THEORETICAL APPROACHES TO MATHEMATICS EDUCATION: THE CONSTRUCT OF RATIONALITY AND ITS INTEGRATION WITH OTHER THEORIES

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RATIONALITY

- The starting point: Habermas' construct of rational behavior in discursive practices

- Three dimensions:
  - knowledge and meta-knowledge at play (epistemic rationality, ER)
  - action and its goals (teleological rationality, TR)
  - communication and related choices (communicative rationality, CR)

- The construct was adapted to analyse mathematical activities like proving and modeling (Boero, 2006; Boero & Morselli, 2009)

- Indeed, all those activities move along between epistemic validity (ER), strategic choices (TR) and communicative requirements (CR)

PME Research Forum

(Boero et alii, 2010)

Argumentation and proof: A contribution to theoretical perspectives and their classroom implementation

Integrated with Toulmin's model of argumentation

- To plan and analyse students' enculturation into the culture of theorems in the context of geometry and elementary theory of numbers
- To analyse argumentations at content and meta level

The "Habermas Group"

- Group of teachers and researchers
- Regular meetings in Turin, starting from 2012

- Workshops on the use of the theoretical tool derived from Habermas

- Integration with other theoretical tools (e.g. Toulmin's model for argumentation, Sfard's commognition approach)

The "Habermas Group"

Rationality in different mathematical domains

Teacher's role in promoting students' rationality

Rationality in strategic games

Rationality and teacher education

Rationality in classroom interaction

Rationality and creativity

PME Research Forum

(Boero, De Simone, Douek, Ferrara, Goizueta, Guala, Martignone, Morselli, Planas, Sabena, 2014)

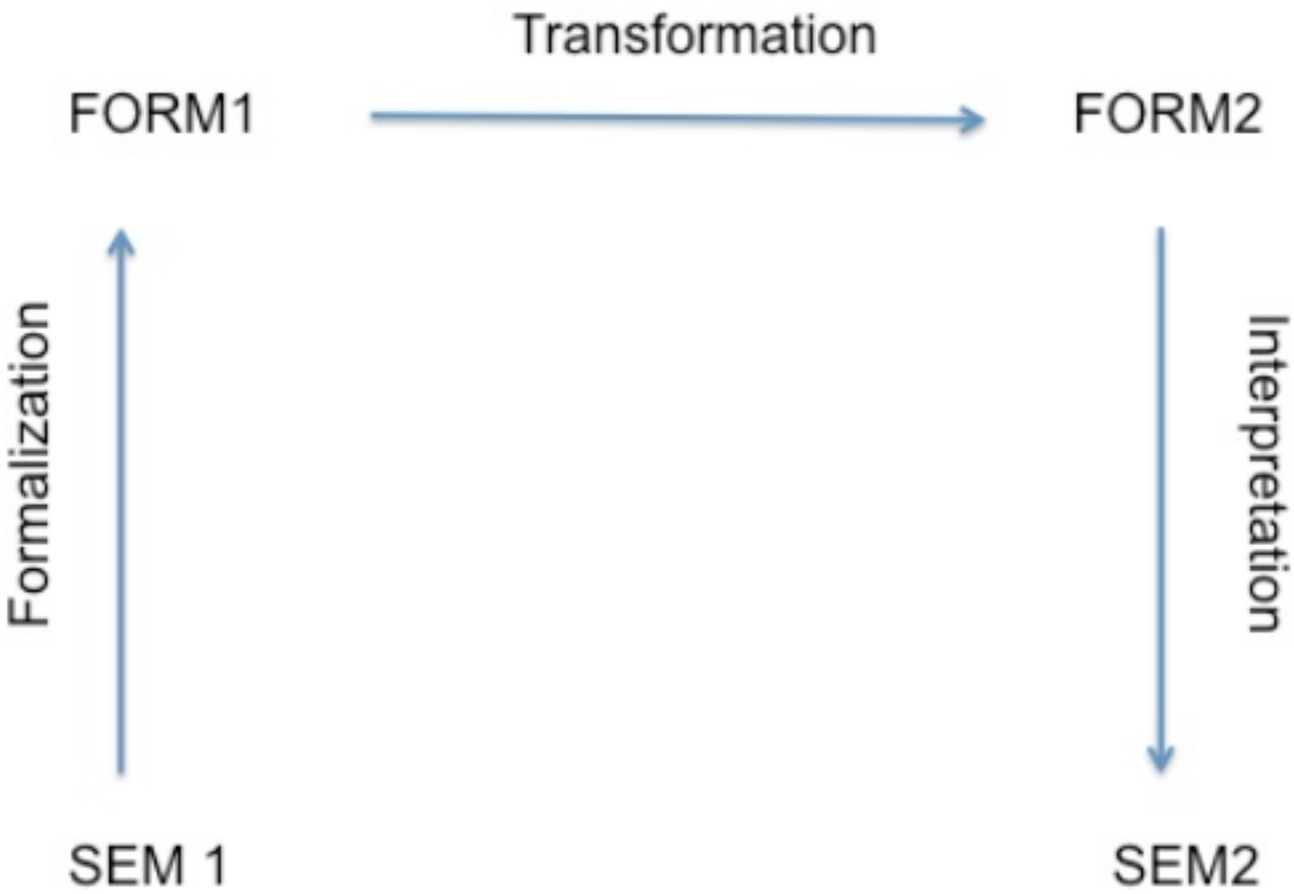
Habermas' construct of rational behavior in mathematics education: new advances and research questions

- Consciousness and creativity in mathematical activity
- The crucial role of the teacher
- Rationality in social interaction

