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Set theory and applications
Special Session B22

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Recent advances in set theory have contributed to the foundations of mathematics, and have created new methods to analyze infinite mathematical objects. The two main topics of the session are:

(1) Infinite combinatorics;
(2) Descriptive set theory.

Schedule and Abstracts

July 25, 2024

11:30–12:30 Topological groups without unitary representations and submeasures
Slawomir Solecki (Cornell University, USA)

Abstract. We give new examples of topological groups that do not have non-trivial continuous unitary representations, the so-called exotic groups. We prove that all groups of the form $L^0(\phi, G)$ where ϕ is a pathological submeasure and G is a topological group, are exotic. This result extends, with a different proof, a theorem of Herer and Christensen on exoticness of $L^0(\phi, \mathbb{R})$ for ϕ pathological. In our arguments, we introduce the escape property, a geometric condition on a topological group, inspired by the solution to Hilbert's fifth problem and satisfied by all locally compact groups, all non-archimedean groups, and all Banach–Lie groups. This is joint work with F. Martin Schneider.

12:30–13:00 On the κ^+-Borel hierarchy of subsets of the generalized Baire space equivalence of graphs
Claudio Agostini (TU Wien, Austria)

Abstract. Descriptive set theory (DST) focuses on the study of definable subsets, particularly Borel subsets, of Polish spaces, which are separable, completely metrizable spaces. Polish spaces are ubiquitous in mathematics, and studying their Borel subsets has deepened our understanding of various phenomena in different fields.

Generalized Descriptive Set Theory (GDST) extends classical descriptive set theory by replacing countable settings with uncountable ones. A significant part of the literature in GDST focuses on the generalized Baire space ${}^\kappa\kappa$ and its κ^+-Borel subsets. Recently, a class of Polish-like spaces was introduced to extend the role of Polish spaces to uncountable settings.

14:30–15:30 Consistency Strength of Generalized Perfect Set Property
Vincenzo Dimonte (University of Udine, Italy)

Abstract. The study of non-separable completely metrizable spaces, like for example λ^ω when λ has cofinality ω, shows that they have many similarities with their separable counterparts, the Polish spaces, and in fact it is possible to build a sensible descriptive set theory on them, and to define an analogue of the Perfect Set Property. It is therefore of interest to understand which sets enjoy such property. As in the separable case, the answer depends on large cardinals. We will show some upper bound consistency result, using Prikry-like forcings, and some lower bound consistency result, that uses what can be considered the first steps in generalized inner model descriptive set theory.

15:30–16:30 Commutativity of cofinal types
Tom Benhamou (Rutgers University, USA)

Abstract. The Tukey order finds its origins in the concept of Moore-Smith convergence in topology, and is especially important when restricted to ultrafilters with reverse inclusion. The Tukey order on ultrafilters over ω was studied intensively by many, but still contains several fundamental unresolved problems. I will present a recently discovered connection to a parallel study at the realm of measurable cardinals, and explain how different the Tukey order is at that levels when compared to the situation on ω. In the second part of the talk, I will demonstrate how ideas and intuition from ultrafilters over measurable cardinals led to new results at the level of ω and present an essentially new method of constructing Tukey-top ultrafilters using Diamond-like Principles on ω.

17:00–18:00 The Borel complexity of proper homotopy equivalence of graphs
Jenna Zomback (University of Maryland, USA)

Abstract. In this talk, we will discuss the relation of proper homotopy equivalence (PHE) of infinite, finite-valence graphs (where we impose a natural Polish topology on the space). We study the Borel complexity of PHE, as well as the homeomorphism relation of infinite type surfaces. This is joint work with Hannah Hoganson.

18:00–18:30 Topological applications of sparse families
Santi Spadaro (University of Palermo, Italy)

Abstract. Let κ be a cardinal. A family of sets $\mathcal{F}$ is called *κ-sparse* if every κ-sized subfamily of $\mathcal{F}$ has an uncountable union. An $\aleph_1$-sparse family in $[\aleph_n]^\omega$, which is in addition cofinal with respect to containment, exists in ZFC for every $n < \omega$. However, the existence of a cofinal $\aleph_1$-sparse family in $([\aleph_\omega]^\omega, \subseteq)$ is independent of ZFC. Indeed, these families exist in the constructible universe, but they are killed by the Chang Conjecture variant $(\aleph_{\omega+1}, \aleph_\omega) \twoheadrightarrow (\aleph_1, \aleph_0)$. On the other hand, an $\aleph_4$-sparse cofinal family in $([\aleph_\omega]^\omega, \subseteq)$ exists in ZFC.

