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Exploiting Low-rank Structures for the Solution of PDEs
Special Session B1

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Exploiting low-rank structures for the solution of partial differential equations (PDEs) has emerged as a powerful technique in the field of numerical simulation and scientific computing. Many real-world systems exhibit inherent low-dimensional patterns among solution variables, allowing for more efficient computation. Techniques like reduced-order modeling, tensor decomposition, matrix factorizations and certain machine learning methods help uncover and exploit these structures, enabling faster simulations and reduced memory usage. This approach not only accelerates the analysis of physical phenomena but also finds applications in fields such as optimal control, uncertainty quantification and computational design. By efficiently approximating complex PDE solutions with fewer degrees of freedom, low-rank methods facilitate scalable algorithms and offer enhanced interpretability, making them invaluable tools for tackling high-dimensional problems in various scientific and engineering domains.

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Low-rank Matrix Solvers for Evolutionary PDEs

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We consider time-dependent partial differential equations (PDEs) of the form

$$\begin{cases} u_t &= \mathcal{L}(u) + f, \quad \text{in } \Omega \times (0, T], \\ u &= g, \qquad\qquad \text{on } \partial\Omega, \\ u(x, 0) &= u_0(x), \end{cases} \tag{1}$$

where $\Omega \subset \mathbb{R}^d$, $d = 1, 2, 3$, and $\mathcal{L}$ is a linear differential operator involving only spatial derivatives.

Many efficient and diverse numerical methods have been developed in the literature for the solution of (1). Roughly speaking, these can be divided into two main categories: *time-marching* schemes and *all-at-once* methods.

In the former class, we first semi-discretized (1), namely we discretize only the space component of the differential problem by applying our favorite scheme. Then, a time integrator is applied to deal with the time component of the problem. The latter step leads to a sequence of linear systems, one at each time step, of the form

$$A_i u_i = f_i, \qquad A_i \in \mathbb{R}^{\bar{n} \times \bar{n}},\ i = 1, \ldots, \ell, \tag{2}$$

where $\bar{n}$ is the number of spatial degrees of freedom and ℓ is the number of time steps.

All-at-once methods significantly differ from time-marching schemes. Indeed, in place of a two-step strategy where we first discretize in space and then in time, the space and time components of (1) are discretized at once whenever an all-at-once approach is adopted. This different point of view in the discretization phase impacts the linear algebra step as well. In particular, rather than the sequence of linear systems in (2), we need to solve a single, yet much larger, linear system of the form

$$\mathcal{A}\mathbf{u} = \mathbf{f}, \tag{3}$$

where $\mathcal{A} \in \mathbb{R}^{\bar{n}\ell \times \bar{n}\ell}$, $\mathbf{u} = [u_1^T, \ldots, u_\ell^T]^T, \mathbf{f} = [f_1^T, \ldots, f_\ell^T]^T \in \mathbb{R}^{\bar{n}\ell}$. The dimensions of $\mathcal{A}$ may make look the all-at-once formulation (3) computationally prohibitive. Nevertheless, the coefficient matrix $\mathcal{A}$ is often extremely structured. This feature, along with other characteristics of the problem at hand, can be fully exploited to design a very efficient numerical linear algebra phase to the point that the all-at-once scheme oftentimes outperforms the time-marching approach (2).

We leverage this structure to reformulate (3) in terms of a single Sylvester matrix equation of the form

$$K\mathbf{U} + M\mathbf{U}B = \mathbf{F}, \tag{4}$$

where $K, M \in \mathbb{R}^{\bar{n} \times \bar{n}}$ are the spatial stiffness and mass matrices, respectively, $B \in \mathbb{R}^{\ell}$ is the discrete time operator stemming from the adopted time integrator, and $\mathbf{U} = [u_1, \ldots, u_\ell], \mathbf{F} = [f_1, \ldots, f_\ell] \in \mathbb{R}^{\bar{n} \times \ell}$. The matrix equation formulation (4) of the discrete problem naturally encodes the separability of the spatial and time derivatives of the underlying differential operator and the tensorized nature of the cylinder $\Omega \times (0, T]$ in (1). We show that this lets us employ different strategies to deal with the spatial and time components of the algebraic problem and combine them in a very efficient solution procedure. In particular, state-of-the-art projection techniques are proposed to tackle the spatial operator while the circulant-plus-low-rank structure of the time discrete operator is exploited to derive effective solution schemes [1]. The resulting algorithm is able to efficiently solve problems with a tremendous number of degrees of freedom while maintaining a low storage demand as we will illustrate in several numerical results.

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Matrix-oriented discretizations for evolutionary PDEs and applications

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Among evolutionary PDEs we focus on reaction-diffusion systems where the so-called Turing instability can give rise to a rich scenario of spatial patterns attained as stationary solutions. To capture the morphological features of Turing patterns is a challenging task because both very fine space discretization and longtime integration are needed. We show that, after semi-discretization in space, the Kronecker structure of the diffusion matrix can be exploited to build matrix-oriented (MO) versions of some classical time integrators. In particular, we con- sider finite differences on square domains and classical Lagrangian FEM on more general domains and special surfaces. In the first case, the fully discrete problem is reformulated as a sequence of Sylvester matrix equations, that we solve by the reduced approach in the associated spectral space [1]. In the second case, at each time step we solve a multiterm Sylvester matrix equation by the matrix form of the Preconditioned Conjugate Gradient (MO-PCG) [2].

As an application, we consider Turing pattern approximation in the DIB morphochemical PDE system for battery modeling [3] and we present encouraging results wrt the classical vector approach (solving large sparse linear systems) in terms of execution times and memory storage. Moreover, we show that the MO approach based on the reduced method can be extended to high order diffusion PDEs like the Cahn-Hilliard equation (4th order) among phase-field models.

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Low rank methods for Turing pattern approximation in reaction-diffusion PDE systems

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In this talk we will discuss two low rank methods for the numerical approximation of Turing patterns, that are stationary solutions of reaction-diffusion PDE (RD-PDE) systems by means of Proper Orthogonal Decomposition (POD) and Dynamic Mode Decomposition (DMD). Both techniques present inaccurate approximations, therefore we will introduce two novel algorithms that aim at stabilizing the studied problem. In the first part of the talk we focus on the stabilization of the POD-DEIM technique. We show that solutions of surrogate models built by classical POD-DEIM exhibit an unstable error behaviour over the dimension of the reduced space. To overcome this drawback, we add a correction term that provides missing information to the reduced model and we apply the POD-DEIM technique to the corrected model. To further improve the computational efficiency, we propose an adaptive version of this algorithm in time that accounts for the peculiar dynamics of the RD-PDE in presence of Turing instability.

In the second part we show that DMD does not provide accurate approximation for datasets describing oscillatory dynamics, like spiral waves, relaxation oscillations and spatio-temporal Turing instability. Inspired by the classical "divide and conquer" approach, we propose a piecewise version of DMD (pDMD) to overcome this problem. The main idea is to split the original dataset in N submatrices and then apply the exact (randomized) DMD method in each subset of the obtained partition. We describe the pDMD algorithm in detail and we introduce some error indicators to evaluate its performance when N is increased.

Throughout the talk, we will show the effectiveness of the proposed methods in terms of accuracy and computational cost for a selection of RD systems.

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Polynomial Approximation for Nonlinear Model Reduction by Moment Matching

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Many physical applications involve modeling the state of a complex system as a high dimensional system of ODEs with algebraic constraints. The high dimensionality of these models introduces significant computational challenges in trying to analyse the behaviour of the system, which is often known as the "curse of dimensionality". One way to alleviate the computational complexity of such problems is by using reduced order modeling to create a simpler system that preserves important characteristics.
This talk focuses on nonlinear model reduction using the technique of moment matching for nonlinear dynamical systems whose input is generated by a signal generator system. Un- der certain assumptions, there exists an invariant manifold associated with the solution of a "Sylvester-like" PDE which can then be related to the steady-state response of the system. This talk introduces a procedure to numerically approximate the solutions to the invariance equa- tions that arise in moment matching. The Galerkin residual method is employed to find an approximate solution to the invariance equation using a Newton iteration on the coefficients of a monomial basis expansion of the solution. These approximate solutions to the invariance equation can then be used to construct reduced order models.

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Residual Data-Driven Variational Multiscale Reduced Order Models for Convection-Dominated Flows

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We investigate the modeling of sub-scale components of proper orthogonal decomposition reduced order models (POD-ROMs) of convection-dominated flows. We propose ROM closure models that depend on the ROM residual. We illustrate the new residual-based data-driven ROM closure within the variational multi-scale (VMS) framework and investigate it in the numerical simulation of a one-dimensional parameter-dependent convection-dominated problem and two-dimensional time-dependent advection-diffusion-reaction problem and flow past a cylinder. Our numerical investigation shows that the new residual-based data-driven VMS-ROM is more accurate than both the coefficient-based data-driven ROMs and the standard Galerkin ROM.

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Structure-Preserving Learning of High-Dimensional Lagrangian Systems

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Lagrangian mechanics is a foundational modeling approach in diverse areas such as structural mechanics, aerospace engineering, wave propagation, and soft robotics. The state equations of these systems are derived from by applying the Euler-Lagrange equations to the Lagrangian energy function, which is specified by the difference of kinetic and potential energy. This results in a highly structured state space and model equations, which are, however, too high-dimensional for fast simulation and control. This talk gives an overview of a few recently developed approaches for learning structure-preserving reduced-order models.

First, we discuss Lagrangian Operator Inference (LOPINF), a data-driven reduced-order model learning framework that respects the Lagrangian structure and learns the associated linear operators for the kinetic and potential energies. We demonstrate the wide applicability and effectiveness of the approach an Euler-Bernoulli beam model from structural mechanics, the sine-Gordon (nonlinear) wave equation, and two large-scale discretization of a soft robot fishtail with 251,000 and 779,232 degrees of freedom, respectively. With much fewer training data than non-structured learning methods require, the Lagrangian learned reduced models provides stability and robustness, and have long-term predictive accuracy.

Second, we present an extension of LOPINF that adds structure-preserving machine learning to learn the nonlinear terms in the Lagrangian models. Through a two-step approach, we first learn the best-fit linear Lagrangian ROM via LOPINF and then use a structure-preserving machine learning method to learn nonlinearities in the reduced space. The proposed approach can learn a structure-preserving nonlinear ROM purely from data, unlike the previously introduced Lagrangian Operator Inference approach that required knowledge about the mathematical form of nonlinear terms (if used on nonlinear models). From a machine learning perspective, it accelerates the training of the structure-preserving neural network by providing an informed prior (i.e., the linear Lagrangian ROM structure), and it reduces the computational cost of the network training by operating on the reduced space. The method is first demonstrated on two simulated examples: a conservative nonlinear rod model and a two-dimensional nonlinear membrane with nonlinear internal damping. Finally, the method is demonstrated on an *experimental dataset* consisting of digital image correlation measurements taken from a lap-joint beam structure from which a predictive model is learned that captures amplitude-dependent frequency and damping characteristics accurately. The numerical results demonstrate that the proposed approach yields generalizable nonlinear ROMs that exhibit bounded energy error, capture the nonlinear characteristics reliably, and provide accurate long-time predictions outside the training data regime.

We highlight that the proposed approaches exploit knowledge of the structure of the governing equations (but not their discretization) to define the form and parametrization of a Lagrangian ROM which can then be learned from projected snapshot data. The method does not require access to FOM operators or computer code.

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Flexible domain decomposition-based couplings of conventional and data-driven models via the Schwarz alternating method

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This talk will describe some recent advancements in developing a rigorous mathematical framework for the domain decomposition-based coupling of arbitrary combinations of first-principles numerical methods (i.e., full order models or FOMs) with data-driven models (i.e., projection-based reduced order models or ROMs) under the flexible Heterogeneous Nu- merical Methods (fHNM) project at Sandia National Laboratories. Specifically, I will present a recent extension of the Schwarz alternating method [1,2] that enables the creation of FOM- ROM and ROM-ROM couplings from nonlinear monolithic problems [3]. This method works by performing an overlapping or non-overlapping domain decomposition (DD) of the physical domain, and solving a sequence of problems on these subdomains, with information propagating through carefully-constructed transmission conditions on the subdomain boundaries. The solution in each subdomain may be characterized by much simpler, localized dynamics, and hence more easily modeled and solved. We will showcase recent results obtained by implementing this method in the open-source Pressio demo-apps library, which demonstrate that the Schwarz alternating method is capable of delivering stable and accurate hybrid models when applied to advection-dominated fluid flow problems with moving shocks, for which monolithic approaches are prohibitively expensive or grossly inaccurate. We will additionally demonstrate that online CPU-time gains are achievable through an implementation of the additive variant of the Schwarz alternating method, which admits more parallelism by solving all subdomain-local problems simultaneously on different processes/threads. Time-permitting, we will describe some ongoing work aimed at developing an automated learning algorithm leveraged to select “optimal” (i.e, accuracy and efficiency maximizing) DDs and ROM vs. FOM placements/assignments.

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Dynamical Low-Rank Approximations for parametric infinite horizon optimal control problems

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Optimal control is a strategy to enhance the performance of complex dynamical systems by minimizing a predefined cost functional, with the overarching goal of steering the system dynamics towards a desired stable configuration. However, its applicability is still limited due to high-dimensional state spaces, especially in parametric settings. To address these challenges, we exploit the Dynamical Low-Rank Approximation (DLRA) methodology for the efficient and precise resolution of high-dimensional feedback control problems, relying on State Dependent Riccati Equations (SDREs) for infinite horizon optimal control. DLRA not only furnishes a compact, low-dimensional representation of the system dynamics but also dynamically evolves alongside the problem, ensuring increased accuracy and enabling effective real-time control settings. In this work, we propose two novel algorithms for tackling feedback control problems:

- Monolithic DLRA (mDLRA-SDREs) for SDRE control, where the SDRE is solved for all parametric instances simultaneously.
- Cascade DLRA (cDLRA-SDREs) for SDRE control, where each SDRE Riccati solution is obtained by leveraging the solution of the previous parameter, enhancing convergence and resulting in a faster evolution of the controlled low-rank system.

We validate the robustness and effectiveness of the algorithms through nonlinear test cases, such as optimal control problems governed by Burgers' equation and the Allen–Cahn equation. Moreover, we compare the DLRA-based control strategies with standard global Proper Orthogonal Decomposition model order reduction, showing advantages in terms of speed-up and accuracy with respect to the ground truth solution.

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Optimal bounds for POD approximations of infinite horizon control problems based on time derivatives

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In this talk we consider the numerical approximation of infinite horizon problems via the dynamic programming approach. The value function of the problem solves a Hamilton-Jacobi-Bellman (HJB) equation that is approximated by a fully discrete method. It is known that the numerical problem is difficult to handle by the so called curse of dimensionality. To mitigate this issue we apply a reduction of the order by means of a new proper orthogonal decomposition (POD) method based on time derivatives. We carry out the error analysis of the method using recently proved optimal bounds for the fully discrete approximations. Moreover, the use of snapshots based on time derivatives allow us to bound some terms of the error that could not be bounded in a standard POD approach. Some numerical experiments show the good performance of the method in practice.

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Using Empirical Tensor Train Approximation for Solving High-Dimensional Optimal Control Problems

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We display two approaches to solve finite horizon optimal control problems. First we solve the Bellman equation numerically by employing the Policy Iteration algorithm. Second, we introduce use open loop methods to learn the value function. To overcome computational infeasability we use tensor trains and multi-polynomials, together with high-dimensional quadrature, e.g. Monte-Carlo. Furthermore, we compare the tensor methods to neural networks and kernel approaches. By controlling a destabilized version of viscous Burgers and a diffusion equation with unstable reaction term numerical evidence is given.

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Functional Analytic Methods in Quantum Many-Body Theory
Special Session B2

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This session aims at presenting the most recent developments in mathematical physics with an emphasis on the derivation of effective theories for describing complex many-body quantum systems at all scales. A paradigmatic example is the N-body Schrödinger equation

$$i\hbar\partial_t\psi = H\psi\ , \quad H = \sum_{i=1}^{N} -\hbar^2\Delta_i + \lambda \sum_{1\leq i<j\leq N} V(x_i - x_j)\ .$$

The particle number N is typically of the order 10^{23}. Due to the enormous number of degrees of freedom, the analysis of the Schrödinger equation is a massive mathematical challenge. However, in certain scaling regimes one may study physical quantities by proving convergence to certain effective (usually non-linear) theories such as Hartree–Fock theory or the Gross–Pitaevskii equation. Other important examples where this approach has proven useful are the quantum Hall effect or spin systems.

In recent years new functional analytic methods have been developed in this context, and our session is meant to provide a platform for the exchange of ideas among researchers working in the field with different mathematical background and research focus.

For more information visit `https://sites.rutgers.edu/umi-ams-joint-meeting`.

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Propagation bounds for 2D many-body fermion systems in a magnetic field

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We construct the infinite-volume dynamics for electrons with short-range two-body interactions in $\mathbb{R}^2$ subject to a periodic potential and a constant magnetic field. The crucial step is to establish propagation bounds of Lieb-Robinson type for the dynamics. We also discuss related results and applications.

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Out-of-time-ordered correlators of mean-field bosons via Bogoliubov theory

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Quantum many-body chaos studies the scrambling of quantum information among large numbers of degrees of freedom. It rests on the prediction that out-of-time-ordered correlators (OTOCs) of the form $\langle [A(t), B] \rangle$, can be connected to classical dynamics. We rigorously prove a variant of this correspondence principle for mean-field bosons. More precisely, we consider N bosons in $\mathbb{R}^3$ with mean-field interactions, that are described by the Hamiltonian H_N, and study the Heisenberg dynamics $A(t) = e^{itH_N} A e^{-itH_N t}$ of an operator A. We show that for suitable operators A, B the OTOC $\langle [A(t), B] \rangle$, with a factorized state $\varphi^{\otimes N}$ as reference state, is in the limit $N \to \infty$ explicitly given by a suitable symplectic Bogoliubov dynamics. The proof uses Bogoliubov theory and extends to higher-order correlators of operators at different times. For these, it yields an out-of-time-ordered analog of the Wick rule. This is joint work with Marius Lemm.

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A Short Proof of BEC in the GP Regime and Beyond

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Cristina Caraci
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In this talk, we present a new and self-contained proof of BEC for the ground state of N bosons moving in the three-dimensional unit torus and interacting through a pair potential with scattering length of order $N^{\kappa-1}$, for a parameter $\kappa \in [0, \frac{1}{20})$. The proof is based on an application of the Schur complement formula and mild a priori bounds on high momentum occupation number operators in the ground state. It improves and significantly simplifies previous results obtained jointly with A. Adhikari and B. Schlein. Compared to previous strategies, our proof avoids both the use of operator exponential expansions and box localization arguments. The talk is based on joint work with M. Brooks, C. Caraci and J. Oldenburg.

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Adiabatic evolution of low-temperature many-body quantum systems

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We consider finite-range, many-body fermionic lattice models and we study the evolution of their thermal equilibrium state after introducing a weak and slowly-varying time-dependent perturbation. Under suitable assumptions on the external driving, we derive a representation for the average of the evolution of local observables via a convergent expansion in the perturbation, for small enough temperatures. Convergence holds for a range of parameters that is uniform in the size of the system. Under a spectral gap assumption on the unperturbed Hamiltonian, convergence is also uniform in temperature. As an application, our expansion allows to prove closeness of the time-evolved state to the instantaneous Gibbs state of the perturbed system, in the sense of expectation of local observables, at zero and at small temperatures. Our strategy is based on a rigorous version of the Wick rotation and fermionic cluster expansion. This talk is based on [1].

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Upper bound on the ground state energy of a dilute gas of hard sphere bosons

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In the last years, there has been substantial progress in the mathematical understanding of the low-energy properties of dilute Bose gases. We discuss a recent result on the energy of a dilute gas of hard sphere bosons of radius a in the thermodynamic limit where the number of particles N and the size of the box L^3 are sent to infinity keeping the density $\rho = N/L^3$ fixed.

More precisely, we prove that in the dilute regime $\rho a^3 \ll 1$ the ground state energy per particle in the thermodynamic limit $e(\rho)$ satisfies the upper bound

$$e(\rho) \leq 4\pi a\rho(1 + C(\rho a^3)^{1/2}).$$

Our result thus resolves the ground state energy up to an error of the order of the so-called Lee-Huang-Yang correction.

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Interacting many-particle systems in the random Kac–Luttinger model and proof of Bose–Einstein condensation

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Following a model originally considered by Kac and Luttinger, we study interacting many-particle systems in a random background. The background consists of hard spherical obstacles with fixed radius and that are distributed via a Poisson point process with constant intensity on R^d, $2 \leq d \in N$. Interactions among the (bosonic) particles are described through repulsive pair potentials of mean-field type. As a main result we prove that in the thermodynamic limit, (complete) Bose–Einstein condensation (BEC), in probability or with probability almost one depending on the strength of the interaction, occurs into the minimizer of a Hartree-type functional. As an important ingredient, we use very recent results obtained by Alain-Sol Sznitman regarding the spectral gap of the Dirichlet Laplacian in a Poissonian cloud of hard spherical obstacles in large boxes. To the best of our knowledge, our paper provides the first proof of BEC, for systems of interacting particles in the Kac–Luttinger model, or in fact for some continuous higher-dimensional random model.

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Quantum logarithmic Sobolev inequalities for quantum many-body systems

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The mixing time of a quantum Markov semigroup describing the dissipative evolution of an open quantum many-body system can be bounded using optimal constants of certain quantum functional inequalities, such as the modified logarithmic Sobolev constant. For classical spin systems, the positivity of such constants follows from a mixing condition for the Gibbs measure, via quasi-factorization results for the entropy.

Inspired by the classical case, we present a strategy to derive the positivity of the modified logarithmic Sobolev constant associated to the dynamics of certain quantum systems from some clustering conditions on the Gibbs state of a local, commuting Hamiltonian. In particular, we show that, for a large class of geometrically-2-local models of Davies generators with commuting Hamiltonians, the mixing time is at most logarithmic in the system size, and this yields the so-called rapid mixing of dissipative dynamics. This is particularly relevant for 1D systems, for which we show rapid thermalization with a system size independent decay rate only from a positive gap in the generator. We also prove that systems in hypercubic lattices of any dimension, and exponential graphs, such as trees, have rapid mixing at high enough temperatures.

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Lower bound for the free energy expansion of low temperature Bose gas

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We consider a system of many interacting bosons in 3D and we show how to derive a lower bound for the expansion, in dilute limit, of the free energy density in thermodynamical regime at low temperature. The particles interact through a pairwise, repulsive potential satisfying a non-increasing condition, covering the case of the hard-core potential. The novelty of the method is in the joint action of the Neumann localization in small boxes, which, differently from other localization techniques, does not change the form of the Hamiltonian, together with a splitting and renormalization of the potential, which allows the rigorous application of the Bogoliubov theory. The expansion, at $T = 0$, corresponds to the so called Lee-Huang-Yang formula, which approximates the energy density up to the second order in the dilute limit.

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Mobility edge, dynamics of the participation ratio, and percolation properties of the landscape function

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We will discuss non-asymptotic two-sided analogues of the Weyl law and Lieb-Thirring inequalities for the Schrodinger operator in terms of the landscape function and some open problems regarding its connection with the mobility edge.

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Wu's correction to the ground state energy of a Bose gas in the Gross-Pitaevskii regime

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We consider a Bose gas trapped on the 3d unit torus in the Gross-Pitaevskii regime and compute the ground state energy up to an error which vanishes faster than $N^{-1} \log N$ as $N \to \infty$. Our result matches the prediction by Wu [1] for the correction to the Lee-Huang-Yang formula in the thermodynamic limit.

The proof involves a renormalization of the Hamiltonian through conjugation with unitary operators that implement correlations. It is well-known that a suitable quadratic and cubic correlation structure allows to rigorously reproduce the predictions of Bogoliubov theory in the Gross-Pitaevskii regime. In order to capture Wu's correction we need to modify such a correlation structure compared to the known results. Yet, our proof shows that no further correlation beyond quadratic and cubic is needed for the accuracy we aim for.

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Kane's Euler Topological Metals

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Topology in condensed matter physics is usually studied in the context of topological -insulators-. These are materials which (in the bulk) do not conduct electric current. The aforementioned "topology" relates to the path-connectedness (or lack thereof) of the abstract space of quantum mechanical Hamiltonians. Remarkably, the path-component itself is indexed by a transport coefficient. Recently, Charles Kane (PRL 22) described a new paradigm to experimentally measure the topology of the Fermi surfaces of metals. We shall describe the mathematics behind connecting Kubo's linear response to the metal's Fermi surface Euler characteristic.

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From decay of correlations to locality and stability of the Gibbs state

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I will show that whenever a Gibbs state satisfies decay of correlations, then it is stable, in the sense that local perturbations influence the Gibbs state only locally, and it is local, namely it satisfies local indistinguishability. These implications hold true in any dimension, only require locality of the Hamiltonian and rely on Lieb-Robinson bounds. A central role in the proofs is played by the quantum belief propagation for Gibbs states, which I will briefly review.

Furthermore, I will discuss how our results can be applied to quantum spin systems in any dimension with short-range interactions at high enough temperature, and to one-dimensional quantum spin chains with translation-invariant and exponentially decaying interactions above a threshold temperature that goes to zero in the limit of finite range interactions.

The talk is based on a joint work with Ángela Capel, Stefan Teufel and Tom Wessel.

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[1] Á. Capel, M. Moscolari, S. Teufel, and T. Wessel, *From decay of correlations to locality and stability of the Gibbs state*, arXiv:2310.09182 (2023).

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The free energy of the Bose gas at low density

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The interacting Bose gas is a system composed of a very large number of quantum particles with totally symmetric wavefunction. Below a critical temperature, a phase transition to a Bose-Einstein condensate is expected to occur, and collective behavior emerges from the underlying many-body theory.

At zero temperature we have precise information on the ground state energy and the low-lying spectrum of excitations (at least in certain scaling limits [1]). However, much less is known close to the transition temperature. In this talk I will discuss how thermal excitations can be described by Bogoliubov theory, allowing us to estimate the free energy of the Bose gas in the Gross-Pitaevskii regime [2].

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Acta Mathematica 222, 219-335 (2019)

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Correlation energy for the low density Fermi gas

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In recent decades, the study of many-body systems has been an active area of research in both physics and mathematics. In this talk we consider a system of N interacting fermions with spin $1/2$ confined in a box in the dilute regime. We are interested in studying the correlation energy, defined as the difference between the ground state energy and that of the free Fermi gas. We will discuss how to derive an asymptotics (see [1], [2]) for the correlation energy in the thermodynamic limit, where the number of particles and the size of the box are sent to infinity while keeping the density fixed. In particular, we will present a result (see [2]) on an upper bound for the correlation energy that is consistent with the well-known Huang-Yang formula of 1957.

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Derivation of Kubo formula for Hall conductance

Martin Fraas, *Martin Fraas*
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I will review derivations of Kubo formula for Hall conductance in quantum Hall effect. The focus will be on two recent results: 1. Derivation of Kubo formula with interactions assuming that the Hamiltonian is gapped in the bulk 2. Derivation of Kubo formula with disorder but without interactions.

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The free energy of the dilute Bose gas at low temperature

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We consider a Bose gas at density $\rho > 0$, interacting through a repulsive potential $0 \le V \in L^2(\mathbb{R}^3)$ with scattering length $\mathfrak{a} > 0$. We prove an expansion for the free energy of the system in the dilute regime $\rho\mathfrak{a}^3 \ll 1$, valid at low temperature $T \lesssim \rho\mathfrak{a}$.
More precisely, we consider the Hamiltonian

$$H_N = \sum_{i=1}^{N} -\Delta_{x_i} + \sum_{1\le i\le j\le N} V(x_i - x_j)$$

acting on symmetric space $L^2_s(\Lambda_L^N)$ where $\Lambda_L = [0, L]^3$. The free energy of the system at temperature $T \ge 0$ is given by

$$F(N,T) = -T \log \mathrm{Tr} e^{-H_N/T} = \inf \left\{ \mathrm{Tr} H_N \Gamma + T \mathrm{Tr} \Gamma \log \Gamma \middle| \Gamma \ge 0,\ \mathrm{Tr}\Gamma = 1 \right\}$$

We prove that in the thermodynamic limit $N \to \infty$, $N/L^3 \to \rho$, when $T \le C\rho\mathfrak{a}$ the following expansion holds

$$\begin{aligned} \frac{1}{L^3} F(N,T) &= 4\pi\mathfrak{a}\rho^2 \left(1 + \frac{128}{15\sqrt{\pi}} (\rho\mathfrak{a}^3)^{1/2}\right) \\ &+ \frac{T^{5/2}}{(2\pi)^3} \int_{\mathbb{R}} \log\left(1 - e^{-\sqrt{|p|^4 + \frac{16\pi\rho\mathfrak{a}}{T} p^2}}\right) dp + \mathcal{O}(\rho\mathfrak{a})^{5/2} \end{aligned}$$

In particular our estimate resolves the free energy per unit volume up to and including the Lee–Huang–Yang order $\mathfrak{a}\rho^2(\rho\mathfrak{a}^3)^{1/2}$.

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Point configurations: energy, designs, and discrepancy
Special Session B3

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Uniformly distributing a large number of points in a domain or on a manifold is a question that arises naturally both in pure mathematics (discrete geometry, probability, analysis) and applications (numerical integration, sampling, frame theory etc), and there are numerous ways to measure the quality of a distribution of points: discrepancy, energy minimization, packing and covering radii, lattices, cubature formulas, designs etc, many of which are closely connected to each other. The methods used to address such problems involve a mixture of a variety of areas of mathematics: discrete geometry (polytopes, equiangular lines), combinatorics (combinatorial discrepancy, combinatorial designs, Latin squares), probability (random point processes, large and small deviation bounds), number theory (lattices, diophantine approximation), approximation theory (cubature formulas, spherical designs, interpolation), applied mathematics (compressed sensing, frames), and others. Moreover, a central role in these topics is played by various branches of analysis, in particular, Fourier, harmonic, and functional analysis, as well as potential theory, orthogonal polynomials, and special functions.

The session will concentrate on numerous problems about distributions of points, with a strong focus on the application of the methods of analysis in this circle of questions.

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Irregularities of distribution for bounded sets and half-spaces

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Let $\mathcal{P}_N$ be a set of N points in $\mathbb{R}^d$ ($d \geqslant 2$) and let $E \subseteq \mathbb{R}^d$. We want to estimate the quality of the distribution of these points with respect to a probability measure μ supported in E. We consider a reasonably large family $\mathcal{R}$ of measurable sets and, for $R \in \mathcal{R}$, we introduce the discrepancy

$$\mathcal{D}_N(R) = \operatorname{card}(\mathcal{P}_N \cap R) - N\mu(R) \ .$$

We prove a few theorems which extend several known results.

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Discrepancy and approximation of absolutely continuous measures with atomic measures

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We prove several results concerning the discrepancy, tested on balls in the d-dimensional torus $\mathbb{T}^d$, between absolutely continuous measures and finite atomic measures.

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Discrete analogues of renowned lower bounds in discrepancy theory

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A well-known result due to H. Montgomery states the following. There exists a constant $c > 0$ such that for every given finite sequence $\{p_n\}_{n=1}^N$ in the 2-dimensional torus $\mathbb{T}^2$ the estimate

$$\int_{\mathbb{T}^2} \Big(\Big| \sum_{n=1}^{N} \chi_{x+B_{\frac{1}{2}}}(p_n) - N|B_{\frac{1}{2}}| \Big|^2 + \Big| \sum_{n=1}^{N} \chi_{x+B_{\frac{1}{4}}}(p_n) - N|B_{\frac{1}{4}}| \Big|^2 \Big) \, dx \geqslant cN^{\frac{1}{2}}.$$

holds. In this talk I will present a discrete version of the above estimate for the d-dimensional torus with $d \not\equiv 1 \bmod 4$. Namely, I will show that the lower bound $cN^{(1-\frac{1}{d})}$ holds true if we replace the integration over all the possible translated balls with an average over a finite number of translated balls. Time permitting, I will present two other results in the spirit of the one described above. One concerning the discrepancy on the torus with respect to squares, the other concerning the discrepancy in the unit cube with respect to anchored boxes.

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Spherical cap $\mathbb{L}_2$-discrepancy for spherical Fibonacci points: recent progress

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The spherical cap $\mathbb{L}_2$-discrepancy measures the irregularity (i.e., the deviation from uniform distribution) of a point set on the unit sphere in $\mathbb{R}^3$ in the $\mathbb{L}_2$-sense with respect to spherical caps.

Lower and upper bounds with matching optimal powers $N^{-3/4}$ are known for minimizing N-point configurations. A proof for the conjectured constant of the large N asymptotics is still missing.

In case of a sequence of constructed N-point configurations the best proven upper bound is of order $N^{-1/2}$; cf. [1].

In this talk, we discuss discrepancy bounds and numerical results for spherical Fibonacci points obtained via Lambert equal area transformation applied to the Fibonacci lattice in the unit square.

The talk is based on joint work with Josef Dick (UNSW, Sydney) and Yuan Xu (University of Oregon, USA).

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The L^2-Discrepancy of Planar Convex Bodies averaged over Affine Transformations

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Let $\mathbb{T}^2$ denote the bi-dimensional torus. Consider a convex body $C \subset \mathbb{T}^2$ with piecewise-$\mathcal{C}^1$ boundary, and write $\mathcal{X}_C$ as its characteristic function. For a set $\mathcal{P}_N \subset \mathbb{T}^2$ of N points, consider the quantity

$$D(C, \mathcal{P}_N) = \sum_{p \in \mathcal{P}_N} \mathcal{X}_C(p) - N|C|.$$

In this talk, we study the L^2-discrepancy of C averaged over affine transformations, that is, including translations, dilations, and rotations. Namely, for an angle $\phi \in (0, 2\pi]$, we intend to give estimates on the asymptotic behavior as $N \to \infty$ of

$$\inf_{\#\mathcal{P}_N = N} \int_{-\frac{\phi}{2}}^{\frac{\phi}{2}} \int_{\frac{1}{2}}^{1} \int_{\mathbb{T}^2} |D(\tau + \delta\sigma_\theta C, \mathcal{P}_N)|^2 \, d\tau \, d\delta \, d\theta,$$

where σ_θ is a rotation by an angle θ, and δ is to be understood as a dilation factor.

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Three problems in discrepancy theory

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I will speak about three of my favourite open problems in discrepancy theory. The first is the problem of the "inverse of the discrepancy": In a unit cube of dimension d, given $\varepsilon > 0$, what is the minimal cardinality of a point set with discrepancy at most ε? The question is very relevant from the perspective of numerical analysis, since the bound for the integration error in Quasi-Monte Carlo integration is proportional to the discrepancy to the set of sampling points. The second problem is about the discrepancy with respect to general measures. The classical case is that of the uniform measure, but it is natural to study also discrepancies with respect to other measures, and consider this as a problem of approximating such a measure by a simple atomic measure. A key question then is: Is the uniform measure the one which is most difficult to approximate? The third problem concerns the spherical cap discrepancy on the 2-sphere. While the upper bound from probabilitic existence proofs essentially matches the lower bound (order approx. $N^{-3/4}$), we do not know of a single deterministic point set whose spherical cap discrepancy is smaller than the square-root cancellation which i.i.d. random points exhibit as well. So the problem is to find a deterministic point set whose spherical cap discrepancy is better than that of a random point set.

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Hyperuniformity and Energy on Projective Spaces

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We study Riesz, Green and logarithmic energy on two-point homogeneous spaces. More precisely we consider the real, the complex, the quaternionic and the Cayley projective spaces. For each of these spaces we provide upper estimates for the mentioned energies using determinantal point processes. Moreover, we determine lower bounds for these energies. Furthermore, we extend the notion of hyperuniformity to the projective spaces and study the connection between energy and the Wasserstein distance.

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Rate of convergence in ergodic transformations

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We discuss the speed of convergence of means of ergodic transformations in the torus.

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Gaps in Kronecker sequences and optimal spherical codes

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The celebrated Three-Gap Theorem states that, if one places first N elements of the Kronecker sequence $\{nx\}$, $n = 1, \ldots, N$, on a unit circle, then distances between consecutive points take no more than three distinct values. I will talk about the higher-dimensional version of this problem asking to find the maximal number of gaps in a high-dimensional Kronecker sequence on a flat torus. Recently, Haynes and Marklof solved this problem in two dimensions by showing that the number of gaps in a two-dimensional Kronecker sequence is no greater than 5. I will show how the problem is connected to the general problem of finding optimal spherical codes and explain several new bounds on the number of gaps in all dimensions confirming, in particular, a weak version of the conjecture of Haynes and Marklof in three dimensions.

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Fourier analysis and signal recovery

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Let $f : \mathbb{Z}_N^d \to \mathbb{C}$ be a signal, and let $\widehat{f}$ denote its Fourier transform. If the Fourier transform of f is sent to the values $\{\widehat{f}(m)\}_{m\in S}$ are missing, we ask what conditions the original signal f can be recovered. We are going to give an elementary exposition of this problem and describe some recent results.

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Minimal logarithmic energy and Sobolev discrepancy

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Given a configuration of points on the sphere $x_1, \ldots, x_N \in \mathbb{S}^2$ the discrete logarithmic energy is given by

$$\mathcal{E}(x_1, \ldots, x_N) = \sum_{i \neq j} \log \frac{1}{|x_i - x_j|}.$$

The asymptotic behavior of the minimum of this energy

$$\mathcal{E}(N) = \min_{x_1, \ldots, x_N \in \mathbb{S}^2} \mathcal{E}(x_1, \ldots, x_N)$$

has been extensively studied, but despite all the efforts, only a few terms of the asymptotic expansion are known, see for example [1]. In this talk I will explain the connection of some recent developments with the study of a Sobolev discrepancy introduced by T. Wolff in an unpublished work.

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Riesz Energy with an External Field: When is the Minimizer a Sphere?

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We will discuss the minimization of Riesz energies with external fields

$$I_{s,V}(\mu) = \int_{\mathbb{R}^d}\int_{\mathbb{R}^d} \Big(\frac{1}{s}\|x-y\|^{-s} + V(x) + V(y)\Big) d\mu(x)d\mu(y)$$

focusing on the cases where $V(x) = \gamma\|x\|^{\alpha}$ for $\alpha, \gamma > 0$. We are particularly interesting in how the choices of s and α, i.e. the strength of repulsion between "electrons" and strength of attraction towards a positively charged source at the origin, respectively, affect the dimension of the support of the minimizing measure, and are able to classify exactly when the support is the uniform measure on a sphere.

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A constrained logarithmic energy problem on the unit circle

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We study the problem of minimizing the logarithmic energy, $\mathcal{E}(\mu) := \iint \log \frac{1}{|z-\zeta|} d\mu(z) d\mu(\zeta)$, of probability measures μ supported on the unit circle with an additional constraint imposed on the mass of a fixed subarc. Namely, for given θ, $0 < \theta < 2\pi$, and given q, $0 < q < 1$, we determine the measure μ^*, such that $\mathcal{E}(\mu^*) = \inf\{\mathcal{E}(\mu) : \mu \in \mathcal{P}(\mathbb{S}^1), \mu(A_\theta) = q\}$, where A_θ is the arc from $e^{-i\theta/2}$ to $e^{i\theta/2}$. The result answers a question raised by E. Meckes in connection with the characterization of behavior of eigenvalues of the kernel of the unitary eigenvalue process. In addition, I will talk about analogous constrained problem for more general energy functionals.

