AMS-UMI International
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Developments in Hyperbolic Geometry
Special Session B6

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The session will focus on recent developments in the theory of hyperbolic manifolds in low and high dimension. For two dimensions there will be speakers with experience in Teichmüller Theory, hyperbolic surface, big and small mapping class groups, and specifically counting problems on surfaces. In three dimensions we plan on having speakers discussing volumes and deformation spaces of hyperbolic 3-manifolds. For higher dimensions we plan on having talks on the recent constructions of fibered higher dimensional hyperbolic manifolds and pro-finite rigidity. The main goals of this session are to bring together a diverse group of European and American academics and to provide ample space for young researchers to present their work and interact with senior faculty.

For more information visit `https://umi.dm.unibo.it/jm-umi-ams/`.

Schedule and Abstracts

July 25, 2024

11:30–12:30 Hyperbolic manifolds and profinite rigidity
Alan Reid (Rice University, USA).

Abstract. In this talk we will survey recent progress on profinite rigidity (both absolute and relative) of the fundamental groups of finite volume hyperbolic manifolds.

12:30–13:00 Fibering of Hyperbolic Manifolds
Giovanni Italiano (University of Oxford, UK).

Abstract. In this talk, based on joint work with Martelli and Migliorini, we are going to present some advancements regarding fibrations of hyperbolic manifold in dimension higher than 3.

14:30–15:00 How many times can two curves on a surface intersect?
Irene Pasquinelli, University of Bristol, UK

Abstract. Given a surface, one might want to understand how "complex" the surface is, in terms of curves. More specifically, we may ask how many times two curves on this surface can intersect. Of course, longer curves might intersect more times. KVol is a quantity measuring how many times curves can intersect, modulo their length. We will give an overview of some cases for which this quantity has been calculated, with particular focus on Veech surfaces, a class of flat surfaces with a rich group of symmetries. This is joint work in progress with Julien Boulanger.

15:00–16:00 Distribution of the components of a random multicurve
Viveka Erlandsson, University of Bristol, UK

Abstract. For random pants decompositions (chosen uniformly at random out of those of total length at most L) on a hyperbolic surface, Mirzakhani studied the distribution of the lengths of the individual components as $L \to \infty$. This result has been generalized independently by Mingkun Liu and Francisco Arana-Herrera to other simple multi-curves. In this talk, using different methods based on convergence of certain measures on the space of geodesic currents, we extend it further to hold for any multicurves, simple or not. This is joint work with Juan Souto.

16:00–16:30 Word-length curve counting on the once-punctured torus
David Fisac (University of Luxembourg, LUXEMBOURG).

Abstract. We will talk about a classification of all the words on $\{a, b\}$ representing a curve with self-intersection one on the once-punctured torus, as an analog for the already-existing classification of simple curves. From here we will discuss how to derive the exact counting of curves of given word-length and self-intersection zero or one, by only combinatorial means. Finally, the consequences of these techniques when giving a hyperbolic structure to the surface. This is part of joint work with Mingkun Liu.

17:00–17:30 Geodesics and norms on the cohomology of hyperbolic 3-manifolds
Cameron Rudd

Abstract. The cohomology of a hyperbolic 3-manifold can be equipped with various geometric and topological norms. One such norm is related to optimizing the Lipschitz constant in a homotopy class of circle valued maps. There is an associated geodesic lamination encoding where the manifold is most stretched by these optimal Lipschitz maps and this lamination depends only on the homotopy class. A natural question then is "what do these laminations look like?" I will discuss how using Dehn surgery and topological and geometric norm comparisons, one can construct examples where these geodesic laminations can be identified.

17:30–18:00 Distance in the pants graph and applications to Teichmüller space
Mehdi Yazdi (Kings College London, UK)

Abstract. Given two pants decompositions of a compact orientable surface S, we give an upper bound for their distance in the pants graph that depends logarithmically on their intersection number and polynomially on the Euler characteristic of S. As a consequence, we find an upper bound on the volume of the convex core of a maximal cusp (which is a hyperbolic structure on $S \times \mathbb{R}$ where given pants decompositions of the conformal boundary are pinched to annular cusps). As a further application, we give an upper bound for the Weil–Petersson distance between two points in the Teichmüller space of S in terms of their corresponding short pants decompositions. The proofs rely on using pre-triangulations, train tracks, and an algorithm of Agol, Hass, and Thurston.

18:00–18:30 Filling Riemann Surfaces by Hyperbolic Schottky Manifolds of Negative Renormalized Volume
Viola Giovannini (University of Luxembourg, LUXEMBOURG).

Abstract. Given a hyperbolizable 3-manifold N with boundary components of genus at least two, the renormalized volume is a real-valued function on the space of convex co-compact hyperbolic structures $CC(N)$ on the interior of N, which always have infinite hyperbolic volume. The simplest examples of convex co-compact hyperbolic 3-manifolds are the handlebodies, and, given a connected Riemann surface X of genus ≥ 2, we call Schottky filling of X a handlebody with boundary at infinity X.

A question attributed to Maldacena asks whether given a connected Riemann surface X of genus at least two, there exists a Schottky filling of X of negative renormalized volume.

