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Dipartimento di Scienze Umane e Sociali
Département des Sciences Humaines et Sociales

Technologies for the learning of algebra with dyscalculic students

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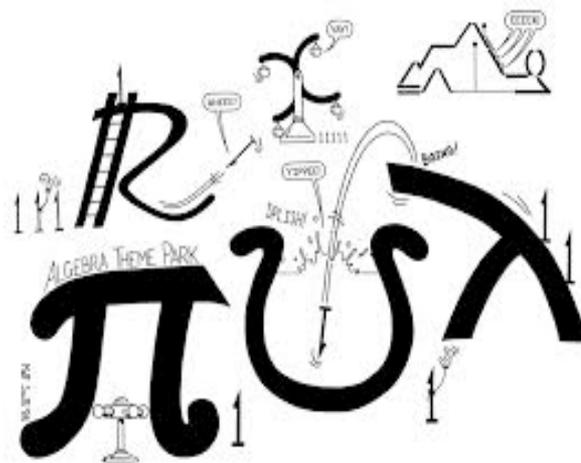
First Joint International Meeting RSME-SCM-SEMA-SIMAI-UMI,
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SPECIAL SESSION ON MATHEMATICS EDUCATION

Dyscalculia?

- The main features of children with dyscalculia are:
- difficulty in learning and remembering arithmetic facts;
 - difficulty in executing calculation procedures, with immature problem-solving strategies, long solution times and high error rates (Butterworth, 2004);
 - fact retrieval and procedural difficulties could be dissociable in dyscalculia Temple (1991).

