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# PDEs Applications to Nonlinear Phenomena
# Special Session B15

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This Special Session is meant to encompass several modern active fields of Applied Mathematics like, for instance, Stochastic Partial Differential Equations, Fluid Dynamics, Nonlinear Elliptic Partial Differential Equations governed by Local as well as Nonlocal Operators, Mathematical Viscoelasticity, Primitive Equations in Climatology, Conservation Laws in Fluid Mechanics and Nonlinear Wave Equations.

**Schedule and Abstracts**

**July 25, 2024**

**11:30–11:50 Vincenzo Vespri (University of Firenze, ITALY)**

**Some preliminary results about anisotropic operators**

*Abstract.* In this talk we will speak about the elliptic or parabolic anisotropic equations whose prototypes are

$$\sum_{i=i}^{N} D_i\big(|D_i u|^{p_i-2} D_i u\big) = 0,$$

$$u_t = \sum_{i=i}^{N} D_i\big(|D_i u|^{p_i-2} D_i u\big),$$

with $1 < p_1 \leq p_2 \leq \cdots \leq p_N < \infty$.

Note that when all the $p_i$ are equal we have the pseudo $p$-Laplacean equation. These equations were initially studied by Giaquinta and Marcellini.

Define

$$\frac{1}{\bar{p}} = \frac{1}{N} \sum_{i=1}^{N} \frac{1}{p_i},$$

and assume $\bar{p} < N$ and $p_{max} < \frac{N\bar{p}}{N-\bar{p}} = p*$: then it is possible to prove that any weak solution is bounded.

The proof of the boundness is based on DeGiorgi and Moser iteration techniques and on a suitable anisotropic Sobolev embedding theorem due to Troisi.

The boundness argument gives also regularity under suitable hypotheses: roughly speaking, assume that the coefficients are differentiable and the derived equation has the same structural conditions. Then, by repeating the boundness argument, one gets the boundness of the derivatives.

Not much is known when the coefficients are depending upon x and not differentiable because the classical De Giorgi theory is not working. In this talk we will speak about what is known till now.

**12:00–12:20 Kazuo Yamazaki (University of Nebraska Lincoln, USA)**

**Stochastic partial differential equations**

*Abstract.* Very recently, there has been much new developments in the research area of stochastic partial differential equations in fluid mechanics. For example, the technique of convex integration led to new results of non-uniqueness in law for various stochastic partial differential equations in fluid mechanics. The technique of convex integration also led to new solution theory of ill-posedness for various locally critical/supercritical singular stochastic partial differential equations in fluid mechanics. Finally, there has been renewed effort to extend local solution theory of singular stochastic partial differential equations to global in time. We discuss some of these recent developments.

**12:30–12:50 Pierluigi Colli (University of Pavia, ITALY)**

**Phase separation and the Cahn–Hilliard–Oono system**

*Abstract.* The talk reports on results for the Cahn–Hilliard–Oono system, which is of interest in the study of pattern formations in phase-separating materials. Well-posedness and optimal control, with the control u located in the mass term, are discussed for the related initial-boundary value problem. General potentials for the phase variable are admitted, in particular a singular potential can be considered. Next, the so-called separation property is shown in some physically relevant cases. The optimal control problem is then addressed and optimality conditions are investigated.

**14:30–14:50 Enzo Vitillaro (University of Perugia, ITALY)**

**On the eigenvalue problem for a bulk/surface elliptic system**

*Abstract.* In this talk we shall deal with the doubly elliptic eigenvalue problem

$$\begin{cases} -\Delta u = \lambda u & \text{in } \Omega, \\ u = 0 & \text{on } \Gamma_0, \\ -\Delta_\Gamma u + \partial_\nu u = \lambda u & \text{on } \Gamma_1, \end{cases}$$

where $\Omega$ is a bonded open open subset of $\mathbb{R}^N$, $N \geq 2$, with $C^r$ boundary $\Gamma = \partial\Omega$, $r \geq 1$, $(\Gamma_0, \Gamma_1)$ is a partition of $\Gamma$ with $\Gamma_1$ being nonempty and relatively open on $\Gamma$, $\mathcal{H}^{N-1}(\overline{\Gamma_0} \cap \overline{\Gamma_1}) = 0$ and $\mathcal{H}^{N-1}(\Gamma_0) > 0$. Here $\Delta$ and $\Delta_\Gamma$ respectively denote the Laplace and the Laplace–Beltrami operators, in $\Omega$ and on $\Gamma$, while $\lambda$ is a real (or complex) scalar.

The eigenvalue problem above, which looks to be new in the mathematical literature, at least when $\Gamma_0 \neq \emptyset$, arises when looking for standing wave solutions of the evolutionary boundary value problem

$$\begin{cases} w_{tt} - \Delta w = 0 & \text{in } \mathbb{R} \times \Omega, \\ w = 0 & \text{on } \mathbb{R} \times \Gamma_0, \\ w_{tt} - \Delta_\Gamma w + \partial_\nu w = 0 & \text{on } \mathbb{R} \times \Gamma_1, \end{cases}$$

which models small free vibrations of the drumhead of a bassdrum.

