AMS-UMI International
Joint Meeting 2024
Palermo, July 23–26, 2024

Representation Theory: Lie Theory and Geometry
Special Session A17

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The session will focus on recent progress in exploring the interconnections between Lie and representation theory. Experts from the areas of geometric, categorical and combinatorial representation theory will be invited with the goal of promoting new connections and collaborations between these areas. Invitations have also been extended to junior mathematicians (postdocs and graduate students) to present their work.

Some of the topics covered in the session will involve tensor triangulated geometry, derived categories and their equivalences, categorification, representations of algebraic groups, quantum groups and Hecke algebras, cluster algebras, and quiver representations. The session will be held on July 23-24.

For more information visit `www.unipa.it`.

Schedule and Abstracts

July 23, 2024

11:00–11:45 Lefschetz operators and representations

Peter Fiebig (FAU Erlangen–Nürnberg, GERMANY)

Abstract. We study vector spaces (over an arbitrary field) that are graded by the weight lattice of a root system, and are endowed with linear operators in each simple root direction. We then explore under which conditions these operators yield representations of an algebraic group or a quantum group associated to the root system. We then try to relax these assumptions and explore the resulting "new representation theories". These might eventually yield candidates for the *higher generation representation theories* that are suggested by the work of Lusztig and Lusztig-Williamson.

12:00–12:45 Flipclasses and combinatorial invariance of Kazhdan-Lusztig polynomials

Francesco Esposito (Università degli Studi di Padova, ITALY)

Abstract. I relate on a new approach to the Combinatorial Invariance Conjecture of Kazhdan–Lusztig polynomials for the symmetric group. We introduce some new combinatorial invariants of intervals in the symmetric group whose analysis leads us to a recipe to compute the coefficients of q^h of the Kazhdan–Lusztig $\widetilde{R}$-polynomials, for $h \leq 6$. This recipe depends only on the isomorphism class (as a poset) of the interval indexing the polynomial and thus provides new evidence for the Combinatorial Invariance Conjecture. If time permits, I will discuss the case of Weyl groups.

14:30–15:15 Linear degenerations of Schubert varieties via quiver Grassmannians

Giulia Iezzi (RWTH Aachen University/Università di Roma Tor Vergata, GERMANY/ITALY)

Abstract. Quiver Grassmannians are projective varieties parametrising subrepresentations of quiver representations. Their geometry is an interesting object of study, due to the fact that many geometric properties can be studied via the representation theory of quivers. For instance, this method was used to study linear degenerations of flag varieties, obtaining characterizations of flatness, irreducibility and normality via rank tuples.

We provide a construction for realising smooth Schubert varieties as quiver Grassmannians and desingularizing non-smooth Schubert varieties. We then exploit this construction to define linear degenerations of Schubert varieties, giving a combinatorial description of the correspondance between their isomorphism classes and the B-orbits of certain quiver representations.

15:30–16:15 Centers and centralizers in (double) affine Hecke algebras

Jonathan Gruber (University of York, UK)

Abstract. The affine Hecke algebra and its center are important objects of study in combinatorial, geometric and categorical representation theory. In this talk, I will discuss a new commutative subalgebra of the affine Hecke algebra of type A, which arises from a centralizer construction in the double affine Hecke algebra. This subalgebra contains the center, and it admits a canonical basis akin to the Kazhdan–Lusztig basis of the affine Hecke algebra. I will explain how the canonical basis can be used as a tool to compute composition multiplicities in Gaitsgory's central sheaves on affine flag manifolds.

17:00–17:45 Wall and chamber structure for finite dimensional algebras and perverse sheaves

Martina Lanini (Università di Roma Tor Vergata, ITALY)

Abstract. The wall and chamber structure of an algebra is a certain polyhedral complex which is in general rich in combinatorics and applications, being, for example, related to tau-tilting theory. Motivated by the desired of understanding the space of Bridgeland stability conditions for the bounded derived category of constructible sheaves on flag varieties, we focus on the case of projective spaces and connect the problem to the study of the wall and chamber structure of a certain finite dimensional algebra.

