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Analysis and Control of Evolutionary Partial Differential Equations
Special Session B11

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This Special Session will feature speakers who have wide-ranging research expertise in the mathematical analysis of dynamical systems, which encompasses a very broad area of research. In this regard, our special session will focus on the topics that fall under the heading of qualitative properties of evolutionary PDEs and their applications to linear/nonlinear equations of parabolic and/or hyperbolic type, particularly those equations coming from physics and mechanics. These PDE systems also arise in (i) fluid dynamics for which the Navier-Stokes (NS) and Euler equations are often invoked as modeling equations, (ii) viscoelasticity, population dynamics, or heat flow in real conductors which have time delays or memory effects, i.e., the dynamics depend on previous states, or they are influenced by the past history of the variables. Then, delay differential equations and equations with memory, where the past history is in play through a convolution integral, will be a topic of interest. In particular, our session will address the wellposedness, long-time behavior (in the sense of global attractors and stability), control, optimization, and regularity properties of the above-mentioned dynamical systems.

For more information visit `www.unipa.it`.

Schedule and Abstracts

July 25, 2024

11:30–11:50 Weak solutions to the heat conducting compressible self-gravitating flows in time-dependent domains

Sarka Necasova (Institute of Mathematics, Academy of Sciences, CZECH REPUBLIC)

Abstract. We consider the heat-conducting compressible self-gravitating fluids in time-dependent domains, which typically describe the motion of viscous gaseous stars. The flow is governed by the 3-D Navier-Stokes-Fourier-Poisson equations where the velocity is supposed to fulfil the full-slip boundary condition and the temperature on the boundary is given by a non-homogeneous Dirichlet condition. We establish the global-in-time weak solution to the system. Our approach is based on the penalization of the boundary behavior, viscosity, and the pressure in the weak formulation. Moreover, to accommodate the non-homogeneous boundary heat flux, the concept of *ballistic energy* is utilized in this work.

12:00–12:20 Nonlocal Cahn-Hilliard fluids

Maurizio Grasselli (Politecnico di Milano, ITALY)

Abstract. The Cahn-Hilliard equation is often used to model the temporospatial evolution of multiphase fluid systems including droplets, bubbles, aerosols, and liquid films. A well-known model consists of the Navier-Stokes system which is nonlinearly coupled with an advective Cahn-Hilliard equation through a capillary force, aka the Korteweg force. I will present some recent results on models where the Cahn-Hilliard equation is not the classical fourth-order equation,

but it is a second-order spatially nonlocal equation. These results are mainly concerned with the existence of strong solutions, uniqueness, and convergence to a single stationary state. Some related open issues will also be mentioned.

12:30–12:50 Exponential stability for a coupled thermoelastic plate–membrane system

Robert Denk (University of Konstanz, GERMANY)

Abstract. Let $\Omega \subset \mathbb{R}^n$ be a bounded sufficiently smooth domain which consists of two sub-domains $\Omega = \Omega_1 \cup \Omega_2 \cup I$, where Ω_2 is the inner part, i.e., $\overline{\Omega_2} \subset \Omega_1$, and $I := \partial\Omega_2$ is the interface between the two parts. We consider a coupled plate-membrane system, where we have a thermoelastic plate equation in the outer sub-domain Ω_1,

$$\begin{aligned} u_{tt} + \Delta^2 u + \Delta\theta = 0 \quad &\text{in} \quad \Omega_1 \times (0,\infty), \\ \theta_t - \Delta u_t - \Delta\theta = 0 \quad &\text{in} \quad \Omega_1 \times (0,\infty) \end{aligned} \tag{1}$$

and a wave equation in the inner sub-domain Ω_2,

$$v_{tt} - \Delta v + m v_t = 0 \quad \text{in} \quad \Omega_2 \times (0,\infty).$$

Here, $m \geq 0$ is a constant parameter describing the damping. On the outer boundary $\partial\Omega$ and on the interface I, we impose appropriate boundary and transmission conditions, respectively.

In an appropriate Hilbert space setting, this problem is well-posed, and the corresponding first-order system generates a C_0-semigroup $(S(t))_{t\geq 0}$ of contractions. Concerning the asymptotic behaviour, we have the following result.

Theorem 1. *The semigroup $(S(t))_{t\geq 0}$ is exponentially stable if $m > 0$ and not exponentially stable if $m = 0$.*

Here, the more difficult case is $m = 0$. For the proof in this case, we use a compactness argument which then needs an estimate for the solution of (1) with boundary data living only in $L^2(I)$. As such rough boundary data are not covered by classical results, we need additional concepts and tools for boundary value problems with rough boundary data.

