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Functional Analytic Methods in Quantum Many-Body Theory
Special Session B2

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This session aims at presenting the most recent developments in mathematical physics with an emphasis on the derivation of effective theories for describing complex many-body quantum systems at all scales. A paradigmatic example is the N-body Schrödinger equation

$$i\hbar\partial_t\psi = H\psi \ , \quad H = \sum_{i=1}^{N} -\hbar^2\Delta_i + \lambda \sum_{1\le i<j\le N} V(x_i - x_j) \ .$$

The particle number N is typically of the order 10^{23}. Due to the enormous number of degrees of freedom, the analysis of the Schrödinger equation is a massive mathematical challenge. However, in certain scaling regimes one may study physical quantities by proving convergence to certain effective (usually non-linear) theories such as Hartree–Fock theory or the Gross–Pitaevskii equation. Other important examples where this approach has proven useful are the quantum Hall effect or spin systems.

In recent years new functional analytic methods have been developed in this context, and our session is meant to provide a platform for the exchange of ideas among researchers working in the field with different mathematical background and research focus.

For more information visit `https://sites.rutgers.edu/umi-ams-joint-meeting`.

Schedule and Abstracts

July 25, 2024

11:30–12:15 Propagation bounds for 2D many-body fermion systems in a magnetic field

Bruno Nachtergaele (University of California, Davis, USA)

Abstract. We construct the infinite-volume dynamics for electrons with short-range two-body interactions in $\mathbb{R}^2$ subject to a periodic potential and a constant magnetic field. The crucial step is to establish propagation bounds of Lieb-Robinson type for the dynamics. We also discuss related results and applications.

12:30–12:50 Out-of-time-ordered correlators of mean-field bosons via Bogoliubov theory

Simone Rademacher (University of Tübingen, GERMANY)

Abstract. Quantum many-body chaos studies the scrambling of quantum information among large numbers of degrees of freedom. It rests on the prediction that out-of-time-ordered correlators (OTOCs) of the form $\langle[A(t), B]\rangle$, can be connected to classical dynamics. We rigorously prove a variant of this correspondence principle for mean-field bosons. More precisely, we consider N bosons in $\mathbb{R}^3$ with mean-field interactions, that are described by the Hamiltonian H_N, and study the Heisenberg dynamics $A(t) = e^{itH_N} A e^{-itH_N t}$ of an operator A. We show that for suitable operators A, B the OTOC $\langle[A(t), B]\rangle$, with a factorized state $\varphi^{\otimes N}$ as reference state, is in the limit $N \to \infty$ explicitly given by a suitable symplectic Bogoliubov dynamics. The proof uses Bogoliubov theory and extends to higher-order correlators of operators at different times. For these, it yields an out-of-time-ordered analog of the Wick rule. This is joint work with M. Lemm.

14:30–14:50 A Short Proof of BEC in the GP Regime and Beyond
Christian Brennecke (University of Bonn, GERMANY)

Abstract. In this talk, we present a new and self-contained proof of BEC for the ground state of N bosons moving in the three-dimensional unit torus and interacting through a pair potential with scattering length of order $N^{\kappa-1}$, for a parameter $\kappa \in [0, \frac{1}{20})$. The proof is based on an application of the Schur complement formula and mild a priori bounds on high momentum occupation number operators in the ground state. It improves and significantly simplifies previous results obtained jointly with A. Adhikari and B. Schlein. Compared to previous strategies, our proof avoids both the use of operator exponential expansions and box localization arguments. The talk is based on joint work with M. Brooks, C. Caraci and J. Oldenburg.

15:00–15:20 Adiabatic evolution of low-temperature many-body quantum systems
Giovanna Marcelli (Università degli Studi Roma Tre, ITALY)

Abstract. We consider finite-range, many-body fermionic lattice models and we study the evolution of their thermal equilibrium state after introducing a weak and slowly-varying time-dependent perturbation. Under suitable assumptions on the external driving, we derive a representation for the average of the evolution of local observables via a convergent expansion in the perturbation, for small enough temperatures. Convergence holds for a range of parameters that is uniform in the size of the system. Under a spectral gap assumption on the unperturbed Hamiltonian, convergence is also uniform in temperature. As an application, our expansion allows to prove closeness of the time-evolved state to the instantaneous Gibbs state of the perturbed system, in the sense of expectation of local observables, at zero and at small temperatures. Our strategy is based on a rigorous version of the Wick rotation and fermionic cluster expansion.

15:30–15:50 Upper bound on the ground state energy of a dilute gas of hard sphere bosons
Giulia Basti (Gran Sasso Science Institute, ITALY)

Abstract. In the last years, there has been substantial progress in the mathematical understanding of the low-energy properties of dilute Bose gases. We discuss a recent result on the energy of a dilute gas of hard sphere bosons of radius a in the thermodynamic limit where the number of particles N and the size of the box L^3 are sent to infinity keeping the density $\rho = N/L^3$ fixed.

