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New Advances in the Mathematical Theory of Control
Special Session B27

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This session brings a variety of Optimal Control theorists to discuss their recent results. Topics include modeling robotics and other systems, the sweeping process, stability issues, and discontinuous dynamics.

Schedule and Abstracts

July 25, 2024

11:30–11:50 Optimal controls over sweeping processes and exponential penalty method for the truncated sweeping sets

Vera Zeidan (Michigan State University, USA)

Abstract. We consider optimal control problems (P) involving a perturbed sweeping process governed by moving, nonsmooth, prox-regular, and possibly *unbounded* sweeping sets, C(t), and having *joint* endpoint constraints. More specifically, $C(t)$ is the intersection of finitely many sub-levels of smooth functions.

When the process is coupled with a standard control system, this general model incorporates different sub-models as particular cases, including second order sweeping processes, a subclass of integro-differential sweeping processes, evolution variational inequalities (EVI), and dynamical variational inequalities (DVI). Note that the presence of the discontinuous and unbounded normal cone $N_{C(t)}$ in the sweeping process renders inapplicable all the known optimality results in optimal control over standard differential inclusions.

In addition to the discrete-time approximation, the method of continuous-time exponential penalty approximation has shown to be instrumental in deriving Pontryagin-type maximum principle for special cases of (P).

The goal of this talk is to demonstrate that when deriving the Pontryagin-type maximum principle for a strong local minimum $(\bar{x}, \bar{u})$ of (P) it is greatly beneficial to develop the exponential penalty approximation not for the sweeping set $C(t)$ but rather for $C(t) \cap \bar{B}_{\bar{\varepsilon}}(\bar{x}(t))$, that is, the truncation of $C(t)$ by a ball centered at the state of the strong local minimum and having a specific radius $\bar{\varepsilon}$ carefully chosen so that the exponential penalty method can me modified successfully. Consequently, the so-obtained Pontryagin-type principle turns out to be valid under minimal local assumptions at or around the optimal state $\bar{x}$, and furthermore, the addition of the extra constraint to $C(t)$ due to the truncation, does not require *any* extra assumption, not even on the constraint qualification.

12:00–12:20 Optimal control pf perturbed sweeping processes with applications to general robotics models

Dao Nguyen (San Diego State University, USA)

Abstract. This talk primarily focuses on the practical applications of optimal control theory for perturbed sweeping processes within the realm of robotics dynamics. By describing these models as controlled sweeping processes with pointwise control and state constraints and by employing necessary optimality conditions for such systems, we formulate optimal control problems suitable to these models and develop numerical algorithms for their solving. Subsequently, we use the Python Dynamic Optimization library GEKKO to simulate solutions to the posed robotics problems in the case of any fixed number of robots under different initial conditions.

12:30–12:50 Applications and simulations of Optimal Control
Norma Ortiz-Robinson (Grand Valley State University, USA)
Abstract. In this talk I will discuss a few recent projects involving modeling of applications with optimal control and their simulation using the Python dynamic optimization library GEKKO.

14:30–14:50 Solutions to the Hamilton-Jacobi equation for dynamic optimization problems with discontinuous time dependence
Piernicola Bettiol (University of Brest, France)
Abstract. We will provide characterizations of the value function as the unique lower semicontinuous solution, appropriately defined, to the Hamilton-Jacobi equation associated with some classes of dynamic optimization problems with discontinuous time dependence. We shall consider optimal control problems of Bolza type in which the running cost is possibly non Lipschitz continuous w.r.t. the state variable and Calculus of Variations problems in which the functional to minimize comprises an end-point cost function and an integral term involving a nonautonomous Lagrangian. We shall give also some illustrative examples.

This talk is based on recent results obtained in collaboration with J. Bernis, C. Mariconda and R. Vinter.

15:00–15:20 An Optimal Control Approach to the Problem of the Longest Self-Supporting Structure
Michele Palladino (University of L'Aquila (Italy)
Abstract. The characterization of the self-supporting slender structure with the furthest length is of interest both from a mechanical and biological point of view. Indeed, from a mechanical perspective, this classical problem was developed and studied with different methods, for example using similarity solutions and stable manifolds. However, none of them led to a complete analytical solution. On the other hand, plant structures such as tree branches or searcher shoots in climbing plants can be considered elastic cantilevered beams. In this paper, we formulate the problem as a non-convex optimisation problem with mixed state constraints. The problem is solved by analysing the corresponding relaxation. With this method, it is possible to obtain an analytical characterization of the cross-section.

