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Study and Research Paths for Teacher Education: a project about teachers' professional development from the Anthropological theory of the didactic

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Study and research paths for teacher education (SRP-TE)

Two main motivations

Study and Research Paths at School (SRP)

- Teach mathematics as a functional tool to solve problems, model situations, develop inquiry
- Important **constraints** are related to the prevailing pedagogical paradigm of 'visiting works', as opposed to the one of 'questioning the world' (Chevallard 2012)
- Teachers education as a field to **study the constraints** and try to **establish new conditions**

The problem of teacher education

- Avoid a “**visiting works**” approach with a set of previously established contents to be learnt
- **Start from teachers' professional concerns** and introduce didactic knowledge as a tool to approach them and to raise new questions: **paradigm of 'questioning the world'**
- Didactics of mathematics as a tool to pose and study questions, not a set of (dogmatic) answers

Outline

- ① The development of teaching (and research) from an institutional perspective
- ② The notion of didactic transposition as a questioning tool
- ③ Study and research paths for teachers education (SRP-TE)
- ④ Two examples of SRP-TE:
 - The sales forecast for in-service secondary school teachers
 - The baker's box for pre-service primary school teachers
- ⑤ Open questions in relation to meta-didactical transposition

1. The development of T and R from an institutional perspective

- **Anthropological Theory of the Didactic (ATD):** importance to the institutional dimension of human activities, including mathematics producing, disseminating, teaching, learning, etc.
- Human activities modelled in terms of **praxeologies** (*praxis* + *logos*): types of questions to solve and techniques to do so, theoretical discourse and assumptions to explain, justify, organize, etc.
- **Dialectics between individuals and institutions:**
 - Praxeologies are collective constructions “living” in **institutions** made of human being sharing common projects
 - Human beings acquire their **praxeological equipment** (knowledge and know-how) by frequenting different institutions they contribute to made evolve

2. The “didactic transposition” as a questioning tool

- The notion of **didactic transposition** (Chevallard 1985) was first introduced as a tool to question the mathematical knowledge involved in teaching and learning processes:
 - What is it? In what mathematical organizations does it appear?
 - Where does it come from? How was it originally? What motivated it?
 - How was it selected and why? How was it transformed in order to become knowledge to be taught, then taught knowledge?
 - Could it be done differently? What are the consequences? Etc.
- **Illusion of transparency** (Bourdieu et al 1968): we think we know what mathematical knowledge is, that what we have learnt is the only and “normal” way of conceiving it. The problem is not the mathematical knowledge but the way it is taught and learnt.
- **Example of integers:** Cid et al (2007), Bosch & Chevallard (2014)

2. The “didactic transposition” as a questioning tool

■ If we apply it to **teachers education**:

- What praxeologies have to be taught? Of what “nature” are they?
- Where do they come from? How were they originally? What motivated them?
- How are they selected and why? How are they transformed in order to become knowledge to be taught, then taught knowledge?
- Could it be done differently? What are the consequences? Etc.

■ **Different levels of questioning:**

- Teaching praxeologies as ‘monuments to visit’ or ‘questioning tools’?
- What relationship with the didactic (researchers’) praxeologies? Same types of questions? Methodologies? Theoretical frames?
- Importance of the mathematical component of these praxeologies: how to help teachers *question* the mathematical knowledge to be taught?

→ **FIRST LINK TO META-DIDACTICAL TRANSPOSITION**

3. Study and Research Paths for Teacher Education

The herbartian schema (Chevallard, 2004)

- Group of **students** X and study **supervisor(s)** Y
- They start from a **question** Q [more or less “big”]
- They should elaborate their own (collective) **answer** A^\heartsuit to Q

$$S(X ; Y ; Q) \rightarrow A^\heartsuit$$

- To elaborate A^\heartsuit , an ‘experimental **milieu**’ is needed

$$[S(X ; Y ; Q) \rightarrow M] \rightarrow A^\heartsuit$$

- The milieu is composed by other bodies of knowledge A_i^\diamond (labeled **answers**) and other **objects** O_k

$$[S(X ; Y ; Q) \rightarrow \{ A_1^\diamond, A_2^\diamond, \dots, A_n^\diamond, O_{n+1}, \dots, O_m \}] \rightarrow A^\heartsuit$$