*Developmental Dyscalculia
(DD) and learning algebra:
Some premises*

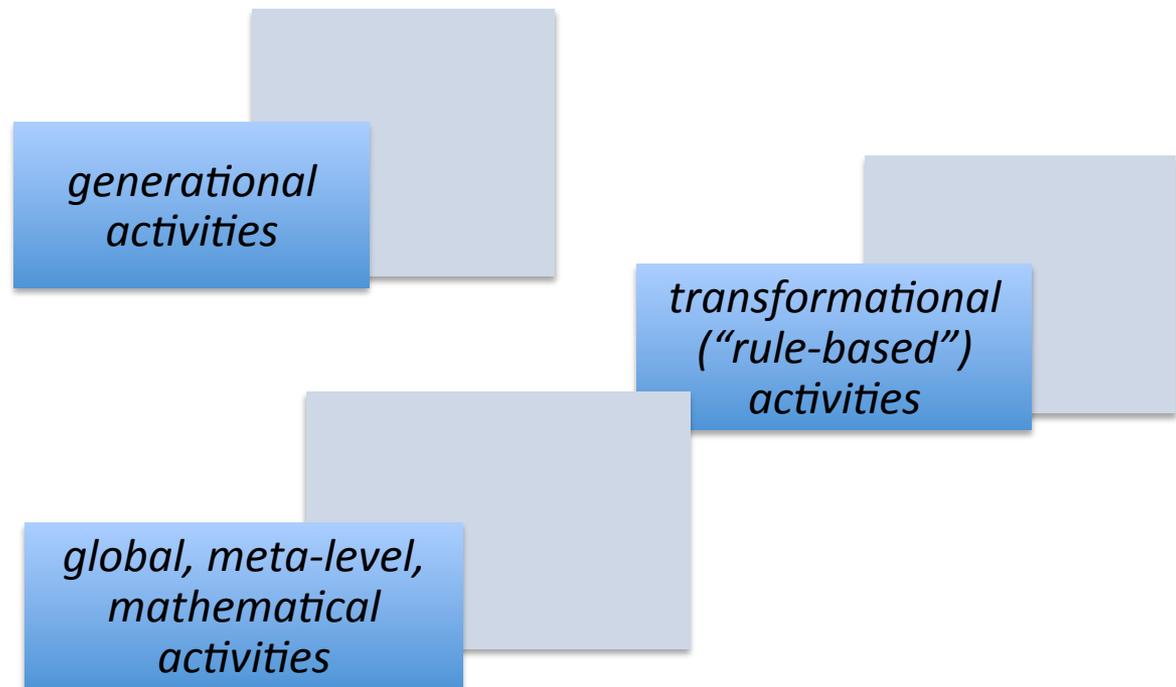


Learning algebra

from educational point of view

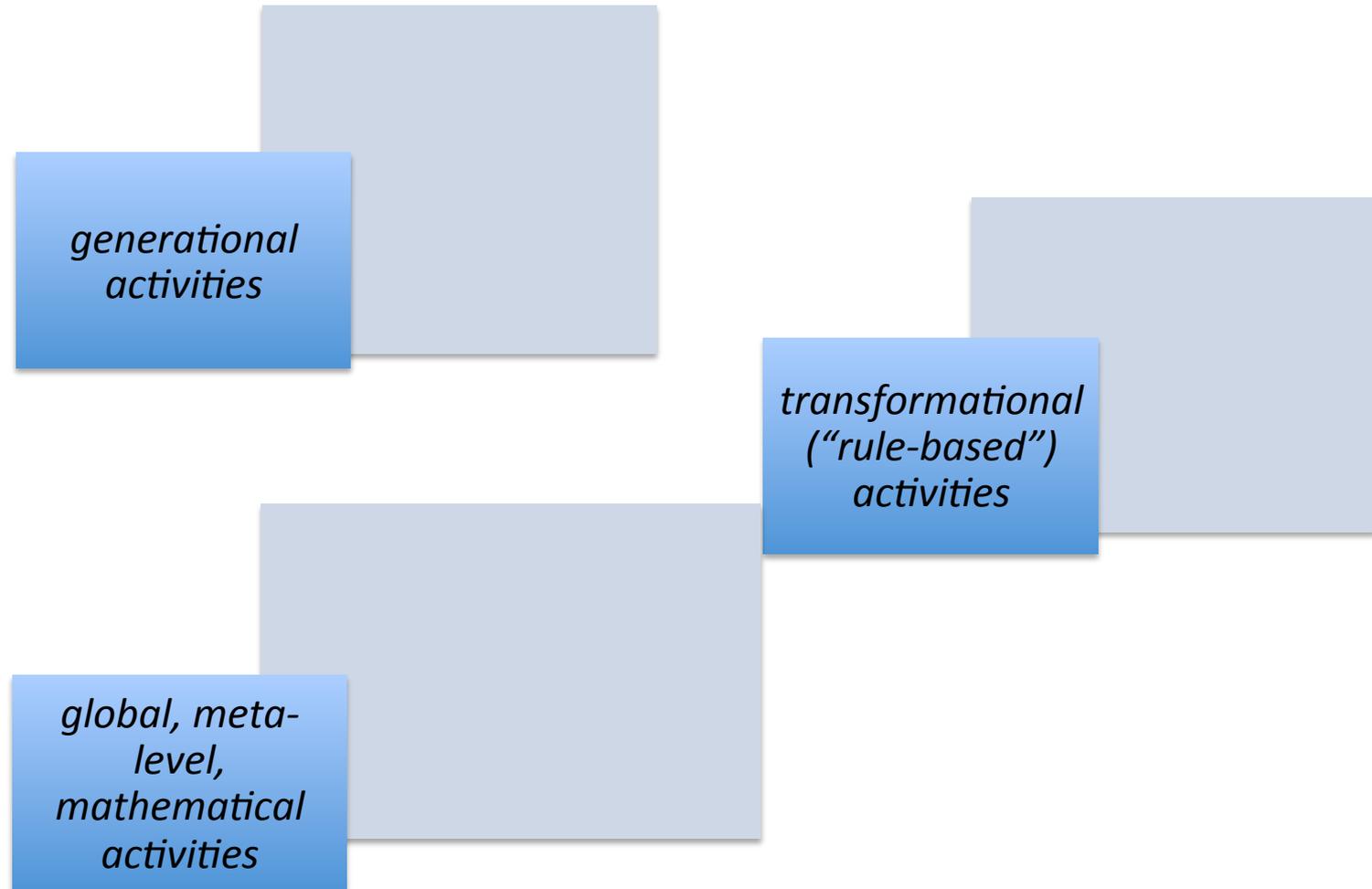
Three different kinds of activities in Learning Algebra

Kieran's
model (1996,
2004)

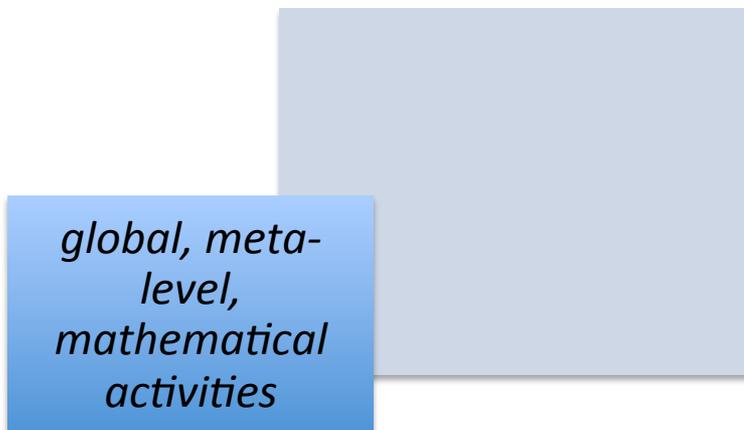
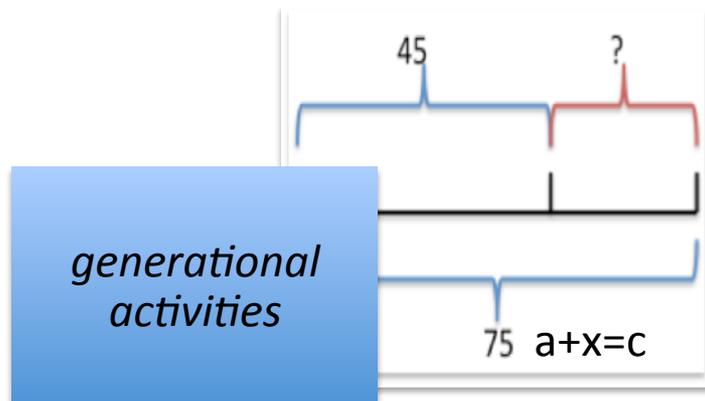


Kieran's model (1996, 2004) :

three different kinds of activities in Learning Algebra



Kieran's model (1996, 2004) : three different kinds of activities in Learning Algebra



