Dynamics of compressible Euler equations and complex flows
Special Session B23

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The topic of this special session is the analysis of partial differential equations (PDE's) arising in physics, which are nonlinear hyperbolic or systems that combine hyperbolic with parabolic features. Such equations are ubiquitous in multiple domains of applied sciences, ranging from high-speed flows in fluids, to flows of complex systems, to plasma physics, and astrophysics. The aim is to bring together people that work on compressible Euler equations (and related subjects) with specialists who work on flows of complex and multiscale systems and to stimulate an exchange between these subjects. We will focus on theory, numerics, as well as modeling and applications, and their interplay. We hope to explore new directions, and to stimulate new collaborations.

Schedule and Abstracts

July 25, 2024

11:30–11:50 An initial-boundary value problem for system of conservation laws with unknown boundary
Debora Amadori (University of L'Aquila, ITALY)

Abstract. In this talk, we study a hyperbolic system of balance laws in one dimension on a domain with a boundary. The shape of the boundary is not prescribed a priori, and it is an unknown of the problem itself. Under appropriate conditions at the boundary and on the source term, we show the existence of a non-characteristic boundary and of a weak entropy solution of small total variation, defined on the resulting domain, that is global in time. We show an application to an inverse problem motivated by the study of planar steady supersonic flow past a wedge.

12:00–12:20 BV weak solutions with bounded support and long-time behavior to an Euler-type flocking model
Cleopatra Christoforou (University of Cyprus, CYPRUS)

Abstract. Recent results on the global in time existence of weak solutions with bounded support to a hydrodynamic model of flocking-type in a one-space dimension and the long time behavior would be presented. An appropriate notion of entropy weak solutions with bounded support is given to capture the behavior of solutions with initial data that has finite total mass confined in a bounded interval and initial density uniformly positive therein without any restrictions on the size of the total variation of the initial data. We show global in time existence of entropy weak solutions with concentration along the interfaces that separate the vacuum with the non-vacuum regions, for any initial data of bounded variation having the structure above. The analysis relies on the front tracking algorithm, a detailed study of the decay of the wave fronts and the influence of the shock discontinuities along the interfaces. In addition, we capture the time-asymptotic limit for such solutions unconditionally, showing the asymptotic decay towards flocking profiles without any further restrictions on the data.

References

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12:30–12:50 A non linear and non local hyperbolic–parabolic system inspired by biology
Elena Rossi (University of Modena and Reggio Emilia, ITALY)

Abstract. Motivated by the description of general predator–prey dynamics and with the specific aim of controlling parasites' propagation in pest control problems, we propose a model consisting of a non linear and non local hyperbolic equation together with a parabolic one. The unknown functions are the densities of the two competing populations. Prey/parasites are assumed to diffuse, while the movement of predators is directed towards the regions where the concentration of prey is greater. This situation is modelled by a non linear and non local function of the prey's density. The two PDEs are further coupled through the source terms, that comprise Lotka–Volterra type interactions and also the presence of external space- and time-dependent controls. We present analytical results on the well posedness of this class of systems, both on the whole space $\mathbb{R}^n$ and on a bounded domain. The presentation is concluded by some numerical simulations, to illustrate the qualitative properties of the solutions.

References

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14:30–14:50 Stability of a numerical method for a transport equation, with the help of noise
Ulrik Skre Fjordholm (University of Oslo, NORWAY)

Abstract. The term "regularization by noise" refers to the phenomenon where the presence of noise may lead to better well-posedness of a differential equation, such as the delay of blowup, higher regularity, or existence/uniqueness for problems whose deterministic counterpart is ill-posed. As a concrete example, I will present a recent paper where we analyse a numerical method for a stochastic transport equation. The velocity field is assumed to be of low regularity (essentially, $V \in L^\infty(\mathbb{R}^d;\mathbb{R}^d)$ with $\mathrm{div}V \in L^p(\mathbb{R}^d)$ for $p > d$) — lower than what is required in the deterministic theory. The stochastic term is of the transport type, with a spatially heterogeneous noise coefficient. We prove that the method satisfies an energy bound, and hence converges weakly to the exact solution. This is joint work with Kenneth Karlsen and Peter Pang (both University of Oslo).