3. Study and Research Paths for Teacher Education

- The herbartian schema as a general model to understand

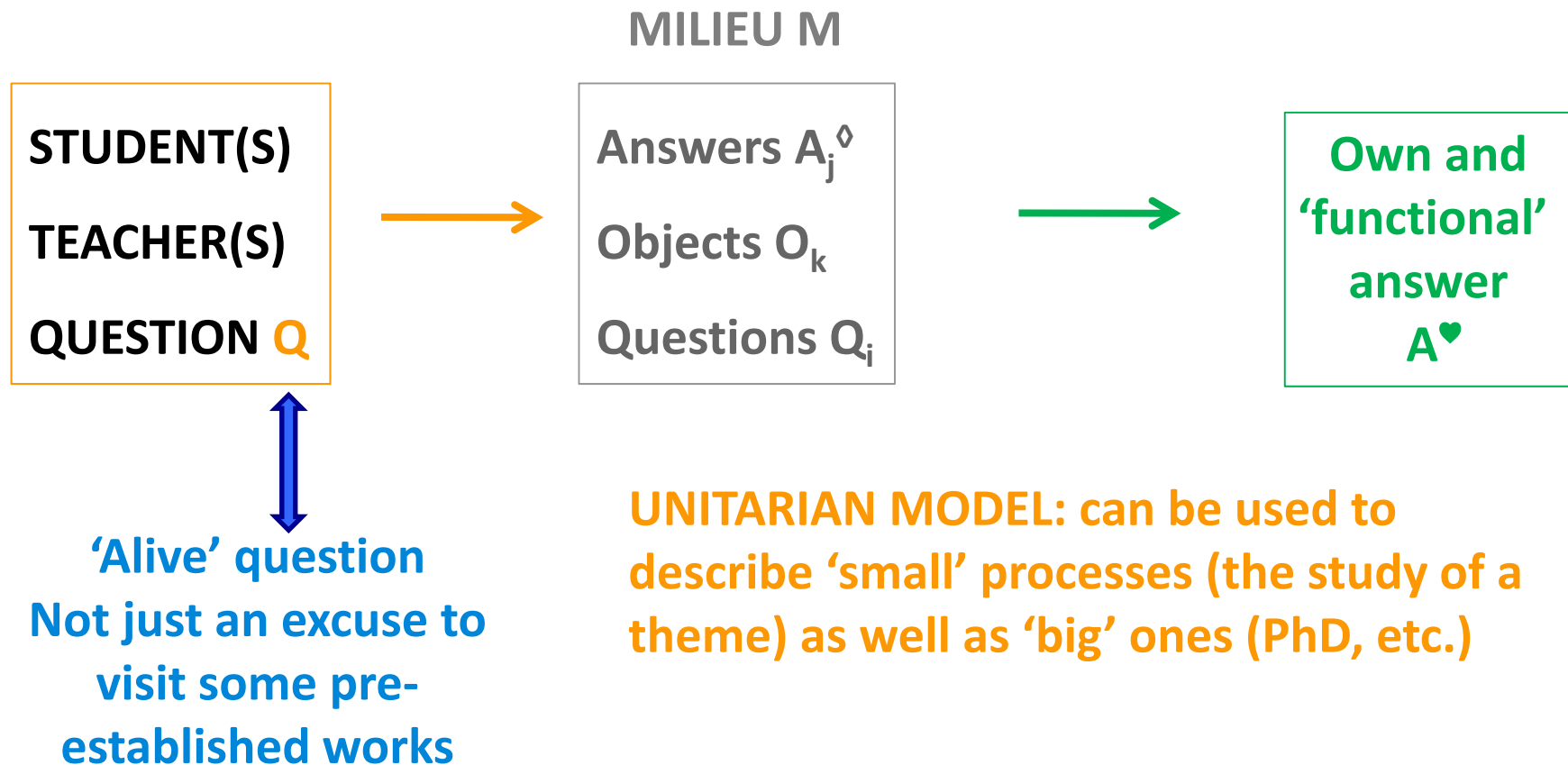
the “transmissive” pedagogy (*monumentalism*)