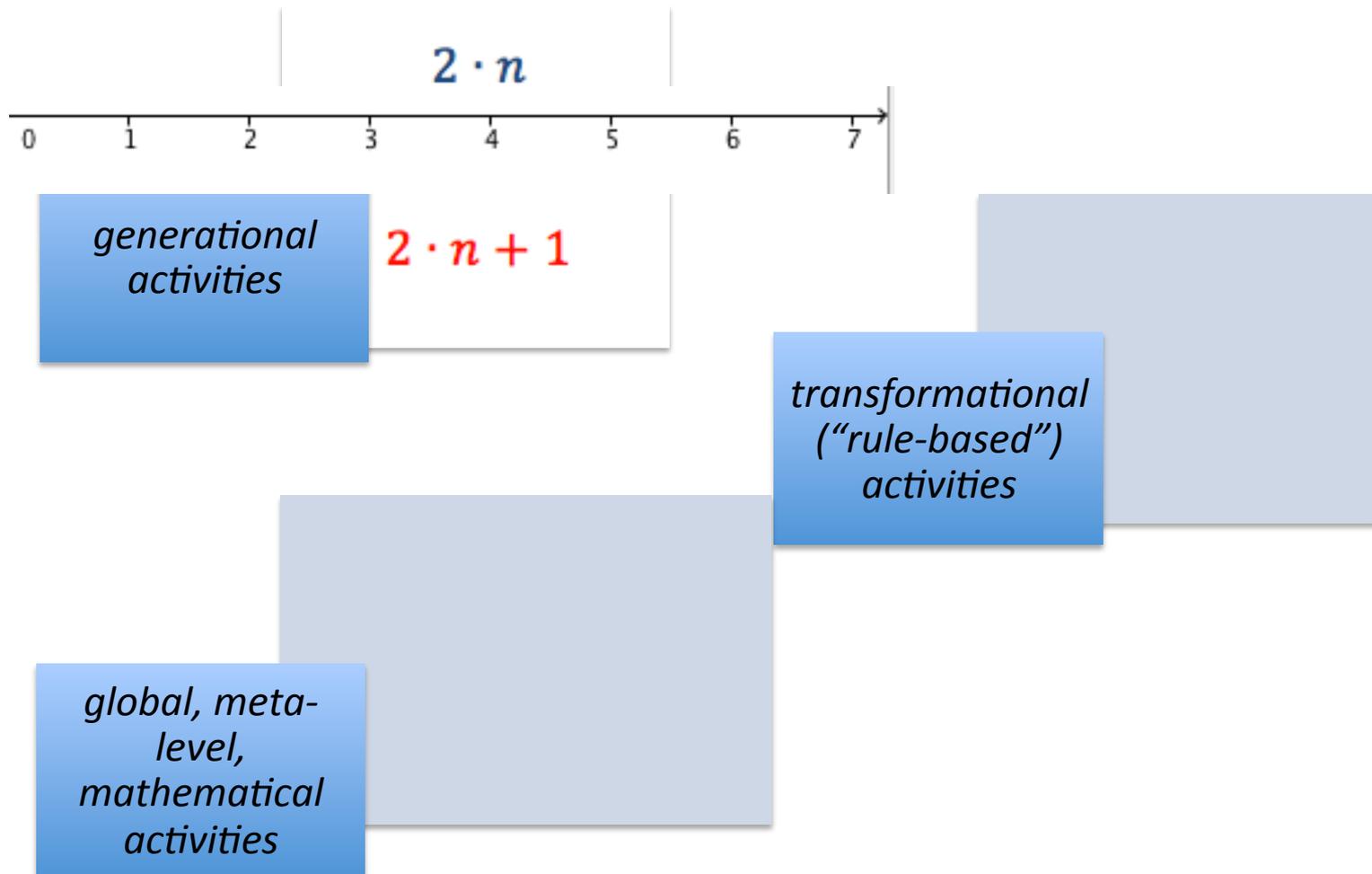
Involve the forming of the expressions and equation :

- i) equations containing an unknown that represent problem situations;*
- ii) expressions of generality arising from geometric patterns or numerical sequences;*
- iii) expressions of the rules governing numerical relationships.*

