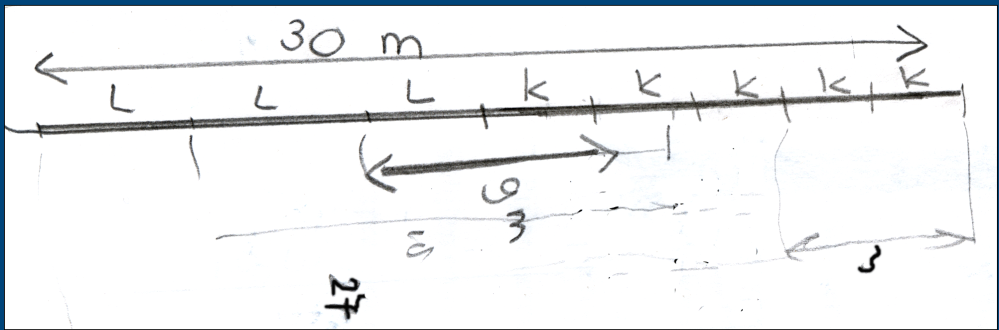
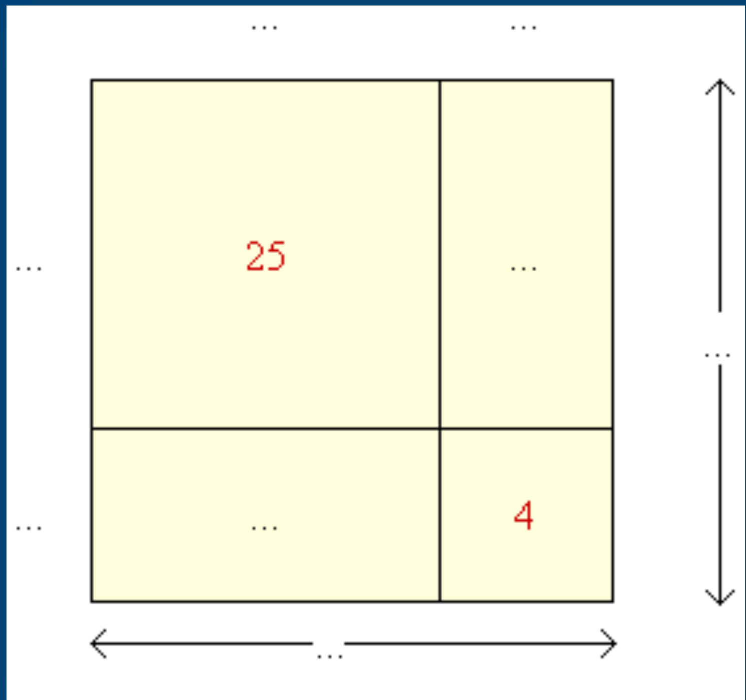
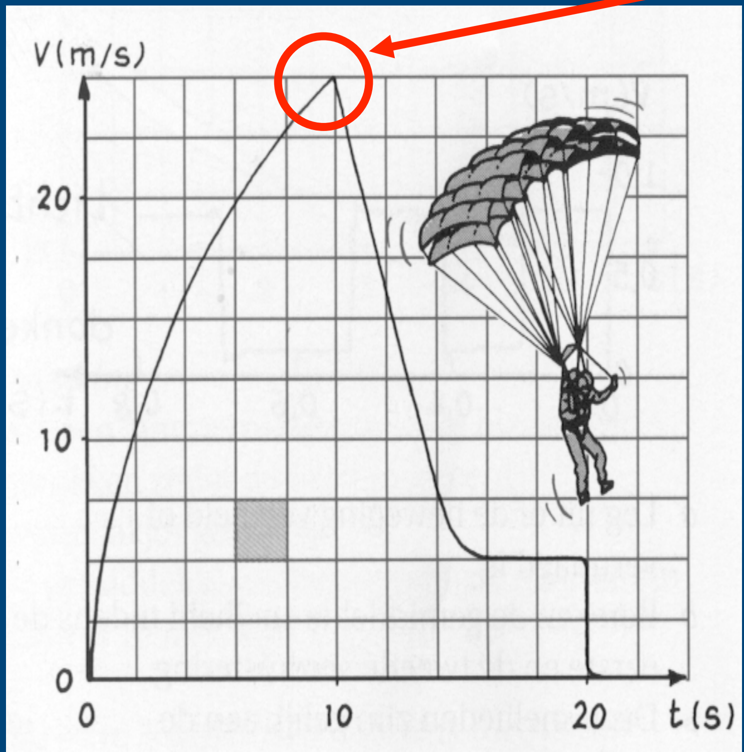


