Topological and Variational Methods for Differential and Difference Equations Special Session A12

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The investigation of ordinary differential equations and systems maintains a constant interest in the mathematical literature. This has never been more apparent than in recent years with the exploding interest in the fractional calculus and fractional differential equations. The great attention towards these equations and systems is mainly due to their role as models in many areas of science and technology such as in diffusion processes, celestial mechanics, flows in porous media, population dynamics, epidemic models, and more recently in social science areas. These equations and systems are also used to obtain solutions to diffusion and transport equations that are often in good agreement with experimental data. In some cases, the underlying process is discretized, which justifies the study of the associated discrete models.

New requests by the scientific community and the development of new mathematical tools serve to motivate interesting recent achievements in several directions.

The main mathematical tools in these studies are topological or variational methods. In some cases, these techniques can be successfully combined with comparison-type techniques such as the upper and lower solution method.

The aim of this special session is to advance knowledge and interest in the study of all these types of problems by inviting researchers to participate and share their current research endeavors.

Schedule and Abstracts

July 23, 2024

11:00–11:25 Uniqueness of Positive Solutions for a Class of Nonlinear Elliptic Equations with Robin Boundary Conditions
Ratnasingham Shivaji (University of North Carolina at Greensboro, USA)

Abstract. We prove uniqueness of positive solutions to the BVP

$$\begin{cases} -\Delta u = \lambda f(u) & \text{in } \Omega, \\ \frac{\partial u}{\partial n} + bu = 0 & \text{on } \partial\Omega, \end{cases}$$

when the parameter λ is large independent of $b \in (0, \infty)$. Here Ω is a bounded domain in $\mathbb{R}^n$ with smooth boundary $\partial\Omega$, $f : [0, \infty) \to [0, \infty)$ is continuous, concave for u large, and sublinear at ∞. Joint work with D.D. Hai & Xiao Wang.

11:30–11:55 Existence results for discrete and differential nonlinear problems
Giuseppina D'Aguì (University of Messina, ITALY)

Abstract. In this talk some results on the existence of two positive solutions to boundary value problems for difference equations and ordinary differential equations are presented. In particular the existence of at least two solutions is obtained by requiring suitable behaviour at infinity and at zero of the primitive of the nonlinear term. Our main tool is a critical point theorem obtained by appropriately combining the powerful classical Ambrosetti-Rabinowitz theorem with a recent non-zero local minimum theorem.

The results presented are part of the research carried out within the project: PNRR-MAD-2022-12376692- PNRR-Missione 6 - Componente 2 Investimento 2.1 Valorizzazione e Potenziamento della Ricerca Biomedica del SSN

12:00–12:25 Existence and approximation of a solution for a two point nonlinear Dirichlet problem
Pasquale Candito (University of Reggio Calabria, ITALY)

Abstract. The existence of at least one positive solution to a second-order nonlinear two-point boundary value problem, is established. Combining difference methods with variational or topological methods, we get a solution as the limit of an appropriate sequence of piecewise linear interpolations. Furthermore, a priori bounds on the infinite norm of a solution and its derivatives are pointed out. Some examples are also discussed to illustrate our results.

References

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12:30–12.55 Periodic solutions in special relativity with singular potentials
Duccio Papini (University of Modena and Reggio Emilia, ITALY)

Abstract. According to special relativity theory, the law of motion $x(t) \in \mathbb{R}^3$ of a particle of mass m and charge q obeys to the Lorentz force equation

$$\frac{d}{dt}\left(\frac{m\dot{x}}{\sqrt{1-|\dot{x}|^2/c^2}}\right) = q[E(t,x) + \dot{x} \times B(t,x)]$$

where c is the speed of light while $E(t,x)$ and $B(t,x)$ are the electric and magnetic fields, respectively, at time t and position x. That equation is formally the Euler-Lagrange equation of the action functional

$$I(x) = \int_0^T mc^2\left(1 - \sqrt{1-|\dot{x}(t)|^2/c^2}\right)dt + \int_0^T q\left[-V(t,x(t)) + A(t,x(t)) \cdot \dot{x}(t)\right]dt$$

where V is the electric potential and A is the magnetic vector potential, in such a way that

$$E(t,x) = -\nabla_x V(t,x) - \partial_t A(t,x) \quad \text{and} \quad B(t,x) = \nabla_x \times A(t,x).$$

For instance, when E and B are generated by charged particles that move along T-periodic orbits in space, the potentials V and A are called Liénard-Wiechert potentials and are T-periodic w.r.t. time and singular w.r.t. space. Moreover, the first term of the functional I is not globally differentiable but is convex and satisfies Szulkin's structural assumptions [5].