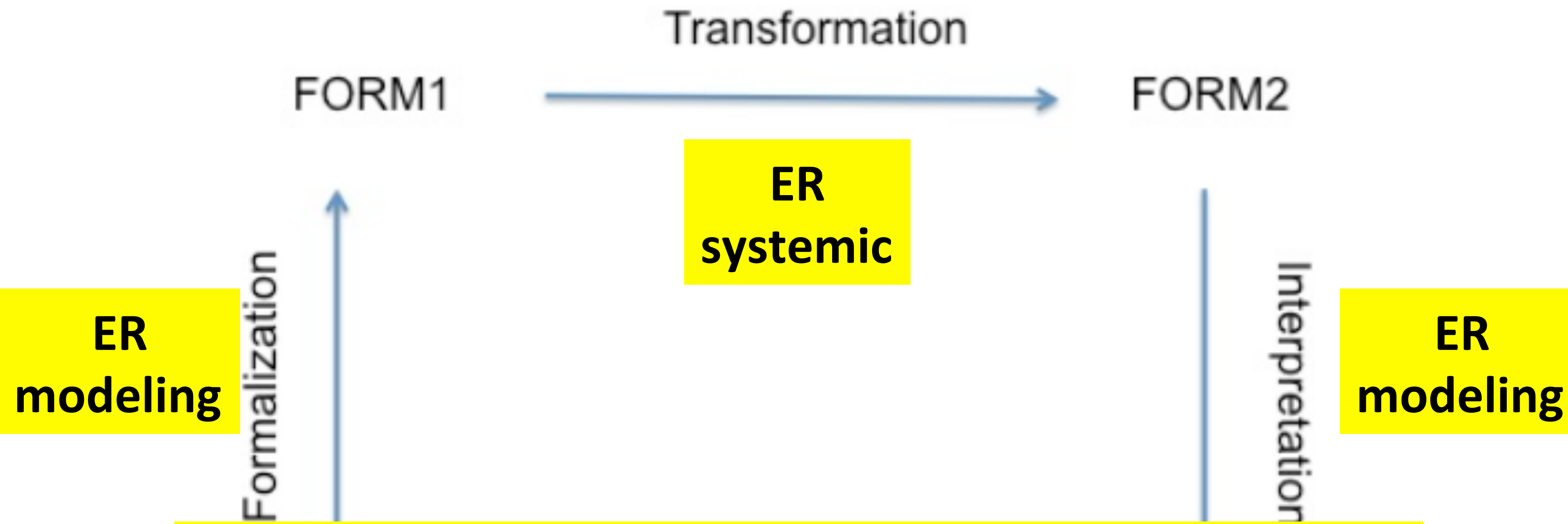
An example of integrated tool

Cycle of algebra + Rationality in proving = Integrated analytical tool

The cycle of algebra (Boero, 2001)

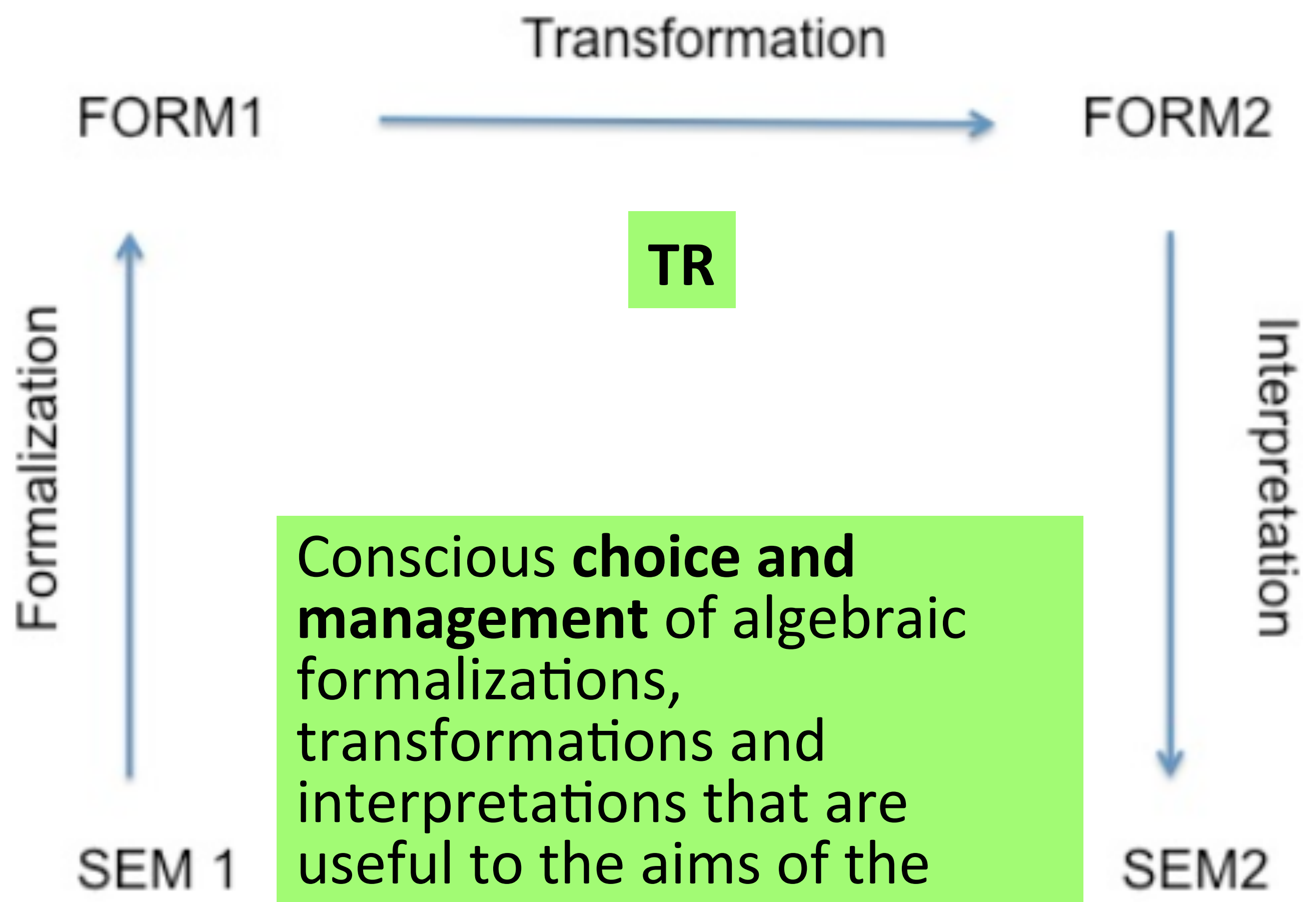


The integrated analytical tool (Morselli, 2013)



Modeling requirements: correctness of algebraic formalizations and interpretation of algebraic expressions
Systemic requirements: correctness of transformation (correct application of syntactic rules of transformation).