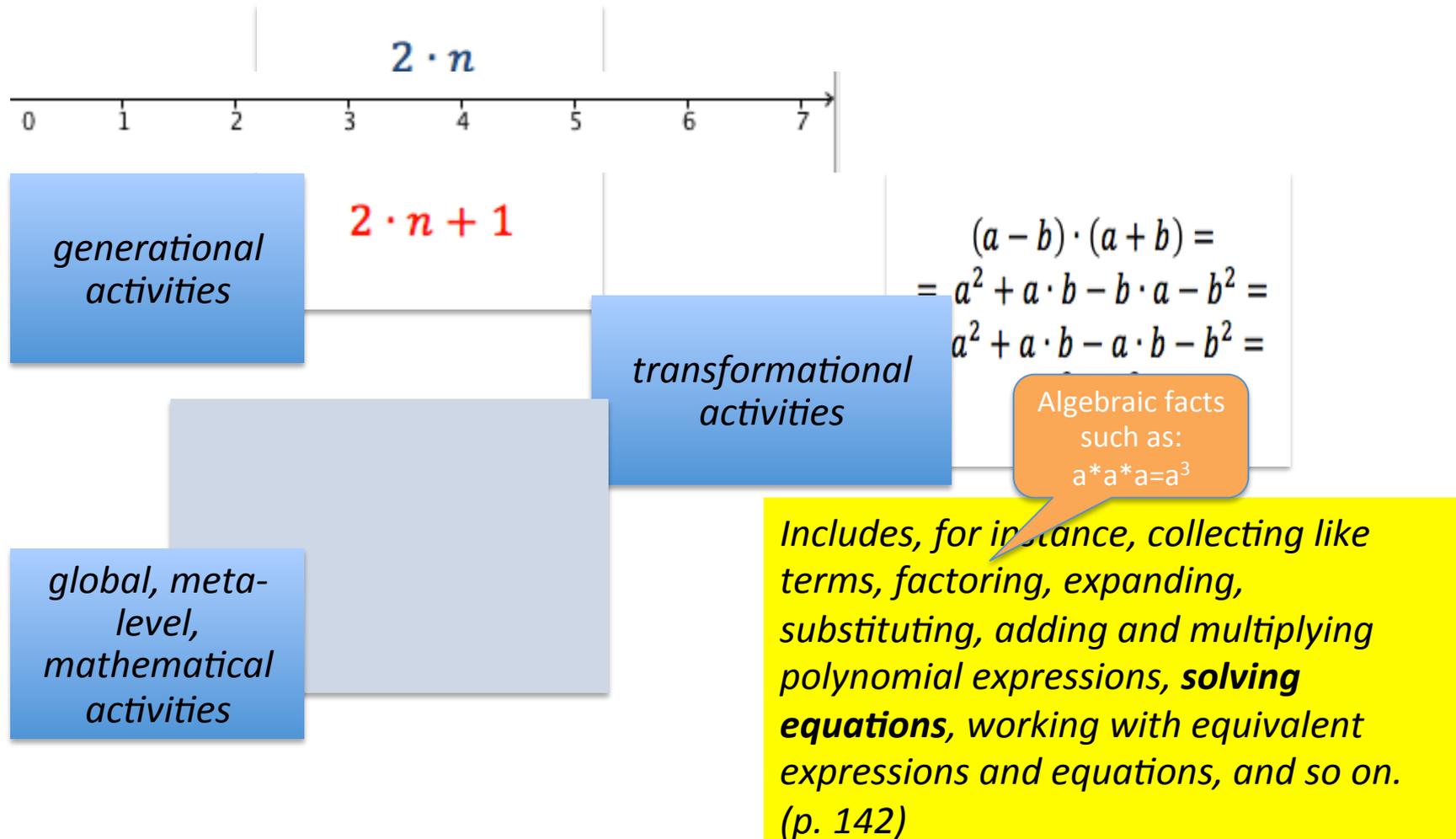
*The underlying objects of expressions and equations are **variables** and **unknowns**, **equal sign** and the **notion of equation solution**.*

*Much of the meaning-building for algebraic objects occurs within the generational activity of algebra
(p.142)*

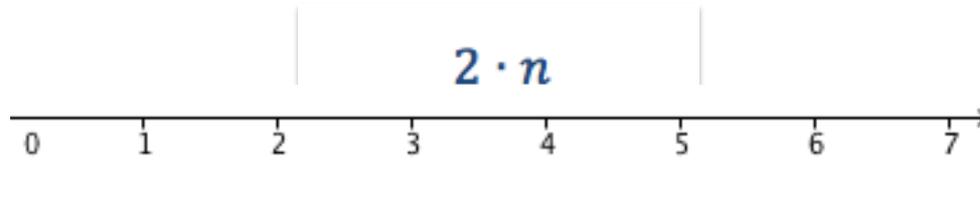
Kieran's model (1996, 2004) : three different kinds of activities in Learning Algebra



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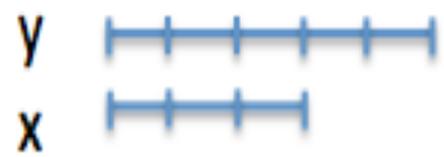


generational
activities

$$2 \cdot n + 1$$

$$(a - b) \cdot (a + b) =$$

tr



global, meta-level,
mathematical
activities

$$\begin{cases} x + y = 8 \\ x = \frac{3}{5}y \end{cases}$$

For which algebra is used as a tool but which are not exclusive to algebra. They include problem solving, **modeling**, generalizing, **analyzing relationships between quantities**, justifying, proving, and predicting - activities that could be engaged in without using any algebra at all. The global meta-level activities are essential to the other activities of algebra, in particular, to the meaning-building generational activities; otherwise all sense of purpose is lost.

Learning algebra

from the point of view of neuroscience

NATURAL LANGUAGE AND ARITHMETIC

Houde &
Tzourio-
Mazoyer, 2003;
Venkatraman,
Ansari, & Chee,
2005; Zago et al,
2001

- Skills related to **arithmetical calculation seem to be related principally to the visuo-spatial area** of the brain (bilateral frontal areas): during the execution of arithmetical tasks the brain areas that manage visuo-spatial memory and mental imagery are activated

Butterworth,
1999

- **There is independence between mechanisms for language and mechanisms for calculation** (parieto-frontal bilateral areas and inferior bilateral temporal areas)