- The initial question remains in the shadow Q
- The teacher brings an answer A^T (‘concept’, praxeology) legitimated by culture and validated by him/herself
- Students assume it as their own answer $A^\heartsuit \approx A^T$
- The relationship to knowledge is a “cultural copying” (*lector - reader*)

$$[S(X ; Y ; Q) \rightarrow \{ A^T \}] \rightarrow A^\heartsuit \approx A^T$$

3. Study and Research Paths for Teacher Education

But also to propose ... **STUDY AND RESEARCH PATHS (SRP)**



Taking into account
a generating
question Q

Meeting other questions
Q', Q'', etc. that seem to
help approach Q
→ *skeleton of the
process*

Search answers A_i^\diamond to
Q', Q'', etc. in the
available *media*

**DIVERSITY OF
POSSIBLE
(study and
research)
PATHS**

Evaluation, diffusion
and development of
the final answer A^\heartsuit

Evaluation (validation) of
answers A_i^\diamond through an
appropriate *milieu*

Elaboration of an
own answer A^\heartsuit

Deconstruction and
reconstruction (development) of
answers A_i^\diamond to adapt them to Q

3. Study and Research Paths for Teacher Education

THE HERBARTIAN SCHEMA TO ANALYSE TE PROCESSES

- **QUESTIONS:** What are the possible initial questions **Q**? Where do they come from? How are they selected? Why are they to be approached? How to formulate them? What assumptions underlie their formulation? What new questions can be derived? Etc.
- **ANSWERS:** What are the available answers A_j^\diamond for (in-service or pre-service) teachers? Where do they come from: didactics research, other disciplines, etc.? How are they made accessible (didactic transposition)? How are they validated? Etc.
- **MILIEUS:** What possible milieus **M** can be used to deconstruct and reconstruct A_j^\diamond till obtaining A^\heartsuit ? Etc.
- **ECOLOGY:** What conditions are needed for SRP-TE to be carried out? How to set them up? What constraints hinder them? Etc.

3. Study and Research Paths for Teacher Education

THE HERBARTIAN SCHEMA TO ANALYSE TE PROCESSES

- **QUESTIONS:** What are the possible initial questions Q ? Where do they come from? How are they selected? Why are they to be approached? How to formulate them? What assumptions underlie their formulation? What new questions can be derived? Etc.
- **ANSWERS:** What are the available answers A_j^\diamond for (in-service or pre-service) teachers? Where do they come from: didactics research, other disciplines, etc.? How are they made accessible (didactic transposition)? How are they validated? Etc.
- **MILIEUS:** What possible milieus M can be used to deconstruct and reconstruct A_j^\diamond till obtaining A^\heartsuit ? Etc.
- **VIABILITY:** What conditions are needed for SRP-TE to be carried out? How to set them up? What constraints hinder them? **What A^\heartsuit is expected?**

→ **SECOND LINK TO META-DIDACTICAL TRANSPOSITION**

3. Study and Research Paths for Teacher Education

Assumptions concerning teacher education programmes

- **TE programmes** should include the questioning of teaching and learning praxeologies, including didactic transposition processes
→ *Professional questions should be at the core of TE*
- **TE programmes** cannot be confined in the “visiting the works” paradigm; they should open towards the new pedagogical paradigm of **questioning the world**
- **CONJECTURE: SRP-TE** could be a useful tool to:
 - (a) Help teachers question and study the dominant paradigm from an external point of view and raise new questions
 - (b) Provide tools for the epistemological and didactic analysis of teaching and learning processes
 - (c) Jointly elaborate new mathematical and didactic infrastructures (or, at least, jointly acknowledge its necessity...)

3. Study and Research Paths for Teacher Education

According to the previous assumptions, we propose to describe SRP-TE in the following stages:

- 1. Let teachers experiment a SRP** close to what could exist in their classes (role-play or real play) *and related to a professional question* (How to teach functions, proportionality, algebra, etc.)
- 2. Analyse the SRP** using didactic tools:
 - i. Mathematical analysis (reference epistemological model)
 - ii. Didactic analysis: Changes in the didactic (and pedagogical) contracts, dialectic media-milieu, questions and answers, etc.
 - iii. Viability of SRP: institutional conditions and constraints
- 3. Design of new SRP adapted to a given group of students**
- 4. Implementation and a posteriori analysis**