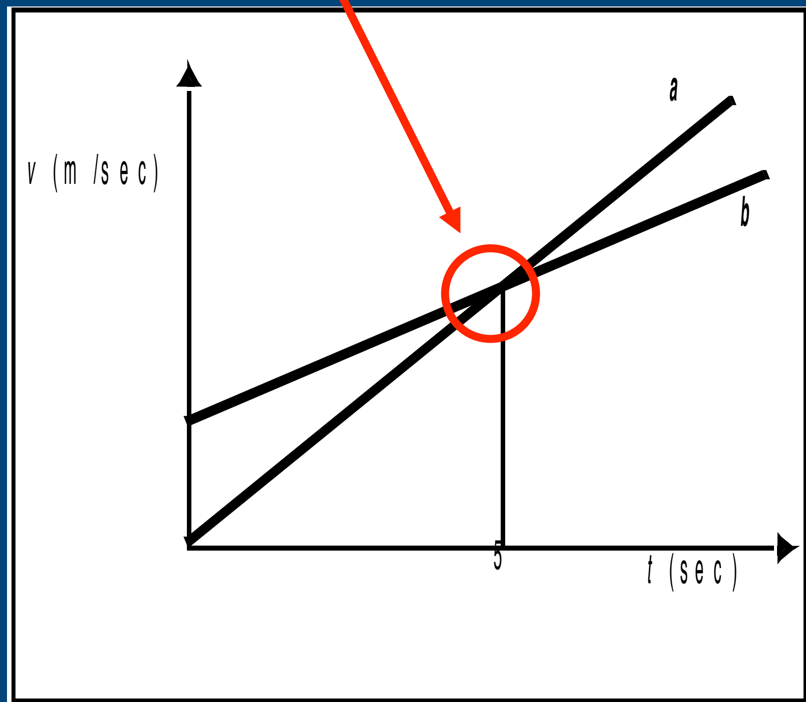


Example 2: Modeling motion

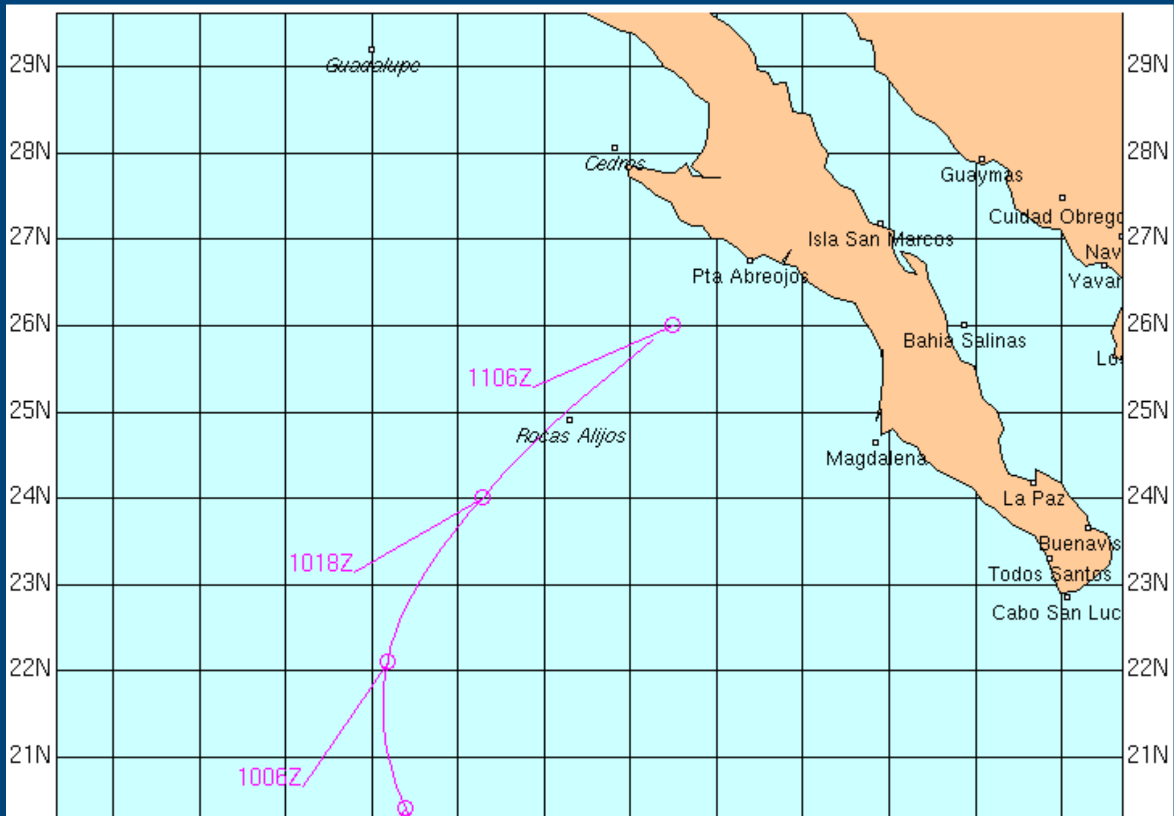
Students' difficulties

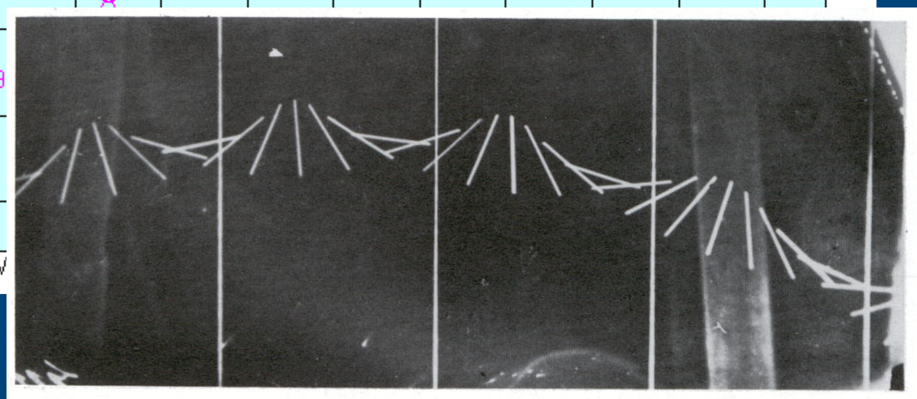
What is happening at these points?