The integrated analytical tool (Morselli, 2013)



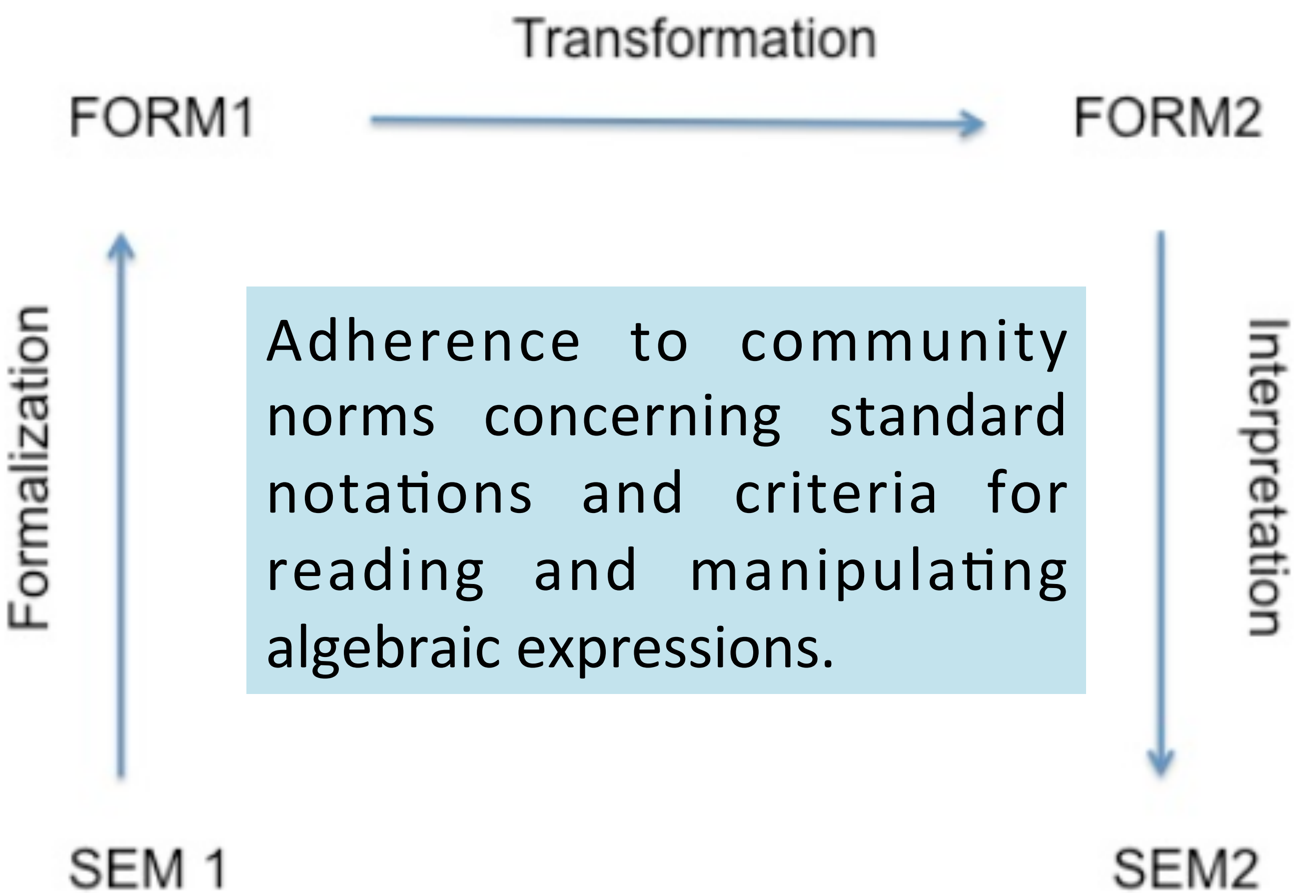
TR

TR

TR

Conscious **choice and management** of algebraic formalizations, transformations and interpretations that are useful to the aims of the activity.

The integrated analytical tool (Morselli, 2013)