15:30–15:50 Strong local optimality in state constrained optimal control problems
Laura Poggiolini (Università degli Studi di Firenze (Italy)
Abstract. In several optimal control problems coming from realistic cases, the state of the system is subject to inequality constraints. In such case, a suitable, non trivial, extension of the celebrated Pontryagin Maximum Principle (PMP) gives a necessary condition for optimality (in the spirit of KKT conditions). However, it is well known that, except for special low-dimension cases, the PMP identifies many non-optimal trajectories as extremals: that is why the optimal control community is active in the search of additional necessary and sufficient optimality conditions.

We address the problem of strong local optimality for state constrained optimal control problems where the dynamics is affine with respect to the control:

$$\begin{aligned}
&\dot{\xi}(t) = f_0(\xi(t)) + u(t) f_1(\xi(t)) \quad \text{a.e. } t \in [0,T], \\
&\xi(0) = x_0, \quad \xi(T) \in \mathcal{N}_f, \\
&c(\xi(t)) \le 0 \quad \forall t \in [0,T], \qquad\qquad |u(t)| \le 1 \quad \text{a.e. } t \in [0,T].
\end{aligned}$$

Here the state space is a smooth manifold M, the function $c\colon M \to \mathbb{R}$ defining the state constraint is assumed to be smooth on a neighborhood of its zero-level set; the vector fields f_0, f_1 are smooth and $\mathcal{N}_f$ is a submanifold of M. The cost to be minimized can be either a Mayer cost or the minimum time to reach $\mathcal{N}_f$, i.e. we consider both the following problems

$$\text{minimize} \quad \psi(\xi(T)), \quad T > 0 \text{ fixed},$$

$$\text{minimize} \quad T, \quad T > 0 \text{ free}.$$

More precisely we aim at giving sufficient conditions for the strong local optimality of an extremal which is given by the concatenation of some internal arcs and a boundary one.

The given conditions are proven to be sufficient via Hamiltonian methods, in the case when the extremal is the concatenation of an internal bang arc, followed by a boundary arc and then by a finite number of internal bang arcs. Currently we are investigating the case when in between the internal bang arcs and the boundary arc there occurs an internal singular arc.

The provided sufficient conditions are regularity conditions on the junction points between the boundary arc and the internal ones, which allow us to use a simpler version of PMP for state-constrained problems plus conditions on the order of the boundary arc, a strengthened version of the necessary conditions along the internal arcs, and the coerciveness of a suitable quadratic form (which is finite-dimensional in the case when no internal singular arc occurs).

16:00–16:20 Can we locally optimize a second order decrease rate of a function for nonlinear and symmetric control systems?

Pierpaolo Soravia (University of Padova (Italy)

Abstract. When a control system has all its vector fields tangent to the level set of a given smooth function u at a point $\hat{x}$, that function can still have a negative rate of decrease with respect to the trajectories of the control system in appropriate sense. In the case when the system is symmetric and u has a decrease rate of the second order, we investigate the existence of a best possible rate in the class of piecewise smooth trajectories. The problem turns out to be purely algebraic, and depends on the eigenvalues of matrices constructed from a basis matrix whose elements are the second order Lie derivatives of u at $\hat{x}$ with respect to the vector fields of the system.

17:00–17:20 Directionality of constrained control systems and discontinuities in clearance

Niles Armstrong (Milwaukee School of Engineering, USA)

Abstract. Control theory studies the evolution of dynamical systems which are actively influenced by some external agent (or controller). We will discuss control systems with kinodynamic constraints on admissible trajectories, wherein one encounters obstacles in state space which must be avoided as the system evolves. In this setting, one defines a system-dependent clearance function quantifying the shortest admissible distance to the obstacle set. We will focus on points of discontinuity in the clearance function and how these discontinuities are experienced as one traverses admissible trajectories. This investigation of discontinuities will leads us to explore a common directionality condition for velocities at a point, characterized by strict positivity of the minimal Hamiltonian.

17:30–17:50 Envelope Generators and Clearance Irregularities in State Constrained Control Systems

Jeremy LeCrone (University of Richmond, USA)

Abstract. We consider nonlinear state–constrained control problems. Specifically, we investigate irregularities of the system–dependent clearance function, i.e. the value function quantifying the minimal cost to reach the constraining obstacle set (or equivalently, the target set).