Learning algebra

from the point of view of neuroscience

NATURAL LANGUAGE AND ALGEBRA

Aphasia

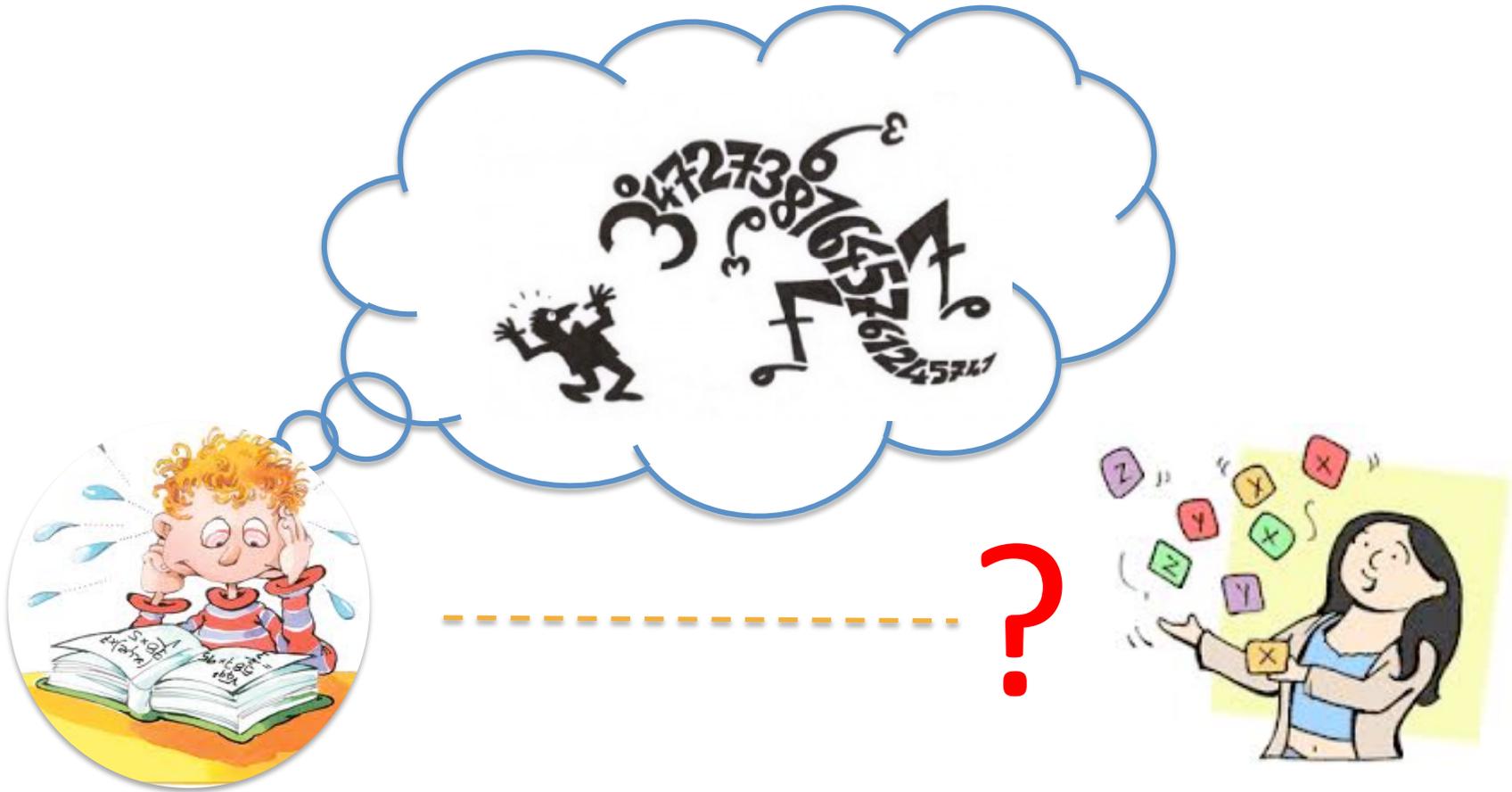
Linguistic abilities are compromised;

The ability to execute arithmetical and algebraic tasks is intact:

- recognize the equivalence between algebraic expressions and transform them
- recall algebraic facts ($a+a = 2a$; $a*a=a^2$),
- execute procedures both in expressions with only algebraic symbols and in numerical expressions (for ex., $8-(3-5) + 3$ or $b-(a-c) + a$).

(Klessinger,
Szczerbinski,
Varley 2007)

DD and Learning algebra



DD and Learning algebra

from the point of view of neuroscience

Some research on dyscalculic learners (Hittmair-Delazer et al., 1995, Dehaene, 1997) showed that there is a dissociation between:

They have difficulties to calculate

2*3, 7-3, 9/3, 5*4

the recovery ability of arithmetic facts, which are compromised

algebraic manipulations, which are intact

They are able to transform and simplify algebraic expressions

$$\frac{a \cdot b}{b \cdot a} = 1$$

$$a \cdot a \cdot a = a^3$$

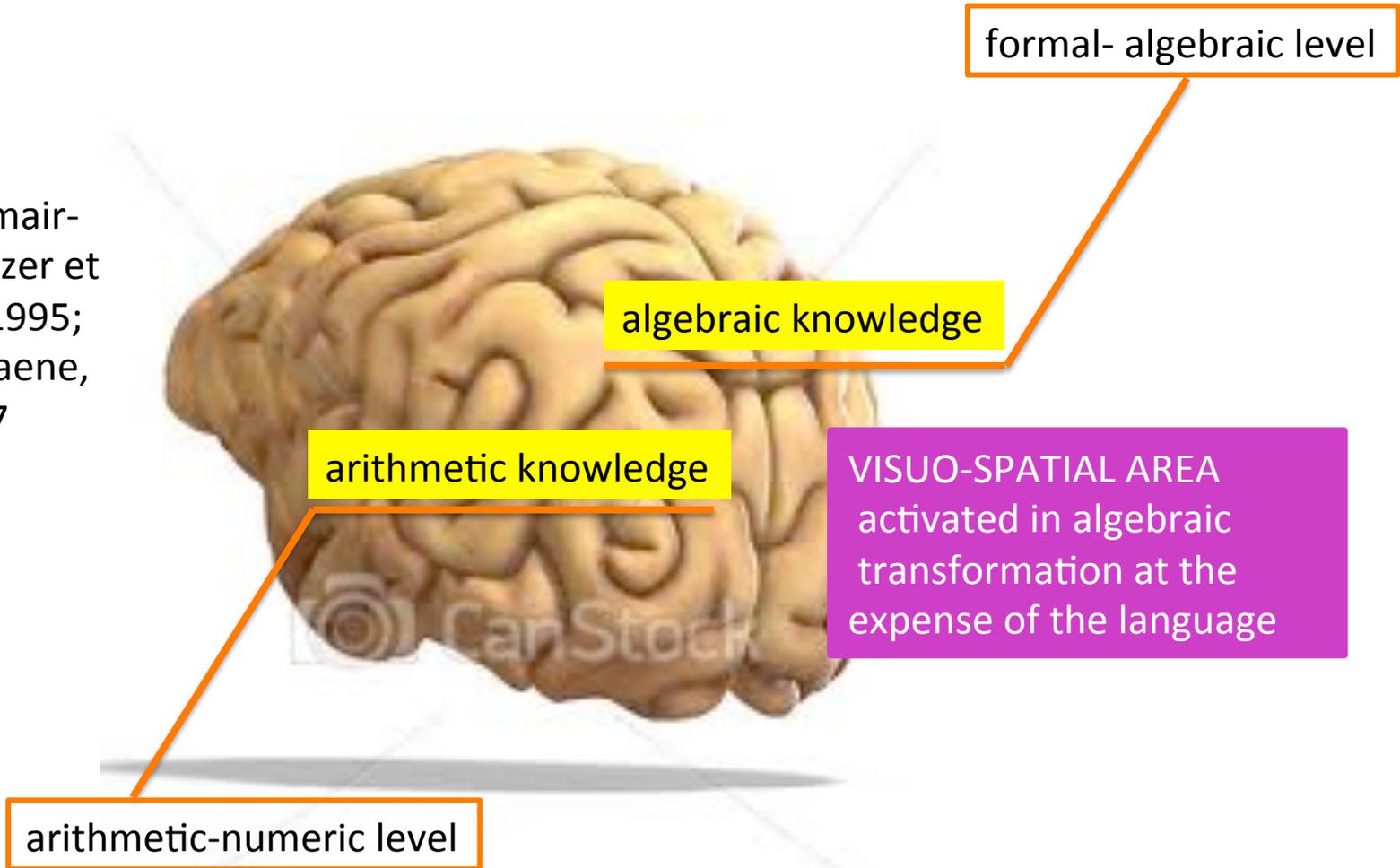
They are able to judge the non equivalence between the algebraic expressions

$$\frac{d}{c} + a = \frac{d+c}{c+a}$$

DD and Learning algebra

from the point of view of neuroscience

Hittmair-Delazer et al., 1995; Dehaene, 1997



arithmetic-numeric level

arithmetic knowledge

algebraic knowledge

formal- algebraic level

VISUO-SPATIAL AREA
activated in algebraic
transformation at the
expense of the language

DD and learning algebra

from the educational point of view



What is the nature of difficulties of students with DD *learning* algebra?