This gives the connection to a general question which has applications also in other fields of PDE, e.g., for SPDEs with boundary noise: Can we solve boundary value problems of the form

$$\begin{aligned} (\lambda - A)u = f \quad &\text{in} \quad \Omega, \\ Bu = g \quad &\text{on} \quad \partial\Omega \end{aligned} \tag{2}$$

with $f \in L^2(\Omega)$ and $g \in L^2(\partial\Omega)$ or with g even belonging to some negative Sobolev space?

Boundary value problems with rough boundary data were considered in [2], even in the L^p-setting. For this, we have to generalize the notion of the boundary trace operator $\gamma_0\colon u \mapsto u|_{\partial\Omega}$. It is known that

$$\gamma_0\colon H_p^s(\Omega) \to B_{pp}^{s-1/p}(\partial\Omega)$$

is continuous if and only if $s > 1/p$. However, one can generalize the trace operator yielding negative Sobolev and Besov spaces on the boundary and develop a theory which solves (2) for such rough boundary data.

This talk is based on joint works with B. Barraza Martínez, J. González Ospino, J. Hernández Monzón (all Barranquilla, Colombia), J. Seiler (Torino, Italy), D. Ploß, and S. Rau (both Konstanz, Germany).

14:30–14:50 On stationary statistical solutions to the Navier–Stokes–Fourier system

Eduard Feireisl (Institute of Mathematics, Czech Academy of Sciences, CZECH REPUBLIC)

Abstract. We show the existence of stationary statistical solution for the Navier-Stokes-Fourier system describing the motion of a general compressible, viscous and heat conducting fluid. The

proof is based on the existence of a bounded absorbing set and asymptotic compactness of global-in-time solutions in suitable topologies.

15:00–15:20 Stabilization of a transmission system controlling a beam equation by heat conduction with memory effect under (Coleman/Pipkin)–Gurtin Thermal Law

Sarah Ismail (University of Bari Aldo Moro, ITALY)

Abstract. Beam-heat coupling constitutes a conventionnal topic in civil, aerospace, and mechanical engineering where thermal loading depicts one of the most decisive loading conditions. On the other hand, since the thermal memory effect is common in some materials, adding hyperbolicity to the heat equation is necessary to take this effect into consideration. For these reasons, we consider the following one-dimensional coupled system wherein Euler-Bernoulli beam equation is interconnected to a heat equation with memory, under Coleman-Gurtin or Gurtin-Pipkin heat conduction law, via transmission conditions established at the interface

$$\begin{cases} u_{tt} + u_{xxxx} = 0, & (x,t) \in (0,1) \times \mathbb{R}_+^*, \\ y_t - c(1-m)y_{xx} - cm \displaystyle\int_0^\infty g(s)\, y_{xx}(x,t-s)\, ds = 0, & (x,t) \in (1,2) \times \mathbb{R}_+^*, \\ u_{xxx}(1,t) = -c(1-m)y_x(1,t) - cm \displaystyle\int_0^\infty g(s) y_x(1,t-s) ds, & t \in \mathbb{R}_+^*, \\ u_t(1,t) = y(1,t), & t \in \mathbb{R}_+^*, \end{cases}$$

After presenting the semigroup setting for the well-posedness, we prove the strong stability of the system using an Arendt-Batty criteria with a diagonalization method. Then, we show that the associated semigroup in the history framework of Dafermos is polynomially stable with decay rate of type t^{-4} . Finally, we achieve exponential stability when Gurtin-Pipkin heat conduction is applied.

15:30–15:50 Qualitative Properties of Composite Structure-Stokes Fluid Interaction PDE Systems

Pelin G. Geredeli (Clemson University, USA)

Abstract. In this work, we analyze the qualitative properties of a composite structure (multilayered) fluid interaction PDE system which arises in multi-physics problems, and particularly in biofluidic applications related to the mammalian blood transportation process and cellular dynamics. The PDE system under consideration consists of the interactive coupling of 3D Stokes flow and 3D elastic dynamics which gives rise to an additional 2D elastic equation on the boundary interface between these 3D PDE systems. We prove that the dynamical system is wellposed. Moreover, we show that the solution to this system satisfies an asymptotic decay to the zero state.