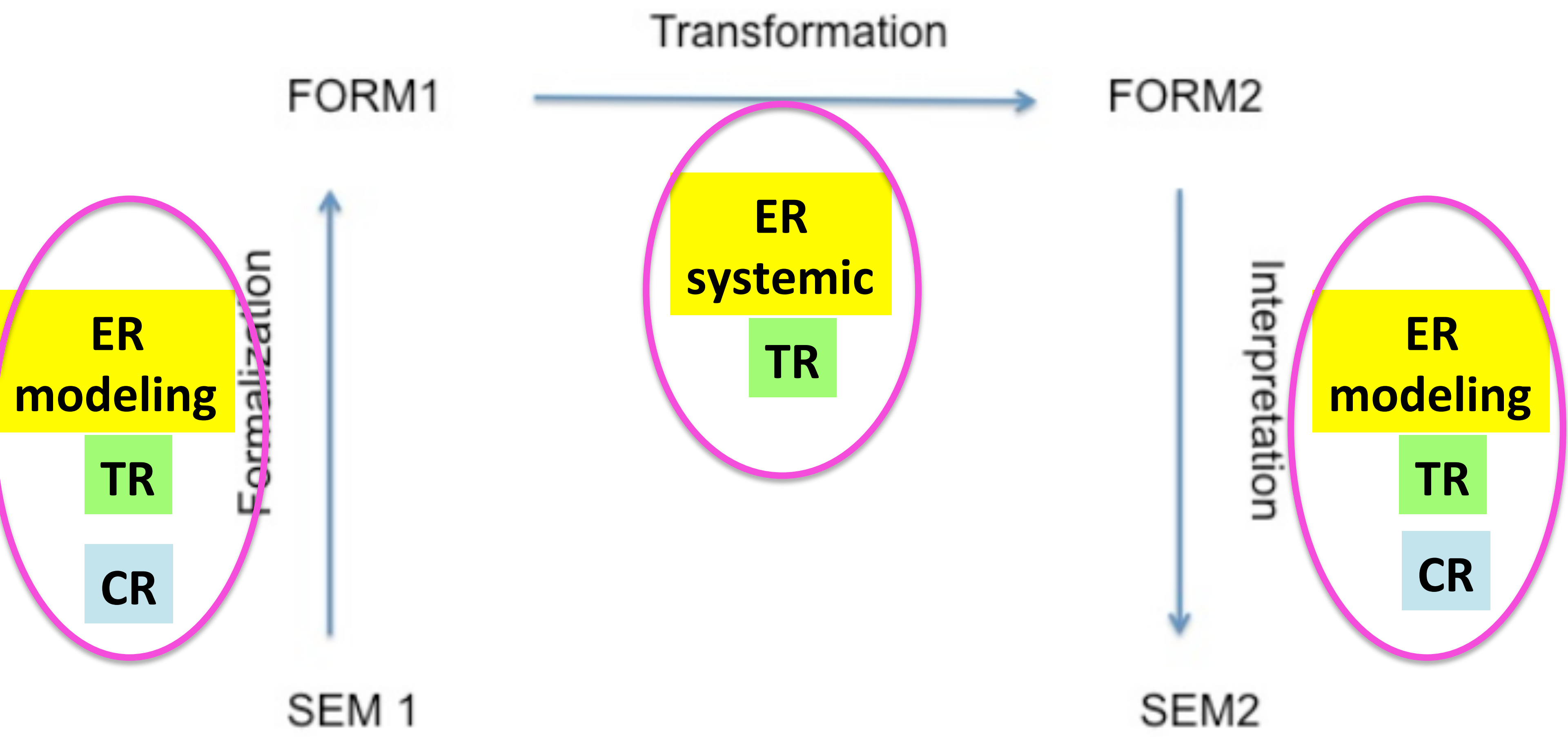


CR

Adherence to community norms concerning standard notations and criteria for reading and manipulating algebraic expressions.

CR

The integrated analytical tool (Morselli, 2013)



In order to justify a new analytic tool in Mathematics Education it is necessary to show how it can be useful:

- in describing and interpreting relevant aspects of the teaching and learning process
- in orienting and supporting teachers' educational choices
- in suggesting new research developments

An episode

- Lower secondary school (12/13 year-old students)

- Experienced teacher

- ***For full details on the context and another example: see Morselli (2013)***

The task

What can you tell about the sum of three consecutive numbers?

Individual work – group work – classroom discussion

Proof by generic example

$$1+2+3=6$$

$+1$

$$7+8+9=24$$

$$51+52+53=156$$

Moreover, if the third number gives a unit to the first number, we have three equal numbers

First algebraic proof

SI PUÓ OSSERVARE CHE LA SOMMA É UN MULTIPLO DI TRE

$1+2+3=6$

$+1$

$7+8+9=24$

$51+52+53=156$

INOLTRE SE IL TERZO NUMERO DÁ UN UNITÁ AL PRIMO, DIVENTANO NUMERI UGUALI SAREBBE

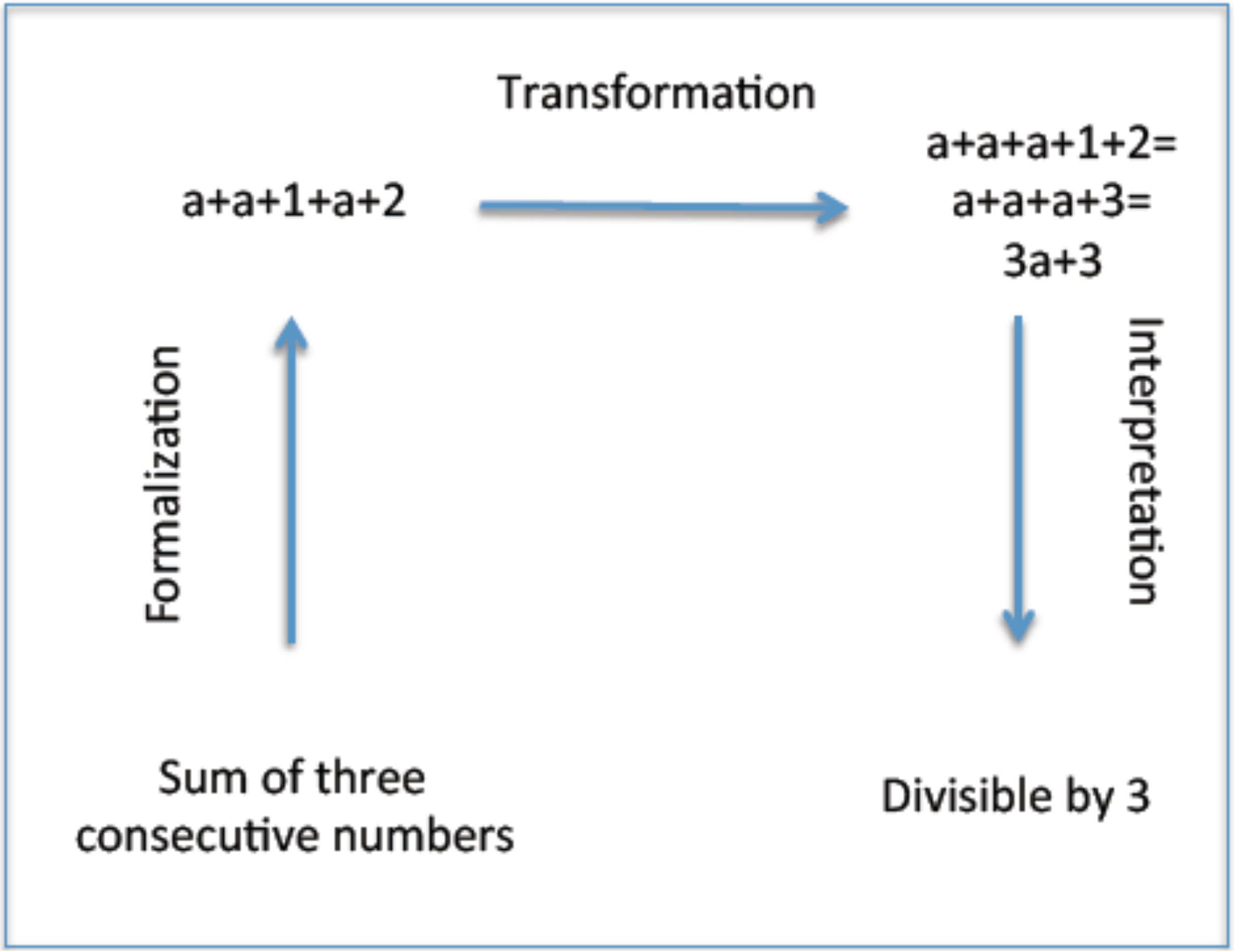
$\boxed{a+(a+1)+(a+2)}$ SI POTREBBE ANCHE FARE $a+a+a+1+2$ PER LA PROPRIETÁ COMMUTATIVA E SAREBBE