More precisely, we prove that in the dilute regime $\rho a^3 \ll 1$ the ground state energy per particle in the thermodynamic limit $e(\rho)$ satisfies the upper bound

$$e(\rho) \leq 4\pi a\rho(1 + C(\rho a^3)^{1/2}).$$

Our result thus resolves the ground state energy up to an error of the order of the so-called Lee-Huang-Yang correction.

16:00–16:20 Interacting many-particle systems in the random Kac–Luttinger model and proof of Bose–Einstein condensation
Maximilian Pechmann (Tennessee Technological University, USA)

Abstract. Following a model originally considered by Kac and Luttinger, we study interacting many-particle systems in a random background. The background consists of hard spherical obstacles with fixed radius and that are distributed via a Poisson point process with constant intensity on R^d, $2 \leq d \in N$. Interactions among the (bosonic) particles are described through repulsive pair potentials of mean-field type. As a main result we prove that in the thermodynamic limit, (complete) Bose–Einstein condensation (BEC), in probability or with probability almost one depending on the strength of the interaction, occurs into the minimizer of a Hartree-type functional. As an important ingredient, we use very recent results obtained by Alain-Sol Sznitman regarding the spectral gap of the Dirichlet Laplacian in a Poissonian cloud of hard spherical obstacles in large boxes. To the best of our knowledge, our paper provides the first proof of BEC, for systems of interacting particles in the Kac–Luttinger model, or in fact for some continuous higher-dimensional random model.

17:00–17:45 Quantum logarithmic Sobolev inequalities for quantum many-body systems

Ángela Capel (University of Cambridge, UK)

Abstract. The mixing time of a quantum Markov semigroup describing the dissipative evolution of an open quantum many-body system can be bounded using optimal constants of certain quantum functional inequalities, such as the modified logarithmic Sobolev constant. For classical spin systems, the positivity of such constants follows from a mixing condition for the Gibbs measure, via quasi-factorization results for the entropy.

Inspired by the classical case, we present a strategy to derive the positivity of the modified logarithmic Sobolev constant associated to the dynamics of certain quantum systems from some clustering conditions on the Gibbs state of a local, commuting Hamiltonian. In particular, we show that, for a large class of geometrically-2-local models of Davies generators with commuting Hamiltonians, the mixing time is at most logarithmic in the system size, and this yields the so-called rapid mixing of dissipative dynamics. This is particularly relevant for 1D systems, for which we show rapid thermalization with a system size independent decay rate only from a positive gap in the generator. We also prove that systems in hypercubic lattices of any dimension, and exponential graphs, such as trees, have rapid mixing at high enough temperatures.

18:00–18:20 Lower bound for the free energy expansion of low temperature Bose gas

Marco Olivieri (University of Copenhagen, DENMARK)

Abstract. We consider a system of many interacting bosons in 3D and we show how to derive a lower bound for the expansion, in dilute limit, of the free energy density in thermodynamical regime at low temperature. The particles interact through a pairwise, repulsive potential satisfying a non-increasing condition, covering the case of the hard-core potential. The novelty of the method is in the joint action of the Neumann localization in small boxes, which, differently from other localization techniques, does not change the form of the Hamiltonian, together with a splitting and renormalization of the potential, which allows the rigorous application of the Bogoliubov theory. The expansion, at $T = 0$, corresponds to the so called Lee-Huang-Yang formula, which approximates the energy density up to the second order in the dilute limit.

July 26, 2024

11:30–12:15 Mobility edge, dynamics of the participation ratio, and percolation properties of the landscape function

Svitlana Mayboroda (U. of Minnesota, USA & ETHZ, SWITZERLAND)

Abstract. We will discuss non-asymptotic two-sided analogues of the Weyl law and Lieb-Thirring inequalities for the Schrodinger operator in terms of the landscape function and some open problems regarding its connection with the mobility edge.

12:30–12:50 Wu's correction to the ground state energy of a Bose gas in the Gross-Pitaevskii regime

Alessandro Olgiati (Politecnico di Milano, ITALY)

Abstract. We consider a Bose gas trapped on the 3d unit torus in the Gross-Pitaevskii regime and compute the ground state energy up to an error which vanishes faster than $N^{-1} \log N$ as $N \to \infty$. Our result matches the prediction by Wu for the correction to the Lee-Huang-Yang formula in the thermodynamic limit.

The proof involves a renormalization of the Hamiltonian through conjugation with unitary operators that implement correlations. It is well-known that a suitable quadratic and cubic correlation structure allows to rigorously reproduce the predictions of Bogoliubov theory in the Gross-Pitaevskii regime. In order to capture Wu's correction we need to modify such a correlation structure compared to the known results. Yet, our proof shows that no further correlation beyond quadratic and cubic is needed for the accuracy we aim for.

14:30–14:50 Kane's Euler Topological Metals
Jacob Shapiro (Princeton University, USA)

Abstract. Topology in condensed matter physics is usually studied in the context of topological -insulators-. These are materials which (in the bulk) do not conduct electric current. The aforementioned "topology" relates to the path-connectedness (or lack thereof) of the abstract space of quantum mechanical Hamiltonians. Remarkably, the path-component itself is indexed by a transport coefficient. Recently, Charles Kane (PRL 22) described a new paradigm to experimentally measure the topology of the Fermi surfaces of metals. We shall describe the mathematics behind connecting Kubo's linear response to the metal's Fermi surface Euler characteristic. Joint with Yichen Hu.