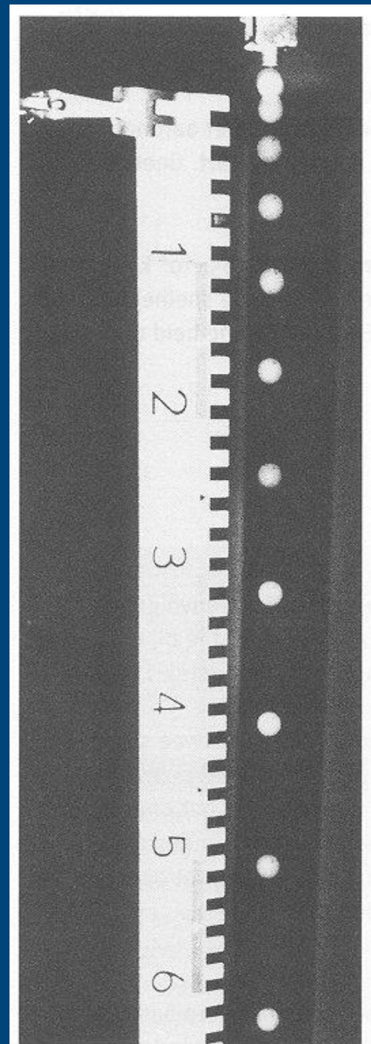




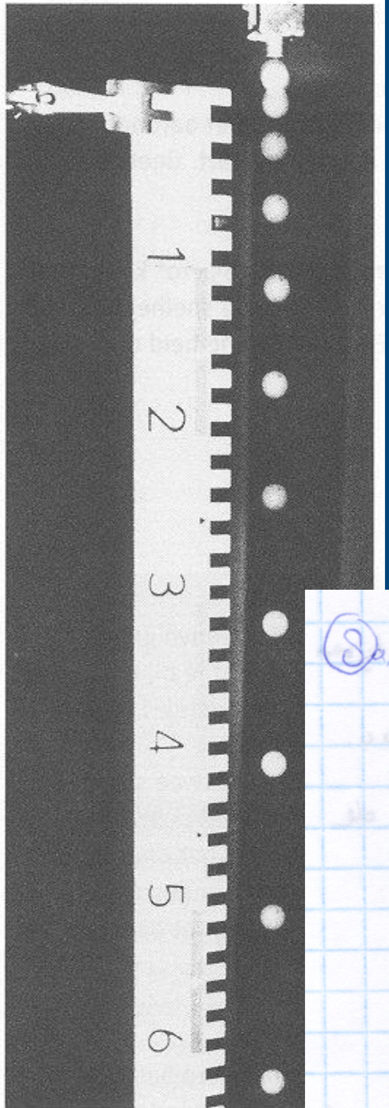
Example 2: When and how far?