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Dyadic operators, weights, and sparse domination in the non-doubling setting

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In joint work with Conde Alonso and Wagner ([1]), we proved a bilinear form sparse domination, and discovered new appropriate weight classes, for bounding certain dyadic shift operators with respect to a class of non-doubling measures. The operators are characterized by a suitable notion of *complexity* which generalize Petermichl's shift operator, featured in [5]. Through examples, we demonstrated that the usual sparse forms and standard Muckenhoupt weight classes are not suitable for these dyadic shifts in the non-doubling setting. The project arose as a natural extension of the work of López-Sánchez, Martell, and Parcet ([4]), where necessary and sufficient conditions on non-doubling measures were derived for bounding a classical dyadic shift operator ([5]). In further joint work with Borges, Conde Alonso, and Wagner, we consider the (recently introduced) dyadic Hilbert transform which Domelevo and Petermichl investigated in connection with its close relationship to the continuous Hilbert transform in [2], and which is featured in their paper (joint with Treil and Volberg) establishing optimal 3/2 bounds for matrix weights ([3]). In particular, we investigate properties necessary and sufficient for the boundedness of this operator in L^p, and similar questions for commutator estimates (with martingale BMO functions). Our work was informed by the results in, and several helpful discussions with Treil on, the non-homogeneous martingale theory established in ([6]).

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The p^{th} frame potentials: An overview

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Finding optimal configurations of point masses under the action of energy functionals defined on $d-$dimensional Euclidian space appears in several fields such as numerical integration, coding theory, quantum information theory, and chemistry. The problem often involves minimizing a functional of the form $\iint_{S^{d-1}\times S^{d-1}} f(x,y)d\mu(x)d\mu(y)$ over all probability measures μ defined on the unit sphere S^{d-1}, or its discrete version, minimizing $\sum_{k\neq\ell} f(\varphi_k,\varphi_\ell)$ over all sets of N vectors $\Phi=\{\varphi_k\}_{k=1}^N \subset S^{d-1}$ for some function f.

In this talk, we will consider the case where $f(x,y)=|\langle x,y\rangle|^p$ for $p\in[0,\infty]$ which is referred to as the p^{th} frame potentials. We focus the interplay between the continuous and the discrete problems, especially when the dimension d is small.

This talk is based on joint works with R. Ben Av, X. Chen, M. Ehler, A. Goldberger, E. Goodman, V. Gonzales, and S. Kang.

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Tchakaloff-like compression of QMC integration

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We present a method based on Tchakaloff-Davis-Wilhelmsen theorems to compress quadrature formulae such as quasi-Monte Carlo (QMC) integration. This means that, when the integral

$$\int_\Omega f(x)\,\mathrm{d}\mu$$

is approximated with QMC method by

$$\frac{|\Omega|}{N}\sum_{i=1}^{N} f(x_i)$$

for certain points uniformly distributed accordingly to the measure μ, we aim to extract some points $t_i \subset \{x_1, \ldots, x_N\}, i = 1, \ldots, M$ with $M < N$ and positive weights w_i, such that the quadrature formula

$$\sum_{i=1}^{M} w_i f(t_i)$$

is a *good* approximation of QMC integration. In particular, the compressed quadrature formula extracted is going to be exact (with respect to QMC approximation) on polynomials in Ω up to a given fixed degree. Therefore, such formulae preserve the approximation power of QMC up to the best uniform polynomial approximation error of a given degree to the integrand, but using a much lower number of sampling points.

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Sampling in inverse problems

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Sampling problems appear whenever a continuous, infinite-dimensional object x, has to be reconstructed from discrete, possibly finite, samples. This requires some assumptions on x. These assumptions can be either expressed by linear conditions, as it happens for Shannon-type results for band-limited functions, or by nonlinear conditions, as with sparsity in compressed sensing or with generative neural networks.

In this talk, I will discuss the case when x is not sampled directly: we sample $F(x)$, where F is the so-called forward map, and wish to reconstruct x. This is the framework of inverse problems, in which an unknown quantity x has to be reconstructed from physical, indirect measurements $F(x)$. Sampling appears in this context since, in practice, it is impossible to directly measure the infinite-dimensional quantity $F(x)$, and only samples are available. I will discuss some abstract results, as well as some examples, including deconvolution and the inverse Radon transform.

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Exact and approximative t-design curves

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In analogy to classical spherical t-design points, we introduce the concept of t-design curves on the sphere $\mathbb{S}^d = \{x \in \mathbb{R}^{d+1} : \|x\| = 1\}$. This means that the line integral along a t-design curve γ integrates exactly all algebraic polynomials f in $d+1$ variables of degree t,

$$\frac{1}{\ell(\gamma)} \int_\gamma f = \int_{\mathbb{S}^d} f \,.$$

For low degrees we construct explicit examples.

We also derive lower asymptotic bounds on the lengths of t-design curves.

Proposition 1. *Assume that a piecewise smooth, closed curve $\gamma : [0,1] \to \mathbb{S}^d$ satisfies*

$$\frac{1}{\ell(\gamma)} \int_\gamma f = \int_{\mathbb{S}^d} f \qquad \text{for all } \; f \in \Pi_t \,.$$

Then its length is bounded from below by

$$\ell(\gamma) \geq C_d t^{d-1}$$

with some constant $C_d > 0$ that may depend on the dimension d but is independent of t and γ.

Our main results prove the existence of asymptotically optimal t-design curves in $\mathbb{S}^2$.

Theorem 2. *In $\mathbb{S}^2$ there exists a sequence of t-design curves $(\gamma_t)_{t\in\mathbb{N}}$ with length $\ell(\gamma_t) \asymp t$.*

We also verify the existence of t-design curves in $\mathbb{S}^d$.

Theorem 3. *In $\mathbb{S}^d$ for $d \geq 3$ there exists a sequence of t-design curves $(\gamma_t)_{t\in\mathbb{N}}$, such that $\ell(\gamma_t) \lesssim t^{d(d-1)/2}$.*

We additionally derive approximative t-design curves that asymptotically match the lower length bounds for t-design curves.

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Distributing points in some Grassmannian manifolds

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The projective logarithmic energy of a collection of points $x_1, \dots, x_N \in \mathbb{P}(\mathbb{R}^3)$ is given by the formula

$$E(x_1, \dots, x_N) = \sum_{i \neq j} \log \frac{1}{\sqrt{1 - \langle x_i, x_j \rangle^2}}.$$

What is the minimal value that this energy can attain? The corresponding question in the sphere has been studied by many authors, see for example [1] for the history and a remarkable collection of results. But in the projective space the question has received much less attention. Upper and lower bounds on that value in the real, complex, quaternionic and octonionic projective spaces were computed by [2], giving quite sharp estimates (sharp from the difference between the upper and lower bounds being small) for that minimum value.

In a recent paper [3] we proved new lower bounds, improving those of [2] for all these cases. In another recent paper [4] we focused on $\mathbb{P}(\mathbb{R}^3)$ showing a constructive sequence of points which attains quite good values of the energy:

Theorem 1. *For any $N = 1, 2, \dots$, the minimal possible value m_N of the projective logarithmic energy in $\mathbb{P}(\mathbb{R}^3)$ satisfies:*

$$W_{log}(\mathbb{P}(\mathbb{R}^3))N^2 - \frac{1}{2}N \log N + cN \leq m_N \leq W_{log}(\mathbb{P}(\mathbb{R}^3))N^2 - \frac{1}{2}N \log N + CN,$$

where

$$W_{log}(\mathbb{P}(\mathbb{R}^3)) = 1 - \log 2$$

is the continuous energy and $c = -0.403426\dots, C = -0.395795\dots$. Moreover, there exists an explicit, contructible set of N points in $\mathbb{P}(\mathbb{R}^3)$, depending on some random parameters, whose expected logarithmic energy is the upper bound above.

Another popular quantity of interest is the so–called spherical discrepancy, see for example [5]. Our result in [4] also shows that the spherical sequence corresponding to the real projective points has order of discrepancy $O(1/\sqrt{N})$ which is certainly not optimal but, to our knowledge, it is if we demand constructivity of the sequence. I will present this result as well as some others, related to the problem of finding well distributed points in Grassmannians, proved in [6].

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Linear statistics of determinantal point processes and norm representations

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We study the asymptotic behaviour of the fluctuations of smooth and rough linear statistics for determinantal point processes on the sphere and on the Euclidean space. The main tool is the generalization of some norm representation results for functions in Sobolev spaces and in the space of functions of bounded variation.

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New Developments in infinite dimensional Lie algebras, vertex operator algebras and the Monster
Special Session B4

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Vertex operator algebras and Borcherds algebras are rich mathematical structures that play a role in many areas of mathematics. They also appear as symmetries of physical models. For example, the Monster Lie algebra was constructed by Borcherds as a quotient of the physical space of the tensor product $V^\natural \otimes V_{1,1}$ of the Moonshine module vertex operator algebra $V^\natural$ and the vertex algebra $V_{1,1}$ for the even unimodular 2-dimensional Lorentzian lattice $\mathrm{II}_{1,1}$. These algebraic structures were central players in the solution of the Conway-Norton Monstrous Moonshine conjecture. Since then, the study of vertex operator algebras, Borcherds algebras, and related structures have remained a source of discovery in both mathematics and physics. Vertex operators algebras and their representations encode the symmetries of two dimensional conformal field theories. Generalized Moonshine has uncovered many connections between number theory and physics. The Monster Lie algebra has recently been shown to be an algebra of gauge symmetries of a compactification of the Heterotic string. Borcherds superalgebras have been linked to the symmetries of supergravity theories. Furthermore, groups associated to Kac–Moody algebras are conjectured to encode symmetries of supergravity theories and there have been recent constructions of Lie group analogs for Borcherds algebras. In this special session, we explore the mathematical developments and possibilities for physical applications raised by these recent discoveries. The list of speakers comprises both mathematicians and physicists, each who have interest in the collaboration between these disciplines.

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Opening remarks: Infinite dimensional Lie algebras and their symmetries

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The Monster Lie algebra $\mathfrak{m}$ and Carnahan's family of Monstrous Lie algebras $\mathfrak{m}_g$, one for each element g of the Monster finite simple group, appear as symmetries in a model of the compactified Heterotic String by Persson, Paquette and Volpato. A recent construction of a Lie group analog for $\mathfrak{m}$ when $g = 1$ may also represent further symmetries. We discuss some conjectures and open questions.

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Opening remarks on infinite dimensional algebras in physical models

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Infinite dimensional algebras play a crucial role in the theoretical description of many physical models, ranging from quantum mechanics to conformal field theory, from integrable models to string theory. In turn, physics has inspired and motivated the construction of a rich class of examples of infinite dimensional algebras, including vertex algebras and Borcherds-Kac-Moody algebras. In this overview talk, I will describe some recent instances of this rich interplay between algebra and physics, and provide some background and physical motivation for some of the topics that will be discussed in this special session.

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Applications of Borcherds's Lie algebra to the uniqueness problem of VOAs of moonshine type

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Theorem 1. *If V is a simple VOA of central charge 24 with a non-singular invariant bilinear form $\langle , \rangle$ and $\sum \dim V_n q^{n-1} = j(\tau) - 744 = q^{-1} + 196884q + \ldots$, then V is C_2-cofinite. More precisely,*

$$C_2(V) = \sum_{n \geq 5} V_n + L(-1)V.$$

Proof. We use the fact $B(V) \cong B(V^\natural)$, where $B(V)$ denotes a Borcherds's Lie algebra of V. □

We next show its application to the space of 1-point functions of V.

Theorem 2. *$\mathcal{F}(V) = \mathcal{F}(V^\natural)$, where $\mathcal{F}(V)$ denotes the space of 1-point functions associated with V.*

About the monstrous moonshine VOA $V^\natural$, it was shown by C. Dong and G. Mason in 2000 that the space $\mathcal{F}(V^\natural)$ of 1-point functions associated to $V^\natural$ is precisely $\mathbb{C}$-linear space spanned by the (meromorphic) modular forms of level 1 and integer weight $k \geq 0$ satisfy holomorphic on $\mathcal{H}$ and has Fourier expansion $q^{-1} + \ldots$.

Proof. Actually, Dong and Mason has shown that for $\theta = e^{2\lambda} + e^{-2\lambda} \in V^\natural_{12}$ with $\lambda \in \Lambda$ and $\langle \lambda, \lambda \rangle = 6$,

$$0 \neq \mathrm{Tr}_{V^\natural} o(\theta) q^{L(0)-1} = \Delta(\tau) = (2\pi)^{12} \eta(\tau)^{24},$$

where $\eta(\tau)$ is the Dedekind eta function, $\Delta(\tau)$ the modular discriminant, and $o(v) := v_{\mathrm{wt}(v)-1}$.

We will reconstruct it by a way which does not depend on the structure of V with the help of Borcherds's Lie algebra and the action of $\mathbb{M}$ on Borcherds's Lie algebra. □

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Vertex operators for imaginary $\mathfrak{gl}_2$ subalgebras in the Monster Lie Algebra

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Elizabeth Jurisich
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The Monster Lie algebra $\mathfrak{m}$ is a quotient of the physical space of the vertex algebra $V = V^\natural \otimes V_{1,1}$, where $V^\natural$ is the Moonshine module vertex operator algebra of Frenkel, Lepowsky, and Meurman, and $V_{1,1}$ is the vertex algebra corresponding to the rank 2 even unimodular lattice $\mathrm{II}_{1,1}$. We discuss vertex algebra elements which project to bases for subalgebras of m isomorphic to $\mathfrak{gl}_2$ and corresponding to imaginary simple roots of $\mathfrak{m}$. The action of the Monster finite simple group $\mathbb{M}$ on $V^\natural$ induces an action of $\mathbb{M}$ on the set of $\mathfrak{gl}_2$ subalgebras corresponding to a fixed imaginary simple root. We will discuss this action and related open questions. (This talk is based on joint work with Lisa Carbone, Elizabeth Jurisich, Maryam Khaqan, and Scott H. Murray.)

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Prosummability in groups for Borcherds algebras

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As part of his work on the solution of the Conway-Norton Monstrous Moonshine conjecture, Borcherds introduced a new class of infinite dimensional Lie algebras, known as generalized Kac-Moody algebras or Borcherds algebras. These Lie algebras have important connections and numerous applications in algebra, number theory, combinatorics and mathematical physics. Therefore, it is important to construct an associated group analogous to the Lie groups in classical Lie theory and Kac-Moody groups in infinite-dimensional Lie theory. However, this is a challenging task because, unlike Kac-Moody algebras, the generators of Borcherds algebras do not act nilpotently or locally nilpotently under their adjoint representation. Carbone, Jurisich and Murray, in their construction of a Lie group analog for the Monster Lie algebra, introduced the notion of prosummability, providing a substitute for local nilpotence and paving the way for group constructions for these infinite-dimensional Lie algebras. We discuss how to extend this notion to all Borcherds algebras and we consider applications to complete adjoint constructions of Kac-Moody groups.

This is a joint work with Lisa Carbone, Elizabeth Jurisich and Scott Murray.

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A Magnus Group construction for a class of Borcherds algebras

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We present a formal power series construction leading to a group for a subclass of Borcherds Lie algebras. We use a Magnus group construction stemming from the Hopf Algebra. This construction is over the rational numbers, but it can be related to the continuous Lie-type group construction over the complex numbers of Carbone, Jurisich, and Murray ([1]).

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Monstrous Lie algebras as Borcherds algebras

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We discuss the construction of Carnahan's non-Fricke monstrous Lie algebras from the point of view of Borcherds algebras.

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Monstrous Moonshine for integral group rings

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We propose a conjecture that unifies and generalizes Monstrous Moonshine and Modular Moonshine, and produce some partial results.

For any group G, and any commutative ring R, we may consider the tensor category of RG-modules that are R-free of finite rank. Homomorphisms from the Grothendieck ring of this category (or some other, similar category) to the complex numbers are called "species" by Benson and Parker. Given a $\mathbb{Z}$-graded RG-module whose graded pieces are R-free of finite rank, any species produces a corresponding formal power series with complex coefficients. We conjecture that when R is a subring of $\mathbb{C}$ and G is a subgroup of the monster, for a distinguished R-form of the moonshine module, any such power series is a hauptmodul. That is, the power series obtained by evaluating any species on the moonshine module is the expansion of a modular function that has discrete stabilizer in $SL_2(\mathbb{R})$, and generates the function field of the corresponding upper half-plane quotient.

For the case that $R = \mathbb{C}$, this conjecture reduces to the Monstrous Moonshine conjecture, proposed by Conway and Norton in 1979 and solved by Borcherds for the Frenkel-Lepowsky-Meurman moonshine module in 1992. When R is isomorphic to a ring of p-adic integers, and G is a cyclic group whose order has p-valuation 1, then this reduces to the Modular Moonshine conjecture of Ryba, proved by Borcherds and Ryba for odd p in 1996–1999, and for $p = 2$ by the speaker in 2017. We have found that our conjecture holds for some additional cases, and furthermore we have classified species for some nonabelian groups G.

This is joint work with Satoru Urano, and combines our paper at arXiv:2111.09404 with newer results.

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Elliptic Curves and Moonshine of the First Janko Group

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We identify distinguished virtual graded modules for the first sporadic simple group discovered by Janko, and establish relationships between these modules and the arithmetic of certain families of elliptic curves.

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Reflective modular varieties and their cusps

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Automorphic forms for orthogonal groups are natural generalisations of elliptic modular forms. An important class of these functions is given by automorphic products. We show that under some natural assumptions there are exactly 11 reflective automorphic products of singular weight. The corresponding modular varieties have a very rich geometry. Surprisingly their 1-dimensional type-0 cusps are naturally parametrised by Schellekens' list. This gives a new complex-geometric proof of this list.

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Conformal blocks of vertex operator algebra and colored parenthesized braid operad

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The following theorem is one of the most important results in the theory of vertex operator algebras, proved by a series of papers by Huang and Lepowsky [1]:

Theorem 1 (Huang-Lepowsky). *The representation category of a regular vertex operator algebra inherits a structure of braided tensor category.*

In the proof of the theorem, a partial operad of tubed Riemann spheres introduced by Huang in his pioneering work plays an important role. However, the proof in [1] is based on a very long formal calculation, since this partial operad is complicated whose product structure is formally defined.

We have given in [2] a more geometric and concise proof of Theorem 1. Huang-Lepowsky's proof is built on the basis of intertwining operators, which is a formal series. However, we use its geometric counterpart, the conformal block. We show that the fundamental groupoid of the E_2 operad (colored parenthesized braid operad) acts naturally on the conformal blocks. This operad is much simpler than Hunag's operad.

Theorem 2 (M). *Let V be a VOA (not necessary regular) and V-mod$_{C_1}$ the category of C_1 cofinite V-modules. Then, the colored parenthesized braid operad lax 2-categorically acts on V-mod$_{C_1}$.*

In this talk, we will discuss these results and an alternative proof of Theorem 1 from Theorem 2 based on [2].

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Classification of Self-Dual Vertex Operator Superalgebras of Central Charge at Most 24

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I will discuss recent joint work with Sven Möller on the construction and classification of self-dual vertex operator superalgebras of central charge up to 24. We employ the 2-neighbourhood graph of the self-dual VOAs of central charge 24 and realize these SVOAs as simple-current extensions of a dual pair. This pair includes a VOA derived from the Leech lattice alongside a lattice VOA. We have identified exactly 969 such SVOAs. The remaining open question concerns the uniqueness of the shorter Moonshine module, which was the subject of my 1995 Ph.D. thesis.

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Genera of Vertex Operator Algebras

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We define and investigate several notions of genera of (suitably regular) vertex operator algebras, building on earlier work in [1,2]. In doing so, we shall see that each notion of equivalence for even lattices (e.g., genus, Witt and rational equivalence) has at least two generalisations to vertex operator algebras, one being a more "classical" analogue, and one being a more honest "quantum" analogue.

We study the relations between these various notions. For example, we give a proof that two vertex operator algebras in the same hyperbolic genus (as defined in [2]) are in the same bulk genus (as defined in [1]).

We also study mass formulae, p-neighbourhood and Hecke operators for hyperbolic genera of vertex operator algebras.

This is work in progress.

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Replicable functions arising from code lattice VOAs fixed by automorphisms

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We ascertain properties of the algebraic structures in towers of codes, lattices, and vertex operator algebras (VOAs) by studying the associated subobjects fixed by lifts of code automorphisms. In the case of sublattices fixed by subgroups of code automorphisms, we identify replicable functions that occur as quotients of the associated theta functions by suitable eta products. We show that these lattice theta quotients can produce replicable functions not associated to any individual automorphisms. Moreover, we show that the structure of the fixed subcode can induce certain replicable lattice theta quotients and we provide a general code theoretic characterization of order doubling for lifts of code automorphisms to the lattice-VOA. Finally, we prove results on the decompositions of characters of fixed subVOAs. This talk is based on joint work with Jennifer Berg, Eva Goedhart, Hussain M. Kadhem, Allechar Serrano López, and Stephanie Treneer.

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Holomorphic vertex operator algebras, Teichmüller modular forms and the monster orbifold

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Almost forty years ago Friedan and Shenker suggested to describe two-dimensional modular invariant conformal field theories in terms of the geometry of the "universal moduli space" of compact complex curves [1]. From this point of view the fundamental problem is the construction of the partition function of the theory on all compact complex curves. This raised many very interesting mathematical problems. In this talk I will describe some recent results in this direction in the mathematical framework of vertex operator algebras [2]. If V is a (complex) simple vertex operator algebra of CFT type having an invariant bilinear form we can define the genus g partition function $\chi_{V,g}$ as 1 if $g = 0$ and as a formal series in $3g-2$ variables defined by through the correlation functions of V, if $g > 0$. The partition function χ_V is the sequence $\{\chi_{V,g}\}_{g \in \mathbb{Z}_{\geq 0}}$ of the genus g partition functions. If V is holomorphic (and strongly rational) with central charge c we show that each genus g partition function gives rise in a natural way to a genus g Teichmüller modular form of wight $c/2$, a higher genus generalization of the well-known modularity of the character of V. This gives strong constraints on the partition functions of holomorphic vertex operator algebras. As an important example comes from a weak form the Harris-Morrison slope conjecture about the geometry of the moduli spaces of Riemann surfaces [3]. If this conjecture holds true then the partition function of any holomorphic V with $c = 24$ and zero weight-one subspace must coincide with the one of the moonshine $V^\natural$. Other results concerns the constraints on V given by its partition function. For a not necessarily holomorphic V we define a the partition function vertex operator subalgebra PV of V. The latter is invariant for $\mathrm{Aut}(V)$ and, when V is unitary, it is a unitary subalgebra of V containing the conformal vector. If V and U are unitary we show that $\chi_V = \chi_U$ if and only if there is a linear isomorphism $\Phi : V \to U$ restricting to a vertex operator algebra isomorphism $\phi : PV \to PU$ and such that $\Phi Y^V(a,z) = Y^U(\phi(a), z)\Phi$ for all $a \in PV$. In particular, if the PV-module V has a unique VOA structure then U and V must be isomorphic. These results open new perspectives on the famous conjecture of Frenkel, Lepowsky and Meurman on the uniqueness of the moonshine vertex operator algebra and relate it to other important conjectures in different areas of mathematics. Assume for example that $PV^\natural$ coincide with the monster orbifold $V^{\natural^{\mathbb{M}}}$ and that the latter is strongly rational. Then, the uniqueness of $V^\natural$ would follow from the weak Harris-Morrison slope conjecture.

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Topological defects in K3 sigma models

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In the context of physics, given a K3 surface, a supersymmetric non-linear K3 sigma model is the internal superconformal field theory (SCFT) in a six dimensional compactification of type IIA superstring on $\mathbb{R}^{1,5} \times K3$. Studying these models, Eguchi, Ooguri and Tachikawa [1] found Mathieu moonshine phenomena for the elliptic genera of K3 surfaces. In [2], Gaberdiel, Hohenegger and Volpato studied symmetries of K3 sigma models, by comparing the isometry group of the Mukai lattice (which is physically interpreted as the D-brane charge lattice) with the Conway group Co_0, the group of automorphisms of the Leech lattice. Recently, symmetries have been reinterpreted in terms of topological operators supported on codimension 1 submanifolds with group-like and invertible fusion rules. This has naturally led to the notion of generalized symmetries as categories of topological defects supported on arbitrary codimension submanifolds with possibly non-invertible fusion rules.

In [3] we apply the same strategy as in [2] to derive a number of general results for the category Top of the topological defect lines preserving $\mathcal{N} = (4,4)$ and spectral flow, studying their fusion with boundary states. We argue that while for certain K3 models infinitely many simple defects, and even a continuum, can occur, at generic points in the moduli space the category is actually trivial, i.e. it is generated by the identity defect. Furthermore, we show that if a K3 model is at the attractor point for some BPS configuration of D-branes, then all topological defects have integral quantum dimension. We also conjecture that a continuum of topological defects arises if and only if the K3 model is a (possibly generalized) orbifold of a torus model. These general results are confirmed by the analysis of a couple of significant examples. Finally, in view of a possible characterization of Top as a subcategory of a maximal category of topological line defects, which would provide a generalized symmetry analogue of Mukai's classical result on finite group symmetries of K3 surfaces, we introduce the study of topological defect lines in the Conway module $V^{f\natural}$, which is the holomorphic SCFT with central charge $c = 12$ and no weight-1/2 fields characterized by a unique $\mathcal{N} = 1$ superconformal structure stabilized by the Conway group Co_0.

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p-adic vertex operator algebras

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We introduce p-adic vertex algebras. Such an object is not quite a VOA in the usual sense, but rather the completion of a VOA with respect to a compatible nonarchimedean norm. In the language of physics, it is a chiral half of a nonarchimedean 2-dimensional bosonic CFT in which a p-adic Banach space replaces the traditional Hilbert space. Our main motivation for considering these objects is that they facilitate the introduction p-adic modular forms *a la* Serre into VOA theory. This gives rise to many new phenomena, and we shall review some of these in the setting of the p-adic Heisenberg VOA.
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Tensor hierarchy algebras and generalised diffeomorphisms

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I will review how certain Borcherds–Kac–Moody superalgebras have been used in physics to encode the bosonic field content of supergravity theories. Any such Borcherds–Kac–Moody superalgebra is a $\mathbb{Z}$-graded extension of a simple finite-dimensional Lie algebra $\mathfrak{g}$ at degree 0 (describing a global internal symmetry of the theory) and includes at degree 1 an integrable lowest-weight module over $\mathfrak{g}$ with lowest weight $-\lambda$. I will show how the construction of the Borcherds–Kac–Moody algebra can be modified, and from the same from the initial data $(\mathfrak{g}, \lambda)$ construct a Lie superalgebra which is not contragredient, so that the $\mathfrak{g}$-module at a degree p is not necessarily dual to the one at degree $-p$. Some well known Lie superalgebras appear as special cases, but also new ones that have not been studied before. These infinite-dimensional Lie superalgebras, known as tensor hierarchy algebras [1], can be used to describe generalised diffeomorphisms, providing a local origin of the global symmetries. The construction can be generalised to cases where $\mathfrak{g}$ is an infinite-dimensional Kac–Moody algebra, but the representation structures that appear in these cases remain to be understood.

The talk is based on collaboration mostly with Martin Cederwall [2] but also with Lisa Carbone [3], Guillaume Bossard, Axel Kleinschmidt, Chris Pope and Ergin Sezgin [4].

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Genus g Zhu Recursion for Vertex Operator Algebras

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We describe Zhu recursion for a vertex operator algebra (VOA) and its modules on a genus g Riemann surface in the Schottky uniformisation. We describe how n-point correlation functions can be naturally expanded in terms $(n-1)$-point functions with universal coefficients given by holomorphic forms and derivatives of the Bers quasiform. We discuss Heisenberg VOA examples where Zhu recursion leads to novel differential equations for the partition function and various classical structures such as the bidifferential of the second kind, holomorphic 1-forms, the prime form and the period matrix.

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On Yangian deformations of $\mathcal{S}$-commutative quantum vertex algebras

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We present the construction of a new class of quantum vertex algebras associated with a normalized Yang R-matrix. They are obtained as Yangian deformations of certain $\mathcal{S}$-commutative quantum vertex algebras and their $\mathcal{S}$-locality takes the form of a single RTT-relation. We establish some preliminary results on their representation theory and then we further investigate their braiding map. In particular we show that the fixed points of the braiding map are related to Bethe subalgebras in the Yangian quantization of the Poisson algebra $\mathcal{O}(\mathfrak{gl}_N((z^{-1})))$, introduced by Krylov and Rybnikov in [2].

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Unitary representations of minimal W-algebras

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I will discuss recent advances on a project joint with Pierluigi Möseneder Frajria (Politecnico di Milano) and Victor Kac (MIT). In [1] we classified the levels k for which the simple affine W-algebra $W_k^{\min}(\mathfrak{g})$ attached to a basic finite dimensional Lie superalgebra $\mathfrak{g}$ and to a minimal even nilpotent element is unitary, i.e. it admits a positive definite invariant Hermitian form. We then started the classification of the unitary representations of these algebras in the Neveu-Schwarz sector. The classification is complete in the so called *non-extremal* cases, and we conjecture that our results in the extremal case provide the complete classification. We also obtained character formulas for the unitary representations, yielding rigorous proofs of the results obtained by physicists (Miki, Eguchi-Taormina) in the 80's for the $N = 3$ and $N = 4$ superconformal algebras, respectively.

We recently moved to study unitary representation in the Ramond sector, which involves studying twisted modules.

The study of unitarity of the Ramond twisted irreducible highest weight modules over the vertex algebra $W^k_{\min}(\mathfrak{g})$, where k is in the unitary range, proceeds along similar lines. The main difference is that in the Ramond sector one has to consider separately two cases: when $\frac{1}{2}\theta$ is not a root of $\mathfrak{g}$, and when it is a root, where θ is the highest root of $\mathfrak{g}$. In both cases the necessary conditions of unitarity are similar to the conditions found in the NS case, except that a canonical constant A has to be replaced by another constant A^{tw}, and the notion of an extremal weight needs to be replaced by that of a Ramond extremal weight.

As in [1] we find sufficient conditions of unitarity of Ramond twisted irreducible highest weight modules over $W^k_{\min}(\mathfrak{g})$ by using its free field realization The parameter space for these representations if given by pairs (ℓ_0, ν), where $\ell_0 \in \mathbb{R}$ and ν is a weight of a certain subalgebra of $\mathfrak{g}$. As a result, we prove unitarity for ℓ_0 larger than an explicit constant B in the cases when ν is not Ramond extremal. It turns out that $B = A^{tw}$ in the cases when $\theta/2$ is a root of $\mathfrak{g}$ which completes the proof of unitarity when ν is not Ramond extremal.

However, in the case when $\theta/2$ is not a root of $\mathfrak{g}$, $B = A^{tw}$ only for some very special weights ν. Generically one has that $B > A^{tw}$, and we need use Euler-Poincaré characters, instead of determinants of ϕ-invariant Hermitian forms for twisted $W^k_{\min}(\mathfrak{g})$-modules, as in the non-twisted sector. At this point we need to use a conjecture, which claims that Arakawa's results on properties of the quantum Hamiltonian reduction functor can be extended to the Ramond twisted case.

The necessary conditions for unitarity of Ramond twisted irreducible highest weight modules over $W^k_{\min}(\mathfrak{g})$ are exhibited in all cases. Moreover, as in [2], any unitary irreducible non-twisted or twisted highest weight module over $W^k_{\min}(\mathfrak{g})$ descends to $W_k^{\min}(\mathfrak{g})$.

Using the character formulas for massless representations in the case of level k_0 when $\dim W_{k_0}^{\min}(\mathfrak{g}) = 1$, we find the denominator identities for $W^k_{\min}(\mathfrak{g})$. As a result, we recover the classical identities of Euler, Gauss and Ramanujan, and find some new identities.

In my talk I will give a quick overview of the above results, with emphasis on the Ramond case and applications to identities.

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Geometric Analysis of Several Complex Variables and PDEs
Special Session B5

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The section, scheduled on July 25-26, will feature talks by leading experts on recent developments in the analysis and geometry of CR manifolds and related partial differential equations. It is expected that the session will promote collaborations between researchers in the U.S. and Italy.

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Dynamics of Fuchsian meromorphic connections with real periods

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A very interesting class of examples of holomorphic maps tangent to the identity at a fixed point in several complex variables is given by the time-1 maps of homogeneous vector fields. In [1] it has been shown that the study of the dynamics of these maps can be reduced to the study of the dynamics of the geodesic field of meromorphic connections on Riemann surfaces. In this talk we shall describe some recent results, obtained in collaboration with Karim Rakhimov, on the dynamics of the geodesic field for Fuchsian meromorphic connections having real periods. The main tools used are: a generalization to general Fuchsian meromorphic connections of a classical formula proved by Teichmüller for quadratic differentials; and the relationship between Fuchsian meromorphic connections with real periods and singular flat Hermitian metrics. In particular, we obtain a description of the possible ω-limit sets of simple geodesics that extends and makes more precise results known for the particular case of Riemann surfaces endowed with a meromorphic k-differential.

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Global hypoellipticity and solvability for evolution operators in time-periodic Gelfand-Shilov spaces

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We consider a class of evolution operators with complex-valued coefficients depending both on time and space variables $(t, x) \in \mathbb{T} \times \mathbb{R}^n$, where $\mathbb{T}$ is the one-dimensional torus. We prove necessary and sufficient conditions for global hypoellipticity and solvability in spaces of time-periodic Gelfand-Shilov functions. The argument of the proof is based on a characterization of these spaces in terms of the eigenfunction expansions given by a fixed self-adjoint, globally elliptic differential operator on $\mathbb{R}^n$, cf. [3]. The results presented are obtained in collaboration with Fernando de Ávila Silva (Federal University of Paraná, Curitiba, Brazil), cf. [1,2].

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On the local behavior of chains on strongly pseudoconvex hypersurfaces in $\mathbb{C}^3$

Giuseppe Della Sala [1]
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We study the behavior of Chern-Moser chains that wind around a small enough neighborhood of a point $p \in M$, where p is a smooth, strongly pseudoconvex hypersurface of $\mathbb{C}^3$. We prove that, if for all $p \in M$ these chains are topologically "tame" in the sense that in appropriate coordinates and almost horizontal initial conditions their images are small perturbations of circles contained in $T^c_p(M)$, then M is locally spherical. This is joint work with Nisrine Bakaev, Seok Ban, Hassan El Bouz, Ahmad Hussein and Jean Moussa.

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Bergman logarithmically flat and obstruction flat CR manifolds

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Let $\Omega \subset \mathbb{C}^n$ be a smoothly bounded, strictly pseudoconvex domain. The boundary $\partial\Omega$ is said to be *Bergman logarithmically flat* if the log singularity in Fefferman's asymptotic expansion of the Bergman kernel vanishes (to infinite order). It is called *obstruction flat* if the log singularity (the obstruction function) of the Cheng–Yau log-potential of the complete Kähler-Einstein metric in Ω vanishes. The Ramadanov Conjecture asserts that if $\partial\Omega$ is Bergman logarithmically flat, then it is spherical. There is a similar conjecture for obstruction flat boundaries. Both conjectures, suitably reformulated, fail for domains in more general complex manifolds in higher dimension ($n \geq 3$), but the situation is still unclear for domains in $\mathbb{C}^n$ (for $n \geq 3$). In this talk, we shall present recent work and open questions concerning these conjectures and the general structure of Bergman logarithmically flat and obstruction flat CR manifolds.

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Equivalence between validity of the p-Poincaré inequality and finiteness of the strict p-capacitary inradius.

Anne-Katrin Gallagher
Gallagher Tool & Instrument

We will talk about some aspects of the equivalence of the validity of the p-Poincaré inequality on an open set Ω in $\mathbb{R}^n$, i.e.,

$$\|f\|_{p,\Omega} \leq C\|\nabla f\|_{p,\Omega} \quad \forall\, f \in \mathcal{C}_c^\infty(\Omega)$$

for some $C > 0$, and the finiteness of the p-capacitary inradius, $\rho_p(\Omega)$, of Ω defined by

$$\rho_p(\Omega) = \sup\{r > 0 \,:\, \forall\, \epsilon > 0 \exists\, x \in \mathbb{R}^n \text{ such that } C_p(\overline{\mathbb{B}_r(x)} \cap \Omega^c) < \epsilon\}$$

Here, $C_p(E)$ denotes the Sobolev p-capacity of E for $E \subset \mathbb{R}^n$.

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A structure theorem for neighborhoods of compact complex manifolds

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Laurent Stolovitch
CNRS and Laboratoire J.-A. Dieudonné, Université Côte d'Azur[2]

We show that the set of holomorphic equivalence classes of holomorphic neighborhoods M of a compact complex manifold C is finite-dimensional, provided $(TM)|_C$ is fixed and the normal bundle of C in M is either *weakly negative* or 2-*positive*.

Our main result is the following.

Theorem 1. *Let C be a compact complex manifold. Assume that N_C is either weakly negative or 2-positive. There is an injective mapping from the set of holomorphic equivalence classes of holomorphic neighborhoods of C into the finite-dimensional space*

$$\mathcal{H}^1(T_C M) := \bigoplus_{\ell \geq 2} H^1(C, T_C M \otimes S^\ell N_C^*),$$

where $S^\ell N_C^*$ is the ℓ-th symmetric power of the dual bundle N_C^* of N_C.

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Semi-classical asymptotics of partial Bergman kernels on R-symmetric complex manifolds with boundary

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Let M be a relatively compact connected open subset with smooth connected boundary of a complex manifold M'. Let (L, h^L) be a positive line bundle over M'. Suppose that M' admits a holomorphic $\mathbb{R}$-action which preserves the boundary of M and lifts to L. In this talk, we will show an asymptotic expansion of a partial Bergman kernel associated to a package of Fourier modes of high frequency with respect to the $\mathbb{R}$-action in the high powers of L. As an application, we establish an $\mathbb{R}$-equivariant analogue of Fefferman's and Bell-Ligocka's result about smooth extension up to the boundary of biholomorphic maps between weakly pseudoconvex domains in $\mathbb{C}^n$. Another application concerns the embedding of pseudoconcave manifolds.

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Bumping and sup norm estimates for $\bar{\partial}$ on smooth pseudoconvex domains in $\mathbb{C}^n$

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The $\bar{\partial}$-Neumann problem was solved in the 60's for pseudoconvex domains, but it took till the 80's for Sobolev estimates to be proven for smooth pseudoconvex domains of finite q-type. Sup norm and Hölder estimates are even harder to obtain. Charlie Fefferman and Joe Kohn handled the problem in $\mathbb{C}^2$ in 1988, and then it took until 2022 for Dusty Grundmeier, Lars Simon, and Berit Stensønes to publish a solution in dimension three for real-analytic pseudoconvex domains of finite D'Angelo 1-type. Their solution involves bumping, namely constructing a bigger domain and a special support function that allows for the uniformity of the estimates to be obtained. I will discuss work in progress with Nicholas Aidoo, John Erik Fornæss, and Berit Stensønes (posthumously) in order to provide the solution in dimension n for smooth pseudoconvex domains of D'Angelo finite 1-type. Using stratifications and a certain amount of real algebraic geometry, we have constructed the bumping, and we are currently working through the estimates.