In this talk, I will present a set of sufficient conditions demonstrating how local directionality in system dynamics, together with corresponding configurations for the obstacle set, guarantee the existence of clearance discontinuities both on the obstacle boundary and propagating out into free space. A selection of applications will be explored. We will discuss further questions regarding interactions between the structure of clearance discontinuities and medial axis structures, which may be associated with sets of irregularities in the gradient of clearance.

18:00–18:20 The minimal time problem of a sweeping process with a discontinuous perturbation

Vinicio Rios (Louisiana State UNiversity, USA)

Abstract. Since their inception in the seventies , sweeping processes have become one of the most popular categories of dynamical systems. Their adaptability to explain nonsmooth phenomena has been a source of motivation for research in areas such as nonsmooth analysis, differential inclusions, control theory, and its associated optimization paradigm, namely, optimal control theory. In this talk, we consider an important instance of the latter framework: the minimal time problem. Specifically, we discuss for the first time the minimal time function of a

sweeping process with a state constraint set that is constant (commonly referred to as the *reflecting boundary problem* and prox-regular, and whose right-hand-side is being perturbed by a discontinuous multifunction satisfying the *Dissipative-Lipschitz property.* Our analysis follows the traditional route based on the invariant properties of the epigraph and hypograph of the minimal time function, which has been successful in characterizing such a function as a proximal solution of a Hamilton-Jacobi equation for different types of dynamics (e.g., Lipschitz differential inclusions , Dissipative-Lipschitz differential inclusions , and sweeping processes with Lipschitz perturbations). Accordingly, we use the prox-regularity of the state constraint set in combination with the dissipative-Lipschitz condition of the perturbing multifunction to establish the first Hamiltonian characterization of strong invariance for sweeping processes with discontinuous perturbations. This result leads to an interpretation of the minimal time function through a pair of Hamilton-Jacobi inequalities, one of which exhibits a limiting component that is intrinsic to the discontinuity of the system.

July 26, 2024

11:30–11:50 Consensus and control for Hegselmann-Krause type models
Cristina Pignotti (University of L'Aquila, Italy)

*Abstract.*We analyze the asymptotic behavior of a Hegselmann-Krause opinion formation model with delayed interactions. Under appropriate assumptions, an exponential consensus estimate is proven without any smallness restrictions on the time delay size. Since the consensus estimates are independent of the agents' number, we are able to extend the stability result to the continuum model obtained as the mean-field limit of the many-individual equation as the number of individuals goes to infinity. Some extensions to Hegselmann-Krause models on network topologies are also discussed. Finally, we illustrate a controllability result for a Hegselmann-Krause opinion formation model with leadership.

This talk is based on joint papers with Young-Pil Choi (Yonsei University, Republic of Korea), Chiara Cicolani (University of L'Aquila), Elisa Continelli (University of L'Aquila), and Alessandro Paolucci.

12:00–12:20 A single player and a mass of agents: a pursuit-evasion-like game
Fabio Bagagiolo (University of Trento, Italy)

Abstract. In this talk I will speak about a finite-horizon differential game of pursuit-evasion type, between a single player and a mass of agents. The player and the mass directly control their own evolution which, for the mass, is given by a first order PDE. By dynamic programming and the use of an adapted concept of non-anticipating strategies, I will derive an infinite dimensional Isaacs equation and address the problem of uniqueness of the value function as viscosity solution on a suitable invariant subset of a Hilbert space.

The novelty of the problem and of the results mainly relies on the derivation of the infinite-dimensional Isaacs equation and on its study in the Hilbert setting. A preliminary study of the controlled evolution of the mass, by the evolution of its density as time-varying state-dependent function, is also interesting. A couple of one-dimensional examples will enlighten the complexity of the game and of the model.

This research is conducted jointly with Rossana Capuani (University of Arizona, USA) and Luciano Marzufero (University of Bolzano, Italy). Some ongoing further research directions will be mentioned.

12:30–12:50 Optimal control of moving sets
Elsa Marchini (Politecnico di Milano, Italy)

Abstract. In the classical Dido's problem we are given a set $V \subset \mathbb{R}^2$ and a number $\ell > 0$: among all curves $\gamma \subset V$ with length ℓ, we seek one which bounds a subset $\Omega \subset V$ with largest possible area. We consider a time-dependent version of this problem, modeling the optimal control of an invasive population. V is interpreted as an island, which is infested by an invasive species (for example, malaria-carrying mosquitoes). By spraying pesticides, we can progressively shrink

the contaminated region $\Omega = \Omega(t)$, until it eventually becomes empty set. The main goal is to describe the optimal strategies $t \mapsto \Omega(t)$, relying on a set of necessary conditions for optimality.