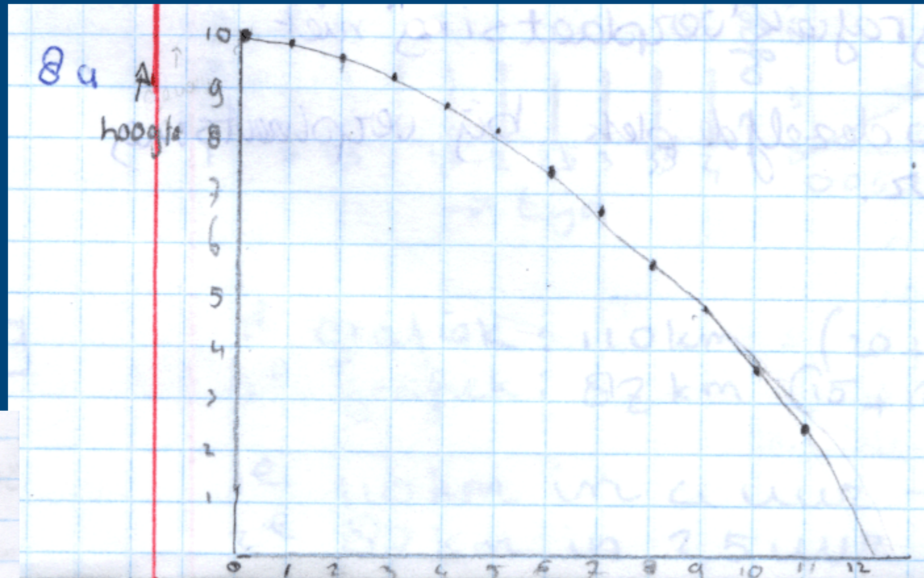


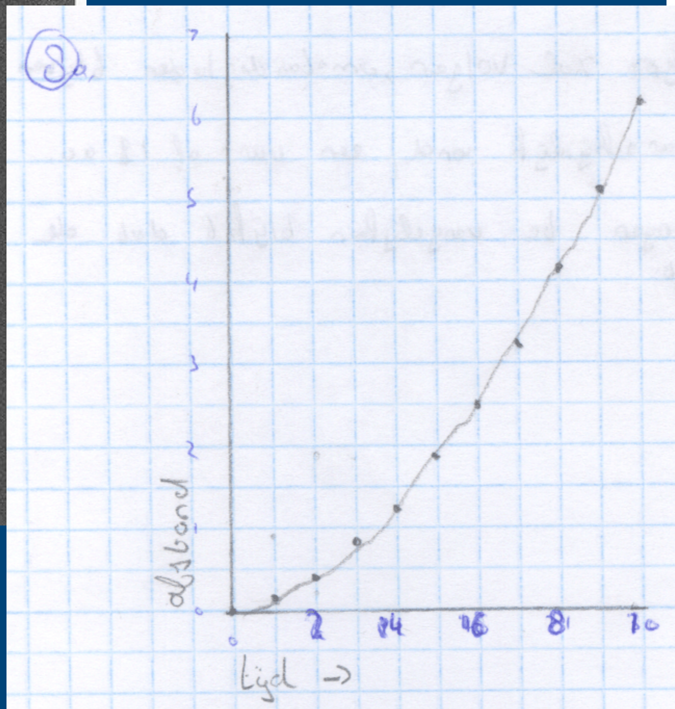


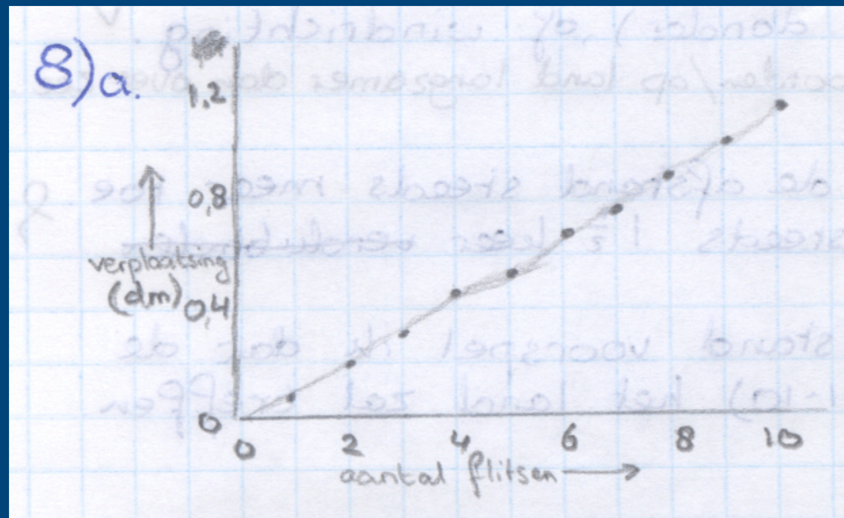


Experiences



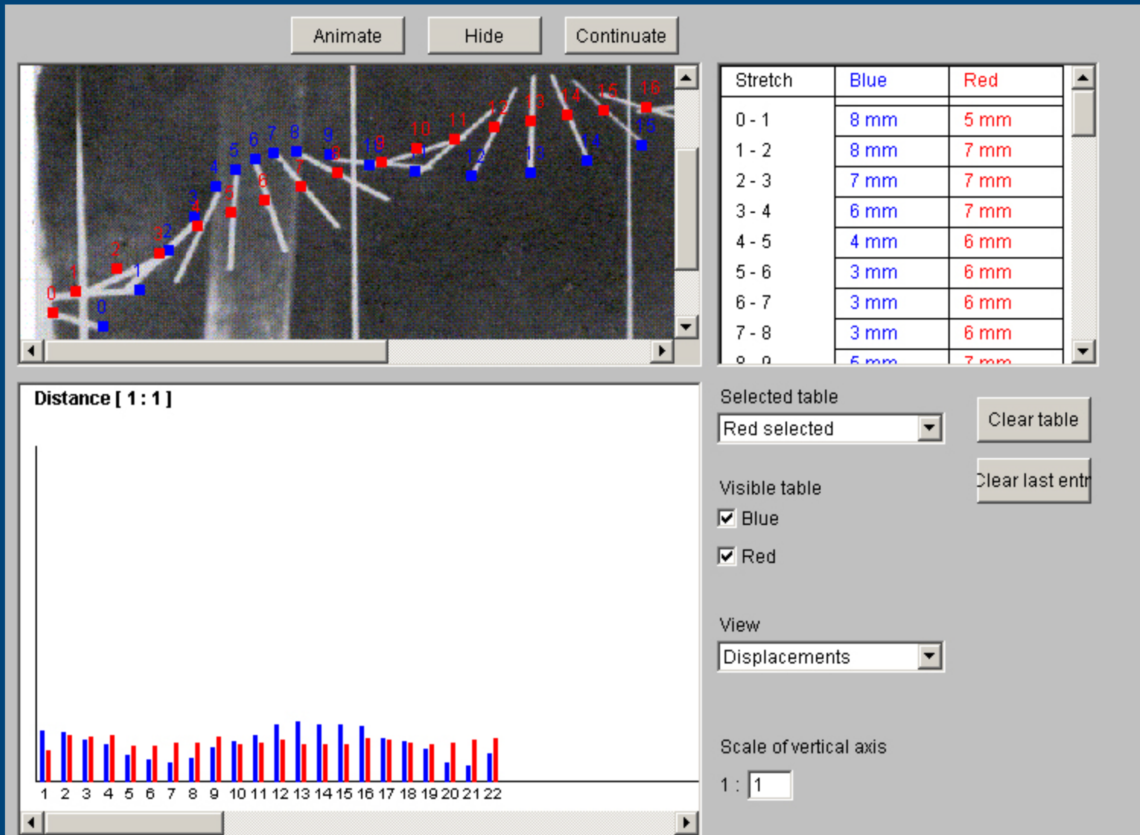


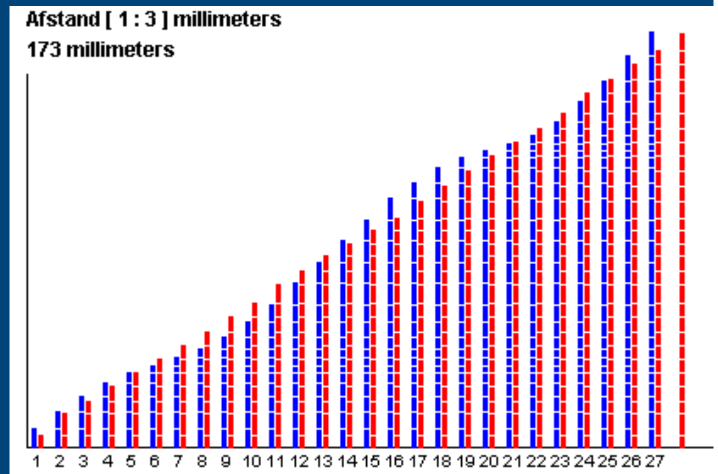




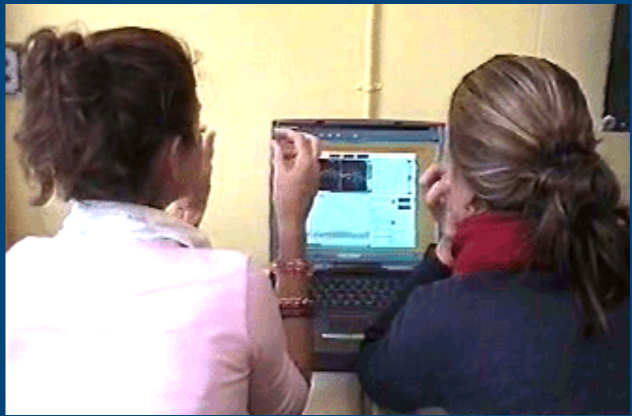
http://www.fisme.science.uu.nl/toepassingen/00197/flits_en/balletje.html

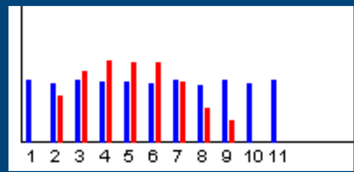
Experiences

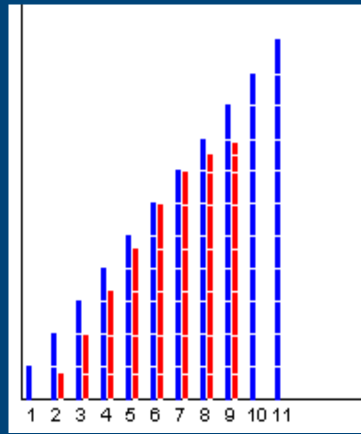




Experiences







O: So why did you choose the one for the total distance?

J: Because it's the total distance that they cover and then you can …

M: Then you can see if they catch up with each other.

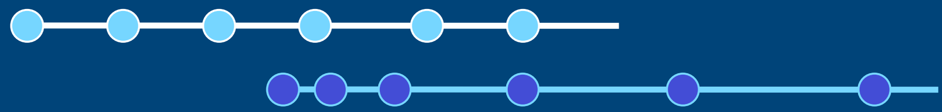
O: And can't you see that in the other? There you can also see that the red catches up with blue?

J: Yes, but …

M: Yes, but that's at one moment. That only means that it's going faster at that moment but not that it'll catch up with the zebra.

Carefully introduced models: A sequence of inscriptions

trace graphs



graphs of displacements

graphs of total distances traveled

Two examples

1. How long? (algebraic expressions)
2. When and how far? (modeling change)

What are similarities?

Both examples show the potential of non-routine problems to support the development of …

1. Reasoning with algebraic expressions
2. Reasoning about (rate of) change

Driving activity: mathematizing

Mathematizing

1. Mathematizing is the major activity of mathematicians
2. Mathematizing fosters applicability by familiarizing students with a mathematical approach to everyday situations
3. Mathematizing relates directly to the idea of reinvention, a process in which students formalize their informal understandings and intuitions

Cobb, P., Qing Zhao & J. Visnovska. (2010). Learning from and Adapting the Theory of Realistic Mathematics education.