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Quasi-finite typeness and 1-regular types on algebraic CR manifolds: global boundedness I

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We present recent results about finite jet determination of CR maps of positive codimension from real-analytic CR manifolds into Nash manifolds (or sets) in complex space. One instance of such results is the unique jet determination of germs of CR maps from minimal real-analytic CR submanifolds in $\mathbb{C}^N$ into Nash subsets of $\mathbb{C}^{N'}$ of D'Angelo finite type, for arbitrary $N, N' \geq 2$. One the ingredients in the proof relies on the global boundedness of the quasi-finite type for Nash maps discussed in the earlier special session "Several Complex Variables: Theory and Applications". This is joint work by B. Lamel, N. Mir and G. Rond.

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Finite jet determination of CR maps in positive codimension II

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We present recent results about finite jet determination of CR maps of positive codimension from real-analytic CR manifolds into Nash manifolds (or sets) in complex space. One instance of such results is the unique jet determination of germs of CR maps from minimal real-analytic CR submanifolds in $\mathbb{C}^N$ into Nash subsets of $\mathbb{C}^{N'}$ of D'Angelo finite type, for arbitrary $N, N' \geq 2$. One the ingredients in the proof relies on the global boundedness of the quasi-finite type for Nash maps discussed in the earlier special session "Several Complex Variables: Theory and Applications". This is joint work by B. Lamel, N. Mir and G. Rond.

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Optimal data spaces for boundary value problems for the d-bar operator on Lipschitz planar domains

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We study the $\overline{\partial}$-equation subject to various boundary value conditions on bounded simply connected Lipschitz domains $D \Subset \mathbb{C}$: for the Dirichlet problem with datum in $L^p(bD, \sigma)$, this is simply a restatement of the fact that members of the holomorphic Hardy spaces are uniquely and completely determined by their boundary values. Here we identify the maximal data spaces and obtain estimates in the maximal p-range for the Dirichlet, Regularity-for-Dirichlet, Neumann, and Robin boundary conditions for $\overline{\partial}$.

This is joint work with W. E. Gryc (Muhlenberg College), J. Xiong (U. of Colorado) and Y. Zhang (Purdue U. - Fort Wayne).

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Some advances in analytic hypoellipticity

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In this talk we discuss the problem of the real analytic regularity of the solutions to sums of squares of vector fields. While the problem of the C^∞ hypoellipticity has been settled from the very beginning by Hörmander, the problem of the analytic hypoellipticity is still open and seems much more involved.
Treves conjecture states that a "sum of squares"-type operator is analytic hypoelliptic if and only if all the Poisson strata of its characteristic set are symplectic. Although this conjecture does not hold in dimension 4 or greater, some model examples would suggest that the analytic regularity still depends on a suitable stratification of the characteristic variety of the operator.
In dimension 3 we also think that the conjecture does not hold, while we think that in dimension 2 it should be true. As a consequence the low dimensional cases seem to offer some perspective into the problem.

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On CMC-immersions of surfaces into Hyperbolic 3-manifolds

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I discuss a parametrization of the moduli space of Constant Mean Curvature (CMC) c-immersions of a closed surface S (orientable and of genus at least 2) into hyperbolic 3-manifolds, by elements of the tangent bundle of the Teichmueller space of S, provided the (prescribed) mean curvature c satisfies $|c| < 1$.

In addition I shall discuss the asymptotic behaviour, as $|c| \to 1$, of those (CMC)-c immersions, and establish a convergence result in terms of the Kodaira map. For example, in case of genus 2, it is possible to catch at the limit (regular) (CMC)-1 immersions, provided we avoid (in a suitable sense) the image by the Kodaira map of the six Weierstrass points of the given Riemann surface. If time permits, I shall mention further progress for higher genus obtained in collaboration with S. Trapani.

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Bergman metrics of constant holomorphic sectional curvature

John N. Treuer
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In 2023, Huang and Li considered complex manifolds admitting a Bergman metric of constant holomorphic sectional curvature. Building on their work, in this talk we show no complex manifold whose Bergman space is base-point free, separates directions and separates points can have a Bergman metric with identically zero holomorphic sectional curvature.

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A new Poincaré type rigidity phenomenon with applications

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In this talk, we discuss a new Poincaré type phenomenon. More precisely, we will present an optimal rigidity theorem for local CR mappings between circle bundles that are defined in a canonical way over (possibly reducible) bounded symmetric domains. We prove such a local CR map, if nonconstant, must extend to a rational biholomorphism between the corresponding disk bundles. We will also talk about some applications of the theorem.

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Global regularity in the $\bar{\partial}$-Neumann problem and finite type conditions.

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The celebrated work of Catlin on global regularity of the $\bar{\partial}$-Neumann operator for pseudoconvex domains of finite type links local algebraic and analytic geometric invariants through potential theory with estimates for $\bar{\partial}$-equation. Yet despite their importance, there seems to be a major lack of understanding of Catlin's techniques, resulting in a notable absence of an alternative proof, exposition or simplification.

The goal of my talk will be to present an alternative proof based on a new notion of a "tower multi-type". The finiteness of the tower multi-type is an intrinsic geometric condition that is more general than the finiteness of the regular type, which in turn is more general than the finite type. Under that condition, we obtain a generalized stratification of the boundary into countably many level sets of the tower multi-type, each covered locally by strongly pseudoconvex submanifolds of the boundary. The existence of such stratification implies Catlin's potential-theoretic "Property (P)", which, in turn, is known to imply global regularity via compactness estimate. Notable applications of global regularity include Condition R by Bell and Ligocka and its applications to boundary smoothness of proper holomorphic maps generalizing a celebrated theorem by Fefferman.

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Heat kernel asymptotics for the $\bar{\partial}$-Neumann Laplacian on manifolds with boundary

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Let M be a compact manifold with boundary smooth boundary X and let L^k be a high power of a hermitian holomorphic line bundle over M. In this talk, I will present our recent work about how to establish the heat kernel asymptotics for the $\bar{\partial}$-Neumann Laplacian on M with values in L^k when k is large. As an application, we give a heat kernel proof of holomorphic Morse inequalities on compact manifolds with boundary. This is a joint work with Chin-Yu Hsiao and George Marinescu.

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On the local Gevrey regularity of local Gevrey vectors of Hörmander's operators of degenerate parabolic type

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My aim in this talk is to report on a common work with Gregorio Chinni about the local Gevrey regularity of local Gevrey vectors of the following operators of second order introduced by L. Hörmander in his famous paper in 1967 (Acta Math. **119**, 147-171):

$$P(x,D) = \sum_{j=1}^{m} (X_j(x,D))^2 + X_0(x,D) + c(x),$$

where $X_j(x,D)$, $j = 0,1,\ldots,m$, are vector fields with real-valued smooth coefficients on an open set Ω in $\mathbb{R}^n$.
The *Hörmander's condition* for hypoellipticity of P in Ω, may be written as follows :
Define the *weight* of X_j : $w(X_j) = 1$, $j \in \{1,\ldots,m\}$, and $w(X_0) = 2$. If $J = (j_1,\ldots,j_r)$ is a multi-index with $j_k \in \{0,1,\ldots,m\}$, $k = 1,\ldots,r$, then define the *weight of the bracket*: $X_J = [X_{j_1}, X_K]$ where $K = (j_2,\ldots,j_r)$, by: $w(X_J) = \sum_{k=1}^{r} w(X_{j_k})$ and it's *length* by r. Then the *Hörmander's condition* is:
for every $x \in \Omega$, there exists an integer $r = r(x)$, such that

$$\dim\left(Span\left(\{X_J(x);\ \text{length of } X_J \text{ less or equal to } r\}\right)\right) = n$$

(i.e. of maximal dimension).
For reason of precision , for any open subset $\widetilde{\Omega}$ in Ω call

$$\tau(\widetilde{\Omega}) = \sup_{x\in\widetilde{\Omega}} \inf w(X_J(x)).$$

The weight, which does not appear in the Hörmander's condition, is useful in the proof where a subelliptic estimate was proved, giving then the hypoellipticity. The index ε of subellipticity given by L. Hörmander was any real less than the inverse of $\tau(\Omega)$. There was also a simple proof by J. J. Kohn of subellipticity but with a smaller ε. Finely the exact ε of subellipticity equal to the inverse of $\tau(\Omega)$ was proved about ten years later by L. Rothshild and E. Stein .

This index is important in order to give precise results in the context of Gevrey regularity of Gevrey vectors. Let us first remark that given $x \in \Omega$ and index $\tau(x)$ given above, then there exists a small neighborhood of x, say ω such that $1/\tau(x)$ is a sub-elliptic index in ω. Recall moreover that an s-Gevrey vector of P, u, in Ω satisfies: for any compact K in Ω there exists C_K such that for any integer k one has

$$u \in L^2(K) \text{ and } \|P^k u\|_{L^2(K)} \le C_K^{2k+1} k^{2sk}.$$

So working with such $\tau(x)$ in ω, we got,

Theorem 1. *Assume the coefficients of P are in s-Gevrey(ω), and P satisfies the above condition. Let u an s-Gevrey vector of P in ω, then u is in $(\tau(x)\cdot s)$-Gevrey space in ω.*

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Developments in Hyperbolic Geometry
Special Session B6

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Gabriele Viaggi
University of Rome La Sapienza, ITALY

Andrew Yarmola
Yale University, USA

The session will focus on recent developments in the theory of hyperbolic manifolds in low and high dimension. For two dimensions there will be speakers with experience in Teichmüller Theory, hyperbolic surface, big and small mapping class groups, and specifically counting problems on surfaces. In three dimensions we plan on having speakers discussing volumes and deformation spaces of hyperbolic 3-manifolds. For higher dimensions we plan on having talks on the recent constructions of fibered higher dimensional hyperbolic manifolds and pro-finite rigidity.

The main goals of this session are to bring together a diverse group of European and American academics and to provide ample space for young researchers to present their work and interact with senior faculty.

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Hyperbolic manifolds and profinite rigidity

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In this talk we will survey recent progress on profinite rigidity (both absolute and relative) of the fundamental groups of finite volume hyperbolic manifolds.

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Fibring of Hyperbolic Manifolds

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Matteo Migliorini
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In 1975, Jørgensen shocked topologists by exhibiting an example of a hyperbolic 3-manifold which fibres over S^1 [3]. This result has opened up a very active field of research, with celebrated contributions of Thurston [4, 5], Agol [1] and Wise [6], which settled a fair understanding of this phenomenon in dimension 3. One of the most important results is the following theorem:

Theorem 1 (Agol, Wise). *Every closed hyperbolic* 3*-manifold* virtually *fibres over* S^1 *(i.e., it has a finite cover which fibres).*

Very few is known in higher dimensions. A simple Euler characteristic argument shows that such fibrations cannot arise when the dimension of the manifold is even, so we will focus on odd-dimensional manifolds.

In this talk, we are going to present some advancements discovered in the previous years.

Theorem 2 ([2]). *There are some (cusped) hyperbolic 5-manifolds which fibre over the circle* S^1.

As corollaries, we get some results in the direction of group theory:

Theorem 3. *There is a hyperbolic group* G *with a finite-type subgroup* H *that is not hyperbolic.*

Corollary 4. *There is a finite type group* H *which does not contain any Baumslag-Solitar group* $BS(m, n)$*, but it is not hyperbolic.*

Inspired by Theorem 1, we ask the following question:

Question: *Is it true that every odd-dimensional hyperbolic manifold virtually fibres over* S^1*?*

If time permits, some techniques and possible direction of research will be discussed.

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How many times can two curves on a surface intersect?

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Given a surface, one might want to understand how "complex" the surface is, in terms of curves. More specifically, we may ask how many times two curves on this surface can intersect. Of course, longer curves might intersect more times. KVol is a quantity measuring how many times curves can intersect, modulo their length. We will give an overview of some cases for which this quantity has been calculated, with particular focus on Veech surfaces, a class of flat surfaces with a rich group of symmetries. This is joint work in progress with Julien Boulanger

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Distribution of the components of a random multicurve

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For random pants decompositions (chosen uniformly at random out of those of total length at most L) on a hyperbolic surface, Mirzakhani studied the distribution of the lengths of the individual components as $L \to \infty$. This result has been generalized independently by Mingkun Liu and Francisco Arana-Herrera to other simple multi-curves. In this talk, using different methods based on convergence of certain measures on the space of geodesic currents, we extend it further to hold for any multicurves, simple or not. This is joint work with Juan Souto.

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Word-length curve counting on the once-punctured torus

David Fisac, *Mingkun Liu*
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This talk will be on the problem of finding a closed formula for the exact number of curves with any given word-length and self-intersection on the once-punctured torus. We will first talk about a prove of the following classic counting theorem by only combinatorial means.

Theorem 1. *Let $\Sigma_{1,1}$ be a once-punctured torus, let ℓ_ω denote the word length in $\pi_1(\Sigma_{1,1})$ with respect to a choice of canonical generators. For $L \in \mathbb{Z}_{\geq 1}$,*

$$|\{\gamma \text{ closed curve} \mid \gamma \text{ simple essential with } \ell_\omega(\gamma) = L\}| = 4\,\varphi(L),$$

where φ is Euler's totient function.

Afterwards, we will present a characterizarion of all curves with a single self-intersection on the once-punctured torus.

Theorem 2. *Let $\Sigma_{1,1}$ be a once punctured torus and $\{a, b\}$ a choice of canonical generators of $\pi_1(\Sigma_{1,1})$. A primitive curve has self-intersection one if and only if, up to renaming the generators in $\{a, b, a^{-1}, b^{-1}\}$, it can be written as one of the following:*

(1) a^2b^2, $aba^{-1}b$, $ab^{-1}a^{-1}b^2$, *or*
(2) $ab^{-1}a^{-1}ba^{n_1}b\cdots a^{n_k}b$, *or* $ab^{-1}a^{-1}ba^{-n_1}b\cdots a^{-n_k}b$, *where the words* $a^{n_1}b\cdots a^{n_k}b$, *and* $a^{-n_1}b\cdots a^{-n_k}b$ *are uniquely determined representatives of primitive simple curves, or*
(3) $a^{n_1}b\cdots a^{n_k}b$, *where* $[n_1, \dots, n_k]$ *satisfies that exists an* $m \in \mathbb{Z}_{\geq 1}$ *such that for all* $i \in \{1, \dots, k\}$, $n_i \in \{m, m+1\}$ *and it is a necklace with* 2*-variation (see Definition in* [1]*), or*
(4) $a^m ba^{m+2}b$, *for some* $m \in \mathbb{Z}_{\geq 1}$.

This will allow us to use the techniques above to prove the counting theorem for the self-intersection 1 case.

Theorem 3. *There are* 8 *primitve closed curves on* $\Sigma_{1,1}$ *of length* 4 *with* 1 *self-intersection. For any* $L \in \mathbb{Z}_{>4}$, *we have*

$$|\{\gamma \text{ primitive closed curve} \mid i(\gamma) = 1,\ \ell_\omega(\gamma) = L\}| = \begin{cases} 8\,\varphi(L-4) & \text{if } L \text{ is odd,} \\ 8\big(\varphi(L-4) + \varphi(L/2)/2\big) & \text{if } L \text{ is even.} \end{cases}$$

With these done, we will discuss asymptotics of the countings, generalizations and applications to hyperbolic geometry.

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Geodesics and norms on the cohomology of hyperbolic 3-manifolds

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The cohomology of a hyperbolic 3-manifold can be equipped with various geometric and topological norms. One such norm is related to optimizing the Lipschitz constant in a homotopy class of circle valued maps. There is an associated geodesic lamination encoding where the manifold is most stretched by these optimal Lipschitz maps and this lamination depends only on the homotopy class. A natural question then is "what do these laminations look like?" I will discuss how using Dehn surgery and topological and geometric norm comparisons, one can construct examples where these geodesic laminations can be identified.

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Distance in the pants graph and applications to Teichmüller space

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Given two pants decompositions of a compact orientable surface S, we give an upper bound for their distance in the pants graph that depends logarithmically on their intersection number and polynomially on the Euler characteristic of S. As a consequence, we find an upper bound on the volume of the convex core of a maximal cusp (which is a hyperbolic structure on $S \times \mathbb{R}$ where given pants decompositions of the conformal boundary are pinched to annular cusps). As a further application, we give an upper bound for the Weil–Petersson distance between two points in the Teichmüller space of S in terms of their corresponding short pants decompositions. The proofs rely on using pre-triangulations, train tracks, and an algorithm of Agol, Hass, and Thurston.

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Filling Riemann Surfaces by Hyperbolic Schottky Manifolds of Negative Renormalized Volume

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Given a hyperbolizable 3-manifold N with boundary components of genus at least two, the renormalized volume is a real-valued function on the space of convex co-compact hyperbolic structures $CC(N)$ on the interior of N, which always have infinite hyperbolic volume. The space $CC(N)$ is parameterized by the Teichmüller $\mathcal{T}(\partial N)$ of the boundary, up to quotient by the group generated by Dehn twists along compressible curves in ∂N. This correspondence is realized by associating to any convex co-compact manifold the conformal structure $X \in \mathcal{T}(\partial N)$ on its boundary at infinity, which is well defined. The simplest examples of convex co-compact hyperbolic 3-manifolds are the handlebodies, and, given a connected Riemann surface X of genus ≥ 2, we call Schottky filling of X a handlebody with boundary at infinity X.

A question attributed to Maldacena asks whether given a connected Riemann surface X of genus at least two, there exists a Schottky filling of X of negative renormalized volume.

In this talk, we will present an upper bound for the renormalized volume in terms of the genus and the hyperbolic curve lengths of a suitable pants decomposition of X, which allows us to positively answer the question of Maldacena for certain classes of Riemann surfaces.

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Efficient cycles for hyperbolic manifolds

Ennio Grammatica, Bruno Martelli, Roberto Frigerio
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The simplicial volume is a homotopy invariant of manifolds introduced by Gromov in his pioneering paper [1]. It is defined as the infimum $\|N\|$ of the ℓ^1-norms of fundamental cycles in the top-dimensional singular homology of N with real coefficients. While being defined only in terms of singular homology, it is deeply related to many geometric properties of N (for example, it vanishes on Riemannian manifolds with non-negative Ricci curvature, while it is positive for negatively curved Riemannian manifolds).

Computing the simplicial volume is usually a very difficult task. Many vanishing theorems are available by now, but positive exact values of the simplicial volume are known only for a few classes of manifolds, notably for complete finite-volume hyperbolic manifolds (thanks to results by Gromov and Thurston [1,2]).

Even when the simplicial volume of a manifold N is known, characterizing (or, at least, exhibiting) *almost minimal* fundamental cycles (i.e. fundamental cycles whose norm is close to $\|N\|$) may be surprisingly difficult. For example, it is known that the simplicial volume of any closed simply connected manifold N vanishes, but there is no recipe, in general, which describes fundamental cycles of N of arbitrarily small norm; in a similar spirit, even if the value of the simplicial volume of the product $\Sigma \times \Sigma'$ of two hyperbolic surfaces has been computed in [5], exhibiting a sequence of fundamental cycles whose norm approximates $\|\Sigma \times \Sigma'\|$ seems very challenging.

The situation is better understood for hyperbolic manifolds: the computation by Gromov and Thurston of the simplicial volume of such manifolds explicitly constructs almost minimal cycles via an averaging operator called *smearing* [2]. A natural question is to which extent this construction is unique, i.e., whether there exist sequences of almost minimal fundamental cycles which do not come from smearing: this problem has been partially addressed by Jungreis and Kuessner in [3,4], where they proved that almost minimal cycles necessarily come from smearing for a huge class of finite volume hyperbolic manifolds, including all the closed ones.

In this paper we improve their results by showing that, in dimension $n \geq 3$, the unique hyperbolic manifolds admitting "exotic" almost minimal fundamental cycles are those which are commensurable with the figure-eight knot complement, or, equivalently, to the Gieseking manifold.

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Integral simplicial volume and triangulation complexity of 3-manifold fibering over the circle

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Triangulation complexity and integral simplicial volume are two topological invariants studied in low-dimensional topology. The first one counts the minimal number of *embedded* simplices that are needed to triangulate a manifold, while the second one deals with the number of *singular* simplices in a fundamental cycle of the manifold. These two invariants share many properties, coincide on surfaces and are similar on many 3-manifolds, and both are very hard to compute.

In this talk, we will study the asymptotic behavior of the triangulation complexity and the integral simplicial volume of cyclic covers of 3-dimensional manifolds fibering over the circle. In particular, we will identify families of 3-manifolds for which these invariants exhibit markedly different behaviors.

This is joint work with Roberto Frigerio.

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Almost maximal $\mathrm{PSL}_2(\mathbb{R})$-representations of punctured surface groups

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Let S be a compact oriented connected surface, $\boldsymbol{x} = \{x_1, \dots, x_n\}$ an n-tuple of distinct points on S, and let $\dot{S} = S \setminus \boldsymbol{x}$.

It follows from Koebe-Poincaré uniformization theorem that, if $\chi(\dot{S}) < 0$, then for every conformal structure J on S there exists a unique J-conformal metric on $\dot{S}$ with curvature -1 and cuspidal ends.

An analogous result holds for metrics with conical ends, due to Heins [1], McOwen [2] and Troyanov [4]. Given $\boldsymbol{\vartheta} = (\vartheta_1, \dots, \vartheta_n)$ in $\mathbb{R}^n_{\geq 0}$ such that $\chi(S, \boldsymbol{\vartheta}) := \chi(\dot{S}) + \sum_i \vartheta_i$ is negative, for every J there exists a unique J-conformal metric $g_{J,\boldsymbol{\vartheta}}$ on $\dot{S}$ with curvature -1 and conical points of angle $2\pi\vartheta_i$ at x_i.

Let $\mathfrak{T}(S, \boldsymbol{x})$ be the Teichmüller space $\mathfrak{T}(S, \boldsymbol{x})$ of conformal structures on S up to isotopies that fix $\boldsymbol{x}$ and let $\mathrm{Rep}_{\boldsymbol{\vartheta}}(\dot{S}, \mathrm{PSL}_2(\mathbb{R}))$ be the space of conjugacy classes of representations $\pi_1(\dot{S}) \to \mathrm{PSL}_2(\mathbb{R})$ that send loops about the puncture x_i to rotations of angle $2\pi\vartheta_i$. Given $\boldsymbol{\vartheta}$ with $\chi(S, \boldsymbol{\vartheta}) < 0$, we obtain a map

$$\mu : \mathfrak{T}(S, \boldsymbol{x}) \longrightarrow \mathrm{Rep}_{\boldsymbol{\vartheta}}(\dot{S}, \mathrm{PSL}_2(\mathbb{R}))$$

defined as $\mu([J]) := [\rho_{J,\boldsymbol{\vartheta}}]$, where $\rho_{J,\boldsymbol{\vartheta}}$ is the monodromy of the unique metric $g_{J,\boldsymbol{\vartheta}}$.

In the cuspidal case (namely $\boldsymbol{\vartheta} = \mathbf{0} := (0, \dots, 0)$), the monodromy representation $\rho_{J,\mathbf{0}}$ allows us to uniquely reconstruct the metric $g_{J,\mathbf{0}}$. The same holds in the conical case, as long as the angles are smaller than π. This implies that in these cases the map μ is a diffeomorphism. The key point is that, when the conical angles are smaller than π, every topological pair of pants decomposition of the punctured surface can be straightened to a geodesic one. Such approach via pair of pants breaks down for larger angles. Even worse, for large angles the map μ is no longer bijective.

Our main result is the following.

Theorem 1 ([3]). *Assume that every $\vartheta_i < 1$ and for every pair of distinct i, j we have $\vartheta_i + \vartheta_j \leq 1$. Then the map μ is a diffeomorphism.*

The hypotheses of Theorem 1 are easily seen to be sharp.

The proof of Theorem 1 uses the function $E_\rho : \mathfrak{T}(S, \boldsymbol{x}) \to \mathbb{R}$, defined for every $\rho \in \mathrm{Rep}_{\boldsymbol{\vartheta}}(\dot{S}, \mathrm{PSL}_2(\mathbb{R}))$, that sends a class $[J]$ to the energy of the unique ρ-equivariant harmonic map $\widetilde{S} \to \mathbb{H}^2$. We study the gradient flow $-\nabla E_\rho$ to show that E_ρ has a stationary point in $\mathfrak{T}(S, \boldsymbol{x})$ and that such stationary point is unique, which will show that μ is bijective.

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Geometry of geodesic currents

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The space of projective, filling currents $\mathbb{P}\mathcal{C}_{fill}(S)$ contains many structures relating to a closed, genus g surface S. For example, it contains the set of all closed curves on S, as well as an embedded copy of Teichmüller space, and many other spaces of metrics on S. It turns out that the Thurston metric on Teichmüller space extends to $\mathbb{P}\mathcal{C}_{fill}(S)$. We will discuss the geometry of $\mathbb{P}\mathcal{C}_{fill}(S)$ with this metric.

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Automorphisms of geodesic currents preserve intersection form

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We will discuss progress in proving Ivanov's meta conjecture in the context of geodesic currents. Ivanov's meta conjecture says that every object naturally associated with a surface and having a 'sufficiently rich' structure has the mapping class group as its group of automorphisms. The conjecture has been proven for various combinatorial objects associated with a surface as well as for the Teichmüller space of a surface. The space of geodesic currents contains many of these structures, such as the set of closed curves up to homotopy and the Teichmüller space. We discuss progress in showing Ivanov's meta conjecture for a natural group of automorphisms of currents.

For more information visit: https://arxiv.org/abs/2309.14532

[1]The speaker would like to thank Eugenia Sapir for all the insightful conversations that lead to this work.
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The space of co-geodesic currents of a hyperbolic group

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We define a notion of hyperplane at infinity for a hyperbolic group G and study G-invariant Radon measures on the space of hyperplanes at infinity, which we call “co-geodesic currents”. Co-geodesic currents are induced by many classical objects such as geodesic currents for surface groups, certain cocompact actions of hyperbolic groups on CAT(0) cube complexes, some actions of hyperbolic groups on real trees, etc. Moreover, there is a natural intersection pairing between co-geodesic currents and geodesic currents, generalizing Bonahon's intersection number when G is a surface group. Furthermore, every co-geodesic current induces natural dual pseudo-metric space with a measured wall structure, in the sense that the intersection of with the current determined by the conjugacy class of an element in G recovers the stable length of that element in the pseudo-metric space. This is joint work in progress with Eduardo Reyes.

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(Random) Hyperbolic surfaces with large systoles

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The systole of a hyperbolic surface X is the least length of a closed geodesic on X. In this talk, I'll discuss the following question: how large can the systole of a hyperbolic surface be? I'll also present some (random) constructions of hyperbolic surfaces with large systoles. This is joint work with Bram Petri.

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Model Theory
Special Session B7

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Model Theory is a branch of mathematical logic that has had many interactions with various areas of mathematics. Over the years Model Theory has produced and developed very sophisticated techniques that apply to (and interact with) various fundamental areas of Mathematics, such as algebra, number theory, dynamical systems, valued fields, and algebraic/analytic geometry. Some notable examples are: o-minimality and its close connections with real analysis and real analytic and algebraic geometry; Hrushovski's work in Diophantine geometry contributing to Mordell-Lang and Manin-Munford Conjectures; Pila and Wilkie's fundamental and startling results on counting rational points on definable sets connecting o-minimality and Diophantine geometry; work first by Macintyre (quantifier elimination) and successively by Denef (cell decomposition) and Loeser for the p-adic fields contributed to motivic integration and rationality of Poincaré series; Zilber's approach to a model theoretic analysis of complex exponentiation. More recently the abstract model theory found applications in combinatorics with significant contributions to the classification theory initiated by Shelah.

This special session will cover both aspects of Model Theory and includes speakers who are leading experts in the research in both areas of model theory, applied and more abstract.

For more information visit `www.matfis.unicampania.it/home-model-theory`

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The Model Theory of Modules over a Ring with Involution

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The model theory of modules over a ring $(R, *)$ with involution has features that enhance the classical theory [3], which includes the familiar theme of positive primitive formulae and how they can be used to define the Ziegler topology on the space of indecomposable algebraically compact representations. The main feature is that the Prest dual of a pp-formula may be composed with the involution to define an operation on the usual modular lattice of pp-formulae in a way that it too becomes endowed with a canonical involution.

We will explain how the *Moore-Penrose inverse,* a notion from the theory of rings of operators [1], is related to elimination of quantifiers. A careful process of formal adjunction of these inverses yields the $*$-*regularization* of $(R, *)$, which is a universal construction that mimics a similar construction due to Olivier [2] for a commutative ring (with involution the identity). A ring $(S, \#)$ with involution in $*$-regular iff the modular pp-lattice with involution is a *quantum logic.*

Another feature of the theory are the $*$-*contravariant forms* that arise in the representation theory of Lie algebras and Chevalley groups [4]. The universal $*$-contravariant form of a module over $(R, *)$ is definite if and only if the modular pp-lattice with involution satisfies the Law of Contradiction, which may be regarded as an axiom schema that partially axiomatizes the finite dimensional representations of certain Lie algebras.

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Equations in the j-invariant and its derivatives

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Zilber's exponential-algebraic closedness conjecture predicts that polynomial-exponential equations must have solutions, unless they have a very good (geometric) reason not to. The same can be asked of other functions having some symmetries, such as abelian exponentials, modular functions, and similar. I'll discuss what Vahagn Aslanyan, Sebastian Eterović and I have done for equations in j, j', j'' in one variable. Even in just one variable, this raises questions that were not addressed in the literature; for instance, we show that the equation $j'' = 0$ has "Zariski dense" many solutions, in a suitable sense. I'll discuss the reasoning behind Zilber's conjecture and the ingredients we used to study j and its derivatives.

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Invariants rings and fields in o-minimal structures

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Jana Marikova [1], was first to study automorphism-invariant groups in o-minimal structures and showed that they can be endowed with a group topology, similarly to the case of definable groups. In this talk I describe results on invariant rings and fields.

While 0-dimensional invariant fields may include the fields of rational or algebraic numbers (clearly not definable), we show that a positive dimensional type-definable or Ind-definable field must be definable. In addition, while an arbitrary invariant field of positive dimension might not be definable we conjecture that it is isomorphic, via an invariant map, to a definable one.

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Model theoretic events

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This talk is motivated by the following two soft questions: How do we sample an infinite sequence from a first order structure? What model theoretic properties might hold on almost all sampled sequences? We advance a plausible framework in an attempt to answer these kinds of questions. The central object of this talk is a proability space. The underlying set of our space is a standard model theoretic object, namely the space of types in countably many variables over a monster model. Our probability measure is an iterated Morley product of a fixed Borel-definable Keisler measure. Choosing a point randomly in this space with respect to our distribution yields a random generic type in infinitely many variables. We are interested in which model theoretic events hold for almost all random generic types. Two different kinds of events will be discussed: (1) The event that the induced structure on a random generic type is isomorphic to a fixed structure; (2) the event that a random generic type witnesses a dividing line.

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Some model theory of quadratic forms

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Vector spaces over finite fields with quadratic forms are an important example in the Cherlin-Hrushovski theory of Lie coordinatization of smoothly approximable theories, which are homogeneous structures that can be approximated by finite homogeneous substructures. These come in two forms: the orthogonal spaces (which have only one quadratic form) and the quadratic geometries (which have a whole family of quadratic forms). We address several basic questions about the model theory of these structures where the field is allowed to be infinite. For example, we classify all pseudo-finite theories of orthogonal spaces and quadratic geometries, axiomatize the model companions of each, and give a reasonably complete neostability-theoretic classification of all of these theories.

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Expansions of $\mathbb{R}$ and $\mathbb{N}$ by k-automatic sets: Choose-your-own-adventure!

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There are compelling and long-established connections between automata theory and model theory, particularly regarding expansions of Presburger arithmetic by sets whose base-k representations are recognized by a finite-state automaton. We call such sets "k-automatic". Büchi automata are the natural extension of finite-state automata to a model of computation that accepts infinite-length inputs. We say a subset X of the reals is "k-regular" if there is a Büchi automaton that accepts (one of) the base-k representations of every element of X, and rejects the base-k representations of each element in its complement. These sets often exhibit fractal-like behavior–e.g., the Cantor set is 3-regular. In this talk we will consider the expansions of Presburger arithmetic and the real additive group by k-automatic and k-regular sets respectively. In the real setting, we will discuss dividing lines in definability from the perspectives of both tame geometry and neostability. In the setting of Presburger arithmetic, we obtain a characterization of expansions of $(\mathbb{N}, +)$ by unary k-automatic sets, and discuss its consequences for the decidability and neostability-related properties of related structures.

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Asymptotics of Skolem's exponential functions

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Let us call Skolem functions the smallest set of functions $f\colon \mathbb{N}^{>0} \to \mathbb{N}^{>0}$ closed under pointwise sum $f+g$, product fg, and exponentiation f^g, and containing the constant function $x \mapsto 1$ and the identity function $x \mapsto x$. We consider the order on the set of Skolem functions given by $f < g$ if $f(x) < g(x)$ for all x large enough. By results of Hardy [4], this is a total order. Skolem [6] conjectured that it is, indeed, a well order of order type ϵ_0. The first part of the conjecture was proved by Ehrenfeucht [3] using Kruskal's tree theorem, while the second part remains unsolved.

In [5], Levitz showed that the order type of the Skolem functions is at most equal to the smallest critical epsilon-number (the least ordinal α such that $\alpha = \epsilon_\alpha$). Furthermore, Levitz's work provides bounds on the order type of certain initial segments of the Skolem functions, for instance the set of Skolem functions below $x \mapsto 2^{2^x}$ has order type at most ϵ_0, and the set of Skolem functions below $x \mapsto 2^{x^x}$ has order type at most ϵ_ω. In [2], van den Dries and Levitz improved the first of these bounds to ω^{ω^ω}.

We will study the asymptotic behaviour of Skolem functions by embedding them in Conway's field of surreal numbers [1]. Our main result is as follows.

Theorem 1. *Let $c \geq 1$ be a surreal number and let Q be a Skolem function. The set of real numbers $r \in \mathbb{R}$ such that there is a Skolem function h satisfying $(h/Q)^c = r + o(1)$ has no accumulation points in $\mathbb{R}$.*

As a consequence, we get a different proof of van den Dries and Levitz's result, and we can improve some of Levitz's upper bounds. For instance, we show that the Skolem functions below $x \mapsto 2^{x^x}$ have order type at most ϵ_0.

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Local coordinatization of projective geometries and explicit bounds in Elekes-Szabó for arbitrary arity and co-dimension

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An influential theorem of Elekes and Szabó indicates that the intersections of a given algebraic variety with large finite grids of points may have maximal size only for varieties that are closely connected to algebraic groups. Techniques from model theory — variants of Hrushovski's group configuration and of Zilber's trichotomy principle — are very useful in recognizing these groups, and led to far reaching generalizations of Elekes-Szabó in the last decade.

In this talk, focusing on the strongly minimal case, we provide a generalization of the earlier result from Chernikov-Peterzil-Starchenko [2] to arbitrary co-dimension, in particular obtaining explicit bounds in a theorem of Bays-Breuillard [4] over the complex numbers.

Our key tool is a very explicit version of the (model theoretic) coordinatization of projective geometries supported on grids with large count, demonstrating that instead of closing under all canonical bases it suffices to add finitely many very explicit ones.

Joint work with Kobi Peterzil and Sergei Starchenko.

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Around first-order rigidity of Coxeter groups

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We survey recent results on the problem of first-order rigidity of Coxeter groups. Specifically, we survey the results of [1] and [4], joint with S. André and with R. Sklinos, respectively. The main result of [4] is that irreducible affine Coxeter groups are first-order rigid, i.e., they are the only finitely generated models of their first-order theory. We deduce from this that irreducible affine Coxeter groups are profinitely rigid in the absolute sense, thus solving an open problem posed in [3]. Paper [1] goes in another direction, specifically, by the work of Sela, for every group G elementary equivalent to the free group on two generators we have that $\mathbb{Z}_2 * \mathbb{Z}_2 * \mathbb{Z}_2$ is elementary equivalent to $\mathbb{Z}_2 * \mathbb{Z}_2 * \mathbb{Z}_2 * G$, and so, without further restrictions, Coxeter groups are very far from being first-order rigid. Our results show that, despite this, to a large extent, from the perspective of Coxeter group theory this is somewhat "accidental", in the sense that if we restrict to models which are generated by involutions the situation is completely different. Our main results in this direction are the following. (1) If W and W' are even Coxeter groups which are elementary equivalent, then they are isomorphic. (2) If W is word hyperbolic and either even, or 2-spherical, or 1-ended, and G is elementary equivalent to W and generated by finitely many involutions, then G is isomorphic to W. Finally, we prove that there are two hyperbolic Coxeter groups which are $\forall\exists$-elementary equivalent but not isomorphic, and so, assuming $\forall\exists$-elimination of quantifiers (which is conjectured to be true by the community), item (2) is not true in the odd case. Result (1) above generalizes the results of [2], where the same was shown for right-angled Coxeter groups.

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Constant power maps on Hardy fields and Transseries

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We study H-fields (certain ordered differential fields generalizing Hardy fields and Transseries) equipped with "constant power maps". We show that this class has a model companion, the models of which include the field of LE-transseries and any maximal Hardy field. We study the induced structure on the constant field, prove a relative decidability result, and give some applications to certain systems of differential equations.

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PDE Theory for Fluid-Structure Interactions
Special Session B8

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This special session focuses on applied problems in the analysis of partial differential equations (PDEs) related to the interactions between a fluid and a solid. We expect contributions from researchers studying coupled models where the structure can be represented through the theory of elasticity (e.g., plates, beams, bulk solids), while the fluid may be included through the Navier-Stokes Equations or alternative theories (e.g. Stokes, Euler, potential flow). The session is also open to talks relating to analytical questions of fluids or solids independently, as they might pertain to applications in coupled fluid-structure systems. For instance, questions about well-posedness, regularity, asymptotic behavior are welcome. Additionally, techniques and results dealing with relevant nonlinear modeling, approximation methods, or computation of numerical solutions and stability are also among the purposes of the session.

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A measure for the stability of structures immersed in a 2D laminar flow

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We introduce a new measure for the stability of structures, such as the cross-section of the deck of a suspension bridge, subject to a 2D fluid force, such as the lift exerted by a laminar wind. We consider a wide class of possible flows, as well as a wide class of structural shapes. Within a suitable topological framework, we prove the existence of an optimal shape maximizing the stability. Applications to engineering problems are also discussed. Based on joint works with Edoardo Bocchi.

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Time-periodic incompressible viscous flow around a translating rigid body

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We consider a time-periodic incompressible three-dimensional Navier-Stokes flow past a translating rigid body. Firstly, we prove existence and uniqueness of strong solutions in the exterior domain Ω, along with pointwise estimates for the velocity and pressure. The fundamental solution of the time-periodic Oseen equations plays a crucial role in the derivation of such estimates. In the second part of the talk, we consider the approximation of the exterior problem in truncated domains $\Omega \cap B_R$ under appropriate artificial boundary conditions at ∂B_R. We prove existence and uniqueness of weak solutions for these bounded domains problems. Finally, the velocity component of the truncation error is estimated in terms of R, showing that, when $R \to \infty$, the velocities of the truncated problems converge, in a suitable norm, to the velocity of the exterior flow. This work generalizes the results of P. Deuring and S. Kračmar for the corresponding steady problem.