July 24, 2024

11:30–12:15 Restricting rational modules to Frobenius kernels

Daniel K. Nakano (University of Georgia, USA)

Abstract. Let G be a connected reductive group over an algebraically closed field of characteristic $p > 0$. Given an indecomposable G-module M, one can ask when it remains indecomposable upon restriction to the Frobenius kernel G_r, and when its G_r-socle is simple (the latter being a strictly stronger condition than the former). In this talk, we investigate these questions for G having an irreducible root system of type A. Using Schur functors and inverse Schur functors as our primary tools, we develop new methods of attacking these problems, and in the process obtain new results about classes of Weyl modules, induced modules, and tilting modules that remain indecomposable over G_r.

This talk represents joint work with Christopher Bendel, Cornelius Pillen and Paul Sobaje.

14:30–15:15 The Lie superalgebra generated by transpositions

Jonathan Kujawa (Oregon State University, USA)

Abstract. The symmetric group has been the object of study since forever. Nevertheless, there are still new things to say. Using the commutator, you can view the group algebra of the symmetric

group as a Lie algebra. In 2003, Marin described this Lie algebra and the subalgebra generated by the transpositions. Since the symmetric group naturally splits into even and odd permutations, you can also ask about the graded version of the commutator. This makes the group algebra into a Lie superalgebra. In 2023, Chris Drupieski and I obtained the super analogue of Marin's results.

15:30–16:15 Seshadri stratifications; An application to matrix Schubert variety

Martina Costa Cesari (University of Bologna, ITALY)

Abstract. Recently Seshadri stratifications on an embedded projective variety have been introduced by R. Chirivì, X. Fang and P. Littelmann. A Seshadri stratification of an embedded projective variety X is the datum of a suitable collection of subvarieties X_τ that are smooth in codimension one, and a collection of suitable homogeneous functions f_τ on X indexed by the same finite set. With such a structure, one can construct a Newton-Okounkov simplicial complex and a flat degeneration of the projective variety into a union of toric varieties. Moreover the theory of Seshadri stratifications provides a geometric setup for a standard monomial theory. In the talk, I will introduce the theory of Seshadri stratification and I will give a Seshadri stratification for matrix Schubert varieties, namely varieties of matrices defined by conditions on the rank of some their submatrices.

17:00–17:45 Deformation of bases for cluster algebras

Salvatore Stella (Università degli Studi dell'Aquila, ITALY)

Abstract. Since their introduction a major problem in the theory of cluster algebras has been to construct bases suitable for them. Indeed, having a basis with some prescribed properties has deep structural consequences on a cluster algebra. Several different constructions were proposed over the years arising from a variety of perspectives (e.g. Lie theory, mirror symmetry, quiver representations, Teichmüller theory, and combinatorics). All these different constructions share a common feature: they consist of pointed elements.

Recently Qin was able to parametrize all basses made out of pointed elements for a large class of cluster algebras. In particular, he proved that all of them are related by upper triangular linear maps. Unfortunately his description of the deformation spaces these transformations span is not explicit limiting the scope of his result in practical applications.

In this talk, after introducing the required notions, we will explain how to construct explicitly these deformation spaces for cluster algebras whose growth is at most linear. Time permitting we will relate this phenomenon to the geometry of certain subvarieties of Kac-Moody groups.

17:55–18:40 Quantum supergroups at root of unity

Milen Yakimov (Northeastern University, USA)

Abstract. In a fundamental sequence of works from the 1990s, De Concini, Kac and Procesi constructed a Poisson geometric framework for the study of the irreducible representations of big quantum groups at roots of unity. We will describe an extension of this framework to all contragredient quantum supergroups at root of unity. The approach of De Concini, Kac, and Procesi relied on a reduction to rank two cases, which is not possible in the super case since there are 13 kinds of additional Serre relations on up to 4 generators. We use a new approach that relies on Nichols algebras and perfect pairings between restricted and non-restricted integral forms. In particular, this gives new proofs of the classical results, independent on Serre relations. The methods apply to a larger, axiomatically defined class of algebras (consisting of the Drinfeld doubles of diagonal pre-Nichols algebras that have 1-parameter deformations). This class includes the quantizations in characteristic 0 of the 34-dimensional Kac-Weisfeiler Lie algebra in characteristic 2 and the 10-dimensional Brown Lie algebra in characteristic 3.

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