14:30–14:50 Lie brackets and state constraints
Franco Rampazzo (University of Padova, Italy)
Abstract. The classical *inward pointing condition* (IPC) for a control system whose state x is constrained in the closure $C := \bar{\Omega}$ of an open set Ω prescribes that at each point of the boundary $x \in \partial\Omega$ the intersection between the dynamics and the interior of the tangent half-space of $\bar{\Omega}$ at x is nonempty. Under this hypothesis, for every system trajectory $x(.)$ on a time-interval $[0, T]$, possibly violating the constraint, one can construct a new system trajectory $\hat{x}(.)$ that satisfies the constraint and whose distance from $x(.)$ is bounded by a quantity proportional to the maximal deviation $d := \text{dist}(\Omega, x([0, T]))$. When (IPC) is violated, the construction of such a constrained trajectory is not possible in general. However, in this paper we prove that a "higher order" inward pointing condition involving Lie brackets of the dynamics' vector fields –together with a non-positiveness curvature-like assumption– allows for a novel construction of a constrained trajectory $\hat{x}(.)$ whose distance from the reference trajectory $x(.)$ is bounded by a quantity proportional to $\sqrt{d}$. As an application, we establish the continuity *up to the boundary* of the value function V of a connected optimal control problem, a continuity that allows to regard V as the *unique* constrained viscosity solution of the corresponding Bellman equation.

15:00–15:20 HJ inequalities involving Lie brackets and feedback stabilizability with cost regulation
Giovanni Fusco (University of Padova, Italy)
Abstract. With reference to an optimal control problem where the state has to approach asymptotically a closed target while paying a non-negative integral cost, we propose a generalization of the classical dissipative relation that defines a Control Lyapunov Function to a weaker differential inequality. The latter involves both the cost and the iterated Lie brackets of the vector fields in the dynamics up to a certain degree $k \geq 1$, and we call any of its (suitably defined) solutions a *degree-k Minimum Restraint Function.* We prove that the existence of a degree-k Minimum Restraint Function allows us to build a Lie-bracket-based feedback which sample stabilizes the system to the target while *regulating* (i.e., uniformly bounding) the cost.

15:30–15:50 The generalized Elvis problem
Claire Pearson (Georgia Tech, USA)
Abstract. We study the problem of minimizing transit time in control systems characterized by region-specific dynamic constraints. The state space is partitioned into polytopes endowed with convex velocity sets. Within each polytope, velocities must satisfy a constant differential inclusion so that time-optimal refraction between regions follows a generalized form of Snell's Law derived via convex duality. This talk focuses on deriving geometric optimality conditions and studying classes of minimal time trajectories between adjacent regions.

16:00–16:20 Computational convex analysis using mathematical software
Clinten Graham (Johns Hopkins, USA)
Abstract. We discuss software approaches to computational convex analysis developed for solving minimal-time problems with piecewise constant dynamics. A symbolic convex analysis toolkit in Mathematica is presented for calculating gauge and support functions of convex sets. Procedures for solving optimality conditions involving normal cones and subgradients are presented. The second part of the talk is devoted to numerical solutions to the multi-region problem using a MATLAB framework. The multi-region problem is challenging to solve exactly due to the combinatorial selection of interfaces. Still, this problem is convex and solvable for fixed sequences of region indices via mathematical programming. While the sequential problem appears intractable by variational techniques, our work leverages the Quickhull algorithm to numerically determine optimal sequences of region transfer. Using this method, we characterize the boundary of the reachable set and describe optimal trajectories.

17:00–17:50 Piecewise constant control with convex bifunctions

Peter Wolenski (Louisiana State University, USA)

Abstract. Convex bifunctions were introduced by Rockafellar as a means to unify generalized convex programs by distinguishing between variables and parameters. They can be viewed as a generalization of a multivalued mapping that assigns a cost to each of its elements. This talk will discuss how minimal time problems can be solved in two mediums, each having its own bifunction $(Fr)(\mathbf{v})$ that records the cost of using the velocity vector $\mathbf{v}$ for the time $r > 0$. The generalized Elvis problem is the case where $(Fr)(\mathbf{v}) := r\left\{1 + \mathcal{I}_F\left(\frac{v}{r}\right)\right\}$, where $\mathcal{I}_F$ is the indicator of the available velocity set.

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