4. Two examples of SRP-TE

SRP-TE_1: Sales forecast for in-service Secondary school teachers

The case of an on-line course in Maths Education for in-service
Secondary school teacher. CICATA (Mexico)

SRP-TE_2: The baker' box for pre-service Primary school teacher

The case of a course in Didactics of Mathematics for pre-service
Primary school teacher. University of Barcelona (UB)

SRP-TE in progress [secondary school teachers education]

- **Saving plans.** Alicia Ruiz Olarría (U Autonoma de Madrid)
- **Real numbers and measure.** Mabel Licera (U Río Cuarto, Argentina):
- **Geometry.** Federico Olivero (U Comahue, Argentina)



4.1. Sales forecast for in-service Secondary school teachers

The screenshot shows a web browser window with the address bar displaying `cursos.cicata.edu.mx/course/info.php?id=19`. The page features a header with a graphic of binary blocks and the text "POSGRADO en línea de MATEMÁTICA EDUCATIVA". Below the header is a breadcrumb trail: "Página Principal (home) ► Cursos ► Maestría en Matemática Educativa ► Maestría 2013 ►". On the left, a "Navegación" sidebar lists links: "Página Principal (home)", "Novedades del sitio", "Curso actual" (with a sub-link "PIME-10"), and "Cursos". On the right, a section titled "Procesos de Institucionalización de la Matemática Escolar" lists several professors: Berta Barquero, Marianna Bosch Casabò, Apolo Castañeda, Josep Gascón, Avenilde Romo Vázquez, Alicia Ruíz, and Mario Sánchez Aguilar.

The case of an on-line course in Maths Education for in-service Secondary school teacher. **CICATA (Mexico)**



4.1. Sales forecast for in-service Secondary school teachers

Q_0 : How to **analyse, adapt, develop** and **integrate** a learning process related to **mathematical modelling** in our teaching practice?

How to institutionally sustain long-term learning processes based on modelling? What difficulties should be overcome? What teaching tools are needed? What new questions arise and how can they be addressed?

During four weeks we approached these issues through a case **study on sales forecast** and considering **four activities** corresponding to four different stages of the didactic analysis:

- epistemological analysis;
- a priori didactic analysis;
- experimentation and a posteriori analysis;
- analysis of the didactic ecology.



4.1. Sales forecast for in-service Secondary school teachers

ACTIVITY 1. Resolution and analysis of 'Forecast sales to Desigual'

Participants' role: mathematical learner ('apprentice')

Aim: Work in the activity to perform a first epistemological analysis

Phase 1: Carry out the assignment to Desigual

Phase 2: Analyse the other participants' reports and contrast with your own

Phase 3: Present a final common report. Accompany it with an analysis of the process followed and the difficulties found



- 9) Evolución de las ventas semanales de las camisetas one-print desde el 15 de febrero de 2010 en la tienda de las Ramblas 136 (Barcelona):

Semana	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11
Camisetas one-print vendidas	233	112	118	130	116	151	159	173	175	230	253

- 10) Evolución de las ventas semanales de las camisetas one-print desde el 1 de marzo de 2010 en la tienda de la Rambla de Catalunya 140 (Barcelona):

Semana	1	2	3	4	5	6	7	8	9	10
Camisetas one-print vendidas	100	101	107	115	125	140	140	164	194	210



4.1. Sales forecast for in-service Secondary school teachers

ACTIVITY 2. Pedagogical guide

Participants' role: traditional teacher going for a pleasure trip

Aim: Design of a (easy readable) pedagogical guide on “Forecast sales”

Phase 1: Elaboration of a first version of the pedagogical guide

Phase 2: Analysis of the group participants' guides

Phase 3: Propose a common guide

ACTIVITY 3. Experimentation

Participants' role: teacher **Aim:** Implement the initial phase of the activity ‘Forecast sales to Desigual’ (first previsions with one variable) with a group of students and analyse it.