16:00–16:20 Matching measures with non-optimal flows

Domènec Ruiz-Balet (Imperial College London, ENGLAND)

Abstract. In this talk we will discuss the problem of matching measures via sub-optimal parameterized flows. The motivation of such problem arises in the context of machine learning, in view of applications in deep learning architectures such as Transformers.

17:00–17:20 Null controllability for non autonomous degenerate parabolic problems

Genni Fragnelli (Universita degli Studi della Tuscia, ITALY)

Abstract. Inspired by a Budyko-Seller model, we consider non-autonomous degenerate parabolic problems. Using Kato's Theorem, we first prove the well-posedness of such problems. Then,

obtaining new Carleman estimates for the non-homogeneous adjoint problems, we deduce null-controllability for the original ones. Some linear and semilinear extensions are also considered, as well as open problem and work in progress.

17:30–17:50 Some inverse problems for fluids

Anna Doubova (University of Sevilla, SPAIN)

Abstract. In this talk we consider inverse problems of the geometric type for the partial differential equations describing the behavior of certain fluids. Our focus lies on determining a portion of the domain where the equations hold true, based on external measurements.

We will consider real-world applications and will explore two crucial aspects: uniqueness and numerical reconstruction. In particular, we will investigate how initial and boundary data influence solution's uniqueness. Among others, we will deal with one-dimensional inverse problems for the Burgers equation and related nonlinear systems, where heat effects, non-constant density and fluid-solid interaction are taken into account. The goal is to determine the size of the spatial interval based on specific boundary observations of the solution. We will explore both analytical and numerical solutions to these problems, employing powerful tools like Carleman estimates and insights from existing research (see [3], [4]). Additionally, we will provide methods to approximate the interval sizes. These works are in collaboration with J. Apraiz, E. Fernández-Cara and M. Yamamoto [1], [2].

July 26, 2024

11:30–11:50 Remarks on the Large-Scale Stabilization in the 2D Kuramoto-Sivashinsky Equation

Vincent Martinez (City University of New York, Hunter College & Graduate Center, USA)

Abstract. This talk will discuss mechanisms for stabilization of large-scale motions in the 2D Kuramoto-Sivashinsky equation, an equation for which the issue of global regularity of smooth solutions remains open. It is on the other hand well-known that large-scale instability is the putative mechanism for finite-time blow-up of solutions. By casting the 2D KSE in structural analogy to the Navier-Stokes equation (NSE), we identify a divergence-based regularity criterion in analogy to the curl-based regularity criterion that exists for the NSE. This regularity criterion depends entirely on the low-mode behavior of the solutions as quantified by negative Sobolev norms that are "almost critical" with respect to the scaling symmetry of the linear part of the KSE. Based on these ideas, we identify a modification of KSE by introducing a low-mode control to the system that positively resolves the issue of global regularity, as well as apply our regularity criterion to the 2D Burgers-Sivashinsky equation to provide an elementary alternative proof of global regularity of solutions. This is joint work with Adam Larios (University of Nebraska–Lincoln).

12:00–12:20 Synthesis results for the linear quadratic problem for evolution equations with finite memory

Francesca Bucci (Universita degli Studi di Firenze, ITALY)

Abstract. This talk is concerned with the optimal control problem with quadratic functionals for certain classes of infinite dimensional linear integro-differential systems. Most studies on optimal control problems for evolutionary partial differential equations (PDE) with memory pertain to more general frameworks – involving, e.g., semilinear PDE and/or non-quadratic functionals – and hence are aimed at establishing the existence of *open-loop* solutions as well as at characterizing them via first (and possibly, second) order optimality conditions. Also, in the presence of an infinite memory, the celebrated "history approach" of C. Dafermos has

proved effective in attaining a reformulation of the nonlocal problem; the equivalent coupled system satisfied by an augmented variable may constitute a starting point for the exploration of a variety of control-theoretic properties.

With focus on the finite horizon linear quadratic problem, in the presence of finite memory, here we offer a Riccati-based approach to the optimization problem that brings about a *closed-loop* representation of the unique optimal control, where the optimal cost operator is determined by solving a corresponding Riccati equation. Our recent advances pertain to an abstract model in which the memory affects the control action, a case interesting enough in itself and which should also serve as a first step for future developments.
(The talk is based on past and ongoing joint work with Paolo Acquistapace (Università di Pisa).)