The aim of the talk is to show that many well–known properties of eingenvalues and eigenfuctions in the case $\Gamma_1 = \emptyset$ extend to this case.

**15:00–15:20 Dimitri Mugnai (University of Tuscia, ITALY)**

**Leray–Lions-type equations with bounded solutions**

*Abstract.* We prove the existence of entire bounded solutions for some classes of quasilinear elliptic equation in $\mathbb{R}^N$ driven by a Leray-Lions operator of $(p, q)-$type. We need two basic tools: a suitable extension of the celebrated convergence lemma of Boccardo-Murat-Puel and a variational approach in intersections of Banach spaces by Candela-Palmieri.

### 15:30–15:50 George Avalos (University of Nebraska Lincoln, USA)

### Continuous and Numerical Analysis of a Biot-Stokes PDE System

*Abstract.* In this talk, we describe our recent work concerning the semigroup well-posedness a of a coupled system of partial differential equations which describes a porous, elastic structure saturated by a given incompressible fluid flow. Moreover, we shall describe a companion finite element method (FEM) by which to numerically approximate the solutions of this particular Biot-Stokes model. The PDE system can be concisely as follows: the Biot system is invoked on a given domain. Moreover, the incompressible Stokes or Navier-Stokes equations evolve within another (distinct) geometry. These respective Biot and Stokes dynamics are coupled by means of interfacing (tangential) Beaver-Joseph-Saffman boundary conditions (BC). These BC are well-known in the literature to constitute the appropriate ones for Stokes or Navier-Stokes flow over a poroelastic surface. Very recently, we established semigroup well-posedness for this Biot-Stokes PDE interaction. Subsequently, we have shown that the arguments which are used to show maximality of the associated Biot-Stokes semigroup generator can be adopted so as to devise a mixed variational formulation for the Biot-Stokes dynamics.

### 16:00–16:20 Raffaella Servadei (University of Urbino, ITALY)

### Fractional Brezis-Nirenberg type equations

*Abstract.* In this talk we deal with a nonlocal fractional Brezis-Nirenberg type problem in presence of jumping nonlinearities and we prove the existence of a nontrivial solution for it, using a recent Linking type theorem due to Perera and Sportelli (see [2]), and some regularity results for weak solutions of nonlocal problems, which are of independent interest.

All these results appeared in a joint paper in collaboration with Giovanni Molica Bisci, Kanishka Perera and Caterina Sportelli (see [1]).

**References**

[1] G. Molica Bisci, K. Perera, R. Servadei, C. Sportelli, *Nonlocal critical growth elliptic problems with jumping nonlinearities*, J. Math. Pures Appl., 183 (2024), 170–196.

[2] K. Perera, C. Sportelli, *New linking theorems with applications to critical growth elliptic problems with jumping nonlinearities*, J. Differential Equations, 349 (2023), 284–317.

### 17:00–17:20 Giovanni Molica Bisci (University of Urbino, ITALY)

### Some minimax results for nonsmooth functionals in the Calculus of Variations

*Abstract.* In the last years, elliptic equations involving a nonsmooth term have attracted several outstanding mathematicians and the interest towards this kind of problems has grown more and more, not only for their intriguing analytical structure, but also in view of their applications in a wide range of contexts. Motivated by this wide interest in the literature, the leading purpose of this talk is to present some recent results on nonsmooth elliptic equations, mainly related to a wide class of functionals defined through multiple integrals of Calculus of Variations. Applications to quasilinear boundary value problems will be presented and some open problems briefly discussed; see [1] and Chapter 8 in [2] for related topics.

**References**

[1] C. Alves, G. Molica Bisci, and S. da Silva, *New minimax theorems for lower semicontinuous functions and applications*, ESAIM: Control, Optimisation and Calculus of Variations. DOI: https://doi.org/10.1051/cocv/2024005 (in press).

[2] G. Molica Bisci and P. Pucci, Nonlinear Problems with Lack of Compactness, De Gruyter Series in Nonlinear Analysis and Applications **36** (2021), i+vii, 1–266.

**17:30–17:50 Vincent Martinez (City University of New York, USA)**

**Determining Modes, Data Assimilation, and the Paradigm of Finite-Dimensional Intertwinements**

*Abstract.* This talk will discuss a new concept of *finite-dimensional intertwinement* which unifies the classical property of Determining Modes, as defined by Foias and Prodi in 1967, and the recent convergence results of Data Assimilation algorithms, by Olson and Titi 2003 and Azouani, Olson, Titi 2014 in the context of the 2D Navier-Stokes equations. We introduce this definition, discuss rigorous results that intimately connect these three ideas, and present various numerical results. This is joint work with Elizabeth Carlson (Caltech), Aseel Farhat (FSU), and Collin Victor (TAMU).