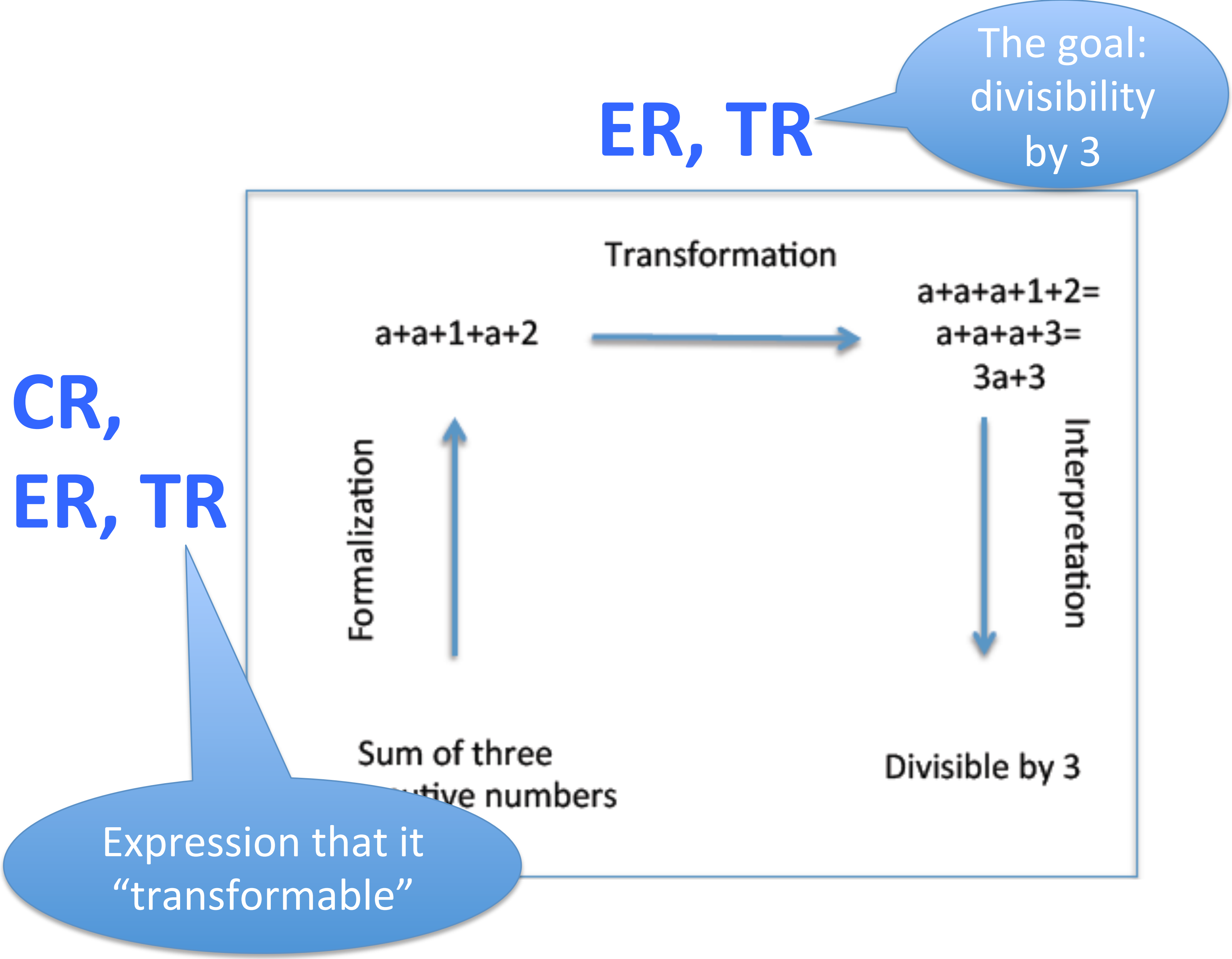
$a \cdot 3+1+2$

$a \cdot 3+3$

$$a+a+1+a+2$$
$$a+a+a+1+2$$
$$a*3+1+2$$
$$a*3+3$$

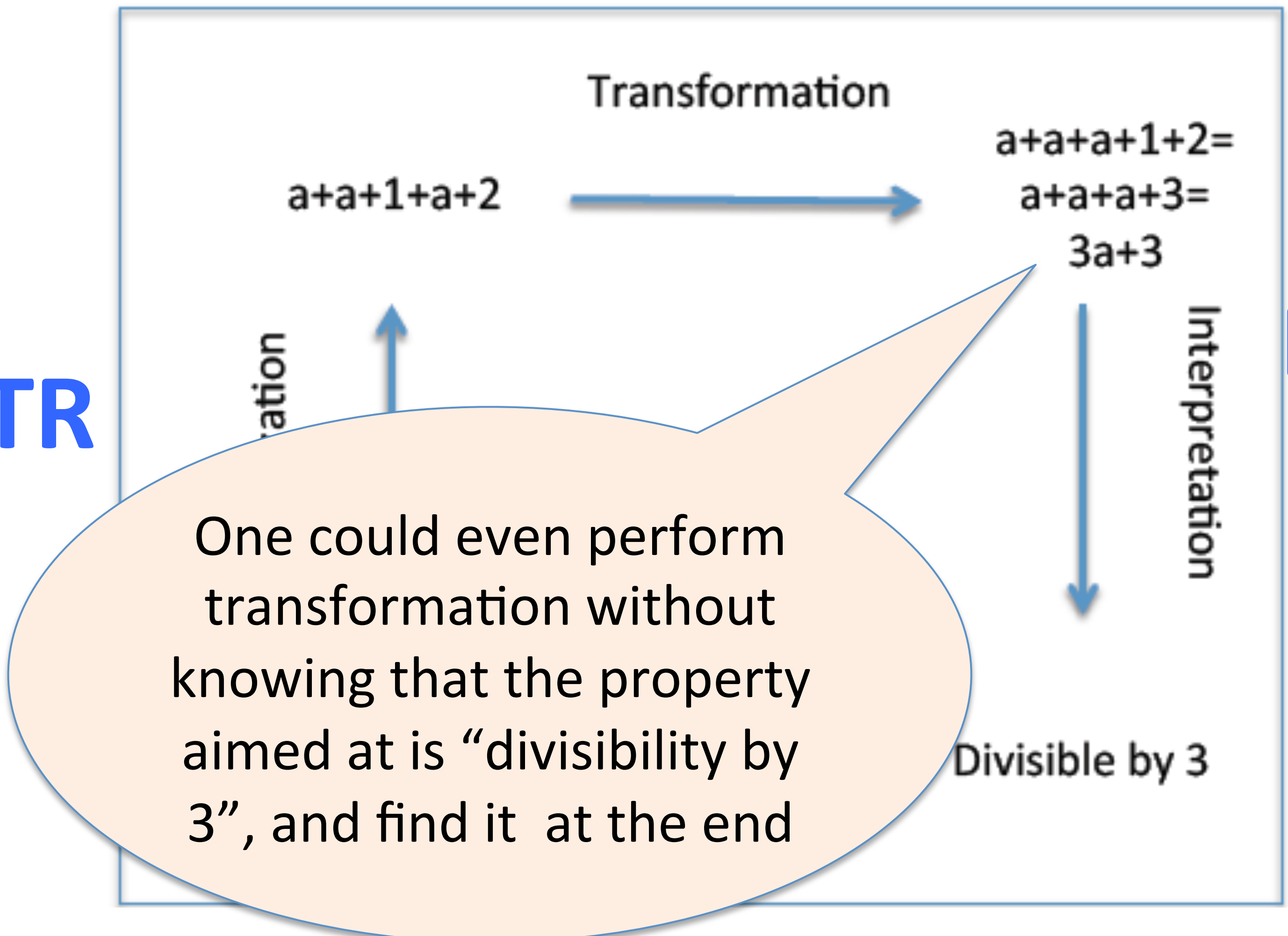
First algebraic proof




ER, TR
The goal: divisibility by 3
CR,
ER, TR
Transformation
a+a+1+a+2
a+a+a+1+2=
a+a+a+3=
3a+3
Formalization
Interpretation
Sum of three
tive numbers
Divisible by 3
Expression that it "transformable"