We use non-smooth critical point theory to prove the existence of infinitely many T-periodic solutions of the differential equation under physically reasonable conditions on V and A: for instance the cases of Liénard-Wiechert potentials [3], of the N-center problem [3] and of the perturbed Kepler problem [2] are included.

We use an adaptation of the non-smooth min-max principle [4,Theorem 3.1] to singular potentials together with arguments developed in [1] and Lusternik-Schnirelman category theory.

References

[1] D. Arcoya, C. Bereanu, P.J. Torres, *Lusternik-Schnirelman theory for the action integral of the Lorentz force equation*, Calc. Var. Partial Differ. Equ., 59 (2020), No. 50.

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[3] A. Boscaggin, W. Dambrosio, D. Papini *Infinitely many periodic solutions to a Lorentz force equation with singular electromagnetic potential*, J. Differential Equations, 383 (2024), 190–213.

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14:30–14:55 Multiple solutions for Kirchhoff double phase problems
Leszek Gasinski (University of the National Education Commission, Cracow, POLAND)

Abstract. Presentation concerns the nonlocal Dirichlet equation of double phase type with the right hand side function of superlinear and subcritical growth. The existence of two constant sign solutions (one positive, the other one negative) and of a sign-changing solution which has exactly two nodal domains and which turns out to be the least energy sign-changing solution is presented. The proof is based on variational tools in combination with the quantitative deformation lemma and the Poincare-Miranda existence theorem.

15:00–15:25 Nearly-circular periodic solutions of a perturbed relativistic Kepler problem: the fixed-period and the fixed-energy problems
Guglielmo Feltrin (University of Udine, ITALY)

Abstract. The motion of a relativistic particle in a Kepler potential can be described by the equation

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{m\dot{x}}{\sqrt{1-|\dot{x}|^2/c^2}}\right) = -\alpha\,\frac{x}{|x|^3}, \quad x \in \mathbb{R}^2 \setminus \{0\},$$

where $m > 0$ is the mass of the particle, c is the speed of light, and $\alpha > 0$ is a constant. Firstly, we illustrate the Hamiltonian formulation of the problem and we focus our attention on the description of the periodic solutions. Secondly, we deal with the perturbed equation

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{m\dot{x}}{\sqrt{1-|\dot{x}|^2/c^2}}\right) = -\alpha\,\frac{x}{|x|^3} + \varepsilon\,\nabla_x U(t,x), \quad x \in \mathbb{R}^2 \setminus \{0\},$$

where $\varepsilon \in \mathbb{R}$. We provide existence of periodic solutions bifurcating, for ε small enough, from the set of circular solutions of the unperturbed system ($\varepsilon = 0$). Both the case of the fixed-period problem (assuming that U is T-periodic in time) and the case of the fixed-energy problem (assuming that U is independent of time) are considered. The same results are also valid in the three-dimensional space. The talk is based on papers written in collaboration with Alberto Boscaggin, Walter Dambrosio, and Duccio Papini under the auspices of the Gruppo Nazionale per l'Analisi Matematica, la Probabilità e le loro Applicazioni (GNAMPA) of the Istituto Nazionale di Alta Matematica (INdAM).

References

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15:30–15:55 Bifurcation of periodic solutions for problems of relativistic mechanics
Alberto Boscaggin (University of Turin, ITALY)

Abstract. We discuss bifurcation of periodic solutions for some problems of relativistic mechanics, such as the relativistic Kepler problem in special relativity and the Schwarzschild problem of general relativity. The main tool is the classical Poincaré-Birkhoff fixed point theorem, together with its higher-dimensional generalizations.