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Long-time behavior of an anistropic rigid body interacting with a Poiseuille flow in an unbounded 2D channel

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We study the long-time behavior of an elliptic rigid body which is allowed to vertically translate and rotate in a 2D unbounded channel under the action of a Poiseuille flow at large distances. The motion of the fluid is modelled by the incompressible Navier-Stokes equations, while the motion of the ellipse is described through Newton's laws in the presence of additional elastic restoring forces but without any structural damping. Our main contributions are a global-in-time existence result and a proof of return to equilibrium. To our knowledge, this represents the first long-time analysis of fluid-solid interaction problems with a given non-trivial final state. Our results are based on two main ingredients: a precise description of the ellipse motion whenever it comes close to the channel boundaries and a stability analysis to compare solutions to fluid-solid problems. To tackle the description of the ellipse motion close to channel boundaries, we provide a novel extensive analysis of fluid-solid interaction asymptotics allowing all possible motions of the solid and non-symmetric configurations. Our return-to-rest result requires to compare different solutions to time-dependent and stationary problems that involve different solid and fluid domains. For this, we provide the first approach on the stability of the stationary problem in a fully eulerian setting.

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On the long-time behaviour of solutions to unforced evolution Navier-Stokes equations under Navier boundary conditions

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We consider the asymptotic behaviour of the solutions to Navier-Stokes unforced equations under Navier boundary conditions in a wide class of merely Lipschitz domains of physical interest. The analysis draws its main motivation from celebrated results by Foias-Saut [2] under Dirichlet conditions; here the choice of the boundary conditions requires carefully considering the geometry of the domain, due to the possible lack of the Poincaré inequality in presence of symmetries. In non-axially symmetric domains we show the validity of the Foias-Saut result about the limit at infinity of the Dirichlet quotient, in axially symmetric domains we provide two invariants of the flow which completely characterize the motion and we prove that the Foias-Saut result holds for initial data belonging to one of the invariants.

Based on the paper [1].

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Well-posedness of a nonlinear shallow water model for oscillating water columns

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We consider a particular wave energy converter, the so-called oscillating water column. Water waves governed by the one-dimensional nonlinear shallow water equations arrive from the off-shore, enter a partially-closed chamber and the consequent variation of air volume activates a turbine. We reformulate the problem as a hyperbolic transmission problem related to the wave-structure interaction at the entrance of the chamber. A time-dependent air pressure inside the chamber is taken into account in the model. First, we impose conservation of the total energy to derive a transmission condition that closes the system. Then, we address its local well-posedness, which is obtained by constructing a Kreiss symmetrizer.

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A fluid-poroviscoelastic structure interaction problem with nonlinear coupling

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We present a well-posedness result for a moving-boundary fluid-poroelastic structure interaction (FPSI) problem involving the coupled dynamics between an incompressible fluid modeled by the Navier-Stokes equation and a multilayered structure consisting of a thin reticular plate and a thick Biot poroelastic medium. The fluid domain and the physical Biot domain, on which the Navier-Stokes and the Biot equations respectively are posed, are time-dependent and a priori unknown domains, which are determined by the a priori unknown reticular plate displacement.

The Biot equations, originally introduced by Biot to model soil consolidation in [1] and [2], model the dynamics of the displacement and pore pressure of a poroelastic medium, which is a medium that is elastic and admits fluid flow through its pores. Such coupled FPSI problems involving incompressible fluids interacting dynamically with poroelastic structures arise in applications to biomedical engineering and geosciences, and are hence of practical importance.

First, we show an existence result for solutions to a regularized form of the FPSI problem, with a regularization parameter $\delta > 0$. This regularization is needed for the analysis, because solutions in the finite energy space for the original problem do not have sufficient regularity for the weak formulation and the moving domains to be well-defined. We give a constructive existence proof for the regularized FPSI problem, by using a splitting scheme which splits the Biot/fluid dynamics and the reticular plate dynamics to define approximate solutions to the FPSI problem. We then pass to the limit in the semidiscrete weak formulations by using compactness arguments, which include compactness arguments of Aubin-Lions type for functions defined on moving domains [3], to obtain strong convergence of the approximate solutions.

Next, upon showing existence of solutions to the regularized problem, we establish a weak-classical consistency result under the additional assumption that the Biot medium is poroviscoelastic, which shows that the weak solutions to the regularized problem are physically reasonable in the following sense. In particular, if the original (non-regularized) FPSI problem has a classical solution on the time interval $[0, T]$, we show that the weak solutions to the regularized problem with regularization parameter $\delta > 0$ converge to the classical solution to the original problem as $\delta \to 0$. We accomplish this by using a new bootstrap argument in addition to energy-type estimates in order to obtain control of geometric quantities that is uniform in the regularization parameter $\delta > 0$.

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On the motion of several rigid bodies in a viscous fluid

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We show that the collective effect of N rigid bodies $(\mathcal{S}_{n,N})_{n=1}^N$ of diameters $(r_{n,N})_{n=1}^N$ immersed in an incompressible non–Newtonian fluid is negligible in the asymptotic limit $N \to \infty$ as long as their total packing volume $\sum_{n=1}^N r_{n,N}^d$, $d = 2, 3$ tends to zero exponentially – $\sum_{n=1}^N r_{n,N}^d \approx A^{-N}$ – for a certain constant $A > 1$. The result is rather surprising and in a sharp contrast with the associated homogenization problem, where the same number of obstacles can completely stop the fluid motion in the case of shear thickening viscosity. A large class of non–Newtonian fluids is included, for which the viscous stress is a subdifferential of a convex potential.

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A proof of Vishik's nonuniqueness theorem for the forced 2D Euler equation

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In this talk we will present a new proof of Vishik's theorem on the non uniqueness of solutions for 2D incompressible Euler equations with forcing. Our proof relies in the construction of an unstable radial vortex into two steps: i) the construction of a piece-wise unstable radial vorticity, ii) the regularization of such a vorticity. The rest of the proof follows the Vishik's strategy and the paper "Instability and nonuniqueness for the 2d Euler equations in vorticity form, after M. Vishik" by D. Albritton et al.

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Homogenization and low Mach number limit for the evolutionary Navier–Stokes–Fourier system

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In this talk, we are going to consider the motion of a compressible, viscous and heat-conducting fluid on a domain perforated by tiny holes. First, we are going to identify a class of dissipative solutions to the Oberbeck-Boussinesq approximation as a low Mach number limit of the primitive system. Secondly, by proving the weak–strong uniqueness principle, we obtain strong convergence to the target system on the lifespan of the strong solution.

This is a joint work with Nilasis Chaudhuri from the Faculty of Mathematics, Informatics and Mechanics of the University of Warsaw.

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Estimates of a possible gap related to the energy equality for Newtonian and non-Newtonian fluids

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We consider the Navier-Stokes and the p-Navier-Stokes equations, and we investigate on the validity of an energy equality.

It is widely acknowledged that a weak solution of the Navier-Stokes equations a priori enjoys an energy inequality. We construct a weak Leray-Hopf solution enjoying a sort of energy equality, which adds a dissipative quantity to the classical energy equality. The additional dissipation, that measures the possible gap with the classical energy equality, is only expressed in terms of energy quantities.

The result is obtained under minimal assumptions on the initial datum for the existence of a weak solution. It is based on a new existence theorem, where our weak solution is the limit of a sequence of solutions to a regularized problem.

Further, we extend our investigation to weak solutions of the chosen model of non-Newtonian fluids within space-periodic domains.

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Long-time analysis of a nonlinear system of PDEs modeling suspension bridges with piers

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We investigate the long-time behavior of the solutions of a nonlinear nonlocal system of evolution PDEs governing the dynamics of suspension bridges with intermediate piers under the action of the wind. The deck of the bridge is modeled as a degenerate plate composed by a central beam which can move vertically and by a continuum of cross sections which can rotate around their center located on the beam. This gives rise to a system with two degrees of freedom: the vertical displacement of the central beam, governed by a beam-type equation, and the torsional displacement of the cross sections, described by a wave-type equation. Due to the presence of the piers, the solutions fail to be smooth and a suitable notion of weak solution is required. The dissipative effects of the structure and the action of external sources (such as the vortex shedding generated by the wind) are explicitly taken into account by inserting appropriate damping and forcing terms. Particular attention is also devoted to the purely longitudinal motion, arising when no torsional displacement occurs.

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Well-posedness of Free Boundary Inviscid Flow-Structure Interaction Systems

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Sarka Necasova
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We obtain the local existence and uniqueness of solutions for a system describing interaction of an incompressible inviscid fluid, modeled by the Euler equations, and an elastic plate, represented by the fourth-order hyperbolic PDE. We provide a priori estimates for the existence with the optimal regularity H^r, for $r > 2.5$, on the fluid initial data and construct a unique solution of the system for initial data $u_0 \in H^r$ for $r \geq 3$.

We also address the compressible Euler equations in a domain with a free elastic boundary, evolving according to a weakly damped fourth order hyperbolic equation forced by the fluid pressure. We establish a priori estimates on local-in-time solutions in low regularity Sobolev spaces, namely with velocity and density initial data in H^3.

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Free boundary problems: viscosity and variational approaches
Special Session B9

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Fausto Ferrari
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This session is scheduled on July 25-26 and it is focused on **Free boundary problems (FBP).** Free boundary problems are a type of differential equation where the domain in which the equation holds, depends itself on the solution. Examples arise in flame propagation, image reconstructions, jet flows, optimal stopping problems in financial mathematics, tumor growth, and in many other different contexts, providing the opportunity for an interplay between the applied sciences and pure mathematical analysis.

A classical example, which we will use here to illustrate the significance of FBP, is the Bernoulli problem. It arises in two dimensional fluid dynamics and it was first studied systematically by Alt and Caffarelli using a variational approach, in the one-phase setting i.e for non-negative solutions. The two-phase case, in which a solution is allowed to change sign, was then investigated by Alt, Caffarelli, and Friedman who devised a fundamental monotonicity formula. The theory was propelled forward by breakthrough results due to Caffarelli and relying on a viscosity formulation of the problem which allowed to take a geometric approach to answer the essential question of the regularity of the free boundary.

Other analogous problems arise when considering models for which the free boundary can occur on a lower dimensional space, for example the *thin Bernoulli problem.* Related problems also appear in the study of cooperative systems of species, in optimization problems for spectral functions, in optimal partition problems, or in the study of harmonic functions with junctions Several evolution problems are also connected to the Bernoulli problem, like the Stefan problem or the Hele-Shaw problem used to describe an incompressible flow lying between two nearby horizontal plates Furthermore, one can consider FBP in non-commutative structures. Other fundamental problems are obstacle-type problems, which also present a vast literature.

Given the interest of the international scientific community for free boundary problems, our special session is dedicated to this topic. In particular, the invited speakers take different approaches to attack important and current questions, whether with variational or viscosity tools.

In the effort of fostering collaborations, especially between senior and junior researchers, we will advertise the session to mathematicians at all levels, at different institutions both in the U.S and in Italy.

This two-day session will feature several lectures of varying length. The special session is upported by the Department of Mathematics of the university of Bologna and INDAM

The list of the speakers who accepted to deliver a seminar and further information are available at the following working in progress web page:
https://math.unibo.it/en/events/free-boundary-problems-viscosity-and-variational-approaches

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For more information visit `https://math.unibo.it/en/events`.

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Sharp order of vanishing for parabolic equations, nodal set estimates and Landis type results

Nicola Garofalo
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I will present a best possible estimate of the order of vanishing of solutions to parabolic equations. For real-analytic leading coefficients, we establish a localised estimate of the nodal set, at a given time-level, that generalises the celebrated one of Donnelly and Fefferman. We also obtain Landis type results for global solutions. This is joint work with Vedansh Arya & Agnid Banerjee.

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Boundary behavior of solutions to fractional elliptic problems

Serena Dipierro
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Solutions of nonlocal equations typically depend rather significantly on their values outside of a given region of interest and, in this sense, it is often convenient to assume "global" conditions to deduce "local" results. In this talk, we present instead a Hopf Lemma for solutions to some integro-differential equations that does not assume any global condition on the sign of the solutions. We also show that non-trivial radial solutions cannot have infinitely many zeros accumulating at the boundary.

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A two-phase free boundary problem for an operator with non-standard growth

Fausto Ferrari
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Claudia Lederman
IMAS - CONICET and Departamento de Matemática, Universidad de Buenos Aires, Argentina

We will present recent results on a two-phase free boundary problem for an operator with non-standard growth.

In fact, we consider viscosity solutions to a free boundary problem for a nonlinear elliptic PDE with non-zero right hand side. We obtain regularity results for solutions and their free boundaries.

The study of PDE's of the type considered is motivated by their application in the modelling of different phenomena, such as non-Newtonian fluids, non-linear elasticity and image reconstruction.

The nonlinear degenerate/singular nature of these equations leads to challenging difficulties that will be discussed in this talk.

The fact that our results hold for merely viscosity solutions allows a wide applicability.

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Boundary unique continuation problems and size estimates of critical sets

Stefano Vita
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In this talk we discuss some recent results on boundary unique continuation problems. In particular, we describe new techniques to obtain size estimates of singular and critical sets of harmonic functions up to a boundary where the functions vanish.

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Free boundary regularity in an optimal partition problem

Roberto Ognibene
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Let us consider a bounded domain, divided into a fixed number of disjoint subdomains and, among all the possible configurations, let us consider the one for which the sum of the first Dirichlet eigenvalues of the subdomains is minimal. In this talk, I will discuss the regularity of the interface which emerges as boundary of such optimal partition and, in particular, I will focus on the regularity up to the fixed boundary. The talk is based on joint works with B. Velichkov.

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Fully nonlinear equations in thin domains: a test function approach

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In this talk I will present a work done in collaboration with Ariela Briani and Hitoshi Ishii, where we extend to fully nonlinear operators the well known result on thin domains of Hale and Raugel. In 1992 they proved that solutions of a Neumann problem in thin domains $\Omega_\epsilon = \{(x, y) \in R^N \times R \mid x \in \Omega,\ 0 < y < \epsilon g(x)\}$, will converge for ϵ going to zero to the solution of a Neumann problem in Ω where the equation itself has an extra first order term coming from the top boundary $y = g(x)$. With a totally new approach, instead of the Laplacian we consider fully nonlinear operators that are proper in the sense of the User's guide but may be degenerate elliptic. The result is more general even in the case of the Laplacian.

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On some segregation models

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Segregation phenomena occur in many areas of mathematics and science: from equipartition problems in geometry, to social and biological processes (cells, bacteria, ants, mammals) to finance (sellers and buyers). Segregation problems model a situation of high competition for resources and involve a combination of diffusion and annihilation between populations. We present three different models: in the first one the competition between species is nonlocal, meaning that the growth of a population at a point is inhibited by all other populations in a full neighbor of that point; in the second one the diffusion terms are given by fully nonlinear operators; in the third one two competing species follow propagation equations, one of them involving a local diffusion while the other one involving a non-local diffusion. These are joint works with Luis Caffarelli, Veronica Quitalo and Monica Torres.

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Weil height and modular Galois representations

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An algebraic field is said to have the *Bogomolov property*, (property (B) for short), if the Weil height is uniformly bounded below outside torsion points. It is well known that property (B) holds for number fields and for potentially abelian extensions; moreover a theorem by Habegger proves property (B) for the field generated by the torsion points of an elliptic curve defined over $\mathbb{Q}$. Together with F. Amoroso we generalized this result, establishing property (B) for the extension cut out by the Galois representation associated to a modular form, assuming the existence of a strong supersingular prime and the fullness of the image of the residual representation; moreover we conjectured that property (B) holds unconditionally for modular Galois representations.
In my talk, I will explain the main idea underlying the proof, and discuss some aspects of particular relevance in the theory of automorphic forms.

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Plateau's laws for surfaces of minimal capacity

Michael Novack
Carnegie Mellon University

Daniel Restrepo
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Anna Skorobogatova
Princeton University

In this talk, we will discuss a novel family of free boundary problems that arise in the study of variational problems with topological constraint in their level sets. More precisely, given a compact set $\mathcal{W} \subset \mathbb{R}^{n+1}$ (a "wire frame"), and a potential $F : [0,1] \to [0,\infty)$, we consider the minimization problems

$$\inf \left\{ \int_\Omega |\nabla u|^2 + F(u) \, : \, u \in C_0^1(\Omega; [0,1]) \, , \, \{u = 1\} \text{ spans } \mathcal{W} \right\}, \tag{1}$$

where $\Omega = \mathbb{R}^{n+1} \setminus \mathcal{W}$. In the case where $F(t)$ is a double-well potential, solutions of (1) corresponds to Allen-Cahn approximations to soap films spanning the wire frame $\mathcal{W}$, whereas if $F \equiv 0$, (1) models surfaces of minimum capacity attached to $\mathcal{W}$.

The heart of the matter in this reformulation of various classical variational problems is the notion of spanning, which corresponds to the so-called homotopic spanning condition introduced in [2] in the context of Plateau's problem -and extended to Sobolev functions in [3]. Homotopic spanning has the advantage of being compatible to the so-called Plateau's laws, meaning that minimizers to Plateau's problem with such boundary conditions are analytic except at some special conical singularities (see [1,4]). We will see that a similar property holds for minimizers to (1): the free boundary $\{u = 1\}$ is a smooth surface possibly except at some special conical singularities including, for instance, triple junctions. Aside of this novel aspect of the singular set, we will discuss in what extent the free boundary problem associated with (1) still shares several interesting features with other well-known models, like the two-phase Bernoulli and optimal partition problems.

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Regularity results for a penalized thin obstacle problem with variable coefficients

Donatella Danielli
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Regularity results for a penalized thin obstacle problem with variable coefficients. Abstract: In this talk we will discuss a two-penalty boundary obstacle problem for a divergence form elliptic operator, motivated by applications to fluid dynamics and thermics. Our goals are to establish regularity properties of the solution and structural properties of the free boundary. The proofs are based on tailor-made monotonicity formulas of Almgren, Weiss, and Monneau type, combined with the classical theory of oblique derivative problems. This is joint work with Brian Krummel (University of Melbourne).

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A capillarity one-phase Bernoulli free boundary problem

Giorgio Tortone
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This seminar explores one-phase Bernoulli free boundary problems within smooth containers, focusing on scenarios where the normal derivative of solutions is prescribed along the containers boundary. We examine the regularity of the free boundary and the structure of the wetting region, characterized by its contact with the fixed boundary.
Key findings include characterizing the contact angle in terms of container permeability and unveiling the smooth $(d-2)$-dimensional nature of the boundary of the wetting region, except for a possible closed set of dimension at most $d-5$.

This is based on a joint work with L. Ferreri and B. Velichkov.

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Regularity for almost minimizers of a degenerate Bernoulli-type functional

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Serena Dipierro, Enrico Valdinoci
Department of Mathematics and Statistics, University of Western Australia
Fausto Ferrari
Dipartimento di Matematica, Università di Bologna

In this talk, we deal with almost minimizers of the energy functional

$$J_p(u,\Omega) := \int_\Omega \Big(|\nabla u(x)|^p + \chi_{\{u>0\}}(x) \Big)\, dx, \quad p > 1, \tag{1}$$

where Ω is a bounded domain in $\mathbb{R}^n$ and $u \geq 0$. The functional J_p is a generalization to each $p > 1$ of the classical one-phase (Bernoulli) energy functional ($p = 2$ in (1)).
Almost minimizers of J_2 were investigated in [2,1]. However, D. De Silva and O. Savin provided in [4] a different approach than [2,1], based on nonvariational techniques, to study almost minimizers of J_2 and their free boundaries. Precisely, inspired by [5], they showed that almost minimizers of J_2 are "viscosity solutions" in a more general sense. This property roughly means that almost minimizers satisfy comparison in a neighborhood of a touching point whose size depends on the properties of the test functions. Once this fact was established, the regularity of the free boundary for almost minimizers followed via the techniques developed by De Silva in [3].
In this talk, we present an optimal Lipschitz continuity result for almost minimizers of J_p, with $p > \max\left\{\frac{2n}{n+2}, 1\right\}$. Our approach is inspired by [4]. Our method mostly relies on using p-harmonic replacements as competitors. The regularity properties of these replacements indeed allow us to infer the Lipschitz continuity of almost minimizers. In particular, we first prove a dichotomy-type result, and next, we improve and iterate one of the two alternatives of the dichotomy. The talk is based on joint work with S. Dipierro, F. Ferrari, and E. Valdinoci, see [6].

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Flipping one sided regularity via a Harnack approach and applications to nonlinear elliptic problems

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In this talk we discuss some recent advances on the regularity theory of non linear elliptic problems showing that weak Harnack type arguments allow the passage from one-side regularity to full regularity in Hölder and Sobolev spaces. As a particular case of these phenomena, we can identify the Caffarelli, Kohn, Nirenberg and Spruck theorem (in the 80s), as well as, some more recent regularity results obtained together with Alessio Figalli (ETH) and Ederson Braga (UFC) both on the regularity of semiconvex supersolutions of uniformly elliptic equations. This problem has some motivations linked to free boundary problems as well. This is a joint work with Edgard Pimentel (University of Coimbra).

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Regularity for almost minimizer with free boundary in Carnot groups of step two

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The regularity of minimizers of the classical one-phase Bernoulli functional was deeply studied after the pioneering work of Alt and Caffarelli. More recently, the regularity of almost minimizers was investigated as well. We present a regularity result for almost minimizers for a one-phase Bernoulli-type functional in Carnot Groups of step two. Our approach is inspired by the methods introduced by De Silva and Savin in the Euclidean setting. Moreover, some recent intrinsic gradient estimates have been employed. Generalizations to the nonlinear framework will be discussed. Some of the results presented are obtained in collaboration with F. Ferrari (University of Bologna) and N. Forcillo (Michigan State University).

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Approximation Theory and Application
Special Session B10

Greg Fasshauer
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Elisa Francomano
University of Palermo, ITALY

This special session is focused on numerical and analytical aspects as well as applications of multivariate approximation problems. Our speakers will discuss kernel-based methods such as applications in high-dimensional interpolation, quasi-interpolation or regression settings. The problems will come from learning-based methods such as deep neural networks or physics-informed neural networks, from signal or image-based analysis settings, as well as computations of partial differential or fractional differential problems. Finally, things might live in cloud-based surfaces, high-dimensional spheres, manifolds, as well as set-valued and vector-valued domains, and our techniques may use analytical, statistical or Bayesian approaches.

For more information visit `https://umi.dm.unibo.it/jm-umi-ams/special-sessions/`.

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Variably Scaled Persistence Kernels (VSPKs) for persistent homology applications

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In recent years, various kernels have been proposed in the context of *persistent homology* to deal with *persistence diagrams* in supervised learning approaches. In this paper, we consider the idea of variably scaled kernels, for approximating functions and data, and we interpret it in the framework of persistent homology. We call them *Variably Scaled Persistence Kernels (VSPKs)*. These new kernels are then tested in different classification experiments. The obtained results show that they can improve the performance and the efficiency of existing standard kernels.

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Products of Matrices and Cascade Networks

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For positive integers $L, N_0, N_1, ..., N_L$, we consider a collection of two-valued matrix functions $\{A_\ell\}_{\ell=1}^L$ on $[0,1]$, called here a *cascade network* of depth L, given by

$$A_\ell(x) := \begin{cases} A_\ell^0, & x \in [0, \frac{1}{2}), \\ A_\ell^1, & x \in [\frac{1}{2}, 1]), \end{cases}$$

where $A_\ell^0, A_\ell^1 \in \mathbb{R}^{N_\ell \times N_{\ell-1}}$, $\ell = 1, \ldots, L$. One can use these matrices to generate matrix-valued functions recursively by

$$y_\ell(x) := A_\ell y_{\ell-1}(\alpha(x)), \quad x \in [0,1], \quad \ell = 1, \ldots, L,$$

where

$$\alpha(x) := \begin{cases} 2x, & x \in [0, \frac{1}{2}), \\ 2x - 1, & x \in [\frac{1}{2}, 1]), \end{cases}$$

and where y_0 is a given input matrix-valued function on $[0,1]$. Cascade networks are closely related to so-called subdivision algorithms and refinement equations of multi-resolution analysis. For example, vector-valued stationary and non-stationary subdivision schemes can be expressed in terms of recursions of the above form, which give rise to products of matrices. In this talk, we will address the problem of under what conditions do these products converge to a continuous limit as $L \to \infty$.

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On the Convergence of Multiscale Kernel Regression under Minimalistic Assumptions

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We analyse the convergence of kernel regression under minimalistic assumptions on the data and on the kernel. To this end, we prove error estimates and convergence rates with respect to the kernel's native space norm. Our results are then transferred to multiscale kernel regression.

[1]The author acknowledges financial support of *Deutsche Forschungsgemeinschaft* (DFG) through the Research Training Group RTG 2583 „Modeling, Simulation and Optimization of Fluid Dynamic Applications“.
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Provable approximations of multivariate functions on smooth manifolds using deep ReLU neural networks

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The expressive power of deep neural networks is manifested by their remarkable ability to approximate multivariate functions in a way that appears to overcome the curse of dimensionality. This ability is exemplified by their success in solving high-dimensional problems where traditional numerical solvers fail due to their limitations in accurately representing high-dimensional structures. To provide a theoretical framework for explaining this phenomenon, we analyze the approximation of Hölder functions defined on a d-dimensional smooth manifold M embedded in R^D, with $d \ll D$, using deep neural networks. Here neural networks are identified with a class of structured parametric functions, consistently with the recent literature. We prove new uniform convergence estimates of the approximation and generalization errors by deep neural networks with ReLU activation functions, showing that such estimates do not depend on the ambient dimension D of the function but only on the lower manifold dimension d, in a precise sense. This result improves existing estimates established in the literature in a similar setting, where approximation and generalization errors were shown to depend weakly on the ambient dimension D.

The result presented in this talk was recently published in [1].

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Kernel-Based Neural Operators

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Operator learning is the task of learning a map from one function space to another. Recently, deep neural networks have been leveraged for this task, resulting in architectures like the Fourier Neural operator (FNO) and the Deep Operator Network (DeepONet). In this talk, we present two kernel-based neural operator architectures. First, we present the Kernel Neural Operator (KNO), an alternative to the FNO that uses one-to-two orders of magnitude fewer trainable parameters to achieve state-of-the-art accuracy on a range of operator learning tasks. Next, we present Ensemble DeepONets, which are kernel-enhanced, locality-aware ensemble operator models that leverage kernel-based partition-of-unity methods to improve the generalization capabilites of DeepONets. While kernels have found their uses in machine learning for over half a century, we believe our new methods represent a novel way of applying modern kernel approximation techniques to machine learning tasks.

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Hyperparameter selection in adaptive kernel-based partition of unity methods by univariate global optimization tools

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Yaroslav D. Sergeyev
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In this talk, in order to detect the kernel shape parameter and the subdomain size utilized in radial kernel-based partition of unity interpolation [4], we have considered univariate global optimization techniques [3]. Particularly, we induced optimistic and pessimistic improvement in the efficient global optimization method [1], and combined it with a leave-one-out cross validation (LOOCV) scheme [2] for each partitioned subdomain. Numerical results show the enhanced performance of the new technique compared to a basic LOOCV scheme.

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Some new perpectives in Korovkin-type approximation theory

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In the last decades, many aspects of Korovkin approximation theory have been deepened and nowadays we have a rather complete theory. A fairly comprehensive treatment of this theory and its main application can be found in [3, 4].

However, some questions have not yet been completely addressed and could increase further the interest toward this theory.

One of the problems which can be surely considered in this framework is the possibility of extending the Korovkin approximation theory in spaces of integrable functions or, in any case, in spaces of not necessarily continuous functions.

Of course, many papers have been devoted to this possibility and we have also the general universal Korovkin-type property (see [3, Theorem 3.2.1]) which allows to consider operators whose ranges are contained in a suitable Banach lattice.

Surely, among the most interesting applications of Korovkin-type approximation theory, we have to consider the convergence of sequences of operators associated with a positive projections (see [3, Chapter 6] and [1, 6, 7]).

In an abstract setting these applications are all framed in spaces of continuous functions and, as far as we know, we find no complete extension to more general spaces.

Some recent contribution in this direction can be found in [2]. The main idea used in [2] is to extend Radon measures to the space of bounded Borel measurable functions. In this way, it is possible to extend Bernstein-Schnabl operators to bounded Borel measurable functions.

However, due to some assumptions used in the paper, this kind of extension does not cover some classical sequences of operators, such as Kantorovich operators.

Another attempt to extend to larger spaces the sequences of operators associated with a positive projection can be found in [5]. In this case, the extension covers the case of Kantorovich operators but only in the interval $[0, 1]$.

In this talk the authors present some further possibilities of considering sequences of positive operators associated with a positive projection in spaces of integrable functions and, in the meantime, to include classical operators such as multi-dimensional Kantorovich or Bernstein-Durrmeyer operators in this extension.

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Variation, approximation, sampling-type operators

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In this talk we will discuss about some approximation results within the spaces of functions of bounded variation by means of sampling-type operators. BV-spaces furnish, indeed, an interesting setting for approximation processes, also suitable to model, from a mathematical point of view, certain problems related to Signal and Image Processing. In this direction, we will present estimates and convergence in variation results focusing the attention on the multidimensional case by means of the classical generalized sampling series, as well as their Kantorovich version, using the concept of variation in the sense of Tonelli. Indeed the geometrical aspects connected to the definition of the Tonelli variation become it suitable to address such kind of problems, also in view of the applicative connections. Some recent results about the nonlinear case will be also presented.

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Some approximation results and estimates for linear operators in Orlicz spaces

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We state and discuss some approximation results and give qualitative estimate on the rate of convergence for a general class of linear operators

$$T_w f(z) = \int_H \chi_w(z - h_w(t)) \cdot L_{h_w(t)} f d\mu_H(t),$$

where H, G are locally compact topological groups, $(\chi_w)_w$ is a family of kernels, $(h_w(\cdot))_w$ is a family of homeomorphisms from H to $h_w(H) \subset G$, and $(L_{h_w(t)})_{t,w}$ are linear operators from $M(G) = \{f : G \to \mathbb{R} \mid f \text{ is measurable}\}$ to $\mathbb{R}$.

Particular choices of H, G and of the operators involved give rise to classical both discrete and integral operators, like the generalized sampling series, the Kantorovich sampling series, the Durrmeyer operators, discrete and integral convolutions and Mellin operators, in both the one and the multidimensional cases, allowing to introduce a general setting where to study both the convergence and the rate of convergence.

We discuss some results concerning the convergence of the operators $T_w f$ to the function f in $C(G)$ (the space of uniformly continuous and bounded functions defined in G) and in Orlicz spaces $L^\varphi(G)$, and give also qualitative estimates for the rate of the convergence, both in $C(G)$ and in $L^\varphi(G)$. These estimates require the introduction of suitable Lipschitz classes and the definition of an appropriate modulus, due to the lack of a metric structure on the topological group G.

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On a sequence of positive linear operators on noncompact real intervals

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The object of this talk is a sequence of positive linear operators of the form

$$C_n(f)(x) := \int_J \int_J \cdots \int_J f\left(\frac{x_1 + \ldots + x_n + rx_{n+1}}{n+r}\right) d\mu_x(x_1) \ldots d\mu_x(x_n) d\mu_n(x_{n+1}),$$

defined for an arbitrary interval J, $x \in J$, $n \geq 1$, $r \geq 0$, probability Borel measures μ_x and μ_n on J and a continuous real-valued function f on J with at most quadratic growth.

Note that if J is a compact interval, then such operators turn into the generalized Kantorovich operators studied in [1]. Furthermore, for $r = 0$, they become the Bernstein-Schnabl operators on noncompact real intervals already studied in [3].

Approximation properties in weighted function spaces of continuous functions on J with respect to wide classes of weights have been investigated, establishing pointwise estimates for uniformly continuous bounded functions as well as with respect to weighted norms. Moreover, a weighted asymptotic formula has also been obtained. The latter result could possibly be used in studying some classes of evolution equations on J as done in [3].

All results presented are taken from [2].

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Vector-Valued Gaussian Processes for Approximating Divergence- or Rotation-free Vector Fields

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Gaussian processes are established tools for approximating unknown functions under the influence of uncertainties. The approximation of Gaussian processes of scalar-valued functions in the so-called regression setting have extensively been studied and are by now well understood.

Less is known about vector-valued Gaussian processes. In this talk, I will discuss vector-valued Gaussian processes for the approximation of divergence-free functions. I will introduce the theory behind such Gaussian processes, link the theory to multivariate approximation theory and give error estimates for the predictive mean in various situations.

[1]Based on joint work with Ian Sloan and Quoc Thong LeGia (UNSW, Sydney)
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The Occupation Kernel Method for Learning Vector Fields with Constraints

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The occupation kernel method (OCK) has proven itself as a robust and efficient method for learning nonparametric systems of ordinary differential equations from trajectories in arbitrary dimensions. Using an implicit formulation provided by vector-valued reproducing kernel Hilbert spaces, we aim to show how the OCK method can be adapted to learn vector fields satisfying various physical constraints. In particular, by choosing an appropriate kernel, we can ensure that the learned vector fields analytically satisfy either solenoidal (divergence-free) and irrotational (curl-free) properties. We validate the proposed method through experiments on a variety of simulated and real datasets. It is shown that the added constraints often lead to better approximations in these application specific problems.

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The shape of the flat radial basis function interpolation limit

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Infinitely smooth radial basis functions (RBFs) $\phi(r)$, where $r = \|x - y\|$ for $x, y \in \mathbb{R}^d$, are in general equipped with a shape parameter ε such that $\phi_\varepsilon(r) = \phi(\varepsilon r)$ is used in approximations. The shape parameter has a significant effect on the accuracy [7] and different methods have been proposed to find the best shape parameter for a given problem [3]. In [1], it was shown that interpolation in the univariate flat RBF limit for distinct node points becomes the Lagrange interpolation polynomial. Some years later it was further shown that for unisolvent node sets, the multivariate flat RBF limit is the unique minimal degree multivariate polynomial interpolant [6, 9, 8]. The form of the interpolant $s_\varepsilon(x)$ was shown to be

$$s_\varepsilon(x) = P_K(x) + \varepsilon^2 P_{K+1}(x) + \varepsilon^4 P_{K+2}(x) + \cdots , \tag{1}$$

where K is the degree of the (multivariate) limit polynomial interpolant, and P_{K+j} is a polynomial of degree $K + 2j$. A recursive algorithm to compute these polynomials was provided in [6], but it can only be used stably for quite small shape parameters and for small numbers of points, which is the same as low values of K. A number of stable evaluation methods have later been derived, such as the so called RBF-QR method class [5, 4, 2], where the linearly dependent flat RBFs are replaced by a more well-conditioned basis. Here we go back to the first recursive formulation, and derive a corresponding non-recursive form that allows us to make some conclusions concerning which properties of the data and the underlying function that determine the best shape parameter. Based on those insights, we propose algorithms that can be used to find function approximations that may be more accurate than polynomial interpolation.

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A new framework for numerical integration

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Numerical integration, or quadrature, is ubiquitous in mathematics, statistics, science, and engineering, with a history dating back to the ancient Babylonians. A standard approach to generating quadrature formulas is to pick a "nice" vector space of functions for which the formulas are exact, such as algebraic or trigonometric polynomials. For integration over intervals, this approach gives rise to Newton-Cotes and Gaussian quadrature rules. However, for geometrically complex domains in higher dimensions, this exactness approach can be challenging, if not impossible since it requires being able to exactly integrate basis functions for the vector space over the domains (or some collection of subdomains). Another challenge with determining good quadrature formulas arises when the integrand is not given everywhere over the domain, but only as samples at predefined, possibly "scattered" points (i.e., a point cloud), which is common in applications involving experimental measurements or when quadrature is a secondary operation to some larger computational effort. In this talk we introduce a new framework for generating quadrature formulas that bypasses these challenges. The framework only relies on numerical approximations of certain Laplace operators and on linear algebra. We show how several classic univariate quadrature formulas can arise from this framework and demonstrate its applicability to generating accurate quadrature formulas for geometrically complex domains (including surfaces) discretized with point clouds.

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A Nyström method for solving fractional relaxation-oscillation equation

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The relaxation-oscillator equation is the primary equation for describing the behaviour of physical phenomena that return to equilibrium after being disturbed. Fractional derivatives have been employed in order to represent slow relaxation and damped oscillation, obtaining the following fractional relaxation-oscillator equation

$$(D^{\alpha}y)(t) + \lambda y(t) = f(t), \quad t > 0, \tag{1}$$

where

$$(D^{\alpha}y)(t) = \frac{1}{\Gamma(m-\alpha)} \int_0^t (t-x)^{m-\alpha-1} y^{(m)}(t)dt \tag{2}$$

is the Caputo's fractional derivative of order α with $m-1 < \alpha < m$ and $m = 1, 2$, $\lambda = \omega_0^2$ with ω_0 the natural frequency of the oscillator and f is an external force. For $0 < \alpha < 1$, (1) under the initial condition

$$y(0) = y_0$$

describes the relaxation with the power law attenuation; for $1 < \alpha < 2$, (1) under the initial conditions

$$y(0) = y_0, \quad y'(0) = y_1$$

represents the damped oscillation with viscoelastic intrinsic damping of oscillation [1, 2].

Several authors have studied the above equation and introduced numerical methods to approximate its solution (see, e.g [3, 4]).

We propose a Nyström method for the global approximation of its solution. Such method is based on the discretization of the integral operator defining (2) by a suitable product quadrature rule obtained using Lagrange interpolation.

Numerical tests showing the performances of the method will be presented, as well as comparisons with other numerical methods.

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An RBF-based Interface Capturing Method for Two-Phase Flows

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Many applications in the natural and applied sciences involve the solution of partial differential equations (PDEs) on surfaces. Application areas for PDEs on static surfaces include image processing, biology, and computer graphics. Applications for PDEs on moving surfaces also occur frequently. Notable examples arise in biology, material science, fluid dynamics, and computer graphics.

There are three main categories of methods for solving PDEs on arbitrary surfaces: the methods rely either on parametrization, on an embedding, or on triangulation. Embedding-type methods have the advantage that the surface-constrained differential operators are identical to their ambient-space analog. One of the most common embedding methods is the closest point method (CPM), [1]. The surface is enclosed inside a thick layer of nodes that belong to a dense three-dimensional grid. Each one of these nodes takes the function value of the one associated with their closest point to the surface, implicitly imposing that the normal derivatives at each node is null. Under that constraint, the surface Laplacian (for instance) is equivalent to its $\mathbf{R^3}$ analog.

Another embedding method called the Radial Basis Functions Orthogonal Gradients method (RBF-OGr) was introduced in [2]. This method is different as every computation is performed on the surface only, including the null normal derivative constraints. By using the RBF method, we take advantage of its meshfree character, which provides the flexibility to represent complex geometries in any spatial dimension while providing a high order of accuracy. Different versions of both the CPM and OGr methods, based on the RBF-based finite difference (RBF-FD) method, have been shown to be powerful tools to solve PDEs on static surfaces, [3,4,5].

This talk bears on the numerical solution of PDEs on *evolving* smooth curves and surfaces. We will introduce an RBF-OGr based method that captures the dynamics of a two-phase flow interface. A number of examples will be presented to illustrate the method.

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On the solution of elliptic PDEs through the multinode Shepard method

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In this talk, we discuss the application of the multinode Shepard method [1] to numerically solve elliptic Partial Differential Equations (PDEs) equipped with various conditions at the boundary of domains of different shapes [2]-[4]. In particular, the multinode Shepard method is proposed to solve elliptic PDEs with Dirichlet and/or Neumann boundary conditions. In the case of discontinuous boundary conditions, it is necessary to design a method which is able to capture the singularities on the boundary. In line with [5], enlargement of the functional space spanned by the multinode Shepard functions is proposed to overcome the difficulties due to the singularity on the boundary. Numerical results are presented to make evidence of the accuracy and efficiency of the method.