Phase 1: Planning and a priori analysis

Phase 2: Experimentation

Phase 3: A posteriori analysis

Phase 4: Experimentation report

4.1. Sales forecast for in-service Secondary school teachers



ACTIVITY 4. Joint analysis and final revision of the pedagogical guide

Participants' role: teacher analyst


Aim: Propose a new version of the pedagogical guide carrying out a mathematical and didactic analysis of the teaching material

Phase 1: Sharing of the experimentations

Phase 2: Revision of the pedagogical guide

Phase 3: Preparation of a joint final version of the pedagogical guide

Retroalimentación




Comentarios a informes y guías

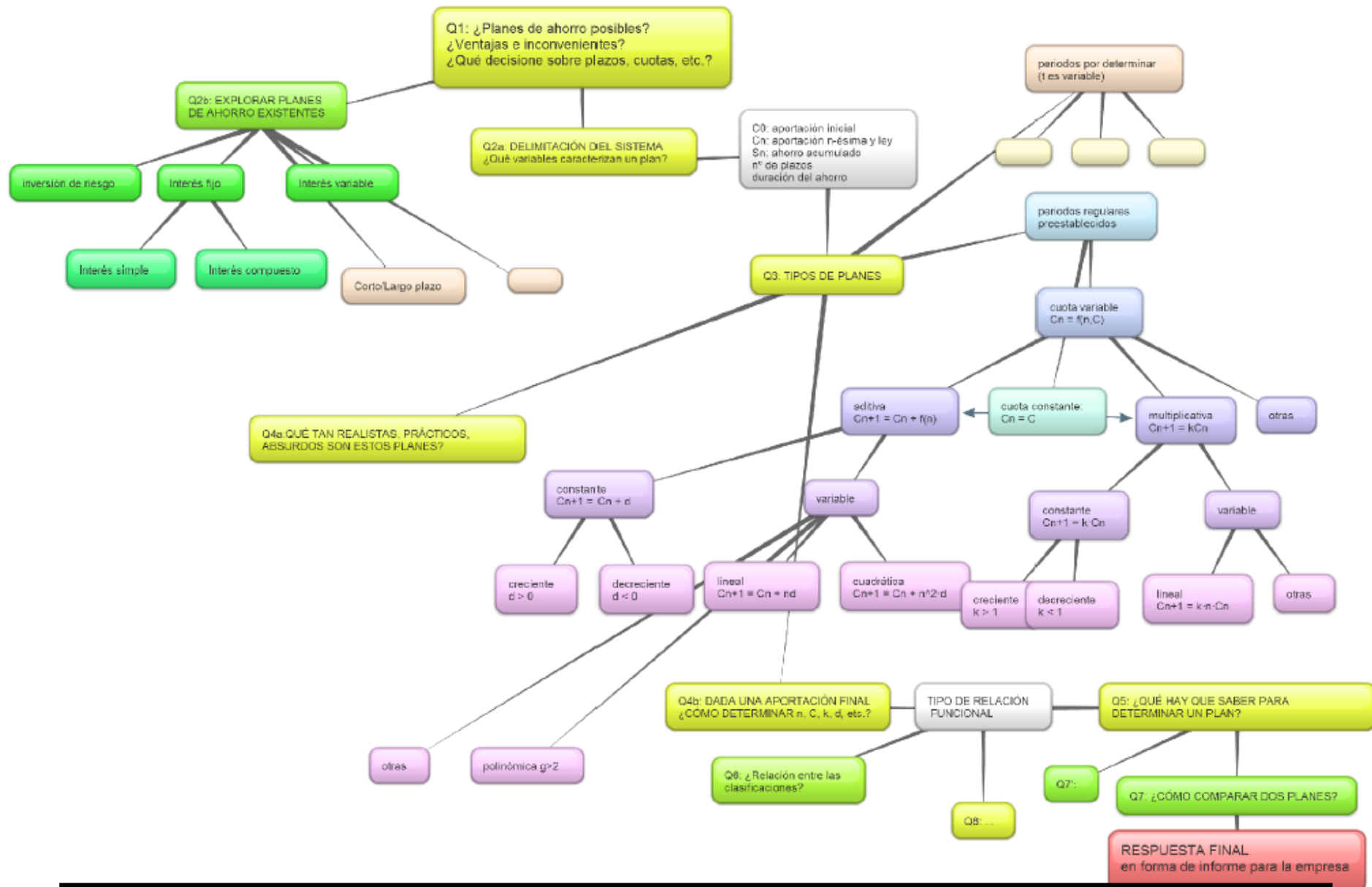
Actividad 3

- Foro de consulta sobre la Actividad 3
- Artículo Serrano, Bosch y Gascón (2013)
- Pautas para realizar y presentar el análisis
- Actividad 3