Mathematizing

The process of formalizing informal understandings

- Students learn to solve problems using representations that make sense to them
- Next, problem situations evoke the need for more sophisticated representations

- This learning process can be described as generating a chain of signification
- Symbols in activity become the signified of new signifiers (Walkerdine, 1988)

From Symbolizing to RME

Challenge for mathematics educators is:

1. To support the development of mathematical symbol systems

2. To prevent that these become isolated pieces of knowledge

Nunes, T., Schliemann, A. D., & Carraher, D. W. (1993). *Street mathematics and school mathematics*. Cambridge : Cambridge University Press.

Realistic Mathematics Education (RME)

4 tenets:

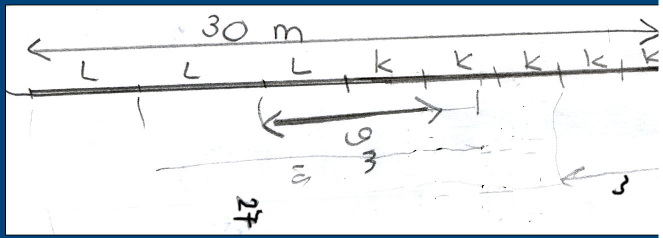
1. Starting in meaningful contexts with non-routine problems to evoke tentative representations
2. Change focus from informal strategies to formal mathematics (mathematizing)
3. Use models to support students´ learning
4. Interactive whole-class teaching

Lange, J. de (1996). *Using and Applying Mathematics in Education*. A.J. Bishop e.a. (eds). International handbook of mathematics education.

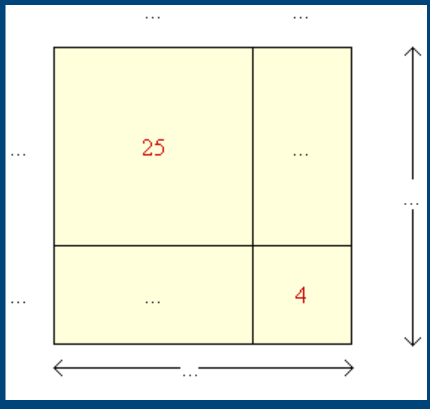
Van den Heuvel-Panhuizen, M. (2003). The didactical use of models in Realistic Mathematics Education. *Educational Studies in Mathematics 54*(1), 9-35.

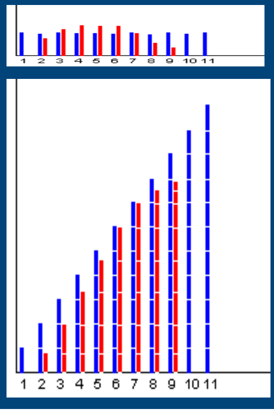
The two examples

- From situational to formal/general



- Non-routine problems evoked tentative representations
- Pre-defined symbols became didactical models that support the development of a mathematical symbol system (connected to the students informal reasoning)



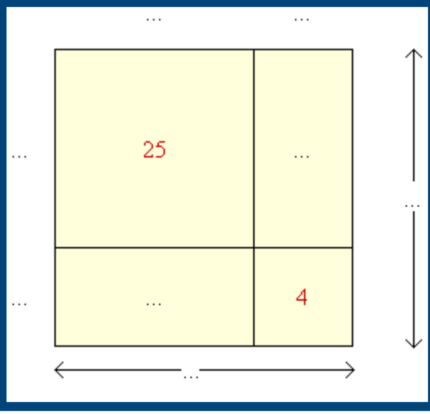


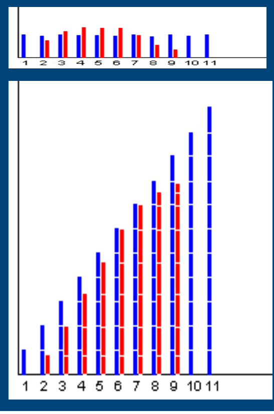
RME tenet 3: Use of models

- Models should not dictate to the students how to proceed, but must be a resource that fits their thinking and supports progress

- A model develops from *a model of a concrete situation* into *a model for more formal mathematical reasoning*

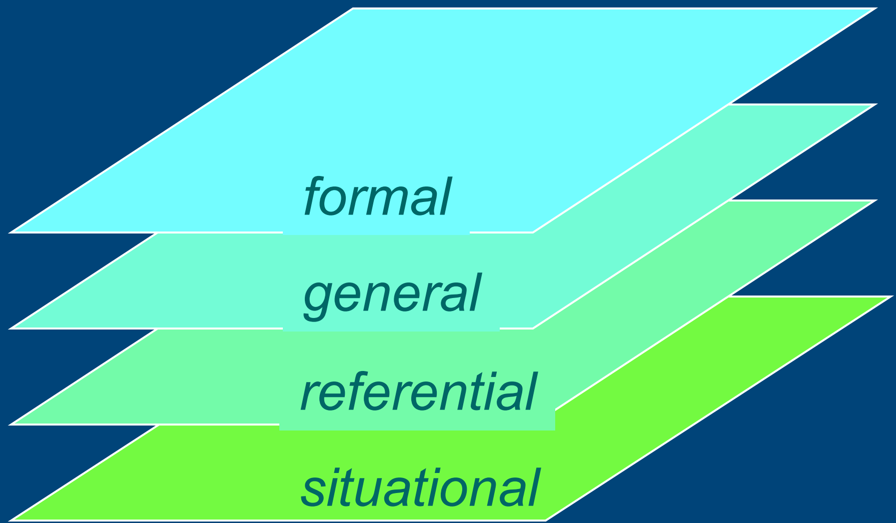
Gravemeijer, K. (1999). How emergent models may foster the constitution of formal mathematics, *Mathematical Thinking and Learning 1*(2), 155–177.





