

Recent Advances in Fluid mechanics: Theory and Computation Special Session B12

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This Special Session will focus on recent advances in the analysis and computation of fluid mechanics models, both compressible and incompressible, Newtonian and non-Newtonian, including recent progress on well-posedness questions, stability, fluid-structure interactions, geophysical models, and boundary layers. Researchers in these areas, junior and senior, primarily from Italy and the US, but also from other geographic areas will be invited to present their work in a fruitful exchange of ideas, with attention to diversity and underrepresented groups.

Schedule and Abstracts

July 25, 2024

11:30–11:50 Singular limits for the Rayleigh-Benard convection problem

Eduard Feireisl (Czech Academy of Sciences, CZECH REPUBLIC)

Abstract. We study the singular limit of the complete Navier-Stokes-Fourier system in the Rayleigh-Benard setting in the low Mach and mild stratification regime. We identify the standard Oberbeck-Boussinesq approximation as the target problem supplemented, however, with an unexpected no-local boundary term.

12:00–12:20 The Role of Dissipation in the Existence of Time-Periodic Solutions to PDE Systems

Boris Muha (University of Zagreb, CROATIA)

Abstract. It is well known that in many mechanical systems where energy is conserved, the phenomenon of resonance can occur, meaning that for certain time-periodic forces, the solution of the system becomes unbounded. Examples of partial differential equations describing such systems include the wave equation and equations of linearized elasticity (Lamé system). On the other hand, resonance will not occur in systems with strong dissipation, such as systems described by the heat equation. More precisely, in such a system, there exists a unique time-periodic solution for each time-periodic right-hand side.

In this lecture, we will address the question "how strong does dissipation need to be to prevent the occurrence of resonance?". We will analyze periodic solutions to the so-called *heat-wave* system, where the wave equation is coupled with the heat conduction equation via a common boundary. In this system, dissipation only exists in the heat component, and the system can be viewed as a simplified model of fluid-structure interaction. We will demonstrate that in certain geometric configurations, there exists a unique time-periodic solution for each time-periodic right-hand side. The proof will require some additional regularity of the right-hand side, and it remains an open question whether this is a technical condition arising from the proof method or if it is indeed necessary.

Finally, we will discuss the open question of whether the result is valid for arbitrary geometry or if there exists a geometry where resonance can occur.

Joint work with Stanislav Mosný, Sebastian Schwarzacher and Justin T. Webster.

12:30–12:50 Conditional regularity for the Navier-Stokes-Fourier system

Anna Abbatiello (University of Campania "L. Vanvitelli", ITALY)

Abstract. A conditional regularity criterion for solutions to a system of partial differential equations in fluid mechanics is a condition involving lower-order norms which, if satisfied, implies that the solutions remains regular; in particular, it can be applied to show that a local strong solution can be extended beyond its maximal time of existence. A direct consequence of the aforementioned result is a blow-up criterion meaning that if a blow-up of solutions occurs then some lower-order norms are not bounded. We proved a blow-up criterion for the compressible Navier-Stokes-Fourier system for general thermal and caloric equations of state with inhomogeneous boundary conditions for the velocity and the temperature. Assuming only that Gibb's equation and the thermodynamic stability hold, we show that solutions in a certain regularity class remain regular under the condition that the density, the temperature and the modulus of the velocity are bounded.

Joint work with Danica Basarić and Nilasis Chaudhuri.

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14:30–14:50 Convergence Analysis for Pseudomonotone Parabolic Problems

Michael Růžička (University of Freiburg, GERMANY)

Abstract. In the talk we discuss several existence proofs for nonlinear elliptic and parabolic problems which contain a pseudomonotone operator. A new notion of non-conforming pseudomonotonicity is introduced and applied. Based on that technique it is shown that numerical approximations based on a spatial non-conforming approximation converge to a weak solution of the original problem.

Joint work with Alex Kaltenbach.

15:00–15:20 Boundary layers equations with an eddy viscosity vanishing at the boundary

Luigi C. Berselli (University of Pisa, ITALY)

Abstract. Following classical models deriving from the analysis of von Kármán and Monin and Obukhov, we introduce a scalar elliptic equation defined on a boundary layer given by $\Pi_2 \times [0, z_{top}]$, where Π_2 is a two dimensional torus, with an eddy vertical viscosity of order z^α , $\alpha \in [0, 1]$, an homogeneous boundary condition at $z = 0$, and a Robin condition at $z = z_{top}$. We show the existence of weak solutions to this boundary problem, distinguishing the cases $0 \leq \alpha < 1$ and $\alpha = 1$. Then we carry out several numerical simulations, showing the ability of our model to accurately reproduce profiles close to those predicted by the Monin-Obukhov theory, by calculating stabilizing functions.

Joint work with François Legeais and Roger Lewandowski.

15:30–15:50 Analysis and numerics for a contactless rebound of elastic bodies in a viscous incompressible fluid

Sebastian Schwarzacher (Uppsala University and Charles University, SWEDEN/CZECH REPUBLIC)

Abstract. This talk is on a joint work with J. Fara, G. Gravina, O. Soucek and K. Tuma. I present some results on the phenomenon of the elastic rebound in a viscous incompressible fluid environment. In the important case of no-slip boundary conditions, it is by now classical that, under certain assumptions, collisions cannot occur in finite time. I will discuss some analytic results and numerical strategies to understand this fascinating yet counter-intuitive fluid-structure interaction. In particular, some arguments are collected that indicate that a physically meaningful rebound is possible even in the absence of a topological contact.