In this talk, we will present an upper bound for the renormalized volume in terms of the genus and the hyperbolic curve lengths of a suitable pants decomposition of X, which allows us to positively answer the question of Maldacena for certain classes of Riemann surfaces.

July 26, 2024

11:30–12:30 Efficient cycles for hyperbolic manifolds
Roberto Frigerio (University of Pisa, ITALY)

Abstract. The simplicial volume is a homotopy invariant of manifolds introduced by Gromov in 1982. It is defined as the infimum $||N||$ of the l_1-norms of fundamental cycles in the top-dimensional singular homology of N with real coefficients. Computing the simplicial volume is usually a very difficult task. Even when the simplicial volume of a manifold N is known, characterizing (or, at least, exhibiting some) fundamental cycles whose norm is close to $||N||$ may be surprisingly difficult. For hyperbolic manifolds the fundamental computation by Gromov and Thurston explicitly constructs such cycles via an averaging operator called smearing. A natural question is to which extent this construction is unique, i.e., whether there exist cycles approximating the simplicial volume which do not come from smearing. In this talk we show that, in dimension $n \geq 3$, the unique hyperbolic manifolds admitting "exotic" almost minimal fundamental cycles are those which are commensurable with the figure-eight knot complement.

12:30–13:00 Integral simplicial volume and triangulation complexity of 3-manifold fibering over the circle
Federica Bertolotti (Scuola Normale Superiore, ITALY)

Abstract. Triangulation complexity and integral simplicial volume are two topological invariants studied in low-dimensional topology. The first one counts the minimal number of *embedded* simplices that are needed to triangulate a manifold, while the second one deals with the number of *singular* simplices in a fundamental cycle of the manifold. These two invariants share many properties, coincide on surfaces and are similar on many 3-manifolds, and both are very hard to compute.

In this talk, we will study the asymptotic behavior of the triangulation complexity and the integral simplicial volume of cyclic covers of 3-dimensional manifolds fibering over the circle. In particular, we will identify families of 3-manifolds for which these invariants exhibit markedly different behaviors.

This is joint work with Roberto Frigerio.

14:30–15:00 Relatively maximal $\mathrm{PSL}_2(\mathbb{R})$-representations of punctured surface groups
Gabriele Mondello (Sapienza Università di Roma, ITALY)

Abstract. It Is well-known that isotopy classes of complete $K = -1$ metrics on a punctured oriented surface bijectively correspond to their monodromy representations. I will report on a joint work with Nicolas Tholozan, in which we show that such correspondence still holds for conical $K = -1$ metrics with conical points of small angles (and we exactly determine which angles). The argument uses the opposite energy-gradient flow on Teichmüller space associated to a representation.

15:00–15:30 Geometry of geodesic currents
Jenya Sapir (Binghamton University, USA)

Abstract. The space of projective, filling currents $\mathbb{P}\mathcal{C}_{fill}(S)$ contains many structures relating to a closed, genus g surface S. For example, it contains the set of all closed curves on S, as well as an embedded copy of Teichmüller space, and many other spaces of metrics on S. It turns out that the Thurston metric on Teichmüller space extends to $\mathbb{P}\mathcal{C}_{fill}(S)$. We will discuss the geometry of $\mathbb{P}\mathcal{C}_{fill}(S)$ with this metric.

15:30–16:00 Automorphisms of geodesic currents preserve intersection form
Meenakshy Jyothis (Binghamton University, USA).

Abstract. We will discuss progress in proving Ivanov's meta conjecture in the context of geodesic currents. Ivanov's meta conjecture says that every object naturally associated with a surface and having a 'sufficiently rich' structure has the mapping class group as its group of automorphisms. The conjecture has been proven for various combinatorial objects associated with a surface as well as for the Teichmüller space of a surface. The space of geodesic currents contains many of these structures, such as the set of closed curves up to homotopy and the Teichmüller space. We discuss progress in showing Ivanov's meta conjecture for a natural group of automorphisms of currents.

16:00–16:30 The space of co-geodesic currents of a hyperbolic group

Dídac Martínez Granado, (University of Luxembourg, LUXEMBOURG).

Abstract. We define a notion of hyperplane at infinity for a hyperbolic group G and study G-invariant Radon measures on the space of hyperplanes at infinity, which we call "co-geodesic currents´´. Co-geodesic currents are induced by many classical objects such as geodesic currents for surface groups, certain cocompact actions of hyperbolic groups on $CAT(0)$ cube complexes, some actions of hyperbolic groups on real trees, etc. Moreover, there is a natural intersection pairing between co-geodesic currents and geodesic currents, generalizing Bonahon's intersection number when G is a surface group. Furthermore, every co-geodesic current induces natural dual pseudo-metric space with a measured wall structure, in the sense that the intersection of with the current determined by the conjugacy class of an element in G recovers the stable length of that element in the pseudo-metric space. This is joint work in progress with Eduardo Reyes.

17:00–17:30 (Random) Hyperbolic surfaces with large systoles

Mingkun Liu (University of Luxembourg, LUXEMBOURG).

Abstract. The systole of a hyperbolic surface X is the least length of a closed geodesic on X. In this talk, I'll discuss the following question: how large can the systole of a hyperbolic surface be? I'll also present some (random) constructions of hyperbolic surfaces with large systoles. This is joint work with Bram Petri.

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