References

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[2] A. Boscaggin, W. Dambrosio and G. Feltrin, *Bifurcation of closed orbits of Hamiltonian systems with application to geodesics of the Schwarzschild metric*, submitted, arXiv:2310.02615

16:00–16:25 Rich dynamics in a model related to suspension bridges

Maurizio Garrione (Polytechnic University of Milan, ITALY)

Abstract. We consider a degenerate plate model for suspension bridge-type structures, encompassing a coupled dynamics involving longitudinal and torsional oscillations. In presence of a time-periodic external force triggering a single longitudinal mode and satisfying suitable assumptions, we show the occurrence of rich and complex dynamics for the corresponding longitudinal oscillation. The goal is achieved by applying a rigorous analytical approach based on the theory of linked twist maps.

References

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17:00–17.25 Birkhoff-Kellogg type results with applications

Gennaro Infante (Calabria University, ITALY)

Abstract. We present some classical and recent results of Birkhoff-Kellogg type in cones. We illustrate their applicability in the context of ordinary, functional and partial differential equations subject to local, nonlocal and functional boundary conditions.
This study was partly funded by: Research project of MIUR (Italian Ministry of Education, University and Research) Prin 2022 “Nonlinear differential problems with applications to real phenomena” (Grant Number: 2022ZXZTN2).

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17:30–17:55 Existence of solutions for some boundary value problems associated with singular equations with Φ-Laplacian operators

Alessandro Calamai (Polytechnic University of Marche, ITALY)

Abstract. We will review some recent results about the existence of solutions for different boundary value problems (BVPs) associated with second- or third-order singular equations involving the Φ-Laplacian operators.

In the recent paper [1] we investigate the following BVP

$$(\mathrm{P})\begin{cases}(\Phi(k(t)u''(t)))' = f(t,u(t),u'(t),u''(t)), \text{ a.e. on } [0,T],\\ \\ u(0)=a,\ u'(0)=b,\ u'(T)=c\end{cases}$$

where f is a Carathédory function, $\Phi : \mathbb{R} \to \mathbb{R}$ is a strictly increasing homeomorphism, and the nonnegative function k may vanish on a set of measure zero.

Under mild assumptions, including a weak form of a Nagumo–Winter growth condition, we prove the existence of solutions of problem (P) in the Sobolev space $W^{2,p}([0,T])$. Our approach is based on fixed point techniques suitably combined to the method of upper and lower solutions.

References

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July 24, 2024

11:30–11:55 Multiplicity of periodic solutions for nearly resonant Hamiltonian systems

Rodica Toader (University of Trieste, ITALY)

Abstract.
Let us first consider a planar Hamiltonian system

$$J\dot{u} = \nabla_u \mathcal{H}(t,u)\,, \tag{1}$$

having a twist dynamics. More precisely, we assume $\mathcal{H}(t,u)$ to be continuous, T-periodic in t, continuously differentiable in $u=(q,p)$, and 2π-periodic with respect to q, and that there exist $a<b$ be such that all solutions $u=(q,p)$ of the system, starting with $p(0)\in[a,b]$, are defined on $[0,T]$ and are such that

$$p(0)=a \implies q(T)-q(0)<0\,, \qquad p(0)=b \implies q(T)-q(0)>0\,.$$

Then, the celebrated Poincaré–Birkhoff Theorem ensures the existence of at least two geometrically distinct T-periodic solutions $u=(q,p)$, with $p(0)\in\,]a,b[\,$. This multiplicity result was proved in [3] and extended to systems in $\mathbb{R}^{2M}$, providing in that case the existence of $M+1$ periodic solutions.

Let us now consider another Hamiltonian system

$$J\dot{z} = \nabla_z H(t,z)\,, \tag{2}$$

which has a completely different dynamics. We assume the function H to be continuous, T-periodic in t, twice continuously differentiable in z, with

$$A \le H''_z(t,z) \le B\,, \quad \text{for every } (t,z)\in[0,T]\times\mathbb{R}^{2N},$$

for some symmetric matrices A,B which satisfy the nonresonance condition

$$\bigcup_{\lambda\in[0,1]} \sigma((1-\lambda)JA+\lambda JB)\cap\frac{2\pi}{T}i\mathbb{Z}=\emptyset\,.$$

Then it was proved in [1] that system (2) has a unique T-periodic solution.