ER, TR

CR,
ER, TR



ER, CR

Second algebraic proof

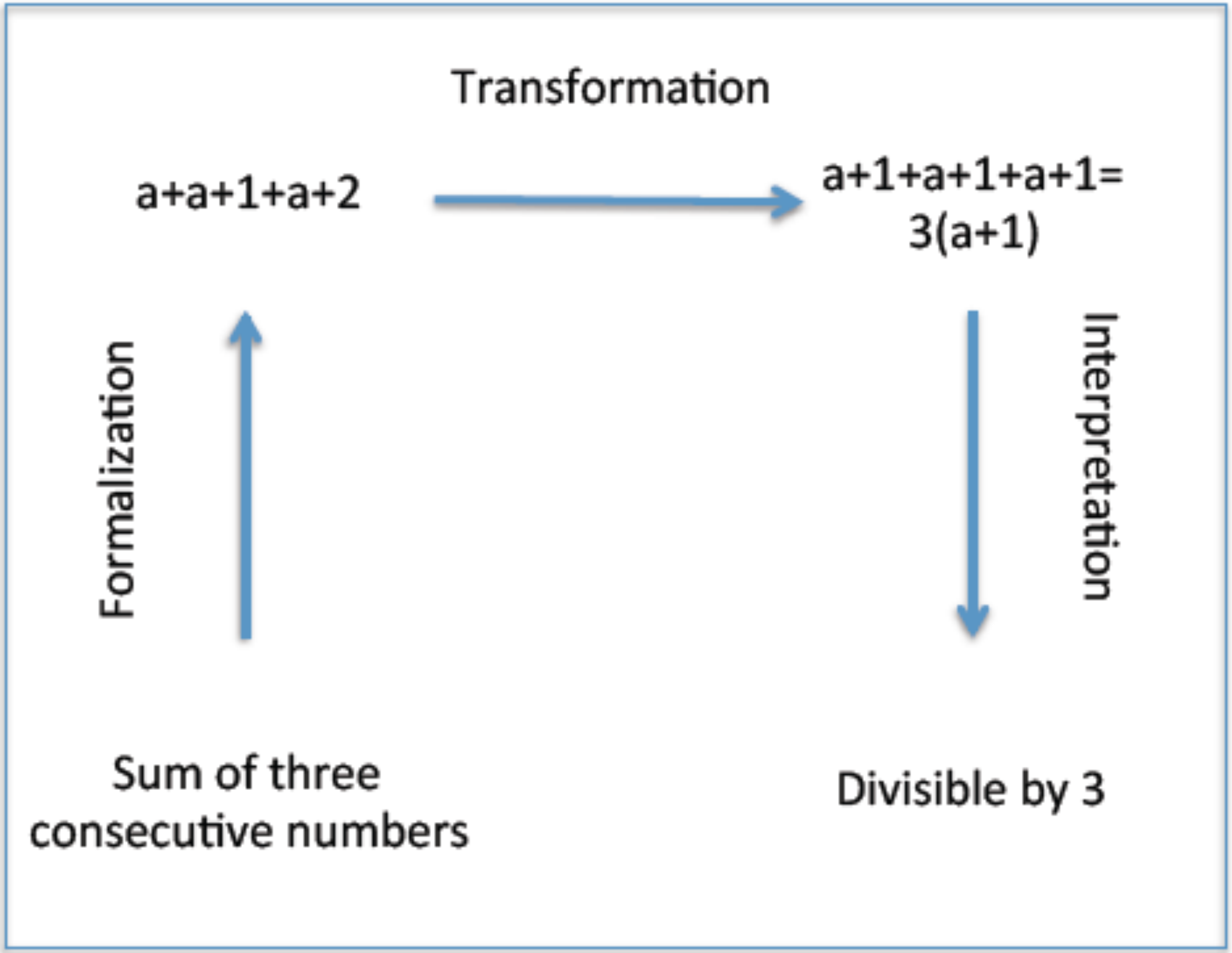
$$a+a+1+a+2=a+1 + a+1+ a+1=3(a+1)$$

$a+1+a+1+a+1=3\cdot(a+1)$

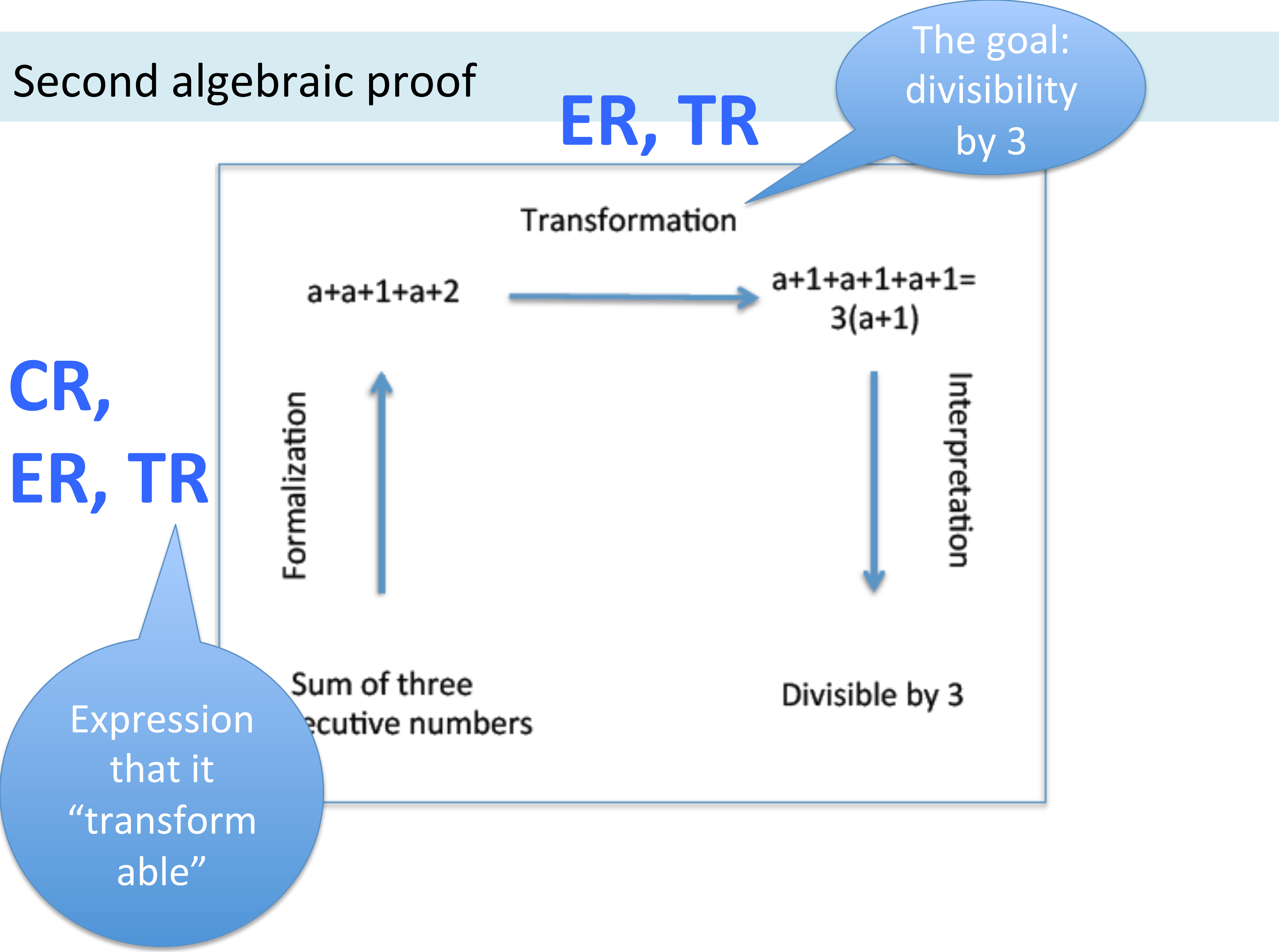
E COSÍ SI DIMOSTRA ANCHE CHE LA SOMMA DI TRE NUMERI CONSECUTIVI È UN MULTIPLO DI 3

And in this way we also prove that the sum of three consecutive numbers is divisible by 3