12:30–12:50 Concerning Higher Regularity Properties of a Certain PDE Interaction of Fluid-Structure Type

George Avalos (University of Nebraska-Lincoln, USA)

Abstract. In this talk we discuss our progress towards (at least partially) verifying a conjecture of the late Igor Chueshov, concerning the qualitative behavior of a coupled partial differential equation (PDE) system which describes a certain fluid-structure interaction (FSI). This particular FSI system comprises a Stokes flow, evolving within a three-dimensional cavity, coupled via a boundary interface, to a two dimensional Euler-Bernoulli (or Kirchhoff) plate, which displaces upon a sufficiently smooth bounded open set; this open set is a portion of the cavity boundary. Previously, I. Chueshov conjectured that, despite the respective (three dimensional) Stokes and (two dimensional) Euler-Bernoulli structural dynamics being coupled via a boundary interface, the strongly continuous semigroup of contractions, which is associated with such FSI dynamics, is in fact analytic in a sector of the complex plane. In other words, the entire FSI system manifests "parabolic-like" behavior, including smoothing effects for positive time.

In this connection, we shall discuss our findings that the modeling fluid-structure strongly continuous semigroup is of a specified Gevrey Class. In other words, the behavior of the strongly continuous semigroup which describes the given interaction between 3D Stokes flow and 2D plate dynamics formally falls in a range between differentiability and analyticity.

14:30–14:50 Stabilization of Degenerate Wave Equations with Drift and with/without Singular Term

Ibtissam Issa (Universita degli studi di Bari Aldo Moro, ITALY)

Abstract. In this talk, we present the stability of a degenerate wave equation featuring localized singular damping, along with a drift term and a leading operator in non-divergence form and with/or without singular term. Exponential stability results are presented under suitable conditions on the degeneracy and singularity coefficients.
The system that will be considered is as follows:

$$\begin{cases} u_{tt} - au_{xx} - \dfrac{\lambda}{d}u - bu_x + \chi_{(x_1,x_2)}u_t = 0, & (x,t) \in (0,1) \times \mathbb{R}_*^+, \\ u(0,t) = u(1,t) = 0, & t \in \mathbb{R}_*^+, \\ u(x,0) = u_0(x), u_t(x,0) = u_1(x), & x \in (0,1), \end{cases} \tag{1}$$

where the damping coefficient is given by $\chi_{(x_1,x_2)}(x)$ such that $0 \le x_1 < x_2 \le 1$ and

$$\begin{cases} a, b, d \in C^0[0,1], \\ a, d > 0 \quad \text{on} \quad (0,1], a(0) = d(0) = 0, \\ \dfrac{b}{a} \in L^1(0,1). \end{cases} \tag{C}$$

15:00–15:20 Rigorous elastohydrodynamic lubrication approximation

Boris Muha (University of Zagreb, CROATIA)

Abstract. Motivated by widespread applications of microfluidic channels and chips, we consider an interaction between a thin layer of an incompressible viscous fluid and an elastic structure. Starting from fluid-structure interaction (FSI) problems, our aim is to rigorously derive corresponding reduced models, which are favorable in engineering applications. We formulate this as a singular limit problem in terms of the vanishing relative fluid thickness, which is carried out based on suitable energy estimates and uniform no-contact results for FSI systems. Reduced models are justified in terms of weak convergence results in the sense that weak limits of solutions to the FSI problem are identified in a relation with solutions of the reduced model.

15:30–15:50 Nonlocal interaction kernels inference in nonlinear gradient flow equations

Jose Antonio Carrillo (University of Oxford, ENGLAND)

Abstract. When applying nonlinear aggregation-diffusion equations to model real life phenomenon, a major challenge lies on the choice of the interaction potential. Previous numerical and theoret- ical studies typically required predetermination of terms and the goal is often to reproduce the observed dynamics qualitatively, not quantitatively. In this talk, we address the inverse problem of identifying nonlocal interaction potentials in nonlinear aggregation-diffusion equations from noisy discrete trajectory data. Our approach involves formulating and solving a regularised varia- tional problem, which requires minimising a quadratic error functional across a set of hypothesis functions. A key theoretical contribution is our novel stability estimate for the PDEs, validating the error functional ability in control- ling the 2-Wasserstein distance between solutions gener- ated using the true and estimated interaction potentials. We demonstrate the effectiveness of the methods through various 1D and 2D examples showcasing collective behaviours.