References

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15:00–15:20 Analysis of traveling waves for Navier–Stokes–Korteweg type models
Corrado Lattanzio (University of L'Aquila, ITALY)

Abstract. We study existence and spectral stability of traveling waves for Navier–Stokes–Korteweg type models, underlying in particular the interplay between capillarity and viscosity effects. Motivated by the theory of superfluidity and the mathematical modeling of semiconductor devices, we first consider the compressible Euler equations with linear and nonlinear viscosity and where

the dispersive term is originated by the quantum effects described through the Bohm potential. The existence of traveling waves (or *viscous dispersive shocks*) is proved for appropriate end states defining Lax shocks for the underlying Euler system, and for arbitrary shock amplitude. The results are obtained by taking advantage of suitable Lyapunov functions and, in the case of large shocks, the framework of existence includes also the case of oscillatory profiles, that is when the effects of the dispersion play a significant role. The model with nonlinear viscosity is formulated in terms of density and velocity and the existence is proved without restrictions for the viscosity and dispersion parameters. The spectral stability of these profiles is also analyzed: we prove stability of the essential spectrum of the linearized operator, provided the end states are subsonic or sonic, and, using the Evans function, we derive estimates for the modulus of possible unstable eigenvalues. The latter allows us to investigate numerically the behavior of the Evans function in sufficiently large region of the unstable half–plane, providing numerical evidence for point spectral stability of arbitrary large, possibly non–monotone profiles. The same results are presently under investigation for the classical Navier–Stokes–Korteweg model, with physical viscosity and general Korteweg terms. This is a joint project with P. Marcati (GSSI), D. Zhelyazov (U. Surrey), R. Folino and R. Plaza (UNAM).

15:30–15:50 Non-uniqueness and energy dissipation for 2D Euler equations with vorticity in Hardy spaces
Stefano Modena (GSSI - Gran Sasso Science Institute, ITALY)

Abstract. We show by convex integration that uniqueness of solutions to the 2D incompressible Euler equations fails in the class of admissible (i.e. energy dissipating), compactly supported, $L^\infty_t L^2_x$ velocity fields having vorticity in the real Hardy space $H^p(\mathbb{R}^2)$, for any $p \in (0, 1)$.

References

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[2] M. Buck, S. Modena, *in preparation.*

16:00–16:20 Error analysis of the θ-method for ODEs with irregular vector fields
Gennaro Ciampa (University of L'Aquila, ITALY)

*Abstract.*The goal of this talk is to investigate the convergence of numerical methods applied to ordinary differential equations with vector fields in the DiPerna-Lions class. We will prove logarithmic rates of convergence (in the mesh-size) of the approximating flows constructed via the so-called θ-method towards the unique regular Lagrangian flow of the given vector field. I will also discuss the applications to the linear transport equation. This is the first result concerning a posteriori error estimate for implicit schemes applied to ordinary differential equations with Sobolev vector fields.

References

[1] T. Cortopassi: *An explicit Euler method for the continuity equation with Sobolev velocity fields.* Preprint, available at https://arxiv.org/abs/2402.04118.

17:00–17:20 On a stochastic fluid-structure interaction problem with compressible fluid and elastic shell
Krutika Tawri (University of California Berkeley, USA)

Abstract. In this talk, we will present our recent results on a non-linearly coupled stochastic fluid-structure interaction problem that involves an isentropic compressible fluid interacting with a thin elastic membrane. The elastodynamics problem is modeled by linear shell equations determining the displacement of a 3D time-dependent domain that contains a fluid whose flow is described by the Navier-Stokes equations. The noise is applied both to the fluid equations as a volumetric body force, and to the structure as an external forcing to the deformable fluid boundary. We will discuss the challenges arising due to the random motion of the time-dependent fluid domain and present our recent result that provides the existence of weak martingale solutions. We will

briefly describe our approach to constructing these solutions which involves a splitting scheme, penalization of boundary behavior and domain extension, construction of an artificial structure variable and a stopping time argument.