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A Novel Two-dimensional Fractional Wavelet Transform

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The subject of fractional integral transforms started in the early 1980's with the publication of Namias's paper on the fractional Fourier transform [1]. The fractional Fourier transform is a generalization of the Fourier transform that depends on an angle α and reduces to the standard Fourier transform for $\alpha = \pi/2$. It is given by

$$\mathcal{F}_\alpha[f](y) = \int_{\mathbb{R}} f(x)\mathcal{K}_\alpha(x,y)dx, \tag{1}$$

where $\mathcal{K}_\alpha(x,y)$ is given by

$$\mathcal{K}_\alpha(x,y) = \begin{cases} \sqrt{\frac{1-i\cot\alpha}{2\pi}} e^{-\frac{i}{2}(x^2+y^2)\cot\alpha + ixy\csc\alpha}, & \alpha \neq n\pi, \\ \frac{1}{\sqrt{2\pi}} e^{ixy}, & \alpha = \frac{\pi}{2}, \\ \delta(x-y), & \alpha = 2n\pi, \\ \delta(x+y), & \alpha = (2n-1)\pi, n \in \mathbb{Z}. \end{cases} \tag{2}$$

Namias's work, which has many applications in optics and imaging [2], was confined to one dimension, but it was later extended to higher dimensions using tensor products of one-dimensional transforms.

Nowadays there are fractional versions of many integral transforms, such as fractional Hankel, fractional Jacobi, fractional Mellin, fractional Radon, and fractional wavelet transforms [3]. Their extensions to higher dimensions were done in different ways.

Recall that a mother wavelet ψ is a function in $L^2(\mathbb{R})$ which satisfies the condition

$$0 \neq C_\psi = \int_{\mathbb{R}} \frac{|\hat{\psi}(w)|^2}{|w|} dw < \infty,$$

where $\hat{\psi}$ is the Fourier transform of ψ. The one-dimensional wavelet transform of $f \in L^2(\mathbb{R})$ with respect to the wavelet ψ is defined by

$$W_\psi f(x,s) = \frac{1}{\sqrt{s}} \int_{\mathbb{R}} f(t) \overline{\psi\left(\frac{t-x}{s}\right)} dt, \forall x \in \mathbb{R}, s \in \mathbb{R} \setminus \{0\},$$

where $\overline{\psi\left(\frac{t-x}{a}\right)}$ is the complex conjugate of $\psi\left(\frac{t-x}{a}\right)$.

In this talk we will discuss a new extension of the fractional wavelet transform to two dimensions that is not a tensor product of two one-dimensional transforms [4].

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Novel wavelet systems for image processing applications

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Wavelets have played a significant role in approximation theory, signal processing, and various other contexts for several decades.

The purpose of this talk is to illustrate the research which has recently been carried out by the author and collaborators in the direction of exploring new strategies for creating novel wavelet systems, particularly in the multivariate setting, with desirable properties in applications.

We present a tensor product argument that enables the explicit construction of filterbanks associated with a non-separable bivariate orthonormal wavelet system and arbitrary scaling matrices [1]. Such constructions are particularly useful in the implementation of a multiple multiresolution analysis [2]. In such a framework, which consists in a generalization of the discrete wavelet transform, the analysis and synthesis steps are implemented in terms of a composition of different (non-separable) filter operators and scaling matrices. This allows for a directional adaptation of the process to the image data and a proper detection of their singularities along curves.

Additionally, we present an extension of the polynomial wavelets proposed in [3] to the bivariate setting and the implementation of a fast algorithm for processing images. Preliminary results [4] demonstrate the potential of such novel bases in image compression problems.

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Accurate quasi interpolation via $m-$harmonic B-splines

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The aim of this talk, is to provide a review on the present possible application of various kinds of polyharmonic B-splines, discuss different constructions and present a simple procedure that provides cardinal quasi-interpolation operators with high approximation order.

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Analysis and Control of Evolutionary Partial Differential Equations
Special Session B11

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This Special Session will feature speakers who have wide-ranging research expertise in the mathematical analysis of dynamical systems, which encompasses a very broad area of research. In this regard, our special session will focus on the topics that fall under the heading of qualitative properties of evolutionary PDEs and their applications to linear/nonlinear equations of parabolic and/or hyperbolic type, particularly those equations coming from physics and mechanics. These PDE systems also arise in (i) fluid dynamics for which the Navier-Stokes (NS) and Euler equations are often invoked as modeling equations, (ii) viscoelasticity, population dynamics, or heat flow in real conductors which have time delays or memory effects, i.e., the dynamics depend on previous states, or they are influenced by the past history of the variables. Then, delay differential equations and equations with memory, where the past history is in play through a convolution integral, will be a topic of interest. In particular, our session will address the wellposedness, long-time behavior (in the sense of global attractors and stability), control, optimization, and regularity properties of the above-mentioned dynamical systems.

This special session is scheduled on July 25-26, 2024.

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Weak solutions to the heat conducting compressible self-gravitating flows in time-dependent domains

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Šárka Nečasová [1]
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We consider the heat-conducting compressible self-gravitating fluids in time-dependent domains, which typically describe the motion of viscous gaseous stars. The flow is governed by the 3-D Navier-Stokes-Fourier-Poisson equations where the velocity is supposed to fulfil the full-slip boundary condition and the temperature on the boundary is given by a non-homogeneous Dirichlet condition. We establish the global-in-time weak solution to the system. Our approach is based on the penalization of the boundary behavior, viscosity, and the pressure in the weak formulation. Moreover, to accommodate the non-homogeneous boundary heat flux, the concept of *ballistic energy* is utilized in this work.

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Nonlocal Cahn-Hilliard fluids

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The Cahn-Hilliard equation is often used to model the temporospatial evolution of multiphase fluid systems including droplets, bubbles, aerosols, and liquid films. A well-known model consists of the Navier-Stokes system which is nonlinearly coupled with an advective Cahn-Hilliard equation through a capillary force, aka the Korteweg force. I will present some recent results on models where the Cahn-Hilliard equation is not the classical fourth-order equation, but it is a second-order spatially nonlocal equation. These results are mainly concerned with the existence of strong solutions, uniqueness, and convergence to a single stationary state. Some related open issues will also be mentioned.

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Exponential stability for a coupled thermoelastic plate–membrane system

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Let $\Omega \subset \mathbb{R}^n$ be a bounded sufficiently smooth domain which consists of two sub-domains $\Omega = \Omega_1 \cup \Omega_2 \cup I$, where Ω_2 is the inner part, i.e., $\overline{\Omega_2} \subset \Omega_1$, and $I := \partial\Omega_2$ is the interface between the two parts. We consider a coupled plate-membrane system, where we have a thermoelastic plate equation in the outer sub-domain Ω_1,

$$\begin{aligned} u_{tt} + \Delta^2 u + \Delta\theta &= 0 \quad \text{in} \quad \Omega_1 \times (0,\infty), \\ \theta_t - \Delta u_t - \Delta\theta &= 0 \quad \text{in} \quad \Omega_1 \times (0,\infty) \end{aligned} \tag{1}$$

and a wave equation in the inner sub-domain Ω_2,

$$v_{tt} - \Delta v + m v_t = 0 \quad \text{in} \quad \Omega_2 \times (0,\infty).$$

Here, $m \geq 0$ is a constant parameter describing the damping. On the outer boundary $\partial\Omega$ and on the interface I, we impose appropriate boundary and transmission conditions, respectively.

In an appropriate Hilbert space setting, this problem is well-posed, and the corresponding first-order system generates a C_0-semigroup $(S(t))_{t\geq 0}$ of contractions. Concerning the asymptotic behaviour, we have the following result.

Theorem 1. *The semigroup $(S(t))_{t\geq 0}$ is exponentially stable if $m > 0$ and not exponentially stable if $m = 0$.*

Here, the more difficult case is $m = 0$. For the proof in this case, we use a compactness argument which then needs an estimate for the solution of (1) with boundary data living only in $L^2(I)$. As such rough boundary data are not covered by classical results, we need additional concepts and tools for boundary value problems with rough boundary data.

This gives the connection to a general question which has applications also in other fields of PDE, e.g., for SPDEs with boundary noise: Can we solve boundary value problems of the form

$$\begin{aligned} (\lambda - A)u &= f \quad \text{in} \quad \Omega, \\ Bu &= g \quad \text{on} \quad \partial\Omega \end{aligned} \tag{2}$$

with $f \in L^2(\Omega)$ and $g \in L^2(\partial\Omega)$ or with g even belonging to some negative Sobolev space?

Boundary value problems with rough boundary data were considered in [2], even in the L^p-setting. For this, we have to generalize the notion of the boundary trace operator $\gamma_0 \colon u \mapsto u|_{\partial\Omega}$. It is known that

$$\gamma_0 \colon H^s_p(\Omega) \to B^{s-1/p}_{pp}(\partial\Omega)$$

is continuous if and only if $s > 1/p$. However, one can generalize the trace operator yielding negative Sobolev and Besov spaces on the boundary and develop a theory which solves (2) for such rough boundary data.

This talk is based on joint works with B. Barraza Martínez, J. González Ospino, J. Hernández Monzón (all Barranquilla, Colombia), J. Seiler (Torino, Italy), D. Ploß, and S. Rau (both Konstanz, Germany).

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On stationary statistical solutions to the Navier–Stokes–Fourier system

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We show the existence of stationary statistical solution for the Navier-Stokes-Fourier system describing the motion of a general compressible, viscous and heat conducting fluid. The proof is based on the existence of a bounded absorbing set and asymptotic compactness of global-in-time solutions in suitable topologies.

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Stabilization of a transmission system controlling a beam equation by heat conduction with memory effect under (Coleman/Pipkin)–Gurtin Thermal Law

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Beam-heat coupling constitutes a conventionnal topic in civil, aerospace, and mechanical engineering where thermal loading depicts one of the most decisive loading conditions. On the other hand, since the thermal memory effect is common in some materials, adding hyperbolicity to the heat equation is necessary to take this effect into consideration. For these reasons, we consider the following one-dimensional coupled system wherein Euler-Bernoulli beam equation is interconnected to a heat equation with memory, under Coleman-Gurtin or Gurtin-Pipkin heat conduction law, via transmission conditions established at the interface

$$\begin{cases} u_{tt} + u_{xxxx} = 0, & (x,t) \in (0,1) \times \mathbb{R}_+^*, \\ y_t - c(1-m)y_{xx} - cm\displaystyle\int_0^\infty g(s)\, y_{xx}(x,t-s)\, ds = 0, & (x,t) \in (1,2) \times \mathbb{R}_+^*, \\ u_{xxx}(1,t) = -c(1-m)y_x(1,t) - cm\displaystyle\int_0^\infty g(s) y_x(1,t-s) ds, & t \in \mathbb{R}_+^*, \\ u_t(1,t) = y(1,t), & t \in \mathbb{R}_+^*, \end{cases}$$

After presenting the semigroup setting for the well-posedness, we prove the strong stability of the system using an Arendt-Batty criteria with a diagonalization method. Then, we show that the associated semigroup in the history framework of Dafermos is polynomially stable with decay rate of type t^{-4} . Finally, we achieve exponential stability when Gurtin-Pipkin heat conduction is applied.

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Qualitative Properties of Composite Structure-Stokes Fluid Interaction PDE Systems

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In this work, we analyze the qualitative properties of a composite structure (multilayered) fluid interaction PDE system which arises in multi-physics problems, and particularly in biofluidic applications related to the mammalian blood transportation process and cellular dynamics. The PDE system under consideration consists of the interactive coupling of 3D Stokes flow and 3D elastic dynamics which gives rise to an additional 2D elastic equation on the boundary interface between these 3D PDE systems. We prove that the dynamical system is wellposed. Moreover, we show that the solution to this system satisfies an asymptotic decay to the zero state.

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Matching measures with non-optimal flows

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In this talk we will discuss the problem of matching measures via sub-optimal parameterized flows. The motivation of such problem arises in the context of machine learning, in view of applications in deep learning architectures such as Transformers.

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Null controllability for non autonomous degenerate parabolic problems

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Inspired by a Budyko-Seller model, we consider non-autonomous degenerate parabolic problems. Using Kato's Theorem, we first prove the well-posedness of such problems. Then, obtaining new Carleman estimates for the non-homogeneous adjoint problems, we deduce null-controllability for the original ones. Some linear and semilinear extensions are also considered, as well as open problem and work in progress.

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Some inverse problems for fluids

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In this talk we consider inverse problems of the geometric type for the partial differential equations describing the behavior of certain fluids. Our focus lies on determining a portion of the domain where the equations hold true, based on external measurements.

We will consider real-world applications and will explore two crucial aspects: uniqueness and numerical reconstruction. In particular, we will investigate how initial and boundary data influence solution's uniqueness. Among others, we will deal with one-dimensional inverse problems for the Burgers equation and related nonlinear systems, where heat effects, non-constant density and fluid-solid interaction are taken into account. The goal is to determine the size of the spatial interval based on specific boundary observations of the solution. We will explore both analytical and numerical solutions to these problems, employing powerful tools like Carleman estimates and insights from existing research (see [3], [4]). Additionally, we will provide methods to approximate the interval sizes. These works are in collaboration with J. Apraiz, E. Fernández-Cara and M. Yamamoto [1], [2].

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Remarks on the Large-Scale Stabilization in the 2D Kuramoto-Sivashinsky Equation

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This talk will discuss mechanisms for stabilization of large-scale motions in the 2D Kuramoto-Sivashinsky equation, an equation for which the issue of global regularity of smooth solutions remains open. It is on the other hand well-known that large-scale instability is the putative mechanism for finite-time blow-up of solutions. By casting the 2D KSE in structural analogy to the Navier-Stokes equation (NSE), we identify a divergence-based regularity criterion in analogy to the curl-based regularity criterion that exists for the NSE. This regularity criterion depends entirely on the low-mode behavior of the solutions as quantified by negative Sobolev norms that are "almost critical" with respect to the scaling symmetry of the linear part of the KSE. Based on these ideas, we identify a modification of KSE by introducing a low-mode control to the system that positively resolves the issue of global regularity, as well as apply our regularity criterion to the 2D Burgers-Sivashinsky equation to provide an elementary alternative proof of global regularity of solutions. This is joint work with Adam Larios (University of Nebraska–Lincoln).

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Synthesis results for the linear quadratic problem for evolution equations with finite memory

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This talk is concerned with the optimal control problem with quadratic functionals for certain classes of infinite dimensional linear integro-differential systems. Most studies on optimal control problems for evolutionary partial differential equations (PDE) with memory pertain to more general frameworks – involving, e.g., semilinear PDE and/or non-quadratic functionals – and hence are aimed at establishing the existence of *open-loop* solutions as well as at characterizing them via first (and possibly, second) order optimality conditions. Also, in the presence of an infinite memory, the celebrated "history approach" of C. Dafermos has proved effective in attaining a reformulation of the nonlocal problem; the equivalent coupled system satistifed by an augmented variable may constitute a starting point for the exploration of a variety of control-theoretic properties.

With focus on the finite horizon linear quadratic problem, in the presence of finite memory, here we offer a Riccati-based approach to the optimization problem that brings about a *closed-loop* representation of the unique optimal control, where the optimal cost operator is determined by solving a corresponding Riccati equation. Our recent advances pertain to an abstract model in which the memory affects the control action, a case interesting enough in itself and which should also serve as a first step for future developments.
(The talk is based on past and ongoing joint work with Paolo Acquistapace (Università di Pisa).)

[1]The research of F. Bucci has been performed in the framework of the MIUR-PRIN Grant 2020F3NCPX "Mathematics for Industry 4.0 (Math4I4)". Bucci's research is currently supported by the Università degli Studi di Firenze under the 2024 Project *"Controllo lineare-quadratico per equazioni integro-differenziali a derivate parziali"*, of which she is responsible. F. Bucci is a member of the Gruppo Nazionale per l'Analisi Matematica, la Probabilità e le loro Applicazioni (GNAMPA) of the Istituto Nazionale di Alta Matematica (INdAM) and participant to the 2024 GNAMPA Project "Controllo ottimo infinito dimensionale: aspetti teorici ed applicazioni".
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Concerning Higher Regularity Properties of a Certain PDE Interaction of Fluid-Structure Type

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In this talk we discuss our progress towards (at least partially) verifying a conjecture of the late Igor Chueshov, concerning the qualitative behavior of a coupled partial differential equation (PDE) system which describes a certain fluid-structure interaction (FSI). This particular FSI system comprises a Stokes flow, evolving within a three-dimensional cavity, coupled via a boundary interface, to a two dimensional Euler-Bernoulli (or Kirchhoff) plate, which displaces upon a sufficiently smooth bounded open set; this open set is a portion of the cavity boundary. Previously, I. Chueshov conjectured that, despite the respective (three dimensional) Stokes and (two dimensional) Euler-Bernoulli structural dynamics being coupled via a boundary interface, the strongly continuous semigroup of contractions, which is associated with such FSI dynamics, is in fact analytic in a sector of the complex plane. In other words, the entire FSI system manifests "parabolic-like" behavior, including smoothing effects for positive time. In this connection, we shall discuss our findings that the modeling fluid-structure strongly continuous semigroup is of a specified Gevrey Class. In other words, the behavior of the strongly continuous semigroup which describes the given interaction between 3D Stokes flow and 2D plate dynamics formally falls in a range between differentiability and analyticity.

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Stabilization of Degenerate Wave Equations with Drift and with/without Singular Term

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Abstract. In this talk, we present the stability of a degenerate wave equation featuring localized singular damping, along with a drift term and a leading operator in non-divergence form and with/or without singular term. Exponential stability results are presented under suitable conditions on the degeneracy and singularity coefficients.
The system that will be considered is as follows:

$$(1)\qquad \begin{cases} u_{tt} - a u_{xx} - \dfrac{\lambda}{d} u - b u_x + \chi_{(x_1,x_2)} u_t = 0, & (x,t) \in (0,1) \times \mathbb{R}^+_*, \\ u(0,t) = u(1,t) = 0, & t \in \mathbb{R}^+_*, \\ u(x,0) = u_0(x), u_t(x,0) = u_1(x), & x \in (0,1), \end{cases}$$

where the damping coefficient is given by $\chi_{(x_1,x_2)}(x)$ such that $0 \le x_1 < x_2 \le 1$ and

$$(C)\qquad \begin{cases} a, b, d \in C^0[0,1], \\ a, d > 0 \quad \text{on} \quad (0,1], a(0) = d(0) = 0, \\ \dfrac{b}{a} \in L^1(0,1). \end{cases}$$

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Rigorous elastohydrodynamic lubrication approximation

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Motivated by widespread applications of microfluidic channels and chips, we consider an interaction between a thin layer of an incompressible viscous fluid and an elastic structure. Starting from fluid-structure interaction (FSI) problems, our aim is to rigorously derive corresponding reduced models, which are favorable in engineering applications. We formulate this as a singular limit problem in terms of the vanishing relative fluid thickness, which is carried out based on suitable energy estimates and uniform no-contact results for FSI systems. Reduced models are justified in terms of weak convergence results in the sense that weak limits of solutions to the FSI problem are identified in a relation with solutions of the reduced model.

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Nonlocal interaction kernels inference in nonlinear gradient flow equations

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When applying nonlinear aggregation-diffusion equations to model real life phenomenon, a major challenge lies on the choice of the interaction potential. Previous numerical and theoretical studies typically required predetermination of terms and the goal is often to reproduce the observed dynamics qualitatively, not quantitatively. In this talk, we address the inverse problem of identifying nonlocal interaction potentials in nonlinear aggregation-diffusion equations from noisy discrete trajectory data. Our approach involves formulating and solving a regularised variational problem, which requires minimising a quadratic error functional across a set of hypothesis functions. A key theoretical contribution is our novel stability estimate for the PDEs, validating the error functional ability in control- ling the 2-Wasserstein distance between solutions generated using the true and estimated interaction potentials. We demonstrate the effectiveness of the methods through various 1D and 2D examples showcasing collective behaviours.

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[1]This work was supported by the Advanced Grant Nonlocal-CPD (Nonlocal PDEs for Complex Particle Dynamics: Phase Transitions, Patterns and Synchronization) of the European Research Council Executive Agency (ERC) under the European Union's Horizon 2020 research and innovation programme (grant agreement No. 883363).

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Energy decay estimates for semilinear evolution equations with memory and delay feedback

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We consider semilinear evolution equations with memory and time-dependent time delay feedback. Under a suitable assumption on the coefficient of the delay feedback, we are able to prove that solutions corresponding to small initial data are globally defined and satisfy an exponential decay estimate. The standard assumption used so far to deal with wave-type equations with time-varying time delay is that the time delay function $\tau(\cdot) : [0, +\infty) \to [0, +\infty)$ must belong to the Sobolev space $W^{1,\infty}(0, +\infty)$ and has to satisfy the following condition: $\tau'(t) \leq c < 1$. Here, instead, we work in a very general setting, namely we only assume the time delay function is continuous and bounded from above.

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Stability Results for Novel Serially-Connected Magnetizable Piezoelectric and Elastic Smart-System Designs

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In this talk, we consider the stability of longitudinal vibrations for transmission problems of two smart-system designs:

(i) a serially-connected elastic–piezoelectric–elastic design with a local damping acting only on the piezoelectric layer

(ii) a serially- connected piezoelectric–elastic design with a local damping acting on the elastic part only.

Unlike the existing literature, piezoelectric layers are considered magnetizable, and therefore, a fully-dynamic PDE model, retaining interactions of electromagnetic fields (due to Maxwell's equations) with the mechanical vibrations, is considered. The design (i) is shown to have exponentially stable solutions. However, the nature of the stability of solutions of the design (ii), whether it is polynomial or exponential, is dependent entirely upon the arithmetic nature of a quotient involving all physical parameters. Furthermore, a polynomial decay rate is provided in terms of a measure of irrationality of the quotient. Noting that this type of result is totally new. The main tool used is the multipliers technique which requires an adaptive selection of cut-off functions together with a particular attention to the sharpness of the estimates to optimize the results.

Keywords: Magnetizable piezoelectric beams · Serially-connected beams · Irrationality measure · Partial viscous damping · Exponential stability · Polynomial stability

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Forward problems for time-fractional degenerate heat equations as possible model for climate sciences

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We discuss initial boundary value problems for time-fractional degenerate heat equations. This type of equations can well simulate characteristic or anomalous heat diffusion in several cases, and one can expect that they are effective mathematical model equations. However, the mathematical backgrounds have not been established yet and we present fundamental theory. We will study also some applications such as inverse problems if possible.

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On integral inequalities with applications to indirect linear or nonlinear stabilization of coupled PDE's

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The purpose of this talk is to present several integral inequalities in the form introduced and stated by the speaker respectively in 1999 [1] for the linear case and in 2004 [5] in a nonlinear framework.

Let $\mathcal{A}$ be the infinitesimal generator of a $\mathcal{C}^0$ semigroup on the Banach space $\mathcal{H}$, $D(\mathcal{A})$ its domain, and $D(\mathcal{A}^k)$ the domain of its powers of order k. The linear integral inequality in Lemma 4.1 of [1] (or in Theorem 3.1 in [2]) states that if for every $T \geqslant 0$ and every U_0 in $D(\mathcal{A}^p)$, $p \geqslant 1$, the integral $\int_0^T ||e^{t\mathcal{A}}U_0||_{\mathcal{H}}^2 dt$ can be controlled by a constant time $||U_0||^2_{D(\mathcal{A}^p)}$, then the norm of $e^{t\mathcal{A}}U_0$ in $\mathcal{H}$ decays polynomially as times goes to ∞. We shall present the first applications of these inequalities which allowed to prove polynomial stability for indirectly damped coupled systems as shown in [1,2] (see also [3]) and other further works [6,7], and so to compensate their lack of exponential stability (see [3]).

The nonlinear integral inequalities as stated in Theorem 2.1 in [5] (see also [4]) is a general property for real-valued nonnegative non increasing functions E defined on the nonnegative real axis. We prove in [5] that if the integral $\int_t^S E(s)N(E(s))ds$ can be controlled up to a positive constant T_0 (independent of t) by $E(t)$ for every $t \geqslant 0$ and under appropriate assumptions on N, then E decays at infinity with a given precise decay rate depending on $E(0)$, T_0 and the function N. No underlying semigroup frame is requested. This stresses the fact that the inequalities presented in [1, 2] and [5] are fundamentally not based on the same mathematical frame. We shall present in a second part how these inequalities allow to derive optimal or quasi-optimal energy decay rates for nonlinearly damped PDE's.

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Recent Advances in Fluid mechanics: Theory and Computation
Special Session B12

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University of Southern California, USA

Anna Mazzucato
Penn State University, USA

This Special Session will focus on recent advances in the analysis and computation of fluid mechanics models, both compressible and incompressible, Newtonian and non-Newtonian, including recent progress on well-posedness questions, stability, fluid-structure interactions, geophysical models, and boundary layers. Researchers in these areas, junior and senior, primarily from Italy and the US, but also from other geographic areas will be invited to present their work in a fruitful exchange of ideas, with attention to diversity and underrepresented groups.

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Singular limits for the Rayleigh-Benard convection problem

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We study the singular limit of the complete Navier-Stokes-Fourier system in the Rayleigh-Benard setiing in the low Mach and mild stratification regime. We identify the standard Oberbeck-Boussinesq approximation as the target problem supplemented, however, with an unexpected no-local boundary term.

[1]The work of E.F. was partially supported by the Czech Sciences Foundation (GAČR), Grant Agreement 24-11034S. The Institute of Mathematics of the Academy of Sciences of the Czech Republic is supported by RVO:67985840.
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The Role of Dissipation in the Existence of Time-Periodic Solutions to PDE Systems

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It is well known that in many mechanical systems where energy is conserved, the phenomenon of resonance can occur, meaning that for certain time-periodic forces, the solution of the system becomes unbounded. Examples of partial differential equations describing such systems include the wave equation and equations of linearized elasticity (Lamé system). On the other hand, resonance will not occur in systems with strong dissipation, such as systems described by the heat equation. More precisely, in such a system, there exists a unique time-periodic solution for each time-periodic right-hand side.

In this lecture, we will address the question "how strong does dissipation need to be to prevent the occurrence of resonance?". We will analyze periodic solutions to the so-called *heat-wave* system, where the wave equation is coupled with the heat conduction equation via a common boundary. In this system, dissipation only exists in the heat component, and the system can be viewed as a simplified model of fluid-structure interaction. We will demonstrate that in certain geometric configurations, there exists a unique time-periodic solution for each time-periodic right-hand side. The proof will require some additional regularity of the right-hand side, and it remains an open question whether this is a technical condition arising from the proof method or if it is indeed necessary.

Finally, we will discuss the open question of whether the result is valid for arbitrary geometry or if there exists a geometry where resonance can occur.

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Conditional regularity for the Navier-Stokes-Fourier system

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A conditional regularity criterion for solutions to a system of partial differential equations in fluid mechanics is a condition involving lower-order norms which, if satisfied, implies that the solutions remains regular; in particular, it can be applied to show that a local strong solution can be extended beyond its maximal time of existence. A direct consequence of the aforementioned result is a blow-up criterion meaning that if a blow-up of solutions occurs then some lower-order norms are not bounded. We proved a blow-up criterion for the compressible Navier-Stokes-Fourier system for general thermal and caloric equations of state with inhomogeneous boundary conditions for the velocity and the temperature. Assuming only that Gibb's equation and the thermodynamic stability hold, we show that solutions in a certain regularity class remain regular under the condition that the density, the temperature and the modulus of the velocity are bounded.

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Convergence Analysis for Pseudomonotone Parabolic Problems

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In the talk we discuss several existence proofs for nonlinear elliptic and parabolic problems which contain a pseudomonotone operator. A new notion of non-conforming pseudomonotonicity is introduced and applied. Based on that technique it is shown that numerical approximations based on a spatial non-conforming approximation converge to a weak solution of the original problem.

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Boundary layers equations with an eddy viscosity vanishing at the boundary

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Following classical models deriving from the analysis of von Kármán and Monin and Obukhov, we introduce a scalar elliptic equation defined on a boundary layer given by $\Pi_2 \times [0, z_{top}]$, where Π_2 is a two dimensional torus, with an eddy vertical viscosity of order z^α, $\alpha \in [0,1]$, an homogeneous boundary condition at $z = 0$, and a Robin condition at $z = z_{top}$. We show the existence of weak solutions to this boundary problem, distinguishing the cases $0 \leq \alpha < 1$ and $\alpha = 1$. Then we carry out several numerical simulations, showing the ability of our model to accurately reproduce profiles close to those predicted by the Monin-Obukhov theory, by calculating stabilizing functions.

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Analysis and numerics for a contactless rebound of elastic bodies in a viscous incompressible fluid

Sebastian Schwarzacher
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This talk is on a joint work with J. Fara, G. Gravina, O. Soucek and K. Tuma. I present some results on the phenomenon of the elastic rebound in a viscous incompressible fluid environment. In the important case of no-slip boundary conditions, it is by now classical that, under certain assumptions, collisions cannot occur in finite time. I will discuss some analytic results and numerical strategies to understand this fascinating yet counter-intuitive fluid-structure interaction. In particular, some arguments are collected that indicate that a physically meaningful rebound is possible even in the absence of a topological contact.

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Instabilities of 3D vortex columns in incompressible inviscid fluids

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Department of Mathematics, Florida State University

We will discuss some new techniques of establishing linear and nonlinear instability of some special inviscid flows, including 3D vortex columns, that is vector fields of the form $u = V(r)e_\theta + W(r)e_z$, where r denotes the distance to the axis of rotation and e_θ and e_z denote the standard cylindrical unit vectors, for a family of profiles V, W. We will demonstrate a construction of infinitely many, genuinely three-dimensional modes of instabilities of some vortex columns, which take the form of "ring modes", localized around $r = r_0$, for some $r_0 > 0$. This is joint work with D. Albritton.

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[1]

On the planar Taylor-Couette system and related exterior problems

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We show that the well-known multiplicity result for the stationary 3D Taylor-Couette flow in the region between two concentric unbounded 3D cylinders cannot be extended to planar flows. This result is complemented with the introduction of a weaker kind of solutions and with connections to some exterior problems. Based on a joint work with Jiri Neustupa and Gianmarco Sperone.

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Finite dimensional approximation of solutions of the 2D incompressible Euler equations

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Gennaro Ciampa, Stefano Spirito
Department of Information Engineering, Computer Science and Mathematics, University of L'Aquila

Gianluca Crippa
University of Basel

In this talk we consider the approximation of solution via a finite dimensional Galerkin methods. In particular, we will focus on the convergence in strong norms of the velocity and the vorticity, and the study of the rates of convergence. We will study the convergence in different regularity settings, including the Yudovich class of weak solutions with bounded vorticity. We compare the results with the analogous ones available for the vanishing viscosity approximation and comment on differences and similarities. The talk is based on joint works with L.C. Berselli (University of Pisa), G. Crippa (University of Basel), and G. Ciampa (University of L'Aquila)

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Energy conservation for fluid flows in an Onsager critical class

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The incompressible Euler equations govern the evolution of an ideal fluid. It is well known that the total kinetic energy is preserved along the time evolution of a regular fluid flow. However, when the motion is very rough, there is theoretical and experimental evidence of formation of chaotic structures that support the dissipation of kinetic energy. Mathematically, this problem translates into finding the critical regularity for weak solutions to the incompressible Euler equations to have conservation or dissipation of kinetic energy (Onsager's conjecture). Currently, the Onsager conjecture is almost solved. It has been been proved that energy is conserved in any subcritical class and there are examples of solutions in any supercritical class violating the energy conservation. In a joint paper with Luigi De Rosa, we gave the first proof of energy conservation for weak solutions to the incompressible Euler system in a critical space, both in absence and presence of physical boundary. This is the first energy conservation result that holds in the incompressible case and fails in the compressible setting.

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Strong non-uniqueness of finite energy solutions to the 3D deterministic and stochastic Navier-Stokes equations

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For any prescribed finite energy divergence-free initial data, we show that there exist infinitely many weak solutions with smooth energy profiles to both the 3D deterministic and stochastic incompressible Navier-Stokes equations. Moreover, we show that every constructed deterministic solution is a vanishing viscosity limit of stochastic solutions.

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On a compressible fluid-structure interaction problem with slip boundary conditions

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We study a system describing the compressible barotropic fluids interacting with (visco) elastic solid shell/plate. In particular, the elastic structure is part of the moving boundary of the fluid, and the Navier-slip type boundary condition is taken into account. Depending on the reference geometry (flat or not), we show the existence of weak solutions to the coupled system provided the adiabatic exponent satisfies $\gamma > \frac{12}{7}$ without damping and $\gamma > \frac{3}{2}$ with structure damping, utilizing the domain extension and regularization approximation. Moreover, via a modified relative entropy method in time-dependent domains, we prove the weak-strong uniqueness property of weak solutions. Finally, we give a rigorous justification of the incompressible inviscid limit of the compressible fluid-structure interaction problem with a flat reference geometry, in the regime of low Mach number, high Reynolds number, and well-prepared initial data.

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Onsager conjecture for SQG

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In analogy with the Onsager conjecture for the Euler equation, the Onsager type of conjecture for the surface quasi-geostrophic (SQG) equation concerns the regularity threshold for the conservation of Hamiltonian. The expected regularity threshold is C^0 by scaling analysis. We prove it rigorously from the flexibility side by constructing solutions in C^{0-} which do not conserve the Hamiltonian.

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Functional Inequalities, shape optimization and elliptic PDEs
Special Session B13

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Optimal functional inequalities often encode important information about the underlying ambient space, which could be critical for both abstract and applied research. Sharp Sobolev type inequalities, for instance, are fundamental tools in analysis, geometry, and mathematical physics. They are used to determine the existence and qualitative behavior of solutions to certain nonlinear PDEs, they are deeply related to isoperimetric problems and many optimal eigenvalue bounds, given in terms of the geometry of the background manifold.

The purpose of this special session is to bring together specialists working on geometric and functional inequalities, shape optimization, and related PDEs, and to encourage lively discussions leading to further developments and open problems.

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Stability for the logarithmic Hardy-Littlewood-Sobolev Inequality with application to the Keller-Segel system

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We prove an optimal stability bound for the Onofri inequality, and then apply a duality method to prove an optimal stability theorem for the logarithmic Hardy-Littlewood-Sobolev inequality. We then apply this to the estimation of the rate of approach to equilibrium for the critical mass Keller-Segel system.

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Some New Inequalities in Analysis and Geometry

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The classical Moser-Trudinger inequality is a borderline case of Sobolev inequalities and plays an important role in geometric analysis and PDEs in general. Aubin in 1979 showed that the best constant in the Moser-Trudinger inequality can be improved by reducing to one half if the functions are restricted to the complement of a three dimensional subspace of the Sobolev space H^1, while Onofri in 1982 discovered an elegant optimal form of Moser-Trudinger inequality on sphere. In this talk, I will present new sharp inequalities which are variants of Aubin and Onofri inequalities on the sphere with or without mass center constraints.

One such inequality, for example, incorporates the mass center deviation (from the origin) into the optimal inequality of Aubin on the sphere, which is for functions with mass centered at the origin. The main ingredient leading to the above inequalities is a novel geometric inequality: Sphere Covering Inequality.

Efforts have also been made to show similar inequalities in higher dimensions. Among the preliminary results, we have improved Beckner's inequality for axially symmetric functions when the dimension $n = 4, 6, 8$. Many questions remain open.

The talk is based on collaboration with Amir Moradifam, Sun-Yung Alice Chang, Yeyao Hu, Weihong Xie, Tuoxin Li, Juncheng Wei, And Zikai Ye.

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Symmetrization results for general nonlocal linear ellipitic and parabolic problems

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We discuss a Talenti-type symmetrization result in the form of mass concentration (*i.e.* integral comparison) for very general linear nonlocal elliptic problems, equipped with homogeneous Dirichlet boundary conditions. In this framework, the relevant concentration comparison for the classical fractional Laplacian can be reviewed as a special case of our main result, thus generalizing previous results obtained in collaboration with B. Volzone. Also a Cauchy-Dirichlet nonlocal linear parabolic problem is considered. The results are contained in a joint paper with G. Piscitelli and B. Volzone.

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Hausdorff measures and Orlicz-Sobolev maps

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A comprehensive theory of the effect of Orlicz-Sobolev maps, between Euclidean spaces, on subsets with zero or finite Hausdorff measure is offered. Arbitrary Orlicz-Sobolev spaces embedded into the space of continuous function and Hausdorff measures built upon general gauge functions are included in our discussion. An explicit formula for the distortion of the relevant gauge function under the action of these maps is exhibited in terms of the Young function defining the Orlicz-Sobolev space. New phenomena and features, related to the flexibility in the definition of the degree of integrability of weak derivatives of maps and in the notion of measure of sets, are detected. Classical results, dealing with standard Sobolev spaces and Hausdorff measures, are recovered, and their optimality is shown to hold in a refined stronger sense. Special instances available in the literature, concerning Young functions and gauge functions of non-power type, are also reproduced and, when not sharp, improved. This is the content of joint work with M.V. Korobkov and J. Kristensen [1].

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Embeddings for Fractional Orlicz-Sobolev spaces

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The optimal target space is exhibited for embeddings of fractional-order Orlicz-Sobolev spaces. Both the subcritical and the supercritical regimes are considered. In the former case, the smallest possible Orlicz target space is detected. In the latter, the relevant Orlicz-Sobolev spaces are shown to be embedded into the space of bounded continuous functions in $\mathbb{R}^n$. Moreover, their optimal modulus of continuity is exhibited.
These results are the subject of a series of joint papers with Andrea Cianchi, Luboš Pick and Lenka Slavíková.

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Blow-up and global solutions for a parabolic problem with Trudinger-Moser nonlinearity

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We consider the Cauchy problem for a 2-space dimensional heat equation with exponential nonlinearity. More precisely, we consider initial data in $H^1(\mathbb{R}^2)$, and a square-exponential nonlinearity, which is critical in the energy space $H^1(\mathbb{R}^2)$ in view of the Trudinger-Moser inequality. By means of energy methods, we discuss the dichotomy between blow-up and global existence for solutions below the ground state energy level. The splitting between blow-up and global existence for low energies is determined by the sign of a suitable functional, and it is related to the corresponding Trudinger-Moser inequality. This is a joint work with Michinori Ishiwata (Osaka University), Bernhard Ruf (Istituto Lombardo Accademia di Scienze e Lettere), and Elide Terraneo (Università degli Studi di Milano).

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On the Maximum Principle for higher order operators

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We discuss a general principle of perturbing higher order operators with lower order derivatives in order to restore the maximum principle in the framework in which it is well known to fail. This is somehow delicate and the main ingredient is a new Harnack-type inequality. We first prove De Giorgi type level estimates for functions in $W^{1,t}$, with $t > 2$. This augmented integrability enables us to establish a new Harnack type inequality for functions which do not necessarily belong to De Giorgi's classes as obtained by Di Benedetto-Trudinger for functions in $W^{1,2}$. As a consequence, we prove the validity of the strong maximum principle for uniformly elliptic operators of any even order, in fairly general domains and in any dimension, provided either lower order derivatives or inertial effects are taken into account.