Actividad 4



Foro de consulta sobre la Actividad 4



Mathematical analysis of a SRP: Tree of questions and answers

4.2. The baker' box for pre-service Primary school teacher

The case of a course in Didactics of Mathematics for pre-service Primary school teacher. **University of Barcelona (UB)**

Conditions for its implementation:

- The course on *Didactics of Mathematics II* runs over the whole first semester for **in-service Primary school teachers**
- The implementation of the SRP-TPD was carried out during one-month and a half (3 hours per week), including **four activities** also corresponding to four different stages of the didactic analysis

Q_0 : How to **design, analyse-evaluate, adapt, develop** and **integrate** a learning process related to real-problems inquiry studies or to mathematical modelling in our teaching practice?

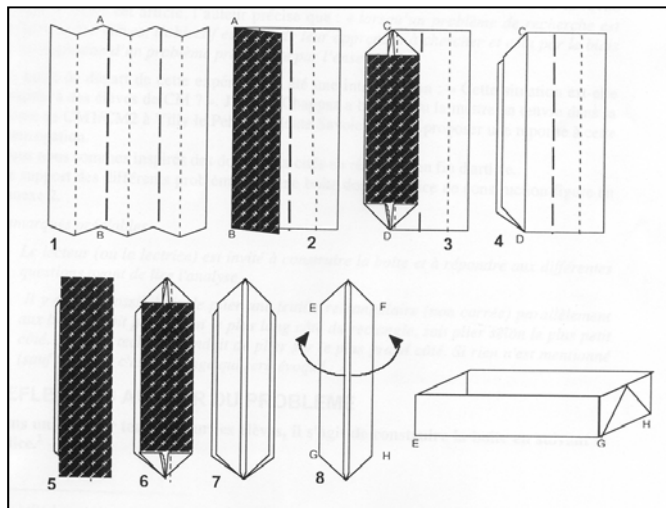
4.2. The baker' box for pre-service Primary school teacher

PHASE 1 – 'Life' an SRP as student

Participants' role: students('apprentice') at Primary school level (or higher)

In the classroom there were some students with the role of 'observers and reporter' (2 mathematical observers and 1 didactic observer).

Aim: Work in the activity to be able to perform a first epistemological analysis



Q_0 : How can we build a box to help a baker to pack her/his cakes? What kind of relation there is between the sizes or dimensions of the sheet and the dimensions of the resulting box?

Chappaz & Michon (2003)

Ruiz-Higueras, L. (2008)

4.2. The baker' box for pre-service Primary school teacher

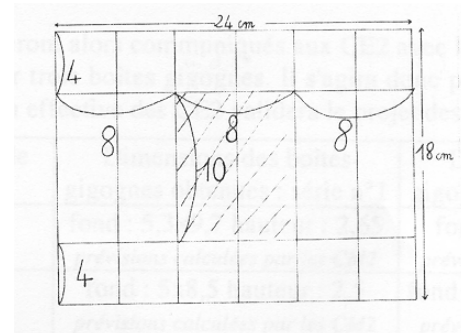
Q_0 : How can we build a box to help a baker to pack her/his cakes? What kind of relation there is between the sizes or dimensions of the sheet and the dimensions of the resulting box?

1st phase:

What are the dimensions of a box obtained from a specific sheet? → Q_1

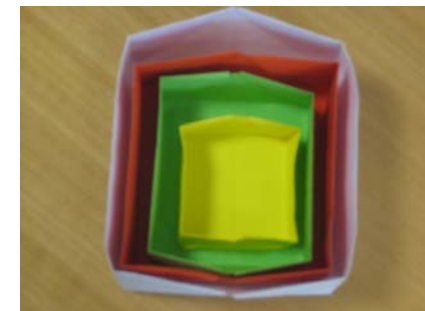


2nd phase: If we need to build a box with some particular box' sizes, what sheet' dimensions do we have to consider? → Q_2



3rd -4rth phases:

From artisans to engineers phase: How can we describe these relations? → Q_3



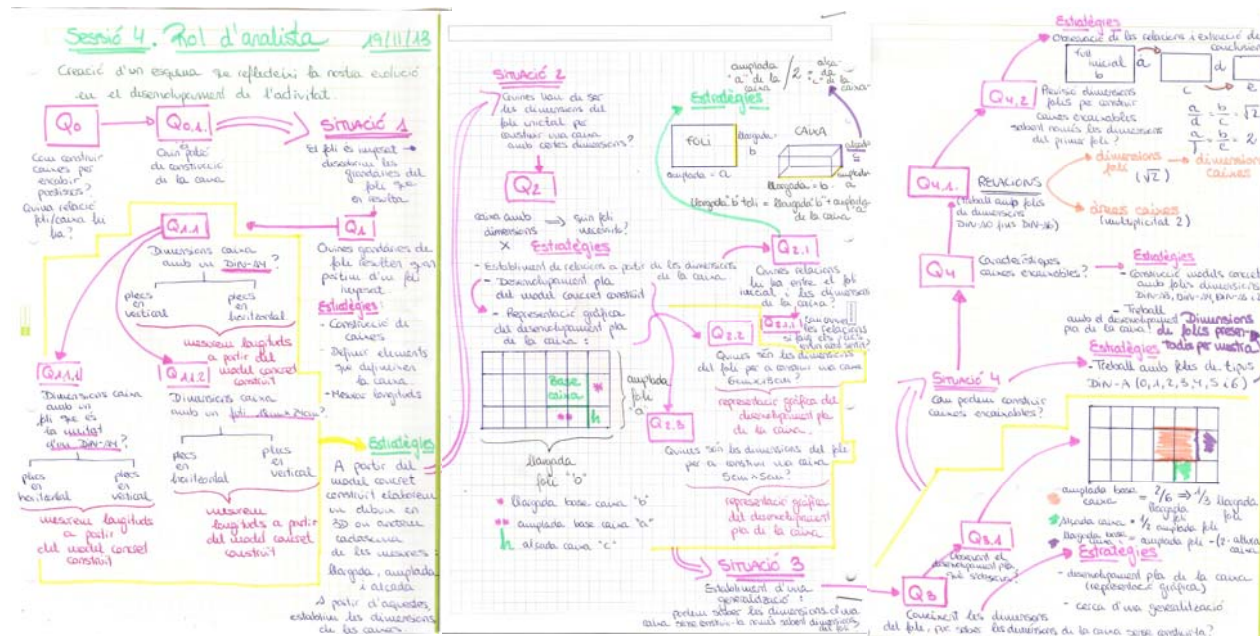
4.2. The baker' box for pre-service Primary school teacher

PHASE 2 – Analyse the SRP as mathematical and didactical analytics

Participants' role: mathematical and didactic analyst

Phase 2.1. Aims and tasks

(A) Build up a 'mathematical skeleton' from the whole mathematical process that students and the teacher came to 'live'.



(B) Us this new mathematical skeleton to: (1) Analyse other groups productions and (2) Define new elements to evaluate their work.

Participants' role: Mathematical and didactic designer (but also Primary school teachers, analysts,...)

- Groups had one month to: (1) Develop their a priori designs; (2) Its experimentation and (3) A posteriori analysis.

SITUACIÓ 0
Sense conèixer el número total de pilotes ni els colors que hi ha a l'ampolla, els alumnes esbrinen quines dues pistes (donades 5 pistes per les mestres) els poden ajudar per poder saber quantes pilotes hi ha de cada color.

Q0
Com podem saber quantes pilotes de cada color hi ha en una ampolla que no podem obrir?

ESTRATÈGIES
- Experimentació amb material manipulatiu
- Raonar a partir de les dades (pistes)
- No hi ha recompte

SITUACIÓ A
5 pilotes
5 colors (blanc, vermell, verd, taronja i groc)

Q0.1
Quins colors heu vist a través del forat de l'ampolla?

Q0.2
De què depèn que surti una pilota o una altra?

Q0.3
Només amb els colors que hem vist, podem predir quantes pilotes hi ha de cada color?

Q0.4
Quines pistes (de les que us oferim) creieu que ens poden ajudar a saber quantes pilotes de cada color hi ha?

Q1
De quin color hi ha més pilotes a l'ampolla?