July 26, 2024

11:30–12:30 Boolean valued semantics for infinitary logics
Matteo Viale (University of Turin, Italy)

Abstract. It is well known that the completeness theorem for $L_{\omega_1\omega}$ fails with respect to Tarski semantics. Mansfield showed that it holds for $L_{\infty\infty}$ if one replaces Tarski semantics with Boolean valued semantics. We use forcing to improve his result in order to obtain a stronger form of Boolean completeness (but only for $L_{\infty\omega}$). Leveraging on our completeness result, we establish the Craig interpolation property and a strong version of the omitting types theorem for $L_{\infty\omega}$ with respect to Boolean valued semantics.

Time permitting we also relate this work to Asperó and Schindler's proof of the forcibility of Woodin's axiom $(*)$ over models of ZFC and large cardinals.

12:30–13:00 The SLO principle for Borel subsets of the generalized Cantor space
Beatrice Pitton (University of Lausanne, Switzerland, and University of Turin, Italy)

Abstract. The Wadge hierarchy establishes a hierarchy of complexity through the comparison of sets via continuous reductions. The Semi-Linear Ordering principle (SLO) asserts that, for any two subsets A and B of a space X, either A can be continuously reduced to B or the complement of B can be continuously reduced to A. While classical descriptive set theory primarily focuses on studying subsets of the space of all countable binary sequences, generalized descriptive set theory aims at developing a higher analogue in which ω is replaced with an uncountable cardinal κ satisfying the condition $2^{<\kappa} = \kappa$. Motivated by understanding the Wadge structure for (various classes of) generalized Borel sets, in this talk we will first discuss the consistency of the failure of the SLO principle for $\mathbf{\Sigma}^0_2(\kappa^+)$ sets and then, starting from the bottom of the Wadge hierarchy, we will analyse the validity of the semi-linear ordering principle as we ascend through the difference hierarchy.

14:30–15:30 Groups without the Generic Point Property
Andrea Vaccaro (University of Muenster, Germany)

Abstract. We show that various groups of homeomorphisms of a strongly locally homogeneous Peano continuum which is not the circle, the sphere, or the real projective plane do not have the generic point property, and it particular have non-metrizable universal minimal flow. This class of spaces includes all closed manifolds of dimension at least three, for which the result was already known, all closed surfaces which are not the sphere or the real projective plane, as well as homogeneous Peano continua such as the Menger curve. This is a joint work with Gianluca Basso and Alessandro Codenotti.

15:30–16:30 Forcing applied to Ramsey theory of Fraïssé structures
Natasha Dobrinen (Notre Dame University, USA)

Abstract. Forcing has been a central tool for unlocking the area of Ramsey theory on Fraïssé structures which have forbidden substructures. The first instance of this appeared in the speaker's work [5] for colorings of finite triangle-free graphs inside the Henson graph. These ideas were quickly extended in [6] and [11]. Subsequent work has expanded applications of forcing to prove Ramsey theorems for a large collection of Fraïssé structures, as exposited in [7]. We will survey the current state of the art in this arena, pointing out where new work has been able to achieve some of the same Ramsey theorems by purely combinatorial methods, starting with [10], as well as instances where the forcing point of view is still optimal.

References

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[10] N. Dobrinen and A. Zucker, *Infinite-dimensional Ramsey theory for binary free amalgamation classes*, arXiv:2303.04246.

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17:00–18:00 Wadge hierarchy on ordinal numbers
Riccardo Camerlo (University of Genoa, Italy)

Abstract. As a contribution to the study of the Wadge hierarchy on general topological spaces, some results and open problems are presented concerning ordinals endowed with their order topology.

18:00–18:30 Developments in Namba Forcing

Maxwell Levine (University of Freiburg, Germany)

Abstract. One way to study the properties of the infinite cardinals is to examine the extent to which they can be changed by forcing. In 1969 and 1970, Bukovský and Namba independently showed that $\aleph_2$ can be forced to be an ordinal of cofinality $\aleph_0$ without collapsing $\aleph_1$. The forcings they used and their variants are now known as Namba forcing. This talk will follow the recent result of the speaker that it is consistent modulo the existence of an inaccessible cardinal that classical Namba forcing for $\aleph_2$ has the weak ω_1-approximation property, answering a question of Cox and Krueger. There are a number of variations of this argument that have implications for weak guessing models, the study of $\aleph_{\omega+1}$, and the minimality of collapsing extensions.

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