From *model-of* to *model-for*

Four levels of the Task Setting:

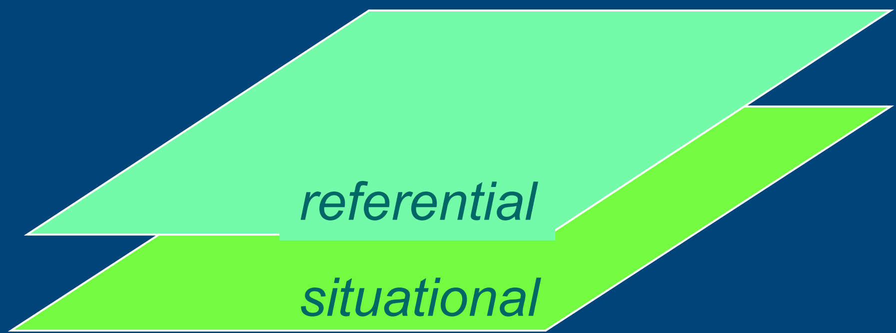


From *model-of* to *model-for*

Situational level
Interpretations and solutions depend on understanding of how to act in the setting (often out of school settings)
Predicting weather

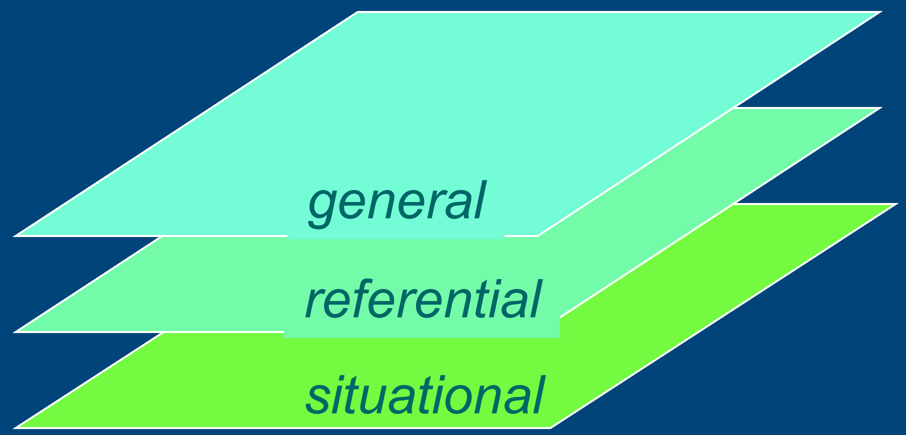
From *model-of* to *model-for*

Referential level
The emerging model derives its meaning from the reference to the situational activity and functions as a *model of* that activity
Reasoning with trace graphs



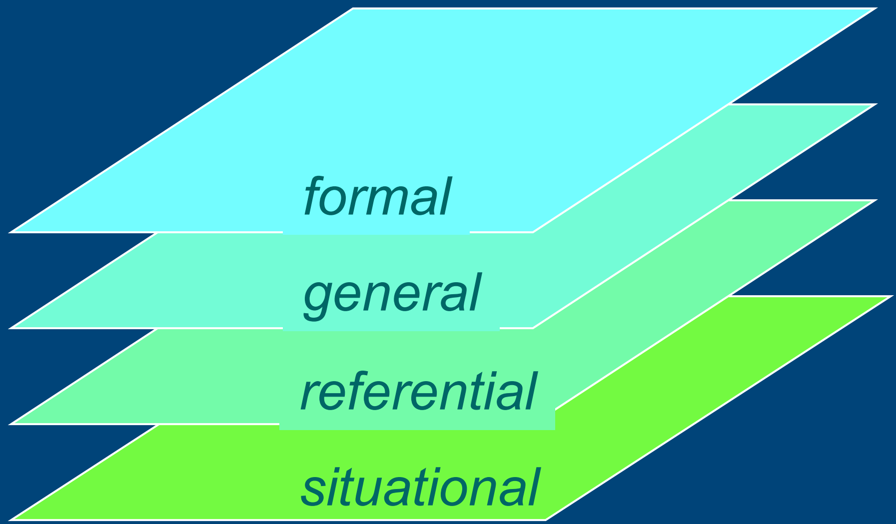
From *model-of* to *model-for*

General level
Attention shifts towards mathematical relations, the model starts to include those mathematical relations, and becomes a *model-for* mathematical reasoning
Reasoning about shapes of discrete graphs & predictions (in various situations)



From *model-of* to *model-for*

Formal level
More formal mathematical reasoning that is no longer dependent on the support of a model
Reasoning about differences and additions for grasping change (slope & area)



RME tenet 1: the use of rich non-routine context problems

As a source for learning,
but also as a goal (learning to apply)

Applications in daily-life,
world of work and/or further study?

A patient is ill. A doctor prescribes a medicine for this patient and advises to take a daily dose of 1500 mg. After taking the dose an average of 25% of the drug leaves the body by secretion during a day. The rest of the drug stays in the blood of the patient.