ESTRATÈGIES
- Experimentació amb material manipulatiu
- Recompte (Freq. Absoluta)

SITUACIÓ 1
Coneixent el número total de pilotes i els colors de les pilotes de l'ampolla, els alumnes esbrinen quin és el color del qual hi ha més pilotes.

SITUACIÓ B
4 pilotes
2 colors (3 blaves-1 rosa)
10 tirades

SITUACIÓ C
4 pilotes
3 colors (2 grogues-1 blava-1 rosa)
10 tirades

Q1.1
Si només podem mirar pel forat, què podem fer per saber quantes pilotes de cada color hi ha?

Q1.2
Què significa que un color surti més cops en el recompte? I que surti menys? I que dos colors surtin el mateix nombre de vegades?

Q1.3
De quin color creieu que hi ha més pilotes a l'ampolla? Per què?

Q1.4
De quin color creieu que hi ha més pilotes a l'ampolla? Per què?

Q2
Quina pilota seria impossible, segur i possible que ens sortís?

ESTRATÈGIES
- Experimentació amb material manipulatiu
- Recompte (Freq. Absoluta)
- Anàlisi de les situacions B i C i dels recomptes- raonar a partir de les dades

SITUACIÓ 2
A partir de les situacions B i C i del recompte, els alumnes diuen quins esdeveniments són impossibles, segurs i possibles.

SITUACIÓ 3
A partir dels resultats del recompte de les situacions B i C, els alumnes busquen les possibles combinacions de colors que hi pot haver en cada situació (B i C), diuen quina combinació de colors és la més probable i així predirien quantes pilotes de cada color hi ha.

Q2.1
Què és impossible que ens surti si mirem per aquesta ampolla? I què és segur? I què és possible?

Q2.2
Què és impossible que ens surti si mirem per aquesta ampolla? I què és segur? I què és possible?

Q3
Quantes pilotes de cada color hi ha a l'ampolla?

ESTRATÈGIES
- Experimentació amb material manipulatiu
- Recompte (Freq. Absoluta)
- Combinació dels diferents colors de cada ampolla
- Anàlisi del recompte i de les combinacions- raonar a partir de les dades
- Elaboració i anàlisi d'un pictograma (Ampliació mostral)

SITUACIÓ B
4 pilotes
2 colors (3 blaves-1 rosa)
10 tirades

SITUACIÓ C
4 pilotes
3 colors (2 grogues-1 blava-1 rosa)
10 tirades

Q3.1
Quines possibles combinacions de colors pot haver a l'ampolla?

Q3.2
Amb els resultats de cada grup, quantes pilotes creieu que hi ha de cada color? Per què?

Q3.3
Si ajuntem els resultats de tota la classe, canviariem d'opinió sobre quantes pilotes hi ha de cada color? Per què?

R0.1: 1
R0.2: 2
R0.3: 3
R0.4: 4
R0.5: 5

R0.1: Sort, atzar
R0.2: Si
R0.3: No

R0.4: Pista 1
R0.5: Pista 2
R0.6: Pista 3
R0.7: Pista 4

R1.1: Blaves
R1.2: Roses
R1.3: Grogues

R2.1: Impossibles
R2.2: Segur
R2.3: Possible

R3.1: SITUACIÓ B
R3.2: SITUACIÓ C
R3.3: SITUACIÓ B
R3.4: SITUACIÓ C

5. Three questions to the audience

(1) The idea of “questioning” is crucial in research and also in teacher education

- How can it be integrated in the conception of a **‘shared praxeology’** between teachers and researches?
- How to overcome the **risk of ‘schizophrenia’** (questioning what you are supposed to be an expert of)? Role of the **‘brokers’**?

(2) What are the elements of the ‘shared praxeology’?

- Type of task or problems? Techniques? Theoretical discourse?

5. Three questions to the audience

(3) Do SRP-TE and the 'double dialectics' go in the same way?

- **Phase 1:** materialisation of the didactic and mathematical levels (including an epistemological model for the analysis)
- **Phase 2:** the study and research process is taken as object of study, thus meta-didactic level
- **Phases 3 and 4:** validation and development of phases 1 and 2



SRP-TE: a project about teachers' professional development
from the Anthropological theory of the didactic

ESKERRIK ASKO

MUCHAS GRACIAS

GRAZIE

THANK YOU VERY MUCH