Second algebraic proof



Second algebraic proof



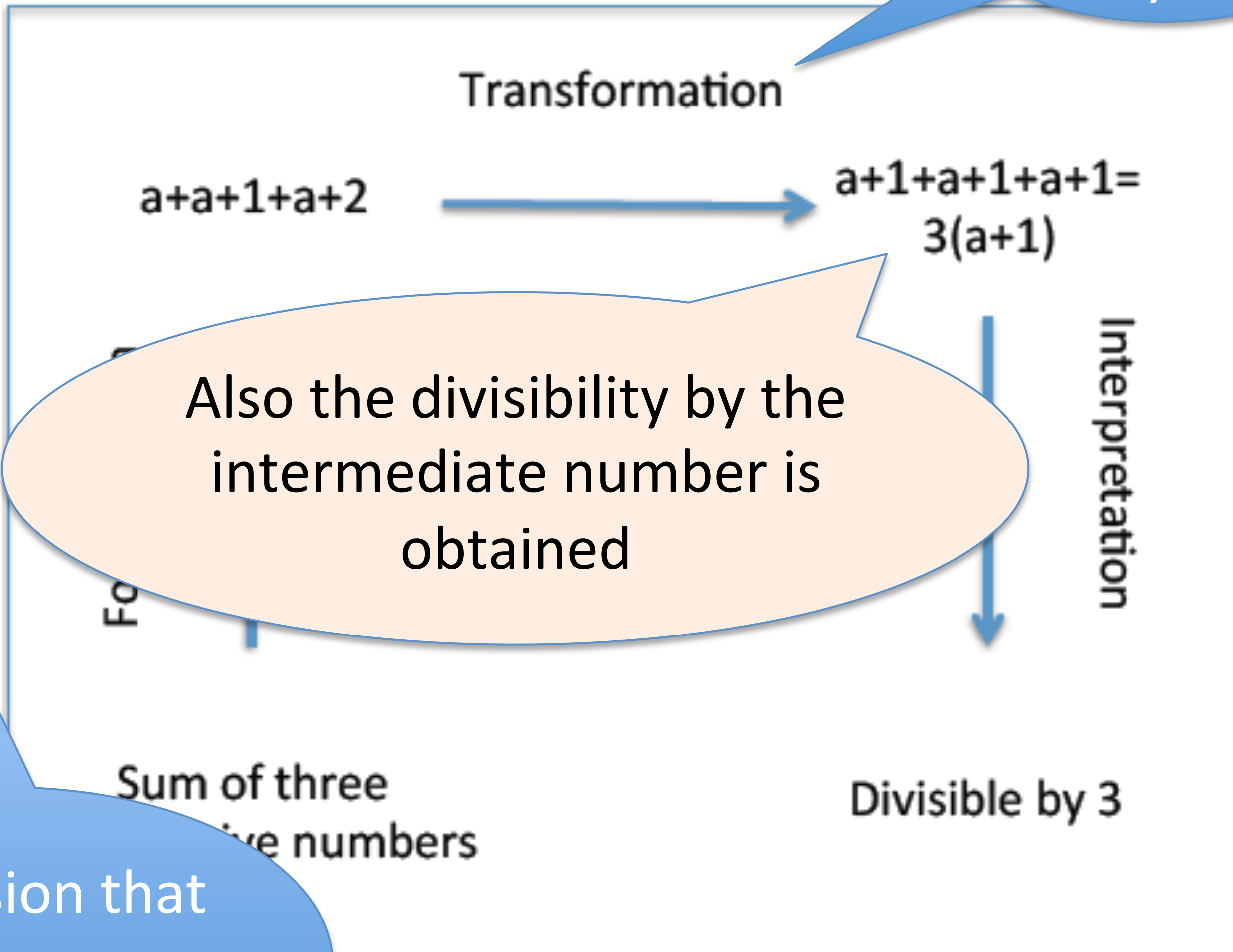
Second algebraic proof

ER, TR

The goal: divisibility by 3

CR,
ER, TR

ER, CR



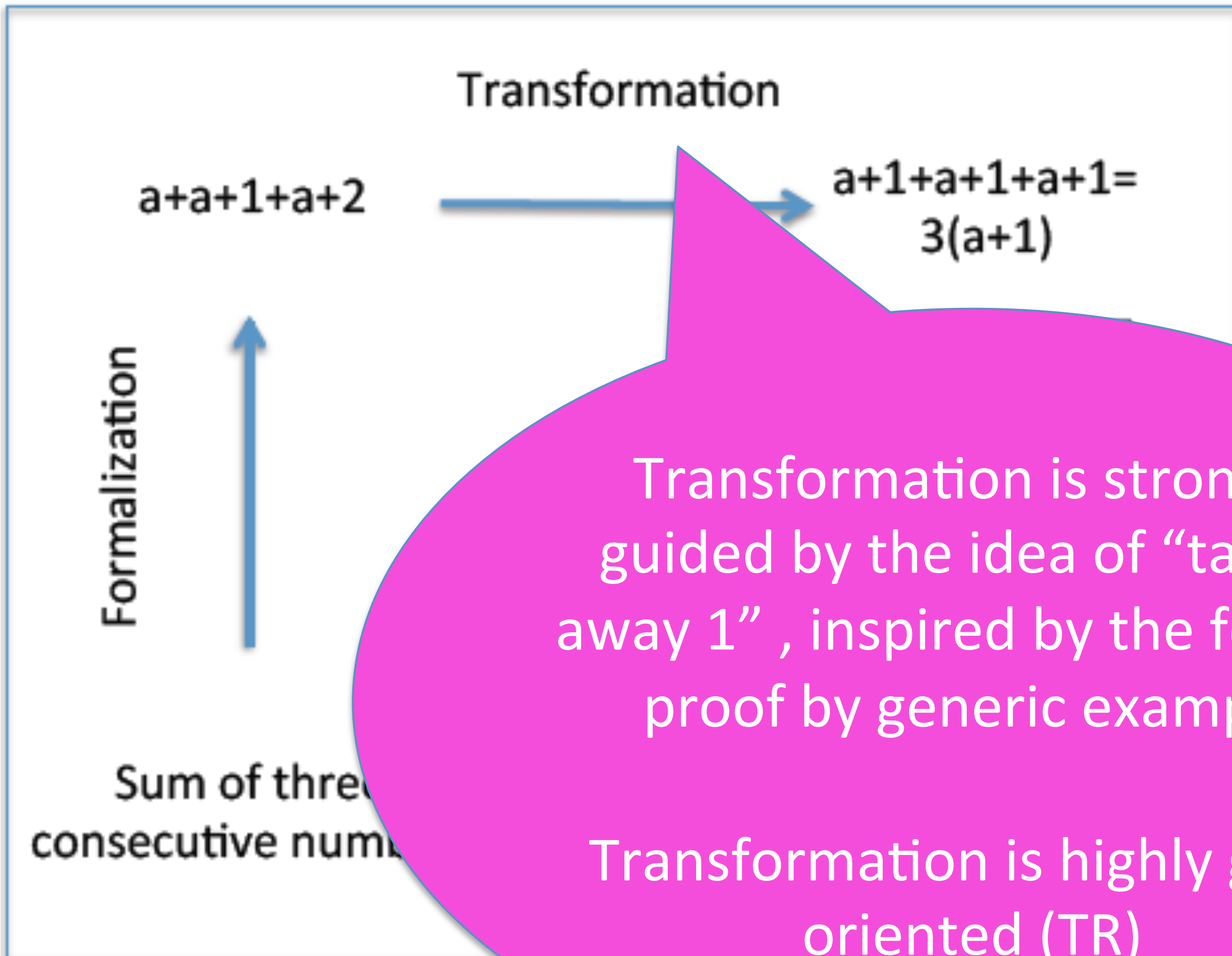
Also the divisibility by the intermediate number is obtained

Expression that is "transformable"

Second algebraic proof

ER, TR

CR,
ER, TR



Transformation is strongly guided by the idea of "taking away 1" , inspired by the former proof by generic example

Transformation is highly goal-oriented (TR)

Comparison between the two proofs:

1. the first one is mainly **syntactical** and could be carried out without having in mind the property to prove;
2. the second one can be performed only under the guide of a strong **anticipation** (one must already have the goal of getting three times the same number);
3. the second algebraic proof seems to be possible only in **continuity** with the argumentation in natural language and numerical examples (proof by generic example).

In order to justify a new analytic tool in Mathematics Education it is necessary to show how it can be useful:

- in describing and interpreting relevant aspects of the teaching and learning process
- in orienting and supporting teachers' educational choices
- in suggesting new research developments

The integrated analytical tool is efficient:

1

in describing and interpreting relevant aspects of the teaching and learning process

Fine-grained analysis of the dialectic between adherence to syntactic rules and goal-oriented management of the formalization, transformation and interpretation processes (ER-TR)

The integrated analytical tool is efficient:

2 in orienting and supporting teachers' educational choices

- Foresee, grasp and manage crucial points (e.g. two possible proofs)
- Outline occasions for meta-mathematical discussions (e.g.proof by generic example)

The integrated analytical tool is efficient:

3 in suggesting new research developments

- Interaction between ER and TR, crucial role of TR in transformation
- Relation between communicative choices (CR) and validity justification (ER) in the context of the classroom
- Design and implementation of task sequences:
  - to propagate the culture of theorems aimed at
  - to create occasions to discuss meta-mathematical aspects of this culture

Thank you for your attention!