In this talk I will present the results obtained in [2] for a Hamiltonian system whose Hamiltonian function has a twisting part and a nonresonant part. We consider a system of the form

$$\begin{cases} J\dot{u} = \nabla_u\mathcal{H}(t,u) + \nabla_u P(t,u,z)\\ J\dot{z} = \nabla_z H(t,z) + \nabla_z P(t,u,z)\,,\end{cases}$$

where $\nabla_u P(t,u,z)$ and $\nabla_z P(t,u,z)$ are bounded, and prove some multiplicity results for the associated T-periodic problem. We begin by assuming a nonresonance condition on the function $H(t,z)$ as the one described above, and then analyse a possible approach to resonance, together with some kind of Landesman–Lazer condition. We propose a new version of this condition, and we also treat the so-called *double resonance* situation. The results are obtained in collaboration with A. Fonda and A. Sfecci (University of Trieste).

References

[1] A. Fonda, J. Mawhin, Iterative and variational methods for the solvability of some semilinear equations in Hilbert spaces, Journal of Differential Equations 98 (1992), 355–375.

[2] A. Fonda, A. Sfecci, R. Toader, Multiplicity of periodic solutions for nearly resonant Hamiltonian systems, preprint 2024.

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12:00–12:25 On semilinear elliptic systems with superlinear boundary conditions

Maya Chhetri (University of North Carolina at Greensboro, USA)

Abstract. We investigate a system of elliptic equations characterized by superlinear and subcritical boundary conditions with a bifurcation parameter. Furthermore, under additional conditions on the nonlinearities near zero, we discuss the existence of a global, connected branch of positive solutions bifurcating from the line of trivial solutions, with a unique bifurcation point from infinity when the bifurcation parameter is zero. We employ bifurcation theory, degree theory, and sub- and super-solution method to obtain our results.

12.30–12.55 On a class of Initial–Boundary Value Problems for renewal equations

Elena Rossi (University of Modena and Reggio Emilia)

Abstract. In biological or epidemiological models, different species are typically described through their densities $u_1, u_2, \ldots, u_k$ and, in general, each u_h depends on time $t \in \mathbb{R}_+$, on age $a \in \mathbb{R}_+$, on a spatial coordinate in $\mathbb{R}^2$ or $\mathbb{R}^3$ and possibly also on some other structural variables. In order to provide a unified treatment of these models, we propose an Initial–Boundary Value Problem (IBVP) for a system of balance laws, considering quite general interaction terms, possibly non linear and/or non local. In particular, we remark that in the present framework diffusion is lacking, thus any movement or evolution described by the IBVP propagates with a finite speed.

We prove well posedness of the solutions, thus local existence, uniqueness and continuous dependence on the initial datum, then we provide conditions ensuring global in time existence. Moreover, we show the stability of the solutions with respect to the functions and the parameters appearing in the IBVP.

References

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14:30–14:55 Existence of nonnegative mild solutions of stochastic evolution inclusions with application to climate change models

Irene Benedetti (University of Perugia, ITALY)

Abstract. We provide sufficient conditions for the existence of mild solutions to stochastic differential inclusions in infinite dimensional Hilbert spaces driven by a cylindrical Wiener process. Inspired by [5] we develop an approximation procedure based on the weak topology. By applying this method, we can accomplish the dual objective of proving the existence of a solution over the entire real half-line while relaxing the commonly assumed hypotheses of Lipschitz continuity and compactness found in the literature on the subject. Moreover, the problem of the non negativity of the solution is addressed. Namely, assuming an additional sign condition that involves both