**18:00–18:20 Alberto Maione (Centre de Recerca Matemàtica of Barcelona, SPAIN)**

**$H$-compactness for nonlocal linear operators in fractional divergence form**

*Abstract.* In this talk we present a new result about the compactness with respect to the H-convergence for a class of non-symmetric and nonlocal linear operators in fractional divergence form, where the oscillations of the matrices are prescribed outside the reference domain. The compactness argument presented today bypasses the failure of the classical localisation techniques, that mismatch with the nonlocal nature of the operators. In the second part of the presentation, we assume symmetry and show an equivalence between the H-convergence of the nonlocal operators and the Γ-convergence of the corresponding energies. At the end of the talk a list of some open problems and new research directions drawn from this work will be presented.

This research is carried out in collaboration with Maicol Caponi (University of Naples - Federico II) and Alessandro Carbotti (University of Salento).

**July 26, 2024**

**11:30–11:50 Genni Fragnelli (University of Tuscia, ITALY)**

**A new look at the beam equation**

*Abstract.* We will discuss a very recent approach to the study of beam-like equations. After introducing the needed mathematical setting for these classes of problems, we will provide some existence results and the description of the behaviour of the solutions for some concrete models. The results presented in this talk are based on a joint work with Dimitri Mugnai.

**12:00–12:20 Aric Wheeler (Duke University, USA)**

**Nonlinear stability of two-dimensional periodic waves in parabolic systems with conservation laws**

*Abstract.* We show that assuming the background periodic wave is diffusively stable, a stronger form of spectral stability, then the wave is nonlinearly stable even in the presence of conservation laws. The key difference with the case without conservation law analyzed by Melinand-Rodrigues is that even for extremely nice perturbations the linearized semigroup decays at a slow rate and so phase modulations play a deeper role. This work is joint with L. Miguel Rodrigues.

**12:30–12:50 Maicol Caponi (University of Naples, ITALY)**

**The viscoelastic paradox in a nonlinear Kelvin-Voigt type model of dynamic fracture**

*Abstract.* In this talk we consider a dynamic model of fracture for viscoelastic materials, in which the constitutive relation, involving the Cauchy stress and the strain tensors, is given in an implicit nonlinear form. We prove the existence of a solution to the associated viscoelastic dynamic system on a prescribed time-dependent cracked domain via a discretisation-in-time argument. Moreover, we show that such a solution satisfies an energy-dissipation balance in which the energy used to increase the crack does not appear. As a consequence, in analogy to the linear case, this nonlinear model exhibits the so-called viscoelastic paradox.

This is a joint work with A. Carbotti and F. Sapio.

**14:30–14:50 Elisabetta Brocchieri (University of Graz, AUSTRIA)**

**Cross-diffusion system driven by fast reaction limit: weak solutions and weak-strong stability**

*Abstract.* Cross-diffusion systems are nonlinear parabolic systems that model the evolution of densities of multi-component populations in interaction. In this talk, we prove the existence of weak solutions for a starvation-driven cross-diffusion system, obtained as the limit of a reaction-diffusion system with linear diffusion and fast reaction. The main tools used to rigorously pass to the limit consist of a priori estimates, given by the analysis of a family of entropy functionals, and a compactness argument. However, we also investigate the regularity of the obtained solution, by improving the entropy a priori estimates using a bootstrap argument. We conclude the analysis with a weak-strong stability result. This is a joint work with L. Corrias, L. Desvillettes and H. Dietert.

**15:00–15:20 Michael Hecht (Center for Advanced System Understanding, GERMANY & University of Wrocław, POLAND)**

**Fast Multivariate Newton Interpolation for Downward Closed Polynomial Spaces and Applications to Numerical Differential Geometry**

*Abstract.* We introduce a fast Newton interpolation algorithm with a runtime complexity of $\mathcal{O}(Nn)$, where $N$ denotes the dimension of the underlying downward closed polynomial space and $n$ its $l_p$-degree, where $p > 1$. We demonstrate that the algorithm achieves the optimal geometric approximation rate for analytic *Bos-Levenberg-Trefethen functions* in the hypercube. In this case, the Euclidean degree ($p = 2$) emerges as the pivotal choice for mitigating the curse of dimensionality. The spectral differentiation matrices in the Newton basis are sparse, enabling the implementation of fast pseudo-spectral methods on flat spaces, polygonal domains, and regular manifolds.

In particular, we revisit our former contribution, entitled *Global Polynomial Level Sets (GPLS) for Numerical Differential Geometry of Smooth Closed Surfaces.*

The GPLS provides an approximation for a wide range of smooth surfaces, which are initially given solely as point clouds, using a global polynomial level set. This enables efficient and accurate computation of various differential-geometric quantities, such as mean and Gauss curvature, or even higher-order terms like the Laplacian of mean curvature, in a straightforward manner. The GPLS significantly reduces the number of surface points required compared to classic alternatives that rely on surface meshes or embedding grids. We sketch extensions to higher dimensions and discuss applications in numerical differential geometry.

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