1. How much mg of the drug is in the blood of the patient after one day?

2. Finish the table.

Day	Mg of drug in blood
0	0
1	1125
2	
3	

3. Explain why you can calculate the amount of drug for the next day with the formula:
   new_amount = (old_amount + 1500) * 0,75

4. After how many days has the patient more than 4 g medicine in the blood? And after how many days 5 g?

5. What is the maximum of amount of the drug that can be reached? Explain your answer.

Improving the WoW connection

A doctor presents the following details about the use of a specific drug:

- An average of 25% of the drug leaves your body by secretion during a day.
- The drug is effective after a certain level is reached.
- Therefore it takes a few days before the drug that you take every day is effective.
- Do not skip a day.
- It can be unwise to compensate a day when you forgot the drug with a double dose the next day.

N.B. These details are a simplification of reality.

Investigation

- Use calculations to investigate how the level of the drug changes when someone starts taking the drug in a daily dose of 1500 mg with for instance three times 500 mg.
- Are the consequences of skipping a day and/or of taking a double dose really so dramatic?
- Can each drug level be reached? Explain your answer.

Product

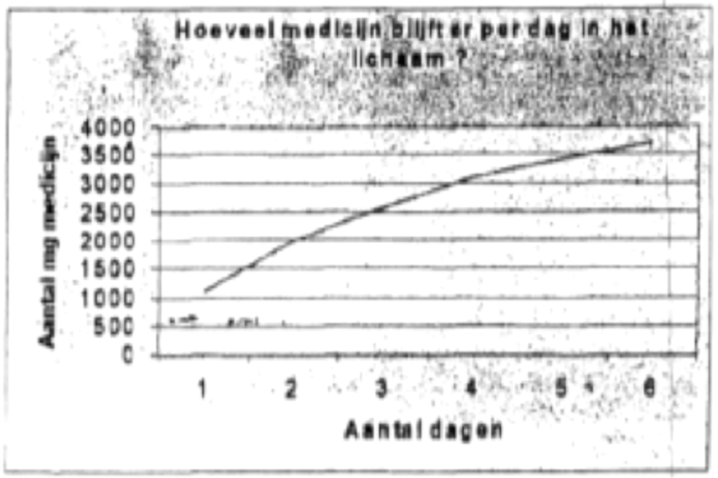
Design a flyer for patients with answers to the above questions. Include graphs and/or tables to illustrate the progress of the drug level over several days.

de Medicijnspiegel

Als je eenmaal per dag naar het toilet gaat verlaat 25 % van de door jou ingenomen medicijnen je lichaam
Dat betekent dat als je eerste dag van je medicijngebruik 3 keer 500mg slikt er daarvan 1500 * 0,75 = 1125mg in je lichaam overblijft

Als je elke dag 3 keer 500mg van het medicijn zou slikken krijg je het volgende resultaat

Dag	*totaal (mg)*	*Vn1*	*Vn2*	*Vn3*
1	1125			
2	1986,75	843,75		
3	2601,5625	632,8125	210,9375	
4	3076,2	474,6375	158,175	52,7625
5	3432,1	355,9	118,7375	39,437
6	3699,09	266,99	88,91	29,8275



De verschilrijen worden niet constant dus is het ook niet mogelijk bij deze rij een directe formule te geven. Wel is er een recusieve formule die luidt : *tn+1 = (1500 + tn) * 0,75*
Dit betekent dat het aantal medicijn in je lichaam geleik is aan het aantal van de vorige dag, daarbij komt 1500 mg en na het plassen blijft er nog 75 % van de totale hoeveelheid over in je lichaam

Het kan gebeuren dat je een dag vergeet je medicijnen in te nemen
Kun je dan zomaar de volgende dag de dubele dosis innemen en heeft dit gevolgen voor het eindpeil ?
Dat is in een tabel duidelijk weer te geven

Dag	*Constant*	*1 keer overslaan*
1	1125	1125
2	1986,75	843,75
3	2601,5625	2882,8125

Tussen de eindhoeveelheden zit niet zo een groot verschil, ongeveer 281,25 mg
Maar als je meerdere dagen overslaat en het later compenseert wordt het verschil steeds groter en krijgt het weldegelijk invloed op het eindpeil. Het is dan ook niet aan te raden dit te doen want hierdoor krijg je een veel te hoog eindpeil

Het kan natuurlijk ook voorkomen dat je een ander eindpeil hebt dan gewenst als je elke dag constant de medicijnen neemt. Dit kan komen doordat je gemiddeld meer of minder dan 25% uitscheid
Maar ook door hoe snel het lichaam de stoffen opneemt e.d

World of Work

- The **context** of the task relates to the WoW
- Students have to take a professional **role**
- Students' **activities** reflect workplace practices
- The task asks for a **product**

http://www.mascil-project.eu/

mascil

Context problems

Non-routine context problems have
the potential to…

- be a resource for learning mathematics
- show relevancy of the discipline
- foster creativity and self confidence
- deal with student differences

Concluding remarks

RME is a dynamic ‘theory’

Current issues:

- What goals for what students?
  (which contexts and to what extent from daily-life, world of work and further study)
- Use of ICT
  (games, simulations, feedback, …)
- Role of the teacher

Thank you

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