17:30–17:50 Existence of weak solutions to the kinetic Cucker-Smale equations with local alignment and stochastic kinetic transport
Jeffrey Kuan (University of Maryland College Park, USA)

Abstract. We present a result on existence of weak martingale solutions for a stochastic kinetic Cucker-Smale equation with local alignment and stochastic kinetic transport. The Cucker-Smale equations, which were introduced in [1] and [2], model the collective dynamics and behavior of agents, such as flocks of birds, schools of fish, and herds, where the velocity of each agent evolves in time and is influenced by the velocities and the spatial locations of other nearby agents in the system. While it is possible to model each agent in the flock or herd individually, when the number of agents is large, it is more tractable to instead provide a kinetic description of the collective dynamics. This involves introducing a function $f(t,x,v)$ for $t \in [0,T]$, $x \in \mathbb{R}^d$, and $v \in \mathbb{R}^d$, which heuristically is a density function describing the relative quantity of agents having position $x \in \mathbb{R}^d$ and velocity $v \in \mathbb{R}^d$ at a given time t. To model the ways in which differences in velocities and spatial positions between agents influence the evolution of $f(t,x,v)$, one can use an interaction kernel to describe pairwise interactions between agents, and a local alignment term, which describes the tendency of agents to align in direction with agents sharing the same position. It is well-known that weak (function-valued) solutions exist to the deterministic Cucker-Smale kinetic model with an interaction term and local alignment, due to work in [3]. The goal of the present work is to extend the well-posedness result for the deterministic kinetic Cucker-Smale dynamics in [3] to the stochastic regime, where the kinetic Cucker-Smale dynamics are perturbed by stochastic kinetic transport. The stochastic kinetic transport represents the effect of random noise, such as random wind or currents, that globally and randomly perturbs the dynamics of the entire system. Such kinetic equations with stochastic kinetic transport have been of recent interest, for example in the case of the Boltzmann equations [4]. We show existence of weak martingale solutions to the kinetic Cucker-Smale equations with stochastic kinetic transport by using approximation parameters to approximate the local velocity and by constructing appropriate approximate solutions. We then establish uniform bounds on the approximate solutions by deriving suitable a priori estimates. Finally, to complete the passage to the limit in the approximate solutions, we develop new stochastic velocity averaging results in the spirit of [4] to pass to the limit in time-dependent and spatially-dependent quantities, such as the density and the momentum, and then we establish tightness of laws of the approximate solutions in order to use the Skorokhod representation theorem.

References

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[2] F. Cucker and S. Smale. On the mathematics of emergence. *Japan J. Math.*, 2:197–227, 2007.

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18:00–18:20 Existence of weak solutions for the unsteady $p(\cdot,\cdot)$-Navier–Stokes equations via a fully-discrete approximation.
Luigi Carlo Berselli (University of Pisa, ITALY)

Abstract. We establish the well-posedness, stability, and (weak) convergence of a fully-discrete approximation of the unsteady $p(\cdot,\cdot)$-Navier–Stokes equations employing an implicit Euler step in time and a discretely inf-sup-stable finite element approximation in space. The result, beside

the interest for the numerical approximation, gives an alternative way to prove existence of weak solutions. Moreover, numerical experiments that supplement the theory are presented.

References

[1] L. C. Berselli and A. Kaltenbach, *Convergence analysis for a finite element approximation of the unsteady* $p(\cdot,\cdot)$*-Navier–Stokes equations*, arXiv:2402.16606, 2024

[2] E. Carelli, J. Haehnle, and A. Prohl, *Convergence analysis for incompressible generalized Newtonian fluid flows with nonstandard anisotropic growth conditions*, SIAM J. Numer. Anal. 48 no. 1 (2010), 164–190.