Difficulties in algebra of students with DD

Sfard, 1992
Thomas & Tall,
1988

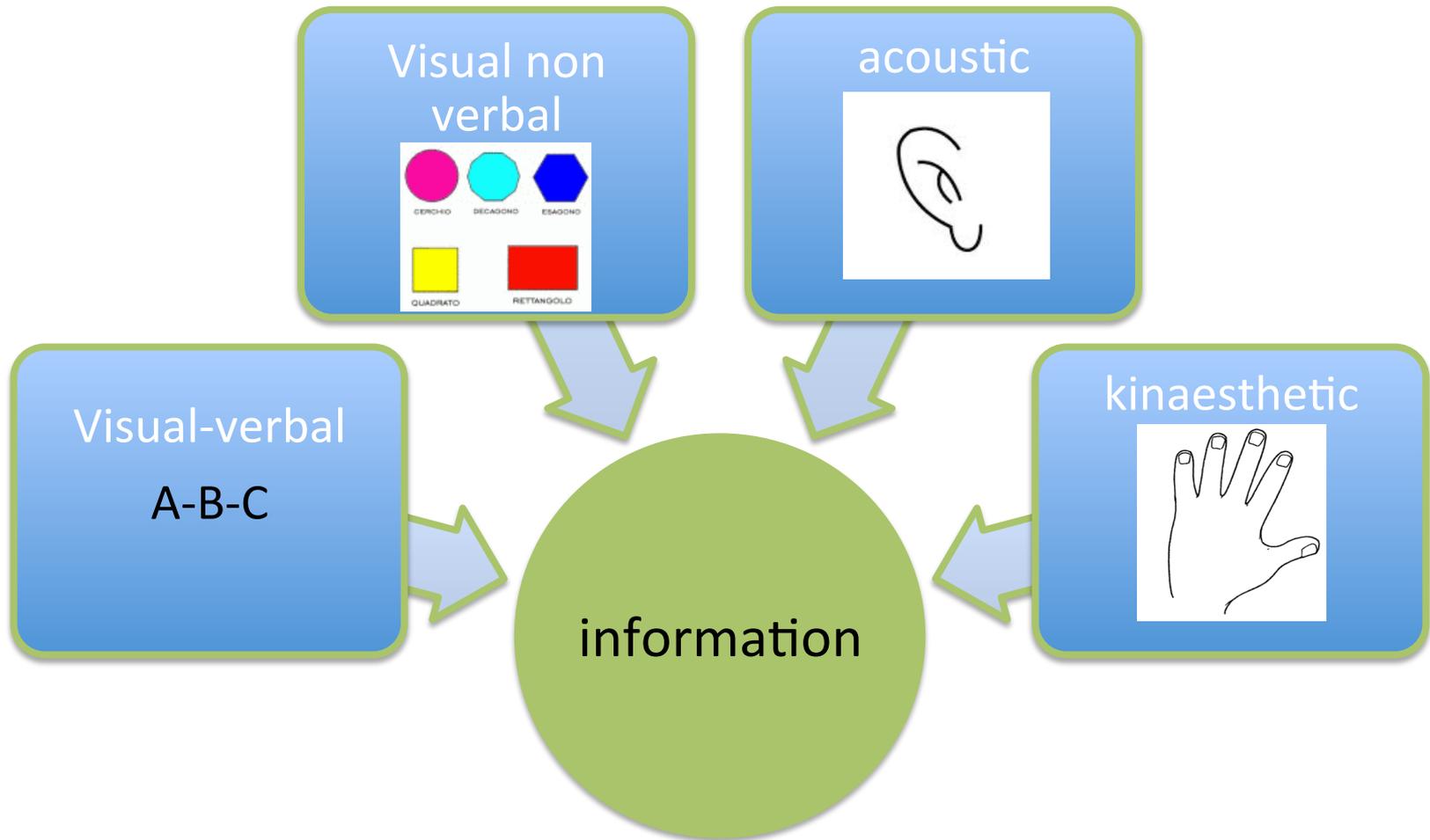
Semantic difficulties: **attribute meaning to algebraic notions**
(variable, unknown, algebraic expression, equation, solution of
an equation, identity ...)

(Baccaglioni-Frank, Robotti 2013; Robotti, Ferrando, 2013)

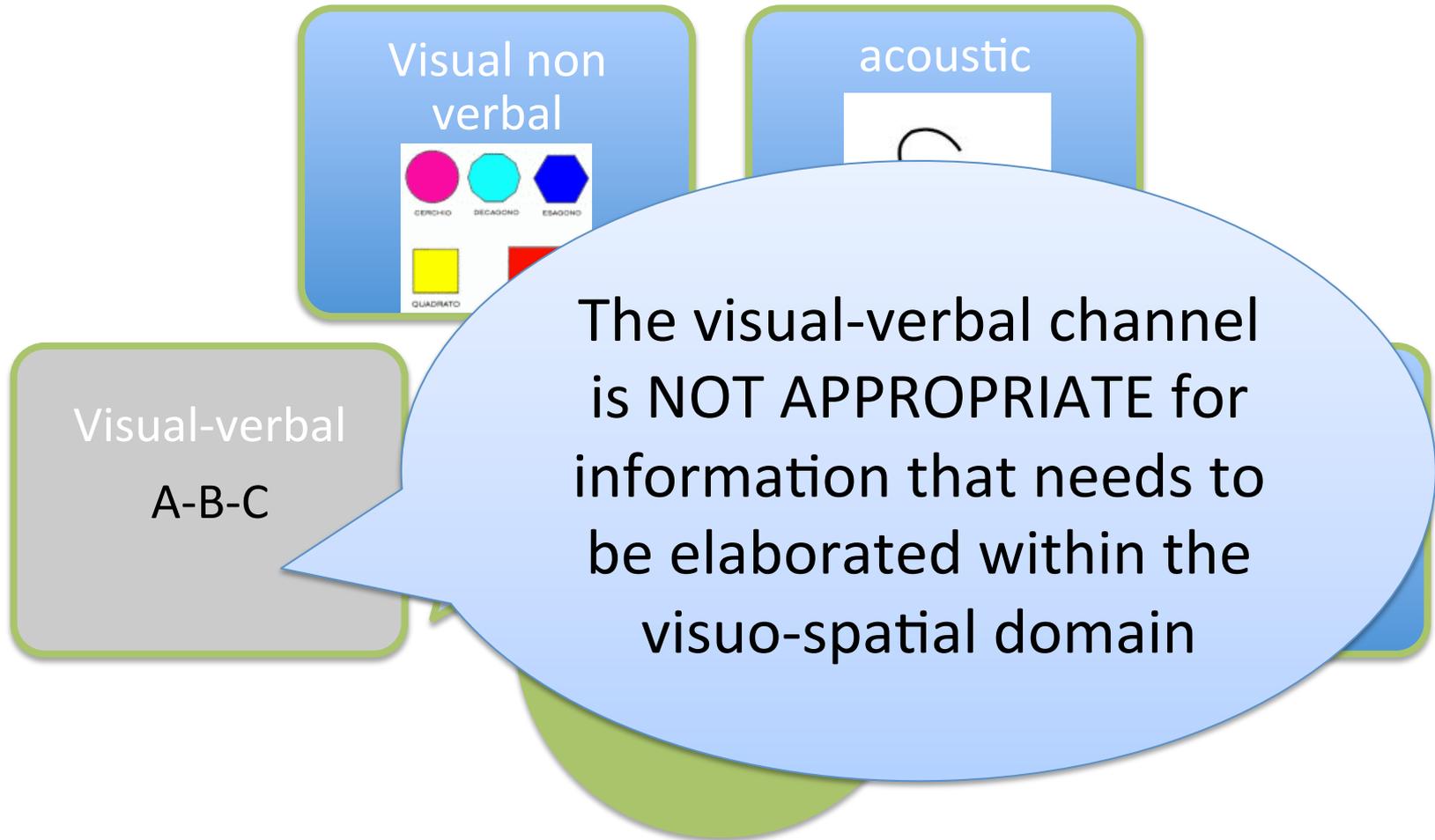
- Construct meaning for the algebraic symbols
- Recover skills of an arithmetical nature (for ex., recovery of number facts)
- Arithmetical difficulties influence algebraic performance, but algebraic performance is of a different nature.
- Develop and use new skills for the rule of algebraic transformation.
- Memory for algebraic rules
- Knowledge of algebraic facts (for ex., $a * a = a^2$)

HOW CAN WE INTERVENE?

Main channels of input/output of information



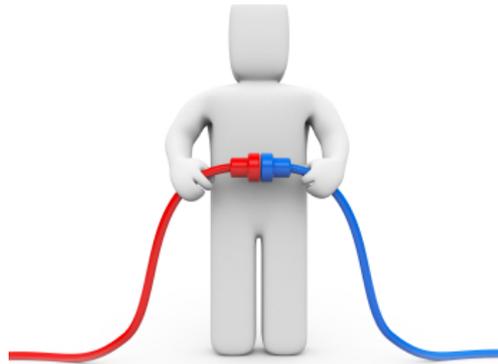
Main channels of input/output of information



HOW CAN WE INTERVENE?

Algebra:

- Variable
- Unknown
- Solution of an equation
- Algebraic transformation...



Meaning (semantics) of algebraic notions

Our hypothesis is that the development of meaning of algebraic notions and symbols is essential in order to manage the different kinds of activities in algebra.

Moreover, the development of meaning of algebraic notions can be fostered efficiently using different forms of representation that include dynamic representations (Arzarello, 2006) and this can be done, for example, with software like AlNuSet.

AlNuSet to foster the development of meaning of algebraic notions



AlNuSet allows teacher to:
design **educational activities characterized by a dynamic perceptive approach to construct the meanings of algebraic notions;**
support students in algebraic activities that involve symbolic manipulation of expressions and propositions.

Case study: Eleonora

- Student Eleonora, 26 years old
- She obtained her first diagnosis of dyscalculia this year

When 3 is added to 3 times a certain number, the sum is 28; find the number.

Generational activity

Transformational activity

Global-meta level activity

What is an equation and what does mean solve an equation?

Case study: Eleonora

- Student Eleonora, 26 years old
- She was diagnosed "dyscalculic" for the first time last year

When 3 is added to 3 times a certain number, the sum is 28; find the number.

$$3x+3=28$$

Transformational activity

Global-meta level activity

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$$3x+3=28$$



$$3x=28-3$$
$$X=25/3$$

Global-meta level activity

What is an equation and what does mean solve an equation?

Case study: Eleonora

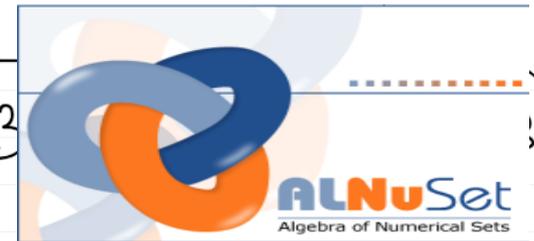
- Student Eleonora, 26 years old
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When 3 is added to 3 times a certain number, the sum is 28; find the number.

Eleonora, is oriented toward computation and not use of algebra, so she subtracts 3 from 28 and then divide 3 – undoing in reverse order, the operations stated in the problem text.

She lost the need to use algebra

How?



Why?

2020



Video concerning case study:

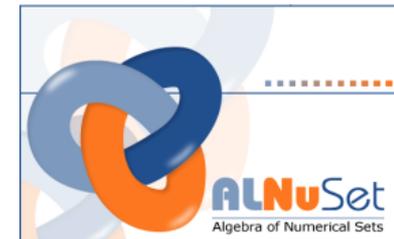
Meaning of algebraic notions:

-variable a unknown

-equation $2 * a = 8$

-solve equation $a = 8/2$

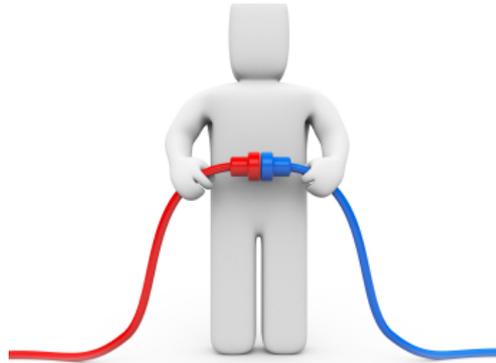
-solution of equation $a = 4$



HOW CAN WE INTERVENE? CONCLUSION

Algebra:

- Variable
- Unknown
- Solution of equation
- Existential quantifier
- Universal quantifier
- ...



**Meaning
(semantics) of
algebraic
notions**

