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Model Theory
Special Session B7

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This special session will concentrate on recent progress in both areas of model theory, applied and more abstract. The speakers are leading figures in the field.

For more information visit `www.matfis.unicampania.it/home-model-theory`.

Schedule and Abstracts

July 25, 2024

11:30–12:15 The Model Theory of Modules over a Ring with Involution
Ivo Herzog (Ohio State University, USA)

Abstract. The model theory of modules over a ring $(R, *)$ with involution has features that enhance the classical theory, which includes the familiar theme of positive primitive formulae and how they can be used to define the Ziegler topology on the space of indecomposable algebraically compact representations. The main feature is that the Prest dual of a pp-formula may be composed with the involution to define an operation on the usual modular lattice of pp-formulae in a way that it too becomes endowed with a canonical involution.

We will explain how the *Moore-Penrose inverse,* a notion from the theory of rings of operators, is related to elimination of quantifiers. A careful process of formal adjunction of these inverses yields the $*$-*regularization* of $(R, *)$, which is a universal construction that mimics a similar construction due to Olivier for a commutative ring (with involution the identity). A ring $(S, \#)$ with involution in $*$-regular iff the modular pp-lattice with involution is a *quantum logic.*

Another feature of the theory are the $*$-*contravariant forms* that arise in the representation theory of Lie algebras and Chevalley groups. The universal $*$-contravariant form of a module over $(R, *)$ is definite if and only if the modular pp-lattice with involution satisfies the Law of Contradiction, which may be regarded as an axiom schema that partially axiomatizes the finite dimensional representations of certain Lie algebras.

This is joint work with S. L'Innocente.

12:30–13:15 Equations in the j-invariant and its derivatives
Vincenzo Mantova (University of Leeds, UK)

Abstract. Zilber's exponential-algebraic closedness conjecture predicts that polynomial-exponential equations must have solutions, unless they have a very good (geometric) reason not to. The same can be asked of other functions having some symmetries, such as abelian exponentials, modular functions, and similar. I'll discuss what Vahagn Aslanyan, Sebastian Eterović and I have done for equations in j, j', j'' in one variable. Even in just one variable, this raises questions that were not addressed in the literature; for instance, we show that the equation $j'' = 0$ has "Zariski dense" many solutions, in a suitable sense. I will discuss the reasoning behind Zilber's conjecture and the ingredients we used to study j and its derivatives.

13:00–14:30 Lunch break

14:30–15:15 Invariants rings and fields in o-minimal structures
Kobi Peterzil (University of Haifa, Israel)

Abstract. Jana Marikova in 2007, was first to study automorphism-invariant groups in o-minimal structures and showed that they can be endowed with a group topology, similarly to the case of definable groups. In this talk I describe results on invariant rings and fields.

While 0-dimensional invariant fields may include the fields of rational or algebraic numbers (clearly not definable), we show that a positive dimensional type-definable or Ind-definable field must be definable. In addition, while an arbitrary invariant field of positive dimension might not be definable we conjecture that it is isomorphic, via an invariant map, to a definable one.

This is joint work with Mirvat Mhameed.

15:30–16:15 Model theoretic events
Kyle Gannon (Peking University, China)

Abstract. This talk is motivated by the following two soft questions: How do we sample an infinite sequence from a first order structure? What model theoretic properties might hold on almost all sampled sequences? We advance a plausible framework in an attempt to answer these kinds of questions. The central object of this talk is a proability space. The underlying set of our space is a standard model theoretic object, namely the space of types in countably many variables over a monster model. Our probability measure is an iterated Morley product of a fixed Borel-definable Keisler measure. Choosing a point randomly in this space with respect to our distribution yields a random generic type in infinitely many variables. We are interested in which model theoretic events hold for almost all random generic types. Two different kinds of events will be discussed: (1) The event that the induced structure on a random generic type is isomorphic to a fixed structure; (2) the event that a random generic type witnesses a dividing line.

16:30–17:00 Coffee break

17:00–17:45 Some model theory of quadratic forms
Nicholas Ramsey (University of Notre Dame, USA)

Abstract. Vector spaces over finite fields with quadratic forms are an important example in the Cherlin-Hrushovski theory of Lie coordinatization of smoothly approximable theories, which are homogeneous structures that can be approximated by finite homogeneous substructures. These come in two forms: the orthogonal spaces (which have only one quadratic form) and the quadratic geometries (which have a whole family of quadratic forms). We address several basic questions about the model theory of these structures where the field is allowed to be infinite. For example, we classify all pseudo-finite theories of orthogonal spaces and quadratic geometries, axiomatize the model companions of each, and give a reasonably complete neostability-theoretic classification of all of these theories.

This is joint work with Charlotte Kestner.

July 26, 2024

11:30–12:15 Expansions of $\mathbb{R}$ and $\mathbb{N}$ by k-automatic sets: Choose-your-own-adventure!
Alexi Block Gorman (Ohio State University, USA)

Abstract. There are compelling and long-established connections between automata theory and model theory, particularly regarding expansions of Presburger arithmetic by sets whose base-k representations are recognized by a finite-state automaton. We call such sets “k-automatic”. Büchi automata are the natural extension of finite-state automata to a model of computation that accepts infinite-length inputs. We say a subset X of the reals is “k-regular” if there is a Büchi automaton that accepts (one of) the base-k representations of every element of X, and rejects the base-k representations of each element in its complement. These sets often exhibit fractal-like behavior–e.g., the Cantor set is 3-regular. In this talk we will consider the expansions of Presburger arithmetic and the real additive group by k-automatic and k-regular sets respectively. In the real setting, we will discuss dividing lines in definability from the perspectives of both tame geometry and neostability. In the setting of Presburger arithmetic, we obtain a characterization of expansions of $(\mathbb{N}, +)$ by unary k-automatic sets, and discuss its consequences for the decidability and neostability-related properties of related structures.