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July 26, 2024

11:30–11:50 On Bayesian data assimilation for PDEs with ill-posed forward problems
Franziska Weber (University of California Berkele, USA)

Abstract. We consider Bayesian data assimilation for time-evolution PDEs, for which the underlying forward problem may be very unstable or ill-posed. We formulate assumptions on the forward solution operator of such PDEs under which stability of the posterior measure with respect to perturbations of the noisy measurements can be proved. We also provide quantitative estimates on the convergence of approximate Bayesian filtering distributions computed from numerical approximations. For the Navier-Stokes equations, our results imply uniform stability of the filtering problem even at arbitrarily small viscosity, when the underlying forward problem may become ill-posed, as well as the compactness of numerical approximations in a suitable metric on time-parametrized probability measures.

12:00–12:20 Asymptotic limits for unipolar and bipolar Euler-type systems
Nuno J. Alves (University of Vienna, AUSTRIA)

Abstract. In this talk, we present some recent developments in the study of asymptotic limits for Euler-type systems via the relative energy method. The first limit that we consider is the high-friction limit. In [1], one establishes this limit for a bipolar Euler-Poisson system towards a bipolar drift-diffusion system, whereas in [3] this limit yields a diffusion-aggregation equation from an Euler-Riesz system. These results extend the ones obtained previously in [4] and [5], and are compared regarding the admissible range of the adiabatic exponents. The high-friction limit connects hyperbolic systems with parabolic equations.

Furthermore, we also consider the zero-electron-mass and quasi-neutral limits for a bipolar Euler-Poisson system [2]. In the former limit, an adiabatic electron system is obtained, while in the combined regime one arrives at an Euler system. In this case, these limiting procedures yield hyperbolic systems from hyperbolic equations.

The technical challenges in these studies are addressed either via the theory of Riesz potentials and related Hardy-Littlewood-Sobolev inequalities, or via standard elliptic regularity theory together with Sobolev embeddings.

The results presented were obtained in collaboration with A. E. Tzavaras, J. A. Carrillo, and Y. P. Choi.

References

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12:30–12:50 Global regularity for the one-dimensional stochastic Quantum-Navier-Stokes equations
Lorenzo Pescatore (University of L'Aquila, ITALY)

Abstract. In this talk I will present some new results concerning the analysis of the stochastically forced 1D Quantum-Navier-Stokes equations. In particular for $x \in \mathbb{T}$, the 1-dimensional flat torus, and $t \in [0, T]$, the system under studying is the following:

$$(1) \qquad \begin{cases} \mathrm{d}\rho + \partial_x(\rho u)\mathrm{d}t = 0 \\ \mathrm{d}(\rho u) + [\partial_x(\rho u^2 + p(\rho))]\mathrm{d}t = [\partial_x(\mu(\rho)\partial_x u) + \rho\partial_x\left(\frac{\partial_{xx}\sqrt{\rho}}{\sqrt{\rho}}\right)]\mathrm{d}t + \mathbb{G}(\rho, \rho u)\mathrm{d}W. \end{cases}$$

The unknowns $\rho > 0$ and $u \in \mathbb{R}$ denote the density and the velocity of the fluid, while

$$p(\rho) = \rho^\gamma, \quad \gamma > 1, \quad \mu(\rho) = \rho^\alpha, \quad \alpha \geq 0,$$

represent the isoentropic pressure and the viscosity coefficient. The stochastic forcing term $\mathbb{G}(\rho, u)\mathrm{d}W$ is a multiplicative noise defined on a stochastic basis with a complete right-continuous filtration $(\Omega, \mathfrak{F}, (\mathfrak{F}_t)_{t\geq 0}, \mathbb{P})$ together with a cylindrical $(\mathfrak{F}_t)$-Wiener process $W(t)$. The related stochastic integral is understood in the Itô sense.
Our analysis is focused on the existence of solutions to (1) which are strong both in PDEs and probability sense. In particular, we prove the local well-posedness of the problem up to a maximal stopping time τ which depends on the $W^{2,\infty}$ norm of the solution (ρ, u) and we derive some a priori estimates in the case of the viscosity exponent $\alpha \in [0, \frac{1}{2}]$, which allow us to extend the local strong solution to a global one by controlling the arising of vacuum states of the density. The analysis is performed for a wide class of density dependent viscosity coefficients and as a byproduct of our results we also get the global well-posedness for the deterministic case.