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From the isoperimetric inequality to functional inequalities under nonnegative curvature

Gioacchino Antonelli
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In this talk I will discuss recent results obtained with several collaborators on isoperimetric and functional inequalities on smooth (or nonsmooth) spaces with nonnegative curvature.

In particular, at first I will discuss a new proof of the sharp and rigid isoperimetric inequality on spaces with nonnegative curvature and maximal volume growth. Then, I will show how this inequality implies sharp and rigid functional inequalities.

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Moser-Trudinger inequalities on complete manifolds with large volume growth

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Liuyu Qin
Department of Mathematics and Statistics, Hunan University of Finance and Economics

In this talk, I will discuss two Moser-Trudinger inequalities on complete Riemannian manifolds with nonnegative Ricci curvature and large volume growth. These inequalities will feature different best constants under different norm conditions. This is joint work with Luigi Fontana and Carlo Morpurgo.

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Decay of Information for the Kac Evolution

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We consider a system of M particles in contact with a heat reservoir of $N \gg M$ particles. The evolution in the system and the reservoir, together with their interaction, are modeled via the Kac's Master Equation. We chose the initial distribution with total energy $N+M$ and show that if the reservoir is initially in equilibrium then the entropy of the system decays exponentially to a very small value. We base our proof on a similar property for the Information.

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New Inequalities for the Low Eigenvalues of the Vibrating Clamped Plate and Buckling Problems with Perturbations

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We discuss new inequalities for the low eigenvalues of the vibrating clamped plate and the buckling of a clamped plate and perturbations of these. Both problems are eigenvalue problems for 4th order partial differential operators where the highest order part of the operator is the biharmonic operator, and in both cases we deal with the case of clamped boundary conditions.

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On a Bliss-Moser type inequality

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We derive a limiting inequality for the integral inequalities by Bliss. We then consider a critical version of this inequality which is of Moser type, and discuss related non-compactness properties. Furthermore, we show that this inequality is related to critical boundary growth for functions on a disk in two dimensions.

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Non-degeneracy of solutions for singular Liouville equations in dimension one

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In this talk, I will discuss existence, classification and non-degeneracy results for solutions to singular Liouville-type equations of the form

$$(-\Delta)^{\frac{n}{2}} u = |x|^{n(\alpha-1)} e^{nu} \quad \text{in } \mathbb{R}^n. \tag{1}$$

In dimension one, the problem has applications in mathematical modelling of galvanic corrosion phenomena for ideal electrochemical cells consisting of an electrolyte solution confined in a bounded domain with an electrochemically active portion of boundary. In higher dimension, Lioville equations have applications to prescribed curvature problems in conformal geometry: solutions correspond to constant Q-curvature metrics on the Euclidean space, with a singular point at the origin.

After a general overview of the existing literature, I will focus on the one-dimensional case and I will prove that solutions of (1) are non-degenerate for $\alpha \in (0,1) \cup (1,2)$. Namely, the space of solutions in $H^{\frac{1}{2}}(\mathbb{R})$ to the linearized equation

$$(-\Delta)^{\frac{1}{2}} \varphi = |x|^{\alpha-1} e^{u} \varphi \quad \text{in } \mathbb{R} \tag{2}$$

has dimension one. The proof relies on the use of harmonic extensions and conformal transformations to rewrite the linearized equation (2) as a Steklov eigenvalue problem on either an intersection or a union of two disks, depending on the values of α.

These results are contained in joint works in collaboration with A. DelaTorre, A. Pistoia, A. Hyder and L. Martinazzi.

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Uniqueness of least-energy solutions to the fractional Lane-Emden equation in the ball

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We prove uniqueness of least-energy solutions to the fractional Lane-Emden equation, under homogeneous Dirichlet exterior conditions, when the underlying domain is a ball $B \subset \mathbb{R}^N$. The equation reads as follows:

$$\begin{cases} (-\Delta)^s u & = & u^p & \text{in } B, \\ u & = & 0 & \text{in } \mathbb{R}^N \setminus B. \end{cases}$$

Here $s \in (0,1)$, and $p \in \left(1, \frac{N+2s}{N-2s}\right)$, which makes the nonlinearity superlinear and subcritical. The proof makes use of Morse theory and is inspired by some results obtained by C. S. Lin in the '90s. A new Hopf's Lemma-type result is an essential ingredient in order to prove nondegeneracy of least-energy solutions.

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Moser-Trudinger inequalities: from local to global

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Given a complete Riemannian manifold, we define the notion of local Moser-Trudinger inequality. We show that by imposing a stronger norm condition or by assuming the validity of the Poincaré inequality, the local Moser-Trudinger inequality implies the global one. Hadamard manifolds provide a significant application.

Joint work with Carlo Morpurgo and Liuju Qin

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Special Geometries and Physics
Special Session B14

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The list of special geometries includes Calabi-Yau manifolds, Quaternionic-Kähler manifolds, Hyper-Kähler manifolds, G2-manifolds and Spin(7)-manifolds. These geometries are studied both for their mathematical sake, and because they provide the mathematical formalism for new developments in Physics. In particular, String theory and M-theory are two of the most influential physical theories in this field: they provide not just motivation and urgency for special geometries and geometries with torsion, but also fundamental insight and conjectures. It is a fair statement that most of recent research on manifolds with special holonomy, such as Calabi-Yau manifolds and G2 manifolds, can be largely attributed to interactions with theoretical Physics. The same is true for certain systems of PDE, such as the Hull-Strominger system.

This special session will bring together a diverse group of geometers working with special structures on manifolds and physicists working in String and M-theory, with the aim of discussing new results and research directions in both fields. The list of speakers includes experts in a variety of very active research areas: Calabi-Yau and G2 geometry, mirror symmetry, generalized geometry, geometric flows and gauge theory.

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Coupled G_2-instantons

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Gauge theory in higher dimensions plays a pivotal role in modern mathematics and theoretical physics. One such gauge theory occurs in 7 dimensions and is associated with the exceptional Lie group G_2: here, the special connections are called G_2-instantons and are analogous to the famous anti-self-dual instantons in 4 dimensions. G_2-instantons play a crucial role in the so-called heterotic G_2 system arising from heterotic String Theory in physics: a complex coupled system on a 7-manifold for an ambient G_2-structure and G_2-instantons on it which is very poorly understood.

Motivated by recent developments in theoretical physics and ideas in generalized geometry, in this talk we introduce the notion of *coupled* G_2*-instantons*. We show that coupled G_2-instantons encode solutions to the heterotic G_2 system and are linked to the notion of generalized Ricci-flatness, thus giving us a new perspective on these challenging areas. We will also discuss some non-trivial examples of coupled G_2-instantons, including on the 7-sphere.

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Mirror of minimal submanifolds and a monotonicity formula

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For Hermitian connections on a Hermitian complex line bundle over a Riemannian manifold, we can define the "volume", which can be considered to be the "mirror" of the standard volume for submanifolds. We call the critical points minimal connections. They can be considered as "mirrors" of minimal submanifolds and analogous to Yang-Mills connections.

In this talk, I will introduce some properties of minimal connections and then state a monotonicity formula. As a corollary, we obtain the vanishing theorem for minimal connections in the odd-dimensional case.

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Deformed Hermitian-Yang-Mills: an example

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Mirror symmetry is a somewhat mysterious phenomenon that relates the geometry of two distinct Calabi-Yau manifolds. In the realm of trying to understand this relationship an equation for a connection on a line bundle in a Kahler manifold appeared. This is commonly called the deformed Hermitian-Yang-Mills equation and I will explain what it is and some current joint work with Benoit Charbonneau and Rosa Sena-Dias which explicitly solves this equation on a specific setting. This helps in understanding the problem of the existence of solutions and explore (or rule out) possible stability conditions.

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CR-twistor spaces over manifolds with G_2 -and $\mathrm{Spin}(7)$-structures

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In 1984 LeBrun constructed a CR-twistor space over an arbitrary conformal Riemannian 3-manifold and proved that the CR-structure is formally integrable [1]. This twistor construction has been generalized by Rossi in 1985 for m-dimensional Riemannian manifolds endowed with a $(m-1)$-fold vector cross product (VCP) [2]. In 2011 Verbitsky generalized LeBrun's construction of twistor-spaces to 7-manifolds endowed with a G_2-structure [3]. In my talk I shall explain how to unify and generalize LeBrun's, Rossi's and Verbitsky's construction of a CR-twistor space to the case when the underlying Riemannian manifold (M, g) has a VCP structure. Then I shall show that the formal integrability of the CR-structure is expressed in terms of a torsion tensor on the twistor space, which is a Grassmanian bundle over (M, g). If the VCP structure on (M, g) is generated by a G_2- or $\mathrm{Spin}(7)$-structure, the vertical component of the torsion tensor vanishes, if and only if (M, g) has constant curvature, and the horizontal component vanishes, if and only if (M, g) is a torsion-free G_2 or $\mathrm{Spin}(7)$-manifold. Finally I shall discuss related open problems. My talk is based on a joint work with Domenico Fiorenza [4].

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Triviality of Frobenius Structures Along Generalized Deformations of Nilmanifolds

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It is known that there exists a natural isomorphism between the Gerstenhaber algebra associated to a primary Kodaira manifold and that associated to any complex manifold obtained as a small generalized deformation of the given primary Kodaira manifold. As a result the Frobenius structure at the primary Kodaira manifolds along the degree-2 direction is trivial. We will present an approach to extend this result on some 2-step nilmanifolds with abelian complex structures.

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T-duality beyond torus bundles

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Target-space duality, or T-duality for short, is a duality that comes from physics [2, 5] in the presence of a torus symmetry but also has very concrete mathematical formulations and consequences. In its simplest form, when the target-space is a Riemannian circle, it is marked by inversion of the radius of the circle and swapping of physical quantities (winding and momentum) to yield equivalent physical theories.

Mathematically, T-duality is made of two ingredients:

- *Topological T-duality* [1], relates the global topology of dual target-spaces: the background form on one side influences the topology of the dual space, there are isomorphisms between the twisted cohomologies of T-dual spaces and also of their twisted K-theories.
- *Geometric T-duality* [3, 4] is an isomorphism of Courant algebroids over T-dual spaces that allows one to transport geometric structures between T-dual spaces.

In this talk we will review the basics of T-duality and delve into progresses in the cases when the torus action has fixed points and when one replaces the torus by non-Abelian objects, extending both topological and geometric T-duality.

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SKT and Kähler-like metrics on complex manifolds

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Several special non-Kähler Hermitian metrics can be introduced on complex manifolds. Among them, SKT metrics deserve particular attention. They can be defined on a complex manifold by saying that the torsion of the Bismut connection associated to the metric is closed. These metrics always exist on compact complex surfaces but the situation in higher dimension is very different. We will discuss several properties concerning these metrics also in relation with the Bismut connection having Kähler-like curvature.

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Generalized Ricci solitons with large symmetry group

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The *generalized Ricci flow* is a geometric flow evolving a family of Riemannian metrics g_t and closed 3-forms H_t as follows

$$\begin{cases} \frac{\partial}{\partial t} g_t = -2\,\mathrm{Ric}_{g_t} + \frac{1}{2}\mathcal{H}_{g_t,H_t}, \\ \frac{\partial}{\partial t} H_t = -\Delta_{g_t} H_t. \end{cases}$$

Here, Δ_g denotes the Hodge Laplacian and $\mathcal{H}_{g,H}(X,Y) = g(\iota_X H, \iota_Y H)$ is a symmetric 2-covariant tensor. In theoretical physics, this flow appears as the renormalization group flow of a nonlinear sigma model arising in string theory. From the mathematical viewpoint, it can be regarded as a generalization of Hamilton's Ricci flow to metric connections with totally skew-symmetric torsion.

Since the RHS of the flow equation is invariant under diffeomorphisms and simultaneous scalings of the pair (g, H), a natural notion of *generalized Ricci solitons* can be introduced. These are pairs (g, H) satisfying the following system of equations

$$\begin{cases} \mathrm{Ric}_g = \lambda g - \frac{1}{2}\mathcal{L}_X g + \frac{1}{4}\mathcal{H}_{g,H}, \\ \Delta_g H = 2\lambda H - \mathcal{L}_X H, \end{cases}$$

for some $\lambda \in \mathbb{R}$ and $X \in \Gamma(TM)$. Solitons give rise to self-similar solutions of the flow and are expected as long time limits and singularity models for it. If $H \equiv 0$, the system above reduces to the classical Ricci soliton equation. On the other hand, if both $\lambda = 0$ and $X = 0$, the soliton reduces to a fixed point of the flow.

In this talk, I will first describe a construction that allows one to obtain infinite families of compact homogeneous spaces admitting invariant fixed points of the generalized Ricci flow [2]. Then, I will discuss the existence of complete rotationally invariant steady ($\lambda = 0$) generalized Ricci solitons on $\mathbb{R}^3$ [3], providing a generalization of the well-known Bryant's construction of the complete SO(3)-invariant steady soliton for the Ricci flow. Time permitting, I will also discuss some related results and open problems in the context of G_2 geometry [1].

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New minimal surfaces in multi-Taub-NUT spaces

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Federico Trinca
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I will describe the construction of new minimal surfaces in hyperkähler 4-manifolds arising from the Gibbons–Hawking Ansatz, i.e. hyperkähler 4-manifolds that admit a triholomorphic circle action. The minimal surfaces we produce are obtained via a gluing construction using well-known surfaces—the Scherk surface in flat space and the holomorphic cigar in the Taub-NUT space—as building blocks. The minimal surfaces we produce are not holomorphic with respect to any complex structure compatible with the metric, they can be parametrised by a harmonic map that satisfies a first-order Fueter-type PDE, and yet are unstable.

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PDEs Applications to Nonlinear Phenomena
Special Session B15

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Paolo Piersanti
Indiana University Bloomington, USA

This Special Session is scheduled on July 25–26. This Special Session is meant to encompass several modern active fields of Applied Mathematics like, for instance, Stochastic Partial Differential Equations, Fluid Dynamics, Nonlinear Elliptic Partial Differential Equations governed by Local as well as Nonlocal Operators, Mathematical Viscoelasticity, Primitive Equations in Climatology, Conservation Laws in Fluid Mechanics and Nonlinear Wave Equations.

We believe that bringing together experts from different applicative fields might constitute an excellent opportunity for learning about each other's recent mathematical achievements in the effort of understanding nonlinear phenomena as well as for fostering new collaborations between US-based Mathematicians and Italy-based Mathematicians.

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Some results about doubly nonlinear equations

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The term doubly nonlinear refers to the fact that the diffusion part depends nonlinearly both on the gradient and the solution itself. Such equations describe several physical phenomena and were introduced by Lions and Kalashnikov. These equations have an intrinsic Mathematical interest because they represent a natural bridge between the more natural generalisation of the heat equation: the p-Laplacian and the Porous Medium equations. Especially in the last few years, many papers were devoted to this topic. The idea is to give an unified approach comprehensive of the Porous Medium and the p-Laplacian equations. The approaches are sometimes not rigorous, sometimes not with sharp assumptions or with unnecessary long proofs. In this talk I will speak about the state-of-the-art, my contribution and the open questions.

[1]We wish to thanks GNAMPA to have partially supported this mission
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Stochastic partial differential equations

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Very recently, there has been much new developments in the research area of stochastic partial differential equations in fluid mechanics. For example, the technique of convex integration led to new results of non-uniqueness in law for various stochastic partial differential equations in fluid mechanics. The technique of convex integration also led to new solution theory of ill-posedness for various locally critical/supercritical singular stochastic partial differential equations in fluid mechanics. Finally, there has been renewed effort to extend local solution theory of singular stochastic partial differential equations to global in time. We discuss some of these recent developments.

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Phase separation and the Cahn–Hilliard–Oono system

Pierluigi Colli
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The talk reports on results for the Cahn–Hilliard–Oono system, which is of interest in the study of pattern formations in phase-separating materials. Well-posedness and optimal control, with the control u located in the mass term, are discussed for the related initial-boundary value problem. General potentials for the phase variable are admitted, in particular a singular potential can be considered. Next, the so-called separation property is shown in some physically relevant cases. The optimal control problem is then addressed and optimality conditions are investigated.

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On the eigenvalue problem for a bulk/surface elliptic system

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In this talk we shall deal with the doubly elliptic eigenvalue problem

$$\begin{cases} -\Delta u = \lambda u & \text{in } \Omega, \\ u = 0 & \text{on } \Gamma_0, \\ -\Delta_\Gamma u + \partial_\nu u = \lambda u & \text{on } \Gamma_1, \end{cases}$$

where Ω is a bonded open open subset of $\mathbb{R}^N$, $N \geq 2$, with C^r boundary $\Gamma = \partial\Omega$, $r \geq 1$, (Γ_0, Γ_1) is a partition of Γ with Γ_1 being nonempty and relatively open on Γ, $\mathcal{H}^{N-1}(\overline{\Gamma_0} \cap \overline{\Gamma_1}) = 0$ and $\mathcal{H}^{N-1}(\Gamma_0) > 0$. Here Δ and Δ_Γ respectively denote the Laplace and the Laplace–Beltrami operators, in Ω and on Γ, while λ is a real (or complex) scalar.

The eigenvalue problem above, which looks to be new in the mathematical literature, at least when $\Gamma_0 \neq \emptyset$, arises when looking for standing wave solutions of the evolutionary boundary value problem

$$\begin{cases} w_{tt} - \Delta w = 0 & \text{in } \mathbb{R} \times \Omega, \\ w = 0 & \text{on } \mathbb{R} \times \Gamma_0, \\ w_{tt} - \Delta_\Gamma w + \partial_\nu w = 0 & \text{on } \mathbb{R} \times \Gamma_1, \end{cases}$$

which models small free vibrations of the drumhead of a bassdrum.

The aim of the talk is to show that many well–known properties of eingenvalues and eigenfuctions in the case $\Gamma_1 = \emptyset$ extend to this case.

[1]This work has been funded by the European Union - NextGenerationEU within the framework of PNRR Mission 4 - Component 2 - Investment 1.1 under the Italian Ministry of University and Research (MUR) programme "PRIN 2022" - 2022BCFHN2 - Advanced theoretical aspects in PDEs and their applications - CUP: J53D23003700006

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Leray–Lions-type equations with bounded solutions

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We prove the existence of entire bounded solutions for some classes of quasilinear elliptic equation in $\mathbb{R}^N$ driven by a Leray-Lions operator of (p, q)-type. We need two basic tools: a suitable extension of the celebrated convergence lemma of Boccardo-Murat-Puel and a variational approach in intersections of Banach spaces by Candela-Palmieri.

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Continuous and Numerical Analysis of a Biot-Stokes PDE System

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In this talk, we describe our recent work concerning the semigroup wellposedness a of a coupled system of partial differential equations which describes a porous, elastic structure saturated by a given incompressible fluid flow. Moreover, we shall describe a companion finite element method (FEM) by which to numerically approximate the solutions of this particular Biot-Stokes model. The PDE system can be concisely as follows: the Biot system is invoked on a given domain. Moreover, the incompressible Stokes or Navier-Stokes equations evolve within another (distinct) geometry. These respective Biot and Stokes dynamics are coupled by means of interfacing (tangential) Beaver-Joseph-Saffman boundary conditions (BC). These BC are wellknown in the literature to constitute the appropriate ones for Stokes or Navier-Stokes flow over a poroelastic surface. Very recently, we established semigroup wellposedness for this Biot-Stokes PDE interaction. Subsequently, we have shown that the arguments which are used to show maximality of the associated Biot-Stokes semigroup generator can be adopted so as to devise a mixed variational formulation for the Biot-Stokes dynamics.

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Fractional Brezis-Nirenberg type equations

Raffaella Servadei
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In this talk we deal with a nonlocal fractional Brezis-Nirenberg type problem in presence of jumping nonlinearities and we prove the existence of a nontrivial solution for it, using a recent Linking type theorem due to Perera and Sportelli (see [2]), and some regularity results for weak solutions of nonlocal problems, which are of independent interest.

All these results appeared in a joint paper in collaboration with Giovanni Molica Bisci, Kanishka Perera and Caterina Sportelli (see [1]).

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Some minimax results for nonsmooth functionals in the Calculus of Variations

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Abstract. In the last years, elliptic equations involving a nonsmooth term have attracted several outstanding mathematicians and the interest towards this kind of problems has grown more and more, not only for their intriguing analytical structure, but also in view of their applications in a wide range of contexts. Motivated by this wide interest in the literature, the leading purpose of this talk is to present some recent results on nonsmooth elliptic equations, mainly related to a wide class of functionals defined through multiple integrals of Calculus of Variations. Applications to quasilinear boundary value problems will be presented and some open problems briefly discussed; see [1] and Chapter 8 in [2] for related topics.

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Determining Modes, Data Assimilation, and the Paradigm of Finite-Dimensional Intertwinement

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Aseel Farhat
Florida State University, USA

<u>*Vincent R. Martinez*</u>
City University of New York, USA

Collin Victor
University of Nebraska at Lincoln, USA

This talk will discuss a new concept of *finite-dimensional intertwinement* which unifies the classical property of Determining Modes, as defined by Foias and Prodi in 1967, and the recent convergence results of Data Assimilation algorithms, by Olson and Titi 2003 and Azouani, Olson, Titi 2014 in the context of the 2D Navier-Stokes equations. We introduce this definition, discuss rigorous results that intimately connect these three ideas, and present various numerical results.

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H-compactness for nonlocal linear operators in fractional divergence form

Alberto Maione
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In this talk we present a new result about the compactness with respect to the H-convergence for a class of non-symmetric and nonlocal linear operators in fractional divergence form, where the oscillations of the matrices are prescribed outside the reference domain. The compactness argument presented today bypasses the failure of the classical localisation techniques, that mismatch with the nonlocal nature of the operators. In the second part of the presentation, we assume symmetry and show an equivalence between the H-convergence of the nonlocal operators and the Γ-convergence of the corresponding energies. At the end of the talk a list of some open problems and new research directions drawn from this work will be presented.

This research is carried out in collaboration with Maicol Caponi (University of Naples Federico II) and Alessandro Carbotti (University of Salento).

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Title of the talk

Genni Fragnelli
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Dimitri Mugnai
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We will discuss a very recent approach to the study of beam-like equations. After introducing the needed mathematical setting for these classes of problems, we will provide some existence results and the description of the behaviour of the solutions for some concrete models.

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Nonlinear stability of two-dimensional periodic waves in parabolic systems with conservation laws

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We show that assuming the background periodic wave is diffusively stable, a stronger form of spectral stability, then the wave is nonlinearly stable even in the presence of conservation laws. The key difference with the case without conservation law analyzed by Melinand-Rodrigues is that even for extremely nice perturbations the linearized semigroup decays at a slow rate and so phase modulations play a deeper role.

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The viscoelastic paradox in a nonlinear Kelvin-Voigt type model of dynamic fracture

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In this talk we consider a dynamic model of fracture for viscoelastic materials, in which the constitutive relation, involving the Cauchy stress and the strain tensors, is given in an implicit nonlinear form. We prove the existence of a solution to the associated viscoelastic dynamic system on a prescribed time-dependent cracked domain via a discretisation-in-time argument. Moreover, we show that such a solution satisfies an energy-dissipation balance in which the energy used to increase the crack does not appear. As a consequence, in analogy to the linear case, this nonlinear model exhibits the so-called viscoelastic paradox.

This is a joint work with A. Carbotti and F. Sapio.

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Cross-diffusion system driven by fast reaction limit: weak solutions and weak-strong stability

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Lucilla Corrias
Département de Mathématiques, Université d'Evry Val-d'Essonne

Laurent Desvillettes, Helge Dietert
UFR de Mathématiques et Informatique, Université Paris Cité

Cross-diffusion systems are nonlinear parabolic systems that model the evolution of densities of multi-component populations in interaction. In this talk, we prove the existence of weak solutions for a starvation-driven cross-diffusion system, obtained as the limit of a reaction-diffusion system with linear diffusion and fast reaction. The main tools used to rigorously pass to the limit consist of a priori estimates, given by the analysis of a family of entropy functionals, and a compactness argument. However, we also investigate the regularity of the obtained solution, by improving the entropy a priori estimates using a bootstrap argument. We conclude the analysis with a weak-strong stability result.

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[1]The authors warmly thank Prof. Y-J. Kim for the fruitful and interesting discussions.
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Fast Multivariate Newton Interpolation for Downward Closed Polynomial Spaces and Applications to Numerical Differential Geometry

Michael Hecht
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We introduce a fast Newton interpolation algorithm with a runtime complexity of $\mathcal{O}(Nn)$, where N denotes the dimension of the underlying downward closed polynomial space and n its l_p-degree, where $p > 1$. We demonstrate that the algorithm achieves the optimal geometric approximation rate for analytic *Bos-Levenberg-Trefethen functions* in the hypercube. In this case, the Euclidean degree ($p = 2$) emerges as the pivotal choice for mitigating the curse of dimensionality. The spectral differentiation matrices in the Newton basis are sparse, enabling the implementation of fast pseudo-spectral methods on flat spaces, polygonal domains, and regular manifolds.

In particular, we revisit our former contribution, entitled *Global Polynomial Level Sets (GPLS) for Numerical Differential Geometry of Smooth Closed Surfaces*.

The GPLS provides an approximation for a wide range of smooth surfaces, which are initially given solely as point clouds, using a global polynomial level set. This enables efficient and accurate computation of various differential-geometric quantities, such as mean and Gauss curvature, or even higher-order terms like the Laplacian of mean curvature, in a straightforward manner. The GPLS significantly reduces the number of surface points required compared to classic alternatives that rely on surface meshes or embedding grids. We sketch extensions to higher dimensions and discuss applications in numerical differential geometry.

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Discrete and Combinatorial Algebraic Topology, Theory and Applications
Special Session B16

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Chris Kapulkin
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Giovanni Paolini
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Extending techniques from algebraic topology to discrete and combinatorial objects such as graphs and finite posets has recently generated an increased amount of interest from researchers working in a variety of areas of mathematics. Several different parts of algebraic topology have undergone at least a partial 'discretization' so far, the most notable being homotopy theory, Morse theory, and sheaf theory, and efforts to make the discrete theories more complete are ongoing. The work has already found numerous applications, including to hyperplane arrangements, dynamical systems, geometric group theory, coarse geometry, configuration spaces, distributed computing, graph colorings, and digital imaging, as well as to network and data analysis. The goal of this session is to bring together researchers who are currently working in discrete algebraic topology with researchers who work in closely related areas in order to expand the mathematical applications of discrete-algebraic-topological techniques and disseminate recent developments in the field.

This special session will take place on July 25th-26th, 2024.

For more information visit `https://sites.google.com/cimat.mx/umi-ams-discrete-alg-top`.

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On Combinatorial Width and the Homeomorphism Problem for 3-Manifolds

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The d-dimensional *Homeomorphism Problem* HP_d asks whether two given closed, orientable, triangulated d-manifolds are homeomorphic. Across the dimensions, the difficulty of HP_d is strikingly different. While HP_2 is easily solved, for any fixed $d \geq 4$ there is no general algorithmic solution to HP_d. In the remaining dimension $d = 3$, the Homeomorphism Problem turns out to be algorithmically decidable, but the known solutions are very complicated and have not been implemented.

Thus, in practice, HP_3 is usually approached with the help of various topological invariants computable from triangulations. (Un)fortunately, those invariants that are fine enough to distinguish a large number of 3-manifold triangulations are often known to be very hard to compute in general. In the last decade, however, it has been shown that some of these invariants can actually be computed very efficiently for triangulations that are "thin" in a combinatorial sense.

In this talk, we present several recent results that relate the key combinatorial "width" parameter in the above context (the treewidth of the dual graph of a 3-manifold triangulation) to classical topological invariants of 3-manifolds (such as the Heegaard genus or the JSJ decompositions). As a consequence of these results, we exhibit infinite families of 3-manifolds that do not admit "thin" triangulations.

Joint work with Jonathan Spreer and Uli Wagner.

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Chromatic Arrangements and Configuration Spaces with Obstacles

Sadok Kallel
American University of Sharjah, UAE, and Laboratoire Painlevé, Université de Lille, France

Given an abstract graph $\Gamma = (V, E)$, define the *chromatic configuration space* to be

$$\mathrm{Conf}_\Gamma(X) = \{(x_1, \cdots, x_n) \in X^{|V|} \mid x_i \neq x_j \text{ if } \{i, j\} \in E(\Gamma)\}$$

When the graph is complete, we get the classical configuration space of pairwise distinct points. We will show that $\mathrm{Conf}_\Gamma(X)$ splits, after only one suspension, into a bouquet of spheres, the number of spheres is computed precisely and is given in terms of some chromatic numbers of the graph. Our tools are poset topology and a geometric description of the homology classes. Similar ideas apply to a related kind of configuration spaces. This work is joint with Moez Bouzouita (University of Tunis).

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TBD

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On the Common Basis Complex and the Partial Decompositions Poset

Volkmar Welker, *Volkmar Welker*
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In this talk we discuss joint work with B. Brück and K. Piterman on generalizations of the common basis complex studied in algebraic K-theory. There for a ring R the common basis complex $\mathrm{CB}(R^n)$ is the simplicial complex whose simplices are collections of free summands $\neq 0, R^n$ for which there is a common basis. The dimension of $\mathrm{CB}(R^n)$ is $2^n - 3$ but there is a conjecture due to Rognes that for a large class of rings the complex if $2n - 4$ connected and it is know that again for a large class of rings the homology above dimension $2n - 3$ vanishes. For fields Rognes conjecture was recently proved by Miller, Patzt and Wilson.

We show in the generality of our setting that the common basis complex is indeed homotopy equivalent to the order complex of partial decompositions. The latter poset for the case of R^n consists of all sets of free submodules that are in direct sum and for which their direct sum has a free complement. It is $2n - 3$ dimensional and hence Rognes conjecture suggests that it could be Cohen-Macaulay. In our generality this is not the case, but it seems to hold for many examples.

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Topological methods in zero-sum Ramsey theory

Florian Frick
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A cornerstone result of Erdős, Ginzburg, and Ziv states that any sequence of $2n-1$ elements in $\mathbb{Z}/n$ contains a zero-sum subsequence of length n. This result has inspired numerous generalizations and variants, collectively known as zero-sum Ramsey theory. In a general form, these results give conditions for any $\mathbb{Z}/n$-labelling of the vertices of an n-uniform hypergraph to have a hyperedge whose labels sum to zero. I will present a topological condition for this to occur in terms of the box complex of the hypergraph. This topological condition for the special case of Kneser hypergraphs is implied by a purely combinatorial criterion in terms of the colorability defect. This provides a unified topological approach to earlier results, such as the original theorem of Erdős, Ginzburg, and Ziv, Olson's generalization to arbitrary finite groups, and zero-sum matchings in hypergraphs. It also yields new zero-sum Ramsey results.

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Homotopy and singular homology groups of finite (di)graphs

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Abstract: We extend classical results in algebraic topology for higher homotopy groups and singular homology groups of pseudotopological spaces. Pseudotopological spaces are a generalization of topological spaces that also include simple directed and undirected graphs. More specifically, we show the existence of a long exact sequence for homotopy groups of pairs of closure spaces and that a weak homotopy equivalence induces isomorphisms for homology and cohomology groups.

Theorem 1. Given a pointed pair of pseudotopological spaces (X, A, x_0), the sequence

$$\cdots \to \pi_n(A, x_0) \xrightarrow{i_*} \pi_n(X, x_0) \xrightarrow{j_*} \pi_n(X, A, x_0) \xrightarrow{\partial} \cdots \to \pi_1(X, A, x_0) \xrightarrow{\partial} \pi_0(A, x_0) \xrightarrow{i_*} \pi_0(X, x_0),$$

is exact, where i_*, j_* are the maps induced my inclusions $i : (A, x_0) \to (X, x_0)$ and $j : (X, x_0, x_0) \to (X, A, x_0)$, respectively and ∂ is the boundary operator described above.

Theorem 2. A weak homotopy equivalence $f : X \to Y$ of pseudotopological spaces induces isomorphisms $f_* : H_n(X; G) \to H_n(Y; G)$ and $f^* : H^n(Y; G) \to H^n(X; G)$ for all n and all coefficient groups G.

With these results we are able to prove out main result, the construction of a weak homotopy equivalences between the geometric realizations of (directed) clique complexes and their underlying (directed) graphs.

Theorem 3. For each finite directed (resp. undirected) graph (X, E) there exist a finite abstract simplicial complex $\overrightarrow{\mathrm{VR}}(X, E)$ (resp. $\mathrm{VR}(X, E)$) and a weak homotopy equivalence $f_X : |\overrightarrow{\mathrm{VR}}(X, E)| \to (X, E)$ (resp. $f_X : |\mathrm{VR}(X, E)| \to (X, E)$) in **PsTop**.

This implies that singular homology groups of finite graphs can be efficiently calculated from finite combinatorial structures, despite their associated chain groups being infinite dimensional. This work is similar to the work McCord [1] did for finite topological spaces, but in the context of pseudotopological spaces. Our results also give a novel approach for studying (higher) homotopy groups of discrete mathematical structures such as digital images.

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Hardness of Promised Colourings and Homotopy of Relational Structures

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The *constrain satisfaction problem* (CSP) is a problem of finding a homomorphism of relational structures and can be stated as follows: Let A be a fixed relational structure (i.e. a set together with a list of relations of various arities), then $\mathrm{CSP}(A)$ asks whether a given relational structure X admits a homomorphism $X \to A$. We remark that the classical *graph colouring problem* can be seen as a special case of CSP and that deciding whether a graph is k-colourable ($\mathrm{CSP}(K_k)$)is NP-complete for $k \geq 3$ [2].

The framework of CSP can further be extended to a *promised* version (PCSP). Here, we fix two strucutes A, B with a known homomorphism $A \to B$. $\mathrm{PCSP}(A, B)$ then asks whether for a given structure X one can either find a homomorphism $X \to A$ or at least show there is no homomorphism $X \to B$. Again, there is a related special case in the theory of graphs:

Given an input graph that is *promised* to be c-colourable, how hard is it to find at least a k-colouring, $k \geq c \geq 3$? This problem (formally $\mathrm{PCSP}(K_c, K_k)$) is conjectured to be NP-hard, with only several cases proven so far [3].

A common technique in the study of a (P)CSP instance is to consider the structure of *polymorphisms* i.e. homomorphisms $A^n \to B$. In recent years, a topological approach for studying the polymorphism was introduced by Krokhin, Opršal, Wrochna, and Živný [4].

In the talk, I'll give an overview of the topological approach, stress the role of various versions of homotopy for relational structures and finally demonstrate an application of the homotopy theory methods in in the study of *linearly ordered* colourings of hypergraphs (joint w. J. Opršal, T. Nakajima, G. Tasinato and U. Wagner [1]):

A linearly ordered (LO) k-colouring of a hypergraph is a colouring of its vertices with colours $1, \ldots, k$ such that each edge contains a unique maximal colour.

We prove that the following promise problem is NP-complete: Given a 3-uniform hypergraph, distinguish between the case that it is LO 3-colourable, and the case that it is not even LO 4-colourable.

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The connectivity of Vietoris-Rips complexes of spheres

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For X a metric space and $r > 0$, the Vietoris-Rips simplicial complex $VR(X;r)$ contains X as its vertex set, and a finite subset of X as a simplex if its diameter is less than r. Some versions of discrete homotopy groups are closely related to the standard homotopy groups of Vietoris–Rips complexes. Interestingly, the Vietoris–Rips complexes of the circle obtain the homotopy types of the circle, the 3-sphere, the 5-sphere, the 7-sphere, ..., as the scale parameter increases. But little is known about Vietoris-Rips complexes of the n-sphere S^n for $n \geq 2$. We show how to control the homotopy connectivity of Vietoris-Rips complexes of spheres in terms of coverings of spheres and projective spaces. For $\delta > 0$, suppose that the first nontrivial homotopy group of $VR(S^n; \pi - \delta)$ occurs in dimension k, i.e., suppose that the connectivity is $k - 1$. Then there exist $2k+2$ balls of radius δ that cover S^n, and no set of k balls of radius $\delta/2$ cover the projective space $\mathbb{R}P^n$.

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Überhomology, dominating sets and the Mayer-Vietoris spectral sequence

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Überhomology is a recently defined triply-graded homology theory of simplicial complexes, which yields both topological and combinatorial information. When restricted to (simple) graphs, a certain specialization of überhomology gives a categorification of the connected domination polynomial at -1; which shows that überhomology of graphs is related to combinatorial quantities such as connected dominating sets. On the topological side, überhomology detects the fundamental class of homology manifolds. In this talk, we present the notion of überhomology, as introduced by D. Celoria, and show some combinatorial and topological properties such as its relation to connected domination. Then, we shall see a more conceptual property: überhomology of simplicial complexes can be identified with the second page of the Mayer-Vietoris spectral sequence, with respect to the anti-star covers.

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Telling apart coarsifications of the integers

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A *coarse homomorphism* between two groups equipped with bi-invariant metrics is a Lipschitz mapping that commutes with the group operation up to bounded error. The group $\mathbb{Z}$ of integers can be given a moltitude of (non proper) invariant metrics that are not bi-Lipschitz equivalent to one another. Now the question is: if we take two copies of $\mathbb{Z}$ equipped with two such metrics, is there a coarse isomorphism between them? In this talk I will discuss an invariant of coarse isomorphism and show how to use it to answer this question for word metrics associated with certain geometric progressions. This is part of more general investigations on the topic of *coarse groups*.

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Symmetry counts: an introduction to equivariant Hilbert and Ehrhart series

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The Ehrhart series of a lattice polytope P is a combinatorial gadget that counts the number of lattice points of P and of its dilations. The Hilbert series of a simplicial complex Σ counts how many monomials supported on faces of Σ exist in each possible degree. The aim of this talk is to introduce equivariant versions of such constructions, where we are not just interested in counting but we also want to record how the action of a finite group affects such collections of lattice points or monomials. Inspired by previous results by Betke–McMullen, Stembridge, Stapledon and Adams–Reiner, we will investigate which extra combinatorial features of the group action give rise to "nice" rational expressions of the equivariant Hilbert and Ehrhart series, and how the two are sometimes related.

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Eulerian magnitude homology

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Magnitude was first introduced by Leinster in 2008 as a notion analogous to the Euler characteristic of a category. Magnitude homology was defined in 2014 by Hepworth and Willerton as a categorification of magnitude in the context of simple undirected graphs, and although the construction of the boundary map suggests that magnitude homology groups are strongly influenced by the graph substructures, it is not straightforward to detect such subgraphs.

In this talk, I will describe the work done by Dr. Chad Giusti and myself toward elucidating the connection between magnitude homology of simple graphs equipped with the hop metric and their combinatorial structure. The approach we take is to observe that a large portion of the magnitude chain complex is redundant, in the sense that the chains reflect combinatorial structure already recorded by chains of lower bigrading. To leverage this observation, we define the subcomplex of *eulerian magnitude chains* $EMC_{k,\ell}(G)$, supported on trails with no repeated landmarks. Focusing on the $k = \ell$ line where the list of landmarks completely determines a trail, we obtain strong relationships between the (k, k)-eulerian magnitude homology groups and the structure of a graph. We accomplish this in Theorem 1 by decomposing such cycles into a generating set described in terms of their *structure graphs*, which encode how terms in the differentials of the constituent chains cancel. We are thus able to characterize subgraphs of a graph that support non-trivial cycles in $EMH_{k,k}(G)$ in terms of the corresponding structure graphs, providing a framework for computing these groups for graphs of interest.