16:00–16:20 Instabilities of 3D vortex columns in incompressible inviscid fluids

Wojciech Ożański (Florida State University, USA)

Abstract. We will discuss some new techniques of establishing linear and nonlinear instability of some special inviscid flows, including 3D vortex columns, that is vector fields of the form $u = V(r)e_\theta + W(r)e_z$, where r denotes the distance to the axis of rotation and e_θ and e_z denote the standard cylindrical unit vectors, for a family of profiles V, W . We will demonstrate a construction of infinitely many, genuinely three-dimensional modes of instabilities of some vortex columns, which take the form of “ring modes”, localized around $r = r_0$, for some $r_0 > 0$. This is joint work with D. Albritton.

July 26, 2024

11:30–11:50 On the planar Taylor-Couette system and related exterior problems
Filippo Gazzola (Politecnico di Milano, ITALY)

Abstract. We show that the well-known multiplicity result for the stationary 3D Taylor-Couette flow in the region between two concentric unbounded 3D cylinders cannot be extended to planar flows. This result is complemented with the introduction of a weaker kind of solutions and with connections to some exterior problems. Based on a joint work with Jiri Neustupa and Gianmarco Sperone.

12:00–12:20 Finite dimensional approximation of solutions of the 2D incompressible Euler equations

Stefano Spirito (University of L'Aquila, ITALY)

Abstract. In this talk we consider the approximation of solution via a finite dimensional Galerkin methods. In particular, we will focus on the convergence in strong norms of the velocity and the vorticity, and the study of the rates of convergence. We will study the convergence in different regularity settings, including the Yudovich class of weak solutions with bounded vorticity. We compare the results with the analogous ones available for the vanishing viscosity approximation and comment on differences and similarities. The talk is based on joint works with L.C. Berselli (University of Pisa), G. Crippa (University of Basel), and G. Ciampa (University of L'Aquila).

12:30–12:50 Energy conservation for fluid flows in an Onsager critical class
Marco Inversi (Universität Basel, SWITZERLAND)

Abstract. The incompressible Euler equations govern the evolution of an ideal fluid. It is well known that the total kinetic energy is preserved along the time evolution of a regular fluid flow. However, when the motion is very rough, there is theoretical and experimental evidence of formation of chaotic structures that support the dissipation of kinetic energy. Mathematically, this problem translates into finding the critical regularity for weak solutions to the incompressible Euler equations to have conservation or dissipation of kinetic energy (Onsager's conjecture). Currently, the Onsager conjecture is almost solved. It has been proved that energy is conserved in any subcritical class and there are examples of solutions in any supercritical class violating the energy conservation. In a joint paper with Luigi De Rosa, we gave the first proof of energy conservation for weak solutions to the incompressible Euler system in a critical space, both in absence and presence of physical boundary. This is the first energy conservation result that holds in the incompressible case and fails in the compressible setting.

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14:30–14:50 Strong non-uniqueness of finite energy solutions to the 3D deterministic and stochastic Navier-Stokes equations

Alexey Cheskidov (University of Illinois at Chicago, USA)

Abstract. For any prescribed finite energy divergence-free initial data, we show that there exist infinitely many weak solutions with smooth energy profiles to both the 3D deterministic and stochastic incompressible Navier-Stokes equations. Moreover, we show that every constructed deterministic solution is a vanishing viscosity limit of stochastic solutions.

15:00–15:20 On a compressible fluid-structure interaction problem with slip boundary conditions

Šárka Nečasová (Academy of Sciences, CZECH REPUBLIC)

Abstract. We study a system describing the compressible barotropic fluids interacting with (visco) elastic solid shell/plate. In particular, the elastic structure is part of the moving boundary of the fluid, and the Navier-slip type boundary condition is taken into account. Depending on the reference geometry (flat or not), we show the existence of weak solutions to the coupled system provided the adiabatic exponent satisfies $\gamma > \frac{12}{7}$ without damping and $\gamma > \frac{3}{2}$ with structure damping, utilizing the domain extension and regularization approximation. Moreover, via a modified relative entropy method in time-dependent domains, we prove the weak-strong uniqueness property of weak solutions. Finally, we give a rigorous justification of the incompressible inviscid limit of the compressible fluid-structure interaction problem with a flat reference geometry, in the regime of low Mach number, high Reynolds number, and well-prepared initial data.

Joint work with Yadong Liu and Sourav Mitra.

15:30–15:50 Onsager conjecture for SQG

Mimi Dai (University of Illinois at Chicago, USA)

Abstract. In analogy with the Onsager conjecture for the Euler equation, the Onsager type of conjecture for the surface quasi-geostrophic (SQG) equation concerns the regularity threshold for the conservation of Hamiltonian. The expected regularity threshold is C^0 by scaling analysis. We prove it rigorously from the flexibility side by constructing solutions in C^{0-} which do not conserve the Hamiltonian.

Joint work with Vikram Giri and Razvan-Octavian Radu.