References

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14:30–14:50 Density of wild data for the Euler system of dynamics
Elisabetta Chiodaroli (University of Pisa, ITALY)

Abstract. We adapt Glimm's approximation method to the framework of convex integration to show density of wild data for the (complete) Euler system of gas dynamics. The desired infinite family of entropy admissible solutions emanating from the same initial data is obtained via convex integration of suitable Riemann problems pasted with local smooth solutions. In addition, the wild data belong to the BV class. The results presented are based on a joint work with Eduard Feireisl [1].

References

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15:00–15:20 Recent developments for the one-dimensional compressible Euler system with local and non-local interactions and dissipation terms
Ewelina Zatorska (University of Warwick, UK)

Abstract. In this talk I will summarise our recent findings on the existence of regular and weak solutions for the compressible Euler equations with nonlocal interaction terms including

attraction-repulsion and alignment [1]. In particular, I will present the relative entropy structure based on the two-velocity reformulation of the system [2], the viscous approximation [4], and the long-time behaviour of solutions [3].

References

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15:30–15:50 Vanishing physical viscosity solutions of characteristic initial-boundary value problems for systems of conservation laws
Laura V. Spinolo (CNR-IMATI, Pavia, ITALY)

Abstract. Consider the viscous approximation of a nonlinear system of conservation laws in one space variable

$$\mathbf{g}(\mathbf{v}^\varepsilon)_t + \mathbf{f}(\mathbf{v}^\varepsilon)_x = \varepsilon\Big(\mathbf{D}(\mathbf{v}^\varepsilon)\mathbf{v}^\varepsilon_x\Big)_x,$$

where $\mathbf{g}, \mathbf{f} : \mathbb{R}^N \to \mathbb{R}^N$ are given functions, $\mathbf{D}$ is also a given function attaining values in the space of $N \times N$ matrices, and the unknown $\mathbf{v}^\varepsilon$ attains values in $\mathbb{R}^N$. The archetypical examples are the compressible Navier-Stokes equations and the viscous MagnetoHydroDynamics (MHD) equations that formally boil down, in the inviscid $\varepsilon \to 0^+$ limit, to the compressible Euler and inviscid MHD equations, respectively. In the case of initial-boundary value problem, it is known that, owing to boundary layers phenomena, different viscous mechanism (that is, different choices of the matrix $\mathbf{D}$) yield different solutions in the vanishing viscosity limit. In my talk I will consider the initial boundary-value problem for the conservation law

$$\mathbf{g}(\mathbf{v}^\varepsilon)_t + \mathbf{f}(\mathbf{v}^\varepsilon)_x = \mathbf{0}$$

and encode information on the underlying viscous mechanism in the formulation of the boundary condition. I will then discuss a new wave front-tracking algorithm providing, for small total variation data, global-in-time existence of admissible solutions of the initial-boundary value problem. The hypotheses cover the most interesting, albeit technically demanding, cases, that is physical (mixed hyperbolic-parabolic) viscosity and characteristic boundary. In particular, the result applies to the inviscid limit of the Navier-Stokes and viscous MHD equations, written in both Eulerian and Lagrangian variables.

References

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16:00–16:20 Renormalized solutions for the Maxwell-Stefan system
Athanasios E. Tzavaras (King Abdullah University of Science and Technology (KAUST), SAUDI)

Abstract. The Maxwell-Stefan system is commonly used to describe diffusion processes of multicomponent systems. In this talk (i) we show how a multicomponent Euler system emerges in the high-friction limit via a process of velocity alignment, and how the Maxwell-Stefan system may be viewed as the zero-mean-motion approximation of the former. (ii) Extend the notion of renormalized solutions to the Maxwell-Stefan system and use it in conjunction with symmetrized relative entropy to obtain uniqueness results.

References

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