Theorem 1. *Let G be a graph and $X = \{\bar{x}^i\}_{i\in[m]} \subseteq EMC_{k,k}(G)$ a collection of trails in G. Then we can decompose EMH cycles supported on X into cycles supported on cliques-trees and cycles supported on circuits of the structure graph $s(X)$.*

In the interest of exploring what features of a graph the (eulerian) magnitude homology groups capture, we turn our attention to classes of random graphs: Erdös-Rényi (ER) random graphs and random geometric graphs on the standard torus. In each context, we derive a vanishing threshold for the limiting expected rank of the (k, k)-eulerian magnitude homology in terms of the density parameter. Further, adapting tools from Kahle and Meckes we develop a characterization of the limiting expected Betti numbers of the (k, k)-eulerian magnitude homology groups in terms of density. In this talk, I will focus on the results in the context of ER graphs.

Theorem 2. *Let $G(n, p)$ be an ER graph and call $\beta_{k,k} = \operatorname{rank}(EMH_{k,k}(G(n, p)))$. If $p = o\left(n^{-1/n}\right)$ then $\lim_{n\to\infty} \frac{\mathbb{E}(\beta_{k,k})}{n^{k+1}p^{2k-1}} = \frac{1}{(k+1)!}$ and $\frac{\beta_{k,k}-\mathbb{E}(\beta_{k,k})}{\sqrt{Var(\beta_{k,k})}} \Rightarrow N(0, 1)$.*

I will finally discuss the homotopy type of the eulerian magnitude chain complex by highlighting its connection with the complex of injective words.

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Bigraded path homology and the magnitude-path spectral sequence

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Magnitude homology, path homology and reachability homology are all homology theories for directed graphs, each satisfying sensible analogues of the Eilenberg–Steenrod axioms. In spite of this, they tend to disagree even on very primitive classes of graphs. For example, consider the directed cycle Z_m of m vertices and m edges, consistently oriented. Whereas magnitude homology distinguishes Z_m from Z_n whenever $m \neq n$, path homology sees Z_1 and Z_2 as "contractible" and all the rest as "circle-like". Meanwhile, from the perspective of reachability homology, every directed cycle appears contractible.

The *magnitude-path spectral sequence* (or *MPSS*) offers a systematic account of these different points of view. Page E^1 of the MPSS is exactly magnitude homology, while (as Asao showed in [1]) path homology can be identified with a single axis of page E^2; the sequence converges, under mild conditions, to reachability homology [3]. In [4] we name page E^2 as a whole the *bigraded path homology* of a directed graph, and study its properties along with those of the other pages of the MPSS. For each $r \in \mathbb{N}$, page E^{r+1} of the MPSS has a homotopy invariance property that holds when maps of directed graphs $f, g\colon G \to H$ are *r-close* with respect to the shortest path metric. (In particular, page E^r sees the directed cycle Z_m as contractible when m is less than r, and distinguishes each of the Z_ms for $m \geq r$.) We demonstrate that each page of the sequence possesses homological properties compatible with this ever-stronger homotopy invariance. Thus, the MPSS encompasses a spectrum of homological perspectives on the category of directed graphs, interpolating between magnitude homology and reachability homology.

Concerning the spectral sequence as a whole, our main results are as follows.

Theorem 1. *Every page of the MPSS has the following homological properties:*

- *It satisfies an excision theorem with respect to the cofibrations introduced in* [2].
- *It satisfies a Künneth theorem with respect to the box product.*
- *It is a finitary functor on the category of directed graphs.*

In particular these hold for bigraded path homology, which also satisfies a Mayer-Vietoris theorem.

Together, these results allow us to show that the cofibration category structure exhibited by Carranza *et al* in [2] admits a natural refinement.

Theorem 2. *The category of digraphs carries a cofibration category structure in which the cofibrations are those of* [2] *and the weak equivalences are maps inducing isomorphisms on bigraded path homology. This is a strictly finer structure than the one exhibited in* [2]*: for instance, bigraded path homology, unlike path homology, distinguishes the directed m-cycles for every $m \geq 2$.*

We speculate that this cofibration category belongs to a nested family of structures, one for each page of the MPSS. Time permitting, I will sketch this idea at the end of the talk.

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Threshold-linear networks, attractors, and oriented matroids

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Threshold-linear networks (TLNs) are common models in theoretical neuroscience that are useful for modeling neural activity and computation in the brain. They are simple, recurrently-connected networks with a rich repertoire of nonlinear dynamics including multistability, limit cycles, quasiperiodic attractors, and chaos. In this talk, I will give a brief introduction to TLNs, and then show how ideas from sheaf theory and oriented matroids provide valuable insights into the connection between network architecture and dynamics.

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The cd-index of semi-Eulerian posets

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The numbers of flags of different ranks of an Eulerian poset are subject to a set of linear relations which have been described by Bayer and Billera. Fine showed that these relations are equivalent to the existence of a certain polynomial in two non-commutative variables, called the cd-index. In our work we show that it is possible to extend the definition of the cd-index from Eulerian to semi-Eulerian posets by a small modification of the flag f-polynomial. In particular we have the following.

Theorem 1. *Let P be a semi-Eulerian poset of rank $n+1$ with flag f-polynomial $\chi_P(a,b)$. The polynomial*

$$\chi'_P(a,b) := \chi_P(a,b) + (\chi(\mathbb{S}^{n-1}) - \chi(P))a\cdots ab$$

admits a cd-index $\Phi_P(c,d)$. Moreover, if P is simplicial we have that

$$\Phi_P(c,d) = \sum_{i=0}^{n-1} h_i(P)\check{\Phi}_i^n(c,d),$$

with $\check{\Phi}_i^n$ as defined by Stanley in [3] and $(h_0(P), h_1(P), \ldots h_n(P))$ is the h-vector of P.

Moreover, the coefficients of the cd-index of many interesting classes of posets (e.g., face posets of regular CW-spheres or Gorenstein* posets) are nonnegative, reflecting linear inequalities in the number of flags. Following partial results and a conjecture of Novik [2], we expect nonnegativity even for face posets of triangulated manifolds. In a joint work with Martina Juhnke and José Samper [1] we show that the coefficients of the cd-index of simplicial semi-Eulerian Buchsbaum posets are nonnegative. This family includes face posets of triangulated manifolds.

Theorem 2. *The coefficients of the cd-index of a semi-Eulerian Buchsbaum simplicial poset are nonnegative.*

This result is obtained by combining a lower bound proved by Novik and Swartz, together with a careful analysis of a family of non-commutative polynomials. This yields to stronger lower bounds on the coefficients than just nonnegativity. As a corollary we obtain nonnegativity of the so-called γ-polynomial of the order complex of Eulerian Buchsbaum simplicial poset of even rank. In particular, we underline the following result.

Corollary 3. *The order complex of any odd-dimensional triangulated manifold satisfies the Charney-Davis conjecture.*

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Shellability of non ω-integral partition poset

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Consider a graph Γ on the vertices $[r] = \{1, 2, \ldots, r\}$, a *flat* π is a flat of the associated graphical matroid. In other words, $\pi \vdash [r]$ is a set partition of the vertices such that for each block π_i of π the subgraph induced by π_i is connected. Let $\omega = (\omega_1, \ldots, \omega_r) \in \mathbb{R}^r$ be an integer vector.

Definition 1. A flat π is ω-integral if for each block π_i the sum $\sum_{j\in\pi_i} \omega_j \in \mathbb{Z}$ is an integer.

Let $\mathcal{L}_{\Gamma,\omega}$ be the poset of all non ω-integral flats of Γ ordered by refinement.

In general the poset $\mathcal{L}_{\Gamma,\omega}$ does not have a maximum, so we add a maximum element denoted by $\hat{1}$ and obtain the poset $\hat{\mathcal{L}}_{\Gamma,\omega} = \mathcal{L}_{\Gamma,\omega} \sqcup \{\hat{1}\}$.

Theorem 1. *The poset $\hat{\mathcal{L}}_{\Gamma,\omega}$ is LEX-shellable. In particular, its order complex is a wedge of spheres.*

The proof of this result is different from the proof of EL-shellability of partition posets and geometric lattices.

From this result we new obtain counting formula for lattice points in translated graphical zonotopes $Z_\Gamma + \omega$. As an application, we solve a problem in topology and representation theory: we determine the Ngo strings of the Hitchin fibration on the reduced locus.

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Tope-pair posets of oriented matroids and hyperplane arrangements

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Oriented matroids encode the combinatorial structure of arrangements of pseudospheres, generalizing arrangements of hyperplanes in real vectorspaces. Salvetti complexes of oriented matroids represent a strictly wider class of homotopy types with respect to complements of hyperplane arrangements in complex vectorspaces (e.g., one obtains fundamental groups that cannot appear in the case of hyperplane arrangements).

To every real hyperplane arrangement and, more generally, to every oriented matroid we associate a “tope-pair poset”. This poset is homotopy equivalent to the Salvetti complex and carries some extra structure that makes it a useful tool for combinatorial topology.

For instance, the tope-pair poset supports a free action of $\mathbb{Z}_4$ that discretizes the diagonal $\mathbb{C}^*$-action on complexified arrangement's complements, and it affords a proof of the fact that the integer cohomology of the Salvetti complex of any oriented matroid is given by the Orlik-Solomon algebra of the underlying matroid.

In the talk I will define the tope-pair poset, survey some of its features, and outline its connections to Artin groups and combinatorial fibrations.

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$K(\pi, 1)$ - and other - conjectures for Artin groups

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In the recent proof of the $K(\pi, 1)$-conjecture for affine Artin groups [1], well-established techniques from Discrete and Combinatorial Algebraic Topology, such as discrete Morse theory and shellability, play a fundamental role. We discuss (besides some aspects of the proof) related conjectures, particularly those stemming from the so-called *dual* structure (see also [2]), as well as explore some related combinatorial approaches.

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Vertex orders: From graphs to complexes

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Many properties of graphs—such as Hamiltonicity and chordality—can be expressed in terms of orders on the vertices of the graph. Many of these definitions have been generalized to simplicial complexes (see [1]). We consider these properties in relation to the following type of shellings of simplicial complexes.

Definition 1. Let Δ be a pure simplicial complex with vertices labeled 1 through n. If the lexicographic order on the facets is a shelling of Δ, we say that Δ is *lex shellable*. Such an order on the vertices is a *lex shelling order*.

For many vertex orders stemming from graph properties, it is straightforward to determine their relationship (or lack thereof) with lex shelling orders. We focus primarily on *unit-interval orders*, for which the connection is more subtle.

Definition 2. Let Δ be a pure d-dimensional simplicial complex with vertices labeled 1 through n. Assume for any facet $F = v_0 v_1 \dots v_d$ of Δ that the d-skeleton of $\{v_0, v_0+1, v_0+2, \dots, v_d\}$ is contained in Δ. Then Δ is a *unit-interval complex* and the order on the vertex set is a *unit-interval order*.

When $d = 1$, we recover a standard definition for unit-interval graphs G. That is, if $a < b < c$ are vertices of G and ac is an edge of G, then both ab and bc are edges of G. For complexes in general, we prove the following.

Theorem 1. *Let Δ be a pure and strongly connected simplicial complex. Then any unit-interval order for Δ is a lex shelling order.*

Time permitting, we will discuss similar vertex order properties and their connections to shelling completable complexes (see [2]).

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Mathematical Methods and Models of Systems Interactions and Network Dynamics Special Session B17

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Networks of dynamical systems exhibit complex global behaviors characterized by non-equilibrium phase transitions, self-organization and criticality, noise-induced patterns and nonlinearity, partial synchronization and desynchronization, and a variety of adaptive responses to internal and external perturbations. A fundamental question is how such collective behaviors emerge from the interplay between the dynamics of individual systems and the network topology in which the systems are embedded.

In recent years numerous investigations have focused on the mathematical properties of adaptive networks of dynamical systems to quantify the laws of time-varying and nonlinear network interactions, to uncover multiple forms of coupling and feedback loops, and to derive principles of coordination and network integration among dynamical systems in association with network states and functions.

Novel analytic and computational methods derived from applied mathematics, nonlinear dynamics, information theory, control and adaptive networks theory as well as high-dimensional network modeling approaches were recently developed to build an adequate theoretical framework and explore how hierarchical organization in network structure (sub-networks and modules) relates to multi-stability and meta-stability of dynamic states with emergent global functions.

While these advances have a broad spectrum of applications to physical, biological, ecological and social systems, investigations of the temporal complexity and emergent global behaviors in adaptive networks of dynamical systems lay the foundation of a new theoretical framework to study living systems. Of particular relevance to physiology and medicine is how physiological systems, processes and functions can be mathematically described and studied within the context of complex networks dynamics.

A new field, Network Physiology, has emerged to address the fundamental question of how physiological systems and sub-systems continuously coordinate, synchronize, and integrate their dynamics as a network to optimize functions and to maintain health. In addition to the traditional approach in biology and physiology that defines health and disease through structural, dynamic, and regulatory changes in individual systems, the new conceptual framework of Network Physiology focuses on the coordination and network interactions among systems as a hallmark of physiological state and function. This poses new challenges in developing generalized methodology adequate to quantify complex dynamics of networks where nodes represent diverse dynamical systems with distinct forms of coupling that continuously change in time.

Novel methods are needed to provide insights into physiological structure and function in health and disease, and across levels of integration from genomic interactions to inter-cellular signaling and metabolic networks, to communications among integrated organ systems.

This special session will provide a venue for leading experts to present and discuss recent advances in the theory of adaptive networks of dynamical systems and their applications to physiology and medicine, as well as to outline key questions and future directions of research aimed to uncover the relations of network topology and dynamics with emerging physiological states and functions in health and disease. This session is scheduled on July $25^{th} - 26^{th}$.

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The New Field of Network Physiology: Building the Human Physiolome

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The human organism is an integrated network where complex physiological systems continuously interact to optimize and coordinate their function. Organ-to-organ interactions occur at multiple levels and spatiotemporal scales to produce distinct physiologic states. Disrupting organ communications can lead to dysfunction of individual systems or to collapse of the entire organism. Yet, we do not know the nature of interactions among systems and sub-systems, and their collective role as a network in maintaining health.

We initiated a new interdisciplinary field, Network Physiology, which aims to address these fundamental questions. Through the prism of concepts and approaches from statistical and computational physics, nonlinear dynamics, and applied mathematics, we will present a new framework to identify and quantify dynamic networks of organ interactions. We focus on inferring coupling and dynamical interactions among organ systems from continuous streams of synchronized recordings of key physiologic variables. In contrast to traditional complex networks theory, where edges/links are constant and represent static graphs of association, novel approaches in Network Physiology aim to establish dynamical aspects of organ communications in real time, to track the evolution of organ network interactions and quantify emerging collective network behaviors in response to changes in physiological state and condition.

We will report first findings utilizing this new framework to (i) investigate brain-brain network interactions across distinct brain rhythms and locations, and their relation to new aspects of neural plasticity in response to changes in physiologic state; (ii) characterize dynamical features of brain-organ communications as a new signature of neuroautonomic control; and (iii) establish basic principles underlying coordinated organ-organ communications. We will demonstrate how physiologic network topology and systems connectivity lead to integrated global behaviors representative of distinct states and functions. The presented investigations are initial steps in building a first Atlas of dynamic interactions among organ systems and the Human Physiolome, a new kind of Big Data of blue-print reference maps that uniquely represent physiologic states and functions under health and disease.

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Analysis of Medical Data with Synolitic Networks

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There's a merging trend in Nonlinear Dynamics, Graph Theory, and Artificial Intelligence. Our focus here is on representing multidimensional data as graphs, even when the network structure is unknown. This method shows promise in handling the complexity of biological systems by linking features through biological and thermodynamic laws. However, it requires prior knowledge of feature connections. Alternatively, correlation graphs for time-series or correlation-prediction graphs can aid in early detection or survival analysis. Another method, pioneered by Zanin and Bocaletti, constructs a "parenclitic" network without prior knowledge of interactions. This approach has been effective in identifying key genes and metabolites in diseases, including cancer using DNA methylation data. Additionally, 2-dimensional kernel density estimation (2DKDE) is proposed for modeling control distribution when linear models are insufficient. We've also introduced "synolitic" networks, an ensemble of classifiers in graph form [1,2], useful for analyzing age-related trajectories in Down's syndrome [3] and predicting survival in severe Covid-19 cases [4,5]. These networks can be considered an ensemble of classifiers in a graph form and thus are a kind of correlation network where the correlation is in the changes between two classes (e.g. disease and non-disease). Notably, Synolitic graphs facilitate the use of Graph Neural Networks for data analysis, hence, enabling Graph Neural Networks to analyse the data which were initially not represented in the form of a network.

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Cytokine-induced coherent structures in a reaction–diffusion-chemotaxis model of Multiple Sclerosis

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In this work, we develop a model for the evolution of the Multiple Sclerosis pathology that considers the modulatory influence of cytokines on the activation rate of macrophages. Our starting point is the reaction diffusion-chemotaxis model proposed in [1], which generalizes the system proposed by Khonsari and Calvez [2, 3] to describe Baló's sclerosis. The model represents the interaction dynamics among three species of cells: macrophages $\tilde{m}(T, X)$, cytokines $\tilde{c}(T, X)$ and oligodendrocytes $\tilde{d}(T, X)$.

$$(1)\quad \begin{cases} \dfrac{\partial \tilde{m}}{\partial T} = D\Delta_{\mathbf{X}}\tilde{m} - \nabla_{\mathbf{X}} \cdot (\Psi(\tilde{m})\nabla_{\mathbf{X}}\tilde{c}) + \lambda \dfrac{\tilde{c}}{k_{\tilde{c}} + \tilde{c}}\tilde{m}(\overline{m} - \tilde{m}), & \Psi(\tilde{m}) = \psi \dfrac{\tilde{m}}{\overline{m} + \tilde{m}} \\ \dfrac{\partial \tilde{c}}{\partial T} = \dfrac{1}{\nu}(\varepsilon \Delta_{\mathbf{X}}\tilde{c} + \mu\tilde{d} + b\tilde{m} - \alpha\tilde{c}), & \\ \dfrac{\partial \tilde{d}}{\partial T} = \kappa\, F(\tilde{m})\, \tilde{m}\, (\overline{d} - \tilde{d}), & F(\tilde{m}) = \dfrac{\tilde{m}}{\overline{m} + \tilde{m}} \end{cases}$$

Through a weakly nonlinear analysis, we focus on the study of the Turing-type instabilities of the nontrivial homogeneous steady state, leading to the settlement of stationary patterns of inflammation and demyelination. Using biologically relevant parameter values, we show that the asymptotic solutions of our model system reproduce the concentric demyelinating rings, confluent plaques, and preactive lesions observed in Balò sclerosis and type III Multiple Sclerosis. Comparing the present model with the model proposed in [1], where the macrophages activation mechanism is due to innate immunity, the cytokine-induced macrophages activation rate encourages pattern formation, and leads to a pattern with smaller wavenumber. Moreover, our findings reveal that the alternative scenario proposed here results in a less aggressive pathology characterized by reduced inflammation levels and significantly slower disease progression.

Under the appropriate regularity conditions on the initial data, we prove the existence of a unique global solution to our proposed system, following the strategy used in [4].

This study provides insights into the role of cytokines in the pathogenesis of Multiple Sclerosis, shedding light on the disease's dynamics and offering potential avenues for therapeutic interventions.

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Nonlinear Schrödinger Equation on Networks and Hybrids

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We review the results obtained in the last decade on the problem of modeling systems described by the Nonlinear Schrödinger Equation, constrained to exotic domains like networks, or metric graphs, and hybrids, i.e. structures made of pieces of different dimensionality. Our results apply to the dynamic of Bose-Einstein condensates in ramified traps, for the case of network, and in magneto and optical traps in the case of hybrids. We will mainly focus on the problem of the existence of Ground States. This is a joint project with Filippo Boni, Raffaele Carlone, Simone Dovetta, Alice Ruighi, Enrico Serra, Lorenzo Tentarelli and Paolo Tilli.

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Effects of anomalous diffusion on pattern formation in the FitzHugh-Nagumo model

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Anomalous diffusion provides a more realistic description of various physical phenomena in different contexts, such as in autocatalytic chemical reactions on porous media, ion channels in the plasma membrane, and also population dynamics. In this talk we shall discuss the influence of anomalous diffusion on the onset of stationary nonhomogeneous structures in the Fitzhugh-Nagumo model, as it is considered a prototype system to describe excitable dynamics both in chemical reactions and population dynamics. [1, 2, 3].

The anomalous diffusion relaxes the classical requirement of a rapidly diffusing inhibitor, allowing spatial segregation of the species in both cases of short-range activation/long-range inhibition or long-range activation/short-range inhibition. Specifically, the anomalous diffusion exponent of the activator small enough, compared to the inhibitor diffusion exponent, enlarges the range of parameters for Turing instability. We will also prove that the presence of anomalous diffusion in the model leads to different phasing of the species maintaining the same kinetics. In particular, the spatial structures induced by long-range activation/short-range inhibition mechanism are always out of phase and subcritical in most of the instability region.

Finally, we shall provide detailed descriptions of the possible emerging patterns in the 1D and 2D rectangular domains via bifurcation analysis.

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Assessment of complexity and dynamical coupling between complex systems using Entropy Rate and Mutual Information Rate Measures: simulations and application to physiological data

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The human organism has been recently described, according to the "Network Physiology" approach, as a complex integrated network composed of multi-component organ systems continuously interacting through various feedback mechanisms to provide homeostatic balance and to react to external stimuli or intrinsic physiological alterations [1]. The dynamical behaviour of a complex system and its pairwise interactions with another system can be respectively evaluated using information-theoretic measures of Entropy Rate (ER) and Mutual Information Rate (MIR). In particular, ER has been widely employed to assess the complexity of a random process, related to nonregularity and unpredictability of its dynamics [2]. On the other hand, MIR is a dynamic measure of the non-directed symmetric interrelationships between coupled systems, and can be expressed as the sum of the individual ERs of the two processes minus their joint entropy rate [3]. Thanks to their suitability for describing short-length data with strong stochastic and noisy components, such measures are of great interest for the practical analysis of physiological time series [2,3].

In this work, after defining the theoretical formulation of ER and MIR dynamical measures, different approaches for their estimation are compared: a linear model-based estimator relying on Gaussian data, two model-free estimators based on discretization of the variables carried out either via uniform quantization through binning or rank ordering through permutations, and a model-free estimator based on direct computation of the differential entropy via k-nearest neighbor searches. The various estimators are first validated and compared on simulated univariate and coupled dynamic systems, including linear autoregressive or mixed non-linear deterministic and linear stochastic dynamics processes. Then, the framework is applied to different datasets of real-world time series describing the dynamics of coupled biomedical physiological systems, including physiological variability series descriptive of cardiovascular and cardiorespiratory interactions assessed at rest and during physiological stress or during controlled breathing conditions.

Our results evidence that statistically significant and physiologically meaningful patterns of the ER and MIR measures can be achieved in the analyzed datasets with a proper selection of the estimation parameters. Simple and fast approaches based on linear parametric or permutation-based model-free estimators allow efficient discrimination of changes in the short-term evolution of complex dynamic systems, while computationally expensive nearest-neighbour method achieves more reliable results in presence of non-linear dynamics [2,3].

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Modeling tumor disease and sepsis in physiological networks

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In this study, we provide a network physiology perspective to the modelling of pathological states induced by tumors or infection. A unified disease model is established using the innate immune system as the reference point. We propose a two-layer network model for carcinogenesis and sepsis based upon the interaction of parenchymal cells (organ tissue) and immune cells via cytokines, and the co-evolutionary dynamics of parenchymal, immune cells, and cytokines [1,2]. Our aim is to show that the complex cellular cooperation between parenchyma and stroma (immune layer) in the physiological and pathological case can be functionally described by a simple paradigmatic model of phase oscillators. By this, we explain carcinogenesis, tumor progression, and sepsis by destabilization of the healthy state (frequency synchronized), and emergence of a pathological state (multifrequency cluster). The coupled dynamics of parenchymal cells (metabolism) and nonspecific immune cells (reaction of innate immune system) are represented by nodes of a duplex layer. The cytokine interaction is modeled by adaptive coupling weights.

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Assessment of cortico-muscular synchronization via complex-system measures

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A deeper understanding of the cortico-muscular synchronisations in the central and muscular component and their interaction will provide insight into how specific coupling characteristics are modulated by age (Graziadio et al., 2010), autonomic regulation in different physiological states and chronic fatigue. Particular attention will be paid to the identification of state and synchronisation measures specific to complex systems, in order to capture the richness of information transfer expressed in the patterns exchanged between the various nodes of the network, and their sensitivity to different behavioural and structural conditions, in particular dominance of manual control networks. We will focus on developing novel analytic and computational methods within the network physiology framework, as well as with attention to fatigue.

The fine-tuning of central networks, which depends on the continuous integration of sensory influxes with programming and executive activities within feedback circuits, supports all behavioural expressions, and in particular the control of hand movement (Tecchio et al., 2020), which are modified by the levels of fatigue (Tecchio et al., 2006; Tomasevic et al., 2013; Padalino et al., 2021). Given the crucial role of hemicorporeal dominance, we studied the synchronisation between brain and muscle electrical activities, depending on the side of the body performing the movement (Tecchio et al., 2006). Furthermore, we manipulated visual feedback during an elementary isometric handgrip to investigate the effects on brain-muscle synchronisations (L'Abbate et al., 2022). We approached this study through the most widely used measure, cortico-muscular spectral coherence (CMC), and a new measure that takes into account the complex nature of the signals involved (the normalised compression distance, CMncd, Pascarella et al., 2024). Modulation of visual information modified the cortico-muscular synchronisations assessed by the two measures and cortical involvement, reflecting the crucial role of gaze in human behaviour. Dominance-dependent features were captured by CMncd more than by CMC, suggesting that signal representation by sinusoids misses an important aspect of neural network communication.

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Normal Form of Turing-Hopf Codimension Two of a Chemotaxis Model of Three-Species Lotka-Volterra with IGP

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In this talk, we explore the codimension two analysis of Turing and Hopf bifurcations within the framework of a three-species Lotka-Volterra model. The model comprises an IG-predator species, an IG-prey species, and a common resource species shared by both. Incorporating Lotka-Volterra type interaction dynamics, coupled with nonlinear diffusion to capture the movement of IG-prey towards lower density areas of IG-predator, our investigation delves into the potential for species extinction within this system [1].

Utilizing linear stability analysis around the coexistence point, we establish the conditions for the occurrence of Hopf instability. Furthermore, we investigate the role of cross-diffusion, which can induce Turing instability in the system [2,3]. Notably, the introduction of cross-diffusion leads to Turing instability, a phenomenon not observed with only classical diffusion terms. Our analysis also examines the influence of individual parameters on both Turing and Turing-Hopf instabilities.

In addition, we employ a perturbation technique based on the method of multiple scales [4] to compute the normal form of the reaction-diffusion system in the proximity of the Turing-Hopf codimension-2 bifurcation point. This allows for a deeper understanding of the system's behavior near critical bifurcation points.

By elucidating the intricate dynamics of Turing and Hopf bifurcations in this Lotka-Volterra model, our study contributes to a deeper understanding of the complex interplay between species interactions, spatial diffusion, and perturbation effects. These findings hold significance for ecological systems where such bifurcations can have profound implications for biodiversity and ecosystem stability.

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Saddle-Node Separatrix-Loops and Neuronal Network Dynamics

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Normal reproductive function and fertility rely on the rhythmic release of gonadotropin-releasing hormone (GnRH), orchestrated by the hypothalamic GnRH pulse generator. The posterodorsal subnucleus of the medial amygdala (MePD), a brain region implicated in processing external environmental cues including stress effects, acts as a key regulator of the GnRH pulse generator. However, the specific neuronal pathways governing the dynamic, stress-induced modulation of GnRH secretion remain largely elusive. Here we employ computational modelling and analysis to investigate the effects of dynamic inputs on GnRH pulse generator activity.

To this end, we develop and analyse a mathematical model representing MePD neuronal circuits comprised of GABAergic and glutamatergic neuronal populations [1], integrating it with our GnRH pulse generator model [2]. Numerical bifurcation analysis enables us to identify critical model parameters and distinct neuronal network dynamic regimes. Moreover, our analysis highlights the significance of saddle-node separatrix-loops in influencing these dynamics. To investigate further the saddle-node separatrix-loops identified in our model, we propose unfolding a generic heteroclinic loop featuring one nonhyperbolic and one hyperbolic saddle using discrete (Poincaré) maps.

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Breaking consensus in kinetic opinion formation models on graphons

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In this work we propose and investigate a strategy to prevent consensus in kinetic models for opinion formation. We consider a large interacting agent system, and assume that agent interactions are driven by compromise as well as self-thinking dynamics and also modulated by an underlying static social network. This network structure is included using so-called graphons, which modulate the interaction frequency in the corresponding kinetic formulation. We then derive the corresponding limiting Fokker Planck equation, and analyze its large time behavior. This microscopic setting serves as a starting point for the proposed control strategy, which steers agents away from mean opinion and is characterised by a suitable penalization depending on the properties of the graphon. We show that this minimalist approach is very effective by analyzing the quasi-stationary solutions mean-field model in a plurality of graphon structures. Several numerical experiments are also provided the show the effectiveness of the approach in preventing the formation of consensus steering the system towards a declustered state.

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How complex is to be a hub? Complexity as a proxy for the network degree distribution

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The relationship between topology and dynamics along the path to synchrony in complex networks has been thoughtfully explored and the knowledge gathered so far has driven crucial applications. However, there are relevant cases in which the system operates in a partial or weakly synchronization regime to preserve the balance between functional integration and parallel processing, whereas full synchronization is pathological. However, even in this incoherent state, each unit is encoding the signature of its structural role in its own dynamics. We explore how this feature can be used to extract information about the network without having to make any reference to pairwise correlations, particularly useful when the structure is unknown [1,2].

To evaluate our hypothesis about the relationship between topological role and node dynamics, we study the evolution of the k-class statistical complexity $\langle C \rangle_k = \sum_{[i|k_i=k]} C_i/N_k$ in large complex networks of dynamical units. We observe that, immediately after coupling, the relative complexity splits as a strongly hierarchical function of k, that persists for all the range of coupling d up to the system synchronization, where all nodes recover the complexity of the uncoupled state. This behaviour suggests a way to rank the nodes according to the complexity of their time series and, therefore, to potentially use this anti-correlation as a proxy for the degree sequence. The fact that this correlation between $\langle C \rangle_k$ and k persists along a large range of coupling d means that the method could be useful in natural systems, where in general, the coupling is not an accessible parameter. We have obtained equivalent results in a large variety of other systems as non phase-coherent chaos as Lorenz model, pulse-coupled neurons [1], delayed systems as Mackey-Glass, higher dymensional systems as Saito [3], and has been experimentally observed in networks of nonlinear electronic circuits [1]. The result is robust against node heterogeneity, noise and dynamical and topological changes.

To conclude, we have shown that a distinctive negative correlation between complexity and degree in a large variety of systems can be observed in the weakly coupled regime. These results suggest that the role played by the topology of a network could be unveiled by just computing the dynamical complexity associated with the time series sampled at each node. The fact that structural information of a network can be inferred without computing pairwise correlations like those commonly performed in functional networks could be exploited in diverse fields as neuroscience, econophysics or power grids.

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Data-Driven Modeling, Inference and Control of Complex Time-Varying Networks: A Neuroscience Application Perspective

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From brain activity dynamics to microbiome, and even chromatin interactions within the genome architecture, the biological processes exhibit a pronounced non-stationary, non-Markovian and non-Gaussian behavior. While the modeling of interactions among neurons and various brain regions builds on the assumption of complete knowledge of the associated complex network (CN) and Markovian (memoryless) assumptions, due to sensing limitations only a part of the complete CN is available at most of the times. In this talk, we will discuss a general and comprehensive mathematical framework for inferring, modeling and controlling adaptive networks of dynamical systems. A special emphasis will be put on capturing the multi-fractal / non-Markovian, non-stationary and non-Gaussian behavior of biological networks and identifying the laws of time-varying and nonlinear network interactions that drive these adaptive networks of dynamical systems. In order to infer the "unknown unknowns" influencing this TVCN, we will discuss techniques for jointly estimating the fractional latent node activities, and unknown drivers, as well as iteratively infer the complete model (latent + observed). In order to infer, characterize, model and efficiently control adaptive networks of dynamical systems, we will discuss the weighted multifractal graph (WMG) generator that allows us not only to deal with scarce and noisy observations but also derive the higher order network statistics of biological systems. This new mathematical framework can uncover the multiple forms of coupling and feedback loops among biological processes that will enable us to derive principles of coordination and network integration among dynamical systems in association with network states and functions. We will review the benefits and remaining challenges related to this framework for adaptive networks of dynamical systems across several neuroscience case studies and highlight several problems to be addressed by our community in order to define the theory of adaptive networks of dynamical systems and their applications to physiology and medicine.

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Brain dynamics in a simple class of adaptive neural networks: from oscillations to avalanches and scaling in collective behaviors

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Brain networks exhibit collective dynamics as diverse as scale-specific oscillations and scale-free neuronal avalanches. Although existing models account for oscillations and avalanches separately, they typically do not explain both phenomena, are too complex to analyze analytically or intractable to infer from data rigorously. Here we propose a feedback-driven Ising-like class of neural networks that captures avalanches and oscillations simultaneously and quantitatively. In the simplest yet fully microscopic model version, we can analytically compute the phase diagram and make direct contact with human brain resting-state activity recordings via tractable inference of the model's two essential parameters. The inferred model quantitatively captures the dynamics over a broad range of scales, from single sensor oscillations to collective behaviors of extreme events and neuronal avalanches. Furthermore, the model reproduces distributions of coarse-grained resting-state activity, which we find to approach a fixed non-Gaussian form with evidence of scaling. Importantly, the inferred parameters indicate that the co-existence of scale-specific (oscillations) and scale-free (avalanches) dynamics, as well as the scaling behaviors observed in coarse-grained activity, occurs close to a non-equilibrium critical point at the onset of self-sustained oscillations.

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On a multiscale mean field spin glass

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We will consider a mean-field disordered system with Sherrington-Kirkpatrick Hamiltonian in the presence of multiple thermal equilibria, namely assuming that the random coupling can be divided into a finite number of families having their own effective equilibrium temperature. The generating functional (thermodynamic pressure) of the model is constructed trough a hierarchical sequence of annealed averages, reminiscent of the Replica Symmetry Breaking interpolation. The above construction can be also seen as a multiscale decomposition of the Hamiltonian viewed as a Gaussian process. We show that the thermodynamic limit of the pressure per particle can be represented as a solution of an infinite dimensional variational principle of the Parisi type. In particular we will show that the multiscale structure acts as constraint in the space of functional order parameters.

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Mapping the dynamics of physiological systems and their interactions: An Information Theoretic Perspective with applications in Network Physiology

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The emerging field of Network Physiology (NP) combines empirical and theoretical knowledge from various disciplines to gain insight into the dynamic interaction of physiological systems as a network [1]. Data-driven network inference methods play a key role in NP and are designed to build a network model from a set of observed time series. Such a model is typically encoded by a graph where nodes constituting dynamical systems are connected by edges representing functional dependencies, describing self-effects and pairwise interactions. Moreover, many physiological systems exhibit high-order interactions, i.e. interactions involving more than two nodes [2].

In this study, we employ various information-theoretic measures across three analytical approaches to characterize: i) node dynamics using the entropy rate (ER) which measures the complexity of a system and quantifies the interactions of order one, i.e. the interaction occurring internally to the analyzed node [3]; ii) pairwise interactions via the Mutual Information Rate (MIR) to measure dynamical coupling and hence quantify the interactions of order two; iii) higher order interactions through the O-Information Rate (OIR) to identify synergistic and redundant behaviors [2]. We show how network interactions can be studied by shifting from the time domain to the frequency domain in the presence of activity rich in oscillatory content, and how these domains can be combined to allow time-resolved and time-frequency analysis when transitions between different physiological states are present [3].

In particular, different approaches and information theoretic measures are used to characterize physiological interactions at different orders to study: i) brain dynamics [4] and brain-heart interactions [3] in humans; ii) respiratory dynamics during sleep apnoea; and iii) transitions in higher order interactions occurring in different sleep stages.

Our results highlight the effectiveness of information-theoretic measures in describing the behavior of dynamical interactions at different orders of interaction thus underpinning their application in NP. Using the NP framework, we elucidate the interplay between different organ systems, taking advantage of recent insights from information theory to unravel the redundancy/synergy balance within brain and physiological networks, facilitating a deeper understanding of diverse physiological mechanisms.

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Entropy measures for long-range correlated sequence

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The talk will address how the *cluster entropy* $\mathcal{S}_{\mathcal{C}}[P]$, based on the Shannon functional of the empirical cluster distribution P and the *relative cluster entropy* $\mathcal{D}_{\mathcal{C}}[P\|Q]$ based on the Kullback-Leibler functional of the empirical cluster distribution P and Q, a model probability distribution of the clusters, can be used to quantify long-range correlated sequences, i.e characterized by probability distributions in the form of power laws.

Case studies in biology (for the characterization of chromosomes, pangenome graphs) and finance (to estimate the Hurst exponent of prices and volatility series build a multiperiod portfolio) will be presented.

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Inverse Problems
Special Session B18

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Inverse problems involve the task of deducing, from observed data, the priorly unknown causal factors that contributed to the data. They are ubiquitous in science and technology, with applications ranging from medical imaging to engineering, as in nondestructive evaluation, to geophysical prospections, just to name a few examples. Mathematically speaking, it entails determining information on a PDE by collecting measurements associated with its solutions. For the applications, it is crucial to have a deep theoretical understanding of inverse problems as well as reliable and efficient numerical reconstruction methods. Both these issues are mathematically very challenging, due to the fact that inverse problems are often nonlinear and ill-posed.

Our special session will focus on key aspects of inverse problems including uniqueness and stability issues as well as reconstruction methods. We will also explore the integration of machine learning tools in this field. We bring together a balanced mix of leading experts in the field and emerging researchers at the Ph.D. or postdoc level. To foster and strengthen the cooperation between the US and Italian inverse problems communities, both groups are well represented in our session.

For more information visit `mate.unipv.it/rondi/AMSUMI-Inverse_Problems.html`.

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The anisotropic fractional Calderón problem

Gunther Uhlmann
University of Washington, USA, and HKUST, Hong Kong, CHINA

We discuss some recent progress on the anisotropic Calderón problem for the fractional Laplacian.

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Stability for two Coefficient Identification Problems

Sonia Foschiatti, *Eva Sincich*
Department of Mathematics, Informatics and Earth Sciences, University of Trieste, Italy
Romina Gaburro
Department of Mathematics and Statistics, University of Limerick, Ireland

In inverse problems, we are interested in analysing the stability of solutions in relation to observed data.

This talk reviews stability estimates for two coefficient identification problems in an anisotropic setting: the Calderón problem (or inverse conductivity problem) and an inverse boundary value problem modeled by the Schrödinger-type equation

$$\operatorname{div}(\sigma\nabla u) + qu = 0.$$

We present two Lipschitz stability estimates derived using the method of singular solutions and quantitative estimates of unique continuation. The first result is obtained in collaboration with Eva Sincich and Romina Gaburro.

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Nonlocality in inverse problems

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We will discuss some general aspects of inverse problems for nonlocal operators. In particular, we will consider the fundamental example of the fractional Calderòn problem, in which an electric potential has to be recovered from nonlocal Dirichlet-to-Neumann data. We will see how the nonlocality of the operator helps in the resolution of the problem, by allowing the use of a surprisingly powerful approximation technique. Finally, we will discuss some interesting applications, results and open problems.