16:00–16:20 Energy decay estimates for semilinear evolution equations with memory and delay feedback

Elisa Continelli (University of L'Aquila, ITALY)

Abstract. We consider semilinear evolution equations with memory and time-dependent time delay feedback. Under a suitable assumption on the coefficient of the delay feedback, we are able to prove that solutions corresponding to small initial data are globally defined and satisfy an exponential decay estimate. The standard assumption used so far to deal with wave-type equations with time-varying time delay is that the time delay function $\tau(\cdot) : [0, +\infty) \to [0, +\infty)$ must belong to the Sobolev space $W^{1,\infty}(0, +\infty)$ and has to satisfy the following condition: $\tau'(t) \leq c < 1$. Here, instead, we work in a very general setting, namely we only assume the time delay function is continuous and bounded from above.

17:00–17:20 Stability Results for Novel Serially-Connected Magnetizable Piezoelectric and Elastic Smart-System Designs

Mohammad Akil (Universite Polytechnique Hauts-de-France, FRANCE)

Abstract. In this talk, we consider the stability of longitudinal vibrations for transmission problems of two smart-system designs:

(i) a serially-connected elastic–piezoelectric–elastic design with a local damping acting only on the piezoelectric layer

(ii) a serially- connected piezoelectric–elastic design with a local damping acting on the elastic part only.

Unlike the existing literature, piezoelectric layers are considered magnetizable, and therefore, a fully-dynamic PDE model, retaining interactions of electromagnetic fields (due to Maxwell's equations) with the mechanical vibrations, is considered. The design (i) is shown to have exponentially stable solutions. However, the nature of the stability of solutions of the design (ii), whether it is polynomial or exponential, is dependent entirely upon the arithmetic nature of a quotient involving all physical parameters. Furthermore, a polynomial decay rate is provided in terms of a measure of irrationality of the quotient. Noting that this type of result is totally new. The main tool used is the multipliers technique which requires an adaptive selection of cut-off functions together with a particular attention to the sharpness of the estimates to optimize the results.

17:30–17:50 Forward problems for time-fractional degenerate heat equations as possible model for climate sciences

Masahiro Yamamoto (University of Tokyo, JAPAN)

Abstract. We discuss initial boundary value problems for time-fractional degenerate heat equations. This type of equations can well simulate characteristic or anomalous heat diffusion in several cases, and one can expect that they are effective mathematical model equations. However, the mathematical backgrounds have not been established yet and we present fundamental theory. We will study also some applications such as inverse problems if possible.

18:00–18:20 On integral inequalities with applications to indirect linear or nonlinear stabilization of coupled PDE's

Fatiha Alabau-Boussouira (Sorbonne Université & Université de Lorraine, FRANCE)

Abstract. The purpose of this talk is to present several integral inequalities in the form introduced and stated by the speaker respectively in 1999 [1] for the linear case and in 2004 [5] in a nonlinear framework.

Let $\mathcal{A}$ be the infinitesimal generator of a $\mathcal{C}^0$ semigroup on the Banach space $\mathcal{H}$, $D(\mathcal{A})$ its domain, and $D(\mathcal{A}^k)$ the domain of its powers of order k. The linear integral inequality in Lemma 4.1 of [1] (or in Theorem 3.1 in [2]) states that if for every $T \geqslant 0$ and every U_0 in $D(\mathcal{A}^p)$, $p \geqslant 1$, the integral $\int_0^T ||e^{t\mathcal{A}}U_0||^2_{\mathcal{H}}dt$ can be controlled by a constant time $||U_0||^2_{D(\mathcal{A}^p)}$, then the norm of $e^{t\mathcal{A}}U_0$ in $\mathcal{H}$ decays polynomially as times goes to ∞. We shall present the first applications of these inequalities which allowed to prove polynomial stability for indirectly damped coupled systems as shown in [1,2] (see also [3]) and other further works [6,7], and so to compensate their lack of exponential stability (see [3]).

The nonlinear integral inequalities as stated in Theorem 2.1 in [5] (see also [4]) is a general property for real-valued nonnegative non increasing functions E defined on the nonnegative real axis. We prove in [5] that if the integral $\int_t^S E(s)N(E(s))ds$ can be controlled up to a positive constant T_0 (independent of t) by $E(t)$ for every $t \geqslant 0$ and under appropriate assumptions on N, then E decays at infinity with a given precise decay rate depending on $E(0)$, T_0 and the function N. No underlying semigroup frame is requested. This stresses the fact that the inequalities presented in [1, 2] and [5] are fundamentally not based on the same mathematical frame. We shall present in a second part how these inequalities allow to derive optimal or quasi-optimal energy decay rates for nonlinearly damped PDE's.

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