12:30–13:15 Asymptotics of Skolem’s exponential functions
Marcello Mamino (University of Pisa, Italy)

Abstract. Let us call Skolem functions the smallest set of functions $f\colon \mathbb{N}^{>0} \to \mathbb{N}^{>0}$ closed under pointwise sum $f+g$, product fg, and exponentiation f^g, and containing the constant function $x \mapsto 1$ and the identity function $x \mapsto x$. We consider the order on the set of Skolem functions given by $f < g$ if $f(x) < g(x)$ for all x large enough. By results of Hardy, this is a total order. Skolem conjectured that it is, indeed, a well order of order type ϵ_0. The first part of the conjecture was proved by Ehrenfeucht using Kruskal's tree theorem, while the second part remains unsolved.

Levitz showed that the order type of the Skolem functions is at most equal to the smallest critical epsilon-number (the least ordinal α such that $\alpha = \epsilon_\alpha$). Furthermore, Levitz's work provides bounds on the order type of certain initial segments of the Skolem functions, for instance the set of Skolem functions below $x \mapsto 2^{2^x}$ has order type at most ϵ_0, and the set of Skolem functions below $x \mapsto 2^{x^x}$ has order type at most ϵ_ω. Van den Dries and Levitz improved the first of these bounds to ω^{ω^ω}.

We will study the asymptotic behaviour of Skolem functions by embedding them in Conway's field of surreal numbers. Our main result is as follows.

Theorem 1. *Let $c \geq 1$ be a surreal number and let Q be a Skolem function. The set of real numbers $r \in \mathbb{R}$ such that there is a Skolem function h satisfying $(h/Q)^c = r + o(1)$ has no accumulation points in $\mathbb{R}$.*

As a consequence, we get a different proof of van den Dries and Levitz's result, and we can improve some of Levitz's upper bounds. For instance, we show that the Skolem functions below $x \mapsto 2^{x^x}$ have order type at most ϵ_0.

This is joint work with Alessandro Berarducci.

13:00–14:30 Lunch break

14:30–15:15 Local coordinatization of projective geometries and explicit bounds in Elekes-Szabó for arbitrary arity and co-dimension

Artem Chernikov (University of Maryland, USA)

Abstract. An influential theorem of Elekes and Szabó indicates that the intersections of a given algebraic variety with large finite grids of points may have maximal size only for varieties that are closely connected to algebraic groups. Techniques from model theory — variants of Hrushovski's group configuration and of Zilber's trichotomy principle — are very useful in recognizing these groups, and led to far reaching generalizations of Elekes-Szabó in the last decade.

In this talk, focusing on the strongly minimal case, we provide a generalization of the earlier result by Chernikov-Peterzil-Starchenko to arbitrary co-dimension, in particular obtaining explicit bounds in a theorem due to Bays and Breuillard over the complex numbers.

Our key tool is a very explicit version of the (model theoretic) coordinatization of projective geometries supported on grids with large count, demonstrating that instead of closing under all canonical bases it suffices to add finitely many very explicit ones.

This is joint work with K. Peterzil and S. Starchenko.

15:30–16:15 Around first-order rigidity of Coxeter groups

Gianluca Paolini (University of Torino, Italy)

Abstract. We survey recent results on the problem of first-order rigidity of Coxeter groups. Specifically, we survey the results of two papers, one jointly written with S. André and the other with R. Sklinos. The main result of the paper with Sklinos is that irreducible affine Coxeter groups are first-order rigid, i.e., they are the only finitely generated models of their first-order theory. We deduce from this that irreducible affine Coxeter groups are profinitely rigid in the absolute sense, thus solving an open problem posed in a recent paper by Möller and Varghese. The paper jointly written with S. André goes in another direction, specifically, by the work of Sela, for every group G elementary equivalent to the free group on two generators we have that $\mathbb{Z}_2 * \mathbb{Z}_2 * \mathbb{Z}_2$ is elementary equivalent to $\mathbb{Z}_2 * \mathbb{Z}_2 * \mathbb{Z}_2 * G$, and so, without further restrictions, Coxeter groups are very far from being first-order rigid. Our results show that, despite this, to a large extent, from the perspective of Coxeter group theory this is somewhat "accidental",

in the sense that if we restrict to models which are generated by involutions the situation is completely different. Our main results in this direction are the following. (1) If W and W' are even Coxeter groups which are elementary equivalent, then they are isomorphic. (2) If W is word hyperbolic and either even, or 2-spherical, or 1-ended, and G is elementary equivalent to W and generated by finitely many involutions, then G is isomorphic to W. Finally, we prove that there are two hyperbolic Coxeter groups which are $\forall\exists$-elementary equivalent but not isomorphic, and so, assuming $\forall\exists$-elimination of quantifiers (which is conjectured to be true by the community), item (2) is not true in the odd case. Result (1) above generalizes results due to Casals-Ruiz, Kazachkov and Remeslennikov, where the same was shown for right-angled Coxeter groups.

16:30–17:00 Coffee break

17:00–17:45 Constant power maps on Hardy fields and Transseries
Elliot Kaplan (McMaster University, Canada)

Abstract. We study H-fields (certain ordered differential fields generalizing Hardy fields and Transseries) equipped with "constant power maps". We show that this class has a model companion, the models of which include the field of LE-transseries and any maximal Hardy field. We study the induced structure on the constant field, prove a relative decidability result, and give some applications to certain systems of differential equations.

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