the deterministic and stochastic nonlinear terms as in [4], we can ensure the non-negativity of the solution starting from a non-negative initial datum. These differential inclusions find applications in climate model studies. Indeed to encompass the broadest possible range in climate change models, it is essential to consider non-deterministic differential equations. For example, cyclones can be treated as a rapidly varying component and represented as a white-noise process. In this context, nonlinear stochastic parabolic differential equations were initially proposed by North and Cahalan [1] to explore non-deterministic variability in energy balance climate frameworks. Climate models based on differential inclusions are characterized by a deterministic part, a stochastic part, or both, described by set-valued maps. Considering set-valued maps allows us to include in the model cases where the exact value of the empirical evidence cannot be calculated, but is known with a certain degree of uncertainty, see e.g. [2, 3].The talk is based on two joint papers, one with Alessandra Cretarola and Lucia Angelini and the second one with Alessandra Cretarola and Lorenzo Guida, University of Perugia.

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15:00–15:25 A non-linear eigenvalue problem for the critical p-Laplace equation in the ball

Matteo Franca (University of Bologna, ITALY)

Abstract. In this talk we show that the number of radial positive solutions of the following critical problem

$$\begin{cases} \Delta_p u(x) + \lambda K(|x|)\, u(x)\, |u(x)|^{q-2} = 0\,, & \\ u(x) > 0 & |x| < 1 \\ u(x) = 0 & |x| = 1. \end{cases}$$

undergoes a bifurcation phenomenon; here $q = \frac{np}{n-p}$, $p > 1$ and $x \in \mathbb{R}^n$. When $K(r)$ is steep enough at $r = 0$ the problem admits (at least) one solution for any $\lambda > 0$, while if $K(r)$ is too flat at $r = 0$ then it admits no solutions for λ small and two solutions for λ large.

The existence of the second solution is new even in the classical Laplace case. The proofs use Fowler transformation and dynamical systems tools: in fact if $1 < p \leq 2$ we can rely on standard tools of invariant manifold theory, while if $p > 2$ we need to develop some ad-hoc box argument to overcome some lack of regularities issues.

15:30–15:55 Periodic solutions of differential equations with oscillating constraints

Marco Spadini (University of Florence, ITALY)

Abstract. This talk is devoted to the investigation of the topological structure of the set of harmonic solutions to a class of implicit ordinary differential equations subjected to periodic perturbations.

In order to that, we study preliminarly the harmonic solutions of periodically perturbed differential equations *in normal form* subjected to a possibly time-dependent constraint; in particular, a so-called *Differential-Algebraic equation* of an appropriate type. Using a combination of techniques from the theory of Topological Degree and Differential-Algebraic Equations, we obtain, for

these systems, a degree theoretic condition ensuring a “global branching” result for the nontrivial periodic solutions.

The latter result actually turns out to be more general than the one sought by the former, original, problem. In fact the investigation of the set of harmonic solutions of our implicit differential equations boils down to the the same problem for Differential-Algebraic equations of the above mentioned type, by the means of the introduction of an extra variable.

16:00–16.25 Multiple solutions for a nonlocal Dirichlet problem driven by the p-Laplacian

Roberto Livrea (University of Palermo, ITALY)

Abstract. The aim of the talk is to study the following problem

$$\begin{cases} -a\left(\int_\Omega u^q dx\right)\Delta_p u = \beta(x)f(t) & \text{in } \Omega, \\ u > 0 & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases} \tag{3}$$

where Ω is a bounded smooth domain in $\mathbb{R}^N$, N is a positive integer, $q \geqslant 1$, $1 < p < +\infty$, while $a \in C([0,\infty))$ is a changing sign function, $\beta \in L^\infty(\Omega)$ and f is suitable continuous function.

As pointed out for example in [2], nonlocal problems having a structure as (3) can be considered for describing biological models of the population diffusion.

The case when the reaction term does not depend on the vectorial variable has been studied first in [3, 4], when $p = 2$, and then, in [1], in the more general case when $1 < p < \infty$.

Here, following [3] and [1], we will assume that a admits a finite number of “positive bumps”; moreover, f is a positive and continuous function in a right neighbourhood of zero, satisfying a monotonicity condition.

The multiplicity of solutions will be achieved by using an auxiliary problem and combining truncation techniques, variational methods and a well known formula, due to Diaz-Saa, with the fixed point theory.

References

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