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Inverse Problems For Third-Order Nonlinear Perturbations Of Biharmonic Operators

Suman Sahoo
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In this talk, we discuss an inverse boundary problems for third-order nonlinear tensorial perturbations of biharmonic operators on a bounded domain in $\mathbb{R}^n$, where $n \geq 3$. By imposing appropriate assumptions on the nonlinearity, we demonstrate that the Dirichlet-to-Neumann map, known on the boundary of the domain, uniquely determines the genuinely nonlinear tensorial third-order perturbations of the biharmonic operator. The proof relies on the inversion of certain generalized momentum ray transforms on symmetric tensor fields. Notably, the corresponding inverse boundary problem for linear tensorial third-order perturbations of the biharmonic operator remains an open question. This is a joint work with Sombuddha Bhattacharyya, Katya Krupchyk and Gunther Uhlmann.

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The stability issue in inverse problems.

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We discuss the issue of stability in inverse problems. Given their ill-posed (and often non-linear) nature, it is necessary to reformulate the issue of stability, the continuous dependence of the unknown relevant physical parameter on the data, within the theory of ill-posed problems. This requires the need to impose *a-priori* information on the unknown parameter that is physically meaningful to the application in mind and that allows to restore stability in the inverse problem in question. As is well known the matter of stability is of fundamental importance in the reliability of any reconstruction procedure of the physical parameter since, in practice, the data/measurements of the problem will be affected by errors.

When the material occupying a domain under investigation $\Omega \subset \mathbb{R}^n$ is anisotropic (its physical property of interest depends on direction), there is a further fundamental obstruction to uniqueness in the inverse problem, due to the fact that any diffeomorphism of $\bar{\Omega}$ that keeps its boundary fixed, changes the physical property of interest in Ω but this changed is not visible through the boundary measurements.

In this talk we will address the above issues in inverse problems and give some positive answers to the questions of uniqueness and stability (and therefore reconstruction) in anisotropic inverse problems.

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Shape reconstruction for a planar conductivity inclusion

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This presentation focuses on the shape reconstruction of a conductivity inclusion. Our goal is to use generalized polarization tensors (GPTs), derived from external measurements, to mathematically reconstruct a homogeneous inclusion with a constant conductivity. The main achievement is developing a formula to represent conformal mapping coefficients by GPTs. To obtain this formula, we establish matrix factorizations for the GPTs.

Theorem 1 (Conformal mapping recovery)**.** *Let Ω be a simply connected planar domain with Lipschitz boundary. Ω is occupied with homogeneous material of conductivity σ_c, i.e., $\lambda = \frac{\sigma_c+\sigma_m}{2(\sigma_c-\sigma_m)}$. The coefficients of the exterior conformal mapping associated with Ω satisfy*

$$\gamma^2 = \frac{\lambda}{2\pi}\left[\left(I-\overline{\mathbb{N}}_{1/2}\mathbb{N}_{1/2}\right)\left(I-4\lambda^2\overline{\mathbb{N}}_{1/2}\mathbb{N}_{1/2}\right)^{-1}\mathbb{N}^{(2)}\right]_{11},$$

$$a_0 = \frac{\left[\left(I-\overline{\mathbb{N}}_{1/2}\mathbb{N}_{1/2}\right)\left(I-4\lambda^2\overline{\mathbb{N}}_{1/2}\mathbb{N}_{1/2}\right)^{-1}\mathbb{N}^{(2)}\right]_{12}}{2\left[\left(I-\overline{\mathbb{N}}_{1/2}\mathbb{N}_{1/2}\right)\left(I-4\lambda^2\overline{\mathbb{N}}_{1/2}\mathbb{N}_{1/2}\right)^{-1}\mathbb{N}^{(2)}\right]_{11}},$$

$$a_m = \frac{\lambda^2}{\pi m}\sum_{n=1}^{m} p_{mn}\left[\mathbb{N}_{1/2}\left(I-\overline{\mathbb{N}}_{1/2}\mathbb{N}_{1/2}\right)\left(I-4\lambda^2\overline{\mathbb{N}}_{1/2}\mathbb{N}_{1/2}\right)^{-1}\mathbb{N}^{(2)}\right]_{n1}, \quad m\geq 1,$$

where $\mathbb{N}^{(1)}$, $\mathbb{N}^{(2)}$ are GPTs and $\mathbb{N}_{1/2} = \mathbb{N}^{(1)}\left(\mathbb{N}^{(2)}\right)^{-1}$. Here, $[\cdot]_{mn}$ denotes an (m,n)-element of the given matrix.

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An inverse problem in monitoring of faults

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Anna Mazzucato[1]
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We discuss recovering the geometry and rock slippage of a buried seismic fault, modeled as an elastic dislocation, from surface displacement measurements. We discuss both the case in which the fault is purely linearly elastic and the case where the fault is assumed viscoelastic. This is also joint work with PhD student Arum Lee.

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Learned regularization by denoising for Limited-Angle Computed Tomography

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Recent advancements in unfolded iterative methods allow for learning the parameters of the optimization algorithm along with a suitable pseudodifferential correction for aspects that cannot be addressed by model-based methods. This supervised learning modality can be extended to the proximal operator, leveraging the modularity of proximal splitting methods and replacing it with a denoiser. More specifically, by considering a particular class of denoisers, defined as a gradient step on a potential function, it is possible to derive a non-convex regularization term, whose proximal operator corresponds to the considered denoiser.
This characterization enables the convergence analysis of the resulting Plug-and-Play (PnP) scheme and the study of potentially accelerated variants, to reduce the number of iterations required to obtain a good solution. The numerical experiments on limited-angle computed tomography (CT) show promising results, demonstrating the benefit of embedding of sophisticated and complex image priors through expressive denoisers.

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Compressed sensing for the sparse Radon transform

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Compressed sensing allows for the recovery of sparse signals from few measurements, whose number is proportional, up to logarithmic factors, to the sparsity of the unknown signal. The classical theory mostly considers either random linear measurements or subsampled isometries. In particular, the case with the subsampled Fourier transform finds applications to undersampled magnetic resonance imaging. In this talk, I will show how the theory of compressed sensing can also be rigorously applied to the sparse Radon transform, in which only a finite number of angles are considered. One of the main novelties consists in the fact that the Radon transform is associated to an ill-posed inverse problem, and the result follows from a new theory of compressed sensing for abstract inverse problems. This is a joint work with G.S. Alberti, A. Felisi and S.I. Trapasso.

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Inverse magnetisation problem in paleomagnetic context

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The process of extraction of relict magnetic information from geosamples and meteorites is a challenging but important task in paleomagnetic research. Due to the weak intensity of the field produced by a magnetised rock, the measurements have to be performed in direct vicinity of the sample and using highly sensitive magnetometric devices such as SQUID and QDM. The basic quantity of interest is the net magnetisation (magnetisation moment vector). Reconstruction of this quantity hinges on effective processing of the experimental data, with the main challenges being the limited measurement area and the noise contamination. Motivated by a concrete experimental setting in the Paleomagnetism lab at EAPS department of MIT (USA), we will focus on constructive issues. Namely, using asymptotic analysis, one can obtain explicit formulas estimating the net magnetisation vector. However, since the measurement area is usually not sufficiently large, we face an intermediate problem of the field extrapolation. We propose and analyse some extrapolation strategies and illustrate them numerically.

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Mathematical analysis of an inverse problem arising in a model of prostate cancer growth

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The availability of cancer measurements over time enables the personalised assessment of tumour growth and therapeutic response dynamics. However, many tumours are treated after diagnosis without collecting longitudinal data, and cancer monitoring protocols may include infrequent measurements. To facilitate the estimation of disease dynamics and better guide ensuing clinical decisions, we investigate an inverse problem enabling the reconstruction of earlier tumour states by using a single spatial tumour dataset and a biomathematical model describing disease dynamics. We focus on prostate cancer, since aggressive cases of this disease are usually treated immediately after diagnosis. We describe the tumour evolution with a phase-field model driven by a generic nutrient ruled by reaction-diffusion dynamics. The model is completed with another reaction-diffusion equation for the local production of prostate-specific antigen, which is a key prostate cancer biomarker. We first improve previous well-posedness results by further showing that the solution operator is continuously Fréchet differentiable. We then analyse the backward inverse problem concerning the reconstruction of earlier tumour states starting from measurements of the model variables at the final time. Since this problem is severely ill-posed, only very weak conditional stability of logarithmic type can be recovered from the terminal data. However, by restricting the unknowns to a compact subset of a finite-dimensional subspace, we can derive an optimal quantitative Lipschitz stability estimate.

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Full discretization and regularization for the Calderón problem

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Luca Rondi
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In this talk, we consider the Calderón problem with discontinuous conductivities. We present a fully discretized variational approach that involves a data fidelity term and a total variation penalty term with a corresponding regularization parameter. The discretization encompasses the boundary measurements, using the complete electrode model, the unknown conductivity and the potential. The two key parameters in the analysis are related to the electrodes size and the mesh resolution. Our analysis establishes a precise method for selecting the discretization and the regularization parameters based on the noise level to ensure that the solution to the discretized problem remains meaningful. Notably, we find that both electrode and mesh size parameters should exhibit a polynomial decay with respect to the noise level. This is joint work with Luca Rondi (Pavia).

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Reduced order models and the Lippmann Schwinger Lanczos method in inverse scattering

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We combine data-driven reduced order models with the Lippmann-Schwinger integral equation to produce a direct nonlinear inversion method. The ROM is viewed as a Galerkin projection and is sparse due to Lanczos orthogonalization. Embedding into the continuous problem, we produce an approximation of the internal solution directly from the data. This internal solution is then used in the Lippmann-Schwinger equation. The approach allows us to process more general transfer functions than the earlier versions of the ROM based inversion algorithms. We give examples of its use for spectral domain MIMO problems and in the time domain given mono static data, targeting synthetic aperture radar. For radar problems, we also show a new technique of data completion of monostatic data to full MIMO, to further improve the internal solution.

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Automorphic forms, Galois representations, and L-functions
Special Session B19

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This session is scheduled on July 25-26. The focus of the proposed scientific session is to present recent developments in a wide research area of modern number theory which sits in the wide framework of the Langlands program. Automorphic forms arise naturally in many different settings of number theory; under the deep web of conjectures that form the Langlands program, they should be related to Galois representations, and discovering properties on one of these gives us information about the other. The natural tool to connect them are complex (and p-adic) L-functions and their relations with Selmer groups, conjecturally described in great generality by Bloch–Kato (in their equivariant Tamagawa number conjecture) as a wide generalization of the Birch and Swinnerton-Dyer Conjecture for elliptic curves, a Milllennium Problem. One of the tools to attack the Bloch–Kato conjecture is the theory of Euler systems and their relation with complex (and p-adic) L-functions. In recent years, there have been many important advances in this area, most notably:

- New constructions of Euler systems via algebraic cycles, which allow one to study automorphic forms and Galois representations for motives of algebraic groups possibly different from GL_2;
- Variation of automorphic forms in families, especially using higher degree coherent cohomology which provides new ways to study Galois representations and their L-functions by deforming them p-adically.

These new developments have exciting applications on many outstanding open conjectures in number theory among which the Bloch–Kato conjectures, as already mentioned, the Iwasawa main conjecture, the study of (completed) cohomology of Shimura varieties.

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p-Adic variantion of de Rham modular sheaves and applications.

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Fabrizio Andreatta
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One of the main topics in this session is the theme of the variation of automorphic forms in p-adic families, especially using higher degree coherent cohomology.

In this talk I will present yet another way of studying Galois representations, L-functions and p-adic L-functions attached to automorphic eigenforms by deforming de Rham cohomology modular sheaves. These are large Banach-sheaves on appropriate strict neighbourhoods of certain components of the ordinary locus in Shimura varieties, with increasing filtrations and integrable connections. By studying the action of the Hecke operators on the de Rham cohomology of these sheaves with connections, one obtains very interesting applications.

These ideas lead so far to new ways of defining triple product p-adic L-functions for automorphic forms of finite slope, instead of ordinary, and studying Katz-type p-adic L-functions in cases in which p is not split in the CM field.

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Balanced triple product p-adic L-functions and classical weight one forms

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The aim of this talk is to introduce a new balanced triple product p-adic L-function and discuss its application to the equivariant Birch & Swinnerton-Dyer conjecture. We state a conjecture in a rank-1 situation analogous to the Elliptic–Stark conjecture formulated by Darmon–Lauder–Rotger in rank-2 and prove it in the CM case; this work fits in the general framework studied by Darmon–Lauder–Rotger and Andreatta–Bertolini–Seveso–Venerucci. Time permitting, we will explain briefly the intriguing technical difficulties behind the construction of this new p-adic L-function whose main feature is to allow classical weight one modular forms in the chosen families. That is joint work with Aleksander Horawa.

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Kolyvagin's conjecture and Perrin-Riou's main conjecture for modular forms, part I

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Maria Rosaria Pati
Laboratoire de Mathématiques Nicolas Oresme, Université de Caen Normandie

Stefano Vigni
Dipartimento di Matematica, Università di Genova

Haining Wang
Shanghai Center of Mathematical Sciences, Fudan University

In this talk, I will state and (time permitting) sketch a proof of an analogue for p-adic Galois representations attached to a higher (even) weight newform f of Kolyvagin's conjecture on the p-indivisibility of derived Heegner points on elliptic curves, where p is a prime number that is ordinary for f. Our strategy, which is inspired by work of W. Zhang in weight 2, builds crucially on results of H. Wang on the indivisibility of Heegner cycles over Shimura curves. In her talk, Maria Rosaria Pati will explain how our work on Kolyvagin's conjecture can be combined with other ingredients to yield a proof of the counterpart for f of Perrin-Riou's Heegner point main conjecture for elliptic curves ("Heegner cycle main conjecture" for f).

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Kolyvagin's conjecture and Perrin-Riou's main conjecture for modular forms, part II

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In this talk, I will state and sketch a proof of the counterpart for a higher (even) weight newform f of Perrin-Riou's Heegner point main conjecture for elliptic curves ("Heegner cycle main conjecture" for f). Our strategy of proof builds on, among other ingredients, our work on Kolyvagin's conjecture for f that was described by Stefano Vigni in his talk. This is joint work with Matteo Longo, Stefano Vigni and Haining Wang.

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The Gross–Kohnen–Zagier theorem via p-adic uniformization

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Let S be a set of rational places of odd cardinality containing infinity and a rational prime p. We can associate to S a Shimura curve X defined over $\mathbb{Q}$. The Gross–Kohnen–Zagier theorem states that certain generating series of Heegner points of X are modular forms of weight $3/2$ valued in the Jacobian of X. We will state this theorem and outline a new approach to prove it using the theory of p-adic uniformization and p-adic families of modular forms of half-integral weight. This is joint work with Lea Beneish, Henri Darmon and Lennart Gehrmann.

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p-adic Waldspurger formulae for non-split p

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Let p be a rational prime. Let f be an eigenform of weight two and χ be a finite order Hecke character over an imaginary quadratic field K. Assume the Rankin pair (f, χ) is conjugate self-dual of sign -1. In this talk, we will discuss the p-adic Waldspurger formula for the triple $(\pi, \chi/K, p)$, particularly when the prime p is non-split in K. This talk is based on a joint work with X. Wan.

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Modularity of mod p Tate–Shafarevich classes

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Let $E/\mathbf{Q}$ be an elliptic curve; modularity of E translates geometrically into the existence of a surjection from the Jacobian of a modular curve to E. A similar modularity property is expected for classes in $\text{Ш}(E/\mathbf{Q})$. More precisely, these cohomology classes correspond to curves of genus one with a point over every completion of $\mathbf{Q}$, whose Jacobian is isomorphic to E. Jetchev and Stein conjectured in [2] that such curves can be realized inside (quotients of) the Jacobians of modular curves over $\mathbf{Q}$. After introducing the conjecture, I will present the following result for p-torsion Tate–Shafarevich classes, whose proof builds on the techniques introduced in [1].

Theorem 1. *([3], Theorem 1.3.1) Assume that E has squarefree conductor and does not have complex multiplication. Let $p > 3$ be a prime of good ordinary reduction. Assume that $\bar{\rho} : \mathrm{Gal}(\bar{\mathbf{Q}}/\mathbf{Q}) \to \mathrm{Aut}_{\mathbf{F}_p}(E[p])$ is surjective and ramified at every prime factor of the conductor congruent to ± 1 modulo p. Then every class in $\text{Ш}(E/\mathbf{Q})[p]$ is modular.*

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Hodge polygons of formal $\mathcal{O}_D$-modules and the p-adic half space

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Let p be a prime number. The p-adic analogue of the complex upper half plane, together with its tower of étale coverings, has long played a prominent role in the p-adic local Langlands program, especially through its cohomology, but also as a uniformising space of p-adic models of certain Shimura curves. Thanks to Drinfeld, this space and its higher dimensional versions admit a moduli interpretation, namely in terms of "formal $\mathcal{O}_D$-modules". In this talk, I will introduce some combinatorial invariants of the latter objects, which help describing the geometry of the p-adic half space and, possibly, of more general moduli of formal groups.

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On p-adic uniformization of abelian varieties and p-divisible groups.

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Given an abelian variety over a finite extension K of $\mathbb{Q}_p$, Fontaine constructed an integration map from the Tate module of A to its Lie algebra. This map gives the splitting of the Hodge–Tate short exact sequence. In recent work with Iovita and Zaharescu, we extended this integration map to the $\overline{K}$-points of the perfectoid universal cover of A, and used this result to give a uniformization of the points of the underlying p-divisible group. Later in joint work with Howe and Wear, we gave a different perspective on this uniformization using 1-motives and extended the uniformization to certain kinds of p-divisible groups. In this talk, I will explain the constructions of each of these results and state some follow up questions concerning these uniformizations.

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Weil height and modular Galois representations

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An algebraic field is said to have the *Bogomolov property*, (property (B) for short), if the Weil height is uniformly bounded below outside torsion points. It is well known that property (B) holds for number fields and for potentially abelian extensions; moreover a theorem by Habegger proves property (B) for the field generated by the torsion points of an elliptic curve defined over $\mathbb{Q}$. Together with F. Amoroso we generalized this result, establishing property (B) for the extension cut out by the Galois representation associated to a modular form, assuming the existence of a strong supersingular prime and the fullness of the image of the residual representation; moreover we conjectured that property (B) holds unconditionally for modular Galois representations.
In my talk, I will explain the main idea underlying the proof, and discuss some aspects of particular relevance in the theory of automorphic forms.

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Equidistribution of CM points on Shimura curves and ternary quadratic forms

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Equidistribution of "special" points is a theme of both analytic and geometric interest in number theory. In particular, this seminar plans to deal with the case of CM points, generalizing the results of [1] to quaternionic Shimura curves over a totally real number field.

The first part will be devoted to a short geometric description of the aforementioned curves and in particular of their special fiber. Subsequently, I plan to describe an equidistribution result of reduction of Galois orbits of CM points in the special fiber of Shimura curves associated both to an unramified and ramified quaternion algebra.

The first results boil down to a correspondence between CM points and primitive representations of certain ternary quadratic forms by the means of optimal embeddings. We then conclude by exploiting subconvexity bounds on the Fourier-Whittaker coefficients of the automorphic theta series (attached to our ternary forms) to obtain the desired equidistribution.

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The algebra $\mathbb{Z}_\ell[[\mathbb{Z}_p^d]]$ and applications to Iwasawa theory

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Ignazio Longhi
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Let ℓ and p be distinct primes, and let Γ be an abelian pro-p-group. I will discuss a structure theorem for the algebra $\Lambda := \mathbb{Z}_\ell[[\Gamma]]$ and its consequences on the structure of Λ-modules. In particular, if $\Gamma \simeq \mathbb{Z}_p^d$ is the Galois group of an extension K/k, with k a global field, one obtains explicit formulae for the orders and ℓ-ranks of certain Iwasawa modules (namely ℓ-class groups and ℓ-Selmer groups) associated with the finite subextensions of K. In the case of ℓ-class groups, this provides different proofs and generalizations of results of Washington and Sinnott.

References

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Drinfeld quasi-modular forms of higher level

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When working with global function fields of characteristic p>0 complex valued (Jacquet-Langlands) automorphic forms are only half of the story, the other part being Drinfeld modular forms. As in the classical characteristic zero setting the space of Drinfeld forms is not stable under differentiation; this is why it is interesting to introduce Drinfeld quasi-modular forms, which indeed behaves well under the action of differential operators. In this talk we will present some new and recent advances on Drinfeld quasi-modular forms for congruence subgroups.

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Dihedral long root A-packets of p-adic G_2 via theta correspondence

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Let G be a split exceptional group of type G_2 over a number field F. According to Arthur's conjectures, the square-integrable automorphic representations of G can be classified into near-equivalence classes, known as global A-packets, indexed by certain homomorphisms known as global A-parameters. For each place v of F, the representations of $G(F_v)$ which can appear as a local component of a representation in a given global A-packet form the corresponding local A-packet.

We will focus on the A-packets attached to a specific type of non-tempered A-parameters of the group G, called dihedral long root A-parameters. These A-parameters can be seen to factor through A-parameters of the group PU_3. Thus, we propose a construction of the non-archimedean local A-packets of G attached to a dihedral long-root A-parameter as theta lifts of the corresponding local A-packets of PU_3, relying on the exceptional theta correspondence between $\mathrm{PU}_3 \rtimes \mathbb{Z}/2\mathbb{Z}$ and G studied in [2].

This talk is based on the joint work [1].

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Recent developments in commutative algebra
Special Session B20

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Alessio Sammartano
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Karen Smith
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Commutative algebra is the field of mathematics that studies commutative rings and their ideals and modules. The subject has deep connections with other fields including algebraic geometry, combinatorics, representation theory, and algebraic topology. In the past few years, there have been several important developments in the field, leading to the solution to many long-standing conjectures and the introduction of new methods.

With this special session, we intend to integrate senior and young active researchers in the field to present their most recent advances and to facilitate the creation of new collaborations. The focus will be on recent progress in the topics of free resolutions, local cohomology, singularities, positive characteristic methods, and interactions of commutative algebra with combinatorics.

For more information visit `https://sites.google.com/view/ca2024palermo/home`.

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Cohomology of line bundles on the incidence correspondence

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A fundamental problem at the confluence of algebraic geometry, commutative algebra and representation theory is to understand the structure and vanishing behavior of the cohomology of line bundles on (partial) flag varieties. Over fields of characteristic zero, this is the content of the Borel–Weil–Bott theorem and is well-understood, but in positive characteristic it remains wide open, despite important progress over the years. In my talk I will describe recent developments obtained over the past couple of years in the case of the incidence correspondence – the partial flag variety consisting of pairs of a point in projective space and a hyperplane containing it.

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Standard monomial theory modulo Frobenius in characteristic two

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Over a field of characteristic zero, De Concini, Eisenbud and Procesi developed a theory of standard monomials, which are a vector space basis for determinantal ideals and provide a tool to study many properties of the ideal. This theory plays a central role in the study of determinantal rings from a representation theoretic approach, and exploits tableaux combinatorics.

In this talk, we present our contribution to a new standard monomial theory for polynomial rings over a field of positive characteristics modulo a Frobenius power, examining the characteristic two case. A main feature of this investigation is given by analogs of semistandard Young tableaux and Schur polynomials in this new context, defined by Gao, Raicu and VandeBogert. Our results agree with one of their conjectures.

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Resolutions of plane curve singularities and Conway–Coxeter friezes

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Conway–Coxeter friezes are arrays of positive integers satisfying a determinantal condition, the so-called diamond rule. Recently, these combinatorial objects have been of considerable interest in algebra, since they encode cluster combinatorics of type A. In this talk I will discuss a new connection between Conway–Coxeter friezes and the combinatorics of a resolution of a plane curve singularity: via the beautiful relation between friezes and triangulations of polygons one can relate each frieze to the so-called lotus of a curve singularity, which was introduced by Popescu-Pampu. This allows to interpret the entries in the frieze in terms of invariants of the curve singularity, and on the other hand, we can see cluster mutations in terms of the desingularization of the curve.

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Generic Distractions and Initial Ideals

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Given a homogeneous ideal I in a polynomial ring S over a field and a monomial order, we iteratively compute initial ideals and ad-hoc monomial distractions in order to construct a distinguished monomial ideal of S associated to I. We call it the distraction-generic initial ideal of I and denote it by D-gin(I). Such an ideal, when char(K)=0, agrees with the usual generic initial ideal of I. Furthermore, it is a strongly stable in any characteristic and when computed with respect to the reverse lexicographic order has properties analogous to $\mathrm{gin}(I)$, for instance it has the same Castelnuovo-Mumford regularity and projective dimension as I. As an application, we can use it to extend to $\mathrm{char}(K) = p$ some work done by Mall in the nineties on strata of the Hilbert schemes defined by fixing a Hilbert series and an upper bound for the Castelnuovo-Mumford regularity.

This is a joint work with Anna-Rose Wolff.

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Componentwise linearity under Gröbner degenerations

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In this talk, we will discuss when Gröbner degenerations preserve the componentwise linearity. More concretely, given that $R = K[X_1, \ldots, X_n]$ is the polynomial ring over a field K and I is a homogeneous ideal of R, a theorem of Conca and Varbaro implies that if the initial ideal $\mathrm{in}(I)$ is square-free, then I has linear resolution if and only if $\mathrm{in}(I)$ has linear resolution. Since each homogeneous ideal with linear resolution is componentwise linear, we aim to explore the conditions under which the property of being componentwise linear can be transferred between I and $\mathrm{in}(I)$. Additionally, we will compare the graded Betti numbers of I and $\mathrm{in}(I)$ under these conditions.

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Source-independence of generalized F-signature.

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Let R be an commutative Noetherian domain containing the finite field $\mathbb{F}_p$. We assume that the Frobenius endomorphism $F\colon R \to R$, $x \mapsto x^p$, is a finite map. This assumption is satisfied if R is a complete local ring with a perfect residue field or if R is localization of a finite type algebra over a perfect field. With this assumption, for any finite R-module M the module $F^e_* M$, obtained by restricting the scalars along the eth iterate of the Frobenius, is still finitely generated.

Definition 1 (Sannai, Huneke – Leuschke, Smith – Van der Bergh)**.** Let Q be the fraction field of R. The dual F-signature of a finitely generated R-module M is defined as

$$\mathrm{s}_{\mathrm{dual}}(M) = \lim_{e\to\infty} \frac{\max\{n \mid \text{there is a surjection } F^e_* M \to \oplus^n M \to 0\}}{\dim_Q(Q \otimes_R F^e_* M)}.$$

In the case $M = R$ this recovers the definition of F-signature introduced by Huneke–Leuschke. Previously, the existence of the limit was established in [4] for $M = R$ and in [3] for $M = \omega_R$, the dualizing module of a Cohen-Macaulay ring. Using the linear algebra approach to building surjections established in [3] we prove that the limit exists in full generality for more general type of limits.

Theorem 1. *For any finitely generated R-modules M, N*

$$\mathrm{s}_{\mathrm{dual}}(M) = \lim_{e\to\infty} \frac{\max\{n \mid \textit{there is a surjection } F^e_* N \to \oplus^n M \to 0\}}{\dim_Q(Q \otimes_R F^e_* N)}$$

and the limit exists.

The fact that one may compute $\mathrm{s}_{\mathrm{dual}}(M)$ from any N is especially useful when R is an invariant subring under an action of a finite group G on a polynomial ring $S = k[x_1, \dots, x_d]$ or, more generally, when there is a finite map $R \to S$ where S is a regular local ring. In this case, $F^e_* S$ is a free S-module by a celebrated theorem of Kunz.

Corollary 2. *Suppose that S is a regular local ring and $R \to S$ is a finite map. Then $\mathrm{s}_{\mathrm{dual}}(M) > 0$ if and only if there exists a surjection of R-modules $\oplus^n S \to M \to 0$ for some $n > 0$.*

We recover a result of Hashimoto obtained by representation theory in [1].

Corollary 3. *Suppose that $S = k[x_1, \dots, x_d]$ is $R = S^G$ where G is a finite subgroup containing no pseudo-reflections. Suppose that R is Cohen-Macaulay and has a dualizing module ω_R. Then the following are equivalent:*

(1) *R is F-rational,*
(2) $\mathrm{s}_{\mathrm{dual}}(\omega_R) > 0$,
(3) $\mathrm{s}_{\mathrm{dual}}(\omega_R) \geq 1/|G|$,
(4) *there exists a surjection of R-modules $S \to \omega_R \to 0$.*

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F-purity of ladder determinantal varieties and their symbolic blowups

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Ladder determinantal varieties are defined by the vanishing of minors in certain subsets of a generic matrix of indeterminates. These varieties were initially introduced to investigate singularities of Schubert varieties, and have since inspired further study due to their rich algebraic structure.

In this talk, we focus on F-singularities of ladder determinantal varieties and their symbolic blowups. After a brief overview of some basic concepts of F-singularity theory, we will present different algebraic techniques to prove that these varieties are F-pure. Time permitting, we will discuss a way to extend these techniques to other classes of ideals.

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Variants of the Buchsbaum property in prime characteristic

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I will discuss some recent refinements of the notion of a Buchsbaum ring in prime characteristic. Recall that a local ring R is said to be *Buchsbaum* if the quantity $e(Q)-\ell_R(R/Q)$ does not depend on the parameter ideal Q, where $e(Q)$ denotes the multiplicity. Recently, Ma and Quy considered a prime characteristic alternative requiring instead for $e(Q)-\ell_R(R/Q^*)$ to be independent of parameter ideals Q, where Q^* is the tight closure of Q.

I will report on some ongoing investigations into this "tight Buchsbaum" notion such as various homological interpretations, its behavior under a section, and a characterization in terms of Rees algebras. Time permitting, I will also discuss yet another alternative which replaces Q^* with the Frobenius closure Q^F in the above quantity.

This is joint work with A. Costantini, K. Goel, K. Maddox, and L.E. Miller.

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Licci ideals

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Linkage was introduced in the nineteenth century as a method to classify projective varieties. Using linkage we can define an equivalence relation and an ideal is called **licci** if it is in the linkage class of a complete intersection. Since linkage preserve many properties of an ideal, licci ideals are particularly nice. Standard examples of licci ideals include perfect ideals of grade 2 and perfect Gorenstein ideals of grade 3. Goals of the subject are to classify linkage classes, to establish properties of licci ideals, to find new classes of licci ideals, and to find necessary and sufficient conditions for ideals to be licci. In this talk I will discuss a surprising conjecture: *licci ideals do not have many minimal generators*! Indeed in collaboration with Huneke and Ulrich we conjectured that the number of generators of a homogeneous licci ideal is bounded above by the greatest last twist in a minimal graded free resolution of the ideal. We proved this conjecture for several classes of ideals, for instance for monomial ideals of finite colenght, ideals containing a maximal regular sequence of quadrics, and licci ideals with nearly pure resolutions. Moreover, we give a sufficient condition for an ideal containing a maximal regular sequence of quadrics to be licci.

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Jet schemes of Pfaffian ideals

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Jet schemes and arc spaces received quite a lot of attention by researchers after their introduction, due to J. Nash, and established their importance as an object of study in M. Kontsevich's motivic integration theory. Several results point out that jet schemes carry a rich amount of geometrical information about the original object they stem from, whereas, from an algebraic point of view, little is know about them.

In this talk we consider the ideal I_{2r} generated by the pfaffians of size $2r$ in an $n \times n$ generic skew-symmetric matrix and, inspired by [2], we study algebraic properties of the corresponding k-th jet schemes ideal $I_r^{n,k}$. In particular we determine under which conditions the corresponding jet scheme varieties are irreducible. Moreover in the case $n = 2r$ we prove that for every k the natural generators of $I_r^{n,k}$ are a Gröbner basis, and that $I_r^{n,k}$ defines a Cohen Macaulay domain of multiplicity r^k. Conjectures and open questions will be stated.

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Lefschetz properties for artinian Gorenstein algebras of low Sperner number

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The cohomology ring A of a smooth projective complex variety has the *strong Lefschetz property* (SLP), i.e., all the multiplication maps, $\times \ell^j \colon A_i \longrightarrow A_{i+j}$, given by powers of a general linear forms have maximal rank. For other artinian Gorenstein algebras in general, i.e., commutative algebras with Poincaré duality, it is well known that this needs not be true, not even for multiplication by general linear forms, $\times \ell \colon A_i \longrightarrow A_{i+1}$, which would be the *weak Lefschetz property* (WLP). However, there are lots of results and some conjectures about when we have the strong or the weak Lefschetz property. In codimension three, we know that artinian Gorenstein algebras satisfy the SLP if the socle degree is at most 5 by [3] and if the *Sperner number*, i.e., the maximal value of the Hilbert function, is at most 6 by [1]. It is also known that artinian complete intersections satisfy the WLP in codimension three [4]. For Gorenstein algebras in codimension four, the smallest known examples of artinian Gorenstein algebras have Sperner number equal to the socle degree plus two [2].

In recent joint work we prove that any artinian Gorenstein algebras with socle degree d and Sperner number at most $d + 1$ satisfies the WLP.

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Lattice Paths, Lefschetz Properties and Almkvist's Conjecture in Two Variables

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Fix a field F, and positive integers m, n. Define the algebra $A_F(m,n) = \dfrac{F[e_1, \ldots, e_n]}{(e_1(m), \ldots, e_n(m))}$, where $e_i = e_i(x_1, \ldots, x_n) = \sum_{1 \leq j_1 < \cdots < j_i \leq n} x_{j_1} \cdots x_{j_n}$ are the i^{th} elementary symmetric function, and $e_i(m) = e_i(x_1^m, \ldots, x_n^m)$. $A(m,n)$ is a graded Artinian complete intersection. In [1], Almkvist conjecture that the Hilbert function of $A(m,n)$ is unimodal for n odd and sufficiently large m, and for n even and any m. Since unimodality is a necessary condition for Lefschetz properties, we conjecture that $A(m,n)$ has the strong Lefschetz property for sufficiently large m, and for even n. In this talk, we consider the case $n = 2$, and we show that $A(m,2)$ has the strong Lefschetz property and the complex Hodge-Riemann property if and only if m is even.

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On the (strong) Koszul property for some Artinian Gorenstein algebras

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A standard graded commutative K-algebra R is *Koszul* when the residue field K has a linear free resolution as an R-module; in particular, the defining ideal of R has to be quadratic. Koszulness is a natural feature of many algebras arising in combinatorics and algebraic geometry. In some cases, an even stronger property holds: namely, there exists a K-basis $\mathfrak{B}$ of R_1 such that, for every choice of a proper subset $\mathfrak{B}' \subsetneq \mathfrak{B}$ and of an element $x \in \mathfrak{B} \setminus \mathfrak{B}'$, the colon ideal $(y \mid y \in \mathfrak{B}') :_R (x)$ is generated by a subset of $\mathfrak{B}$. Standard graded K-algebras R with this property were called *strongly Koszul* by Herzog, Hibi and Restuccia.

In this talk I will discuss Koszulness for some classes of Artinian Gorenstein rings, possibly including some ongoing work about the strong Koszul property for some Artinian Gorenstein rings related to determinantal objects and to secant varieties of Severi varieties.

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Commutative algebra of polyominoes

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Polyominoes are two-dimensional objects obtained by joining edge by edge squares of same size. Originally, polyominoes appeared in mathematical recreations, but it turned out that they have applications in various fields, for example, theoretical physics and bio-informatics. Among the most popular topics in combinatorics related to polyominoes one finds enumerating polyominoes of given size, including the asymptotic growth of the numbers of polyominoes, tiling problems, and reconstruction of polyominoes. Recently Qureshi [5] introduced a binomial ideal induced by the geometry of a given polyomino, called polyomino ideal, and its related algebra. From that moment different authors studied algebraic properties and invariants related to this ideal (see [3, 6, 4, 2, 7, 1]). In this talk, we provide a comprehensive overview of the state-of-the-art results that have been obtained on polyomino ideals and their related algebra. In the first part of the talk, we discuss questions about the primality and the Gröbner bases of the polyomino ideal. In the second part, we talk over some algebraic invariants such as Castelnuovo-Mumford regularity, Hilbert series, and Gorensteinnes and related properties of the polyomino ideal and its coordinate ring.

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Binomial edge ideals and Serre's condition (S_2)

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A binomial edge ideal is an ideal of a polynomial ring, generated by binomials corresponding to the edges of a finite simple graph. It can be also viewed as the ideal generated by *some* minors of a generic matrix with two rows.

The aim of this talk is to point out the connections between the combinatorics of finite simple graphs and the algebraic properties of binomial edge ideals. In particular, after reviewing some basic results, I will focus on a combinatorial characterization of binomial edge ideals satisfying Serre's condition (S_2).

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A constructive approach for graphs whose binomial edge ideal is Cohen-Macaulay

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Binomial edge ideals have been introduced in [5] and, independently, in [7]. They are associated to finite simple graphs, in fact they arise from the 2-minors of a $2\times n$ matrix related to the edges of a graph with n vertices. The problem of finding a characterization of Cohen–Macaulay binomial edge ideals has been studied intensively by many authors (e.g. [4],[1]). In this talk we present a computational approach to find (see [6]) or construct graphs (see [2]) whose binomial edge ideal is Cohen-Macaulay by a library of the author. Thanks to this computation we obtained graphs with nice properties within the Cohen-Macaulay ones. Moreover, we verify a recent conjecture of D. Bolognini, A. Macchia and F. Strazzanti (see [3]).

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Recent Trends in Stochastic analysis
Special Session B21

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Mykhaylo Shkolnikov
Carnegie Mellon University & Princeton University, USA

Stochastic analysis methods and robust techniques have recently led to a variety of advances in the theory of (stochastic) partial differential equations (PDEs). The spectrum of applications is very wide and includes stochastic homogeneization, fluid mechanics, mathematical finance, statistical mechanics and the stochastic quantization of quantum field theories. The goal of the session is to bring together mathematicians who have applied stochastic analysis to (stochastic) PDEs of many different kinds, in particular singular stochastic PDEs such as KPZ and Φ^4, Fisher-KPP PDE, Navier-Stokes PDE, and Stefan problems among others.

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Hyperbolic Anderson model with Lévy white noise: fluctuations of the spatial average

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n this talk, we study the hyperbolic Anderson model in dimension 1 driven by a space-time Lévy white noise with finite variance. Motivated by recent active research on limit theorems for SPDEs driven by Gaussian noise, we present the first study in this Lévy noise setting. The goal of the talk is to show that, with appropriate normalization and centering, the spatial average of the solution converges in distribution to the standard normal distribution, and to estimate the speed of this convergence in the Wasserstein (or Kolmogorov) distance. This talk is based on joint work with Guangqu Zheng (University of Liverpool).

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Weak coupling scaling of critical SPDEs

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The study of stochastic PDEs has known tremendous advances in recent years and, thanks to Hairer's theory of regularity structures and Gubinelli and Perkowski's paracontrolled approach, (local) existence and uniqueness of solutions of subcritical SPDEs is by now well-understood. The goal of this talk is to move beyond the aforementioned theories and present novel tools to derive the scaling limit (in the so-called weak coupling scaling) for some stationary SPDEs at the critical dimension. Our techniques are inspired by the resolvent method developed by Landim, Olla, Yau, Varadhan, and many others, in the context of particle systems in the supercritical dimension. Time allowing, we will explain how it is possible to use