15:00–15:20 From decay of correlations to locality and stability of the Gibbs state
Massimo Moscolari (Politecnico di Milano, ITALY)

Abstract. I will show that whenever a Gibbs state satisfies decay of correlations, then it is stable, in the sense that local perturbations influence the Gibbs state only locally, and it is local, namely it satisfies local indistinguishability. These implications hold true in any dimension, only require locality of the Hamiltonian and rely on Lieb-Robinson bounds. A central role in the proofs is played by the quantum belief propagation for Gibbs states, which I will briefly review.

Furthermore, I will discuss how our results can be applied to quantum spin systems in any dimension with short-range interactions at high enough temperature, and to one-dimensional quantum spin chains with translation-invariant and exponentially decaying interactions above a threshold temperature that goes to zero in the limit of finite range interactions.

15:30–15:50 The free energy of the Bose gas at low density
Chiara Boccato (University of Pisa, ITALY)

Abstract. The interacting Bose gas is a system composed of a very large number of quantum particles with totally symmetric wavefunction. Below a critical temperature, a phase transition to a Bose-Einstein condensate is expected to occur, and collective behavior emerges from the underlying many-body theory.

At zero temperature we have precise information on the ground state energy and the low-lying spectrum of excitations (at least in certain scaling limits). However, much less is known close to the transition temperature. In this talk I will discuss how thermal excitations can be described by Bogoliubov theory, allowing us to estimate the free energy of the Bose gas in the Gross-Pitaevskii regime.

16:00–16:20 Correlation energy for the low density Fermi gas
Emanuela L. Giacomelli (LMU Munich, GERMANY)

Abstract. In recent decades, the study of many-body systems has been an active area of research in both physics and mathematics. In this talk we consider a system of N interacting fermions with spin $1/2$ confined in a box in the dilute regime. We are interested in studying the correlation energy, defined as the difference between the ground state energy and that of the free Fermi gas. We will discuss how to derive an asymptotics for the correlation energy in the thermodynamic limit, where the number of particles and the size of the box are sent to infinity while keeping the density fixed. In particular, we will present a result on an upper bound for the correlation energy that is consistent with the well-known Huang-Yang formula of 1957.

17:00–17:45 Derivation of Kubo formula for Hall conductance
Martin Fraas (University of California, Davis, USA)

Abstract. I will review derivations of Kubo formula for Hall conductance in quantum Hall effect. The focus will be on two recent results:

(1) Derivation of Kubo formula with interactions assuming that the Hamiltonian is gapped in the bulk
(2) Derivation of Kubo formula with disorder but without interactions.

18:00–18:20 The free energy of the dilute Bose gas at low temperature
Arnaud Triay (LMU Munich, GERMANY)

Abstract. We consider a Bose gas at density $\rho > 0$, interacting through a repulsive potential $0 \leq V \in L^2(\mathbb{R}^3)$ with scattering length $\mathfrak{a} > 0$. We prove an expansion for the free energy of the system in the dilute regime $\rho\mathfrak{a}^3 \ll 1$, valid at low temperature $T \lesssim \rho\mathfrak{a}$.

More precisely, we consider the Hamiltonian

$$H_N = \sum_{i=1}^{N} -\Delta_{x_i} + \sum_{1 \leq i < j \leq N} V(x_i - x_j)$$

acting on symmetric space $L^2_s(\Lambda_L^N)$ where $\Lambda_L = [0, L]^3$. The free energy of the system at temperature $T \geq 0$ is given by

$$F(N,T) = -T \log \mathrm{Tr} e^{-H_N/T} = \inf \left\{ \mathrm{Tr} H_N \Gamma + T\, \mathrm{Tr} \Gamma \log \Gamma \;\middle|\; \Gamma \geq 0,\ \mathrm{Tr}\Gamma = 1 \right\}$$

We prove that in the thermodynamic limit $N \to \infty$, $N/L^3 \to \rho$, when $T \leq C\rho\mathfrak{a}$ the following expansion holds

$$\begin{aligned} \frac{1}{L^3} F(N,T) = 4\pi\mathfrak{a}\rho^2 &\left(1 + \frac{128}{15\sqrt{\pi}}(\rho\mathfrak{a}^3)^{1/2}\right) \\ &+ \frac{T^{5/2}}{(2\pi)^3} \int_{\mathbb{R}^3} \log\left(1 - e^{-\sqrt{|p|^4 + \frac{16\pi\rho\mathfrak{a}}{T} p^2}}\right) dp + \mathcal{O}(\rho\mathfrak{a})^{5/2} \end{aligned}$$

In particular our estimate resolves the free energy per unit volume up to and including the Lee–Huang–Yang order $\mathfrak{a}\rho^2(\rho\mathfrak{a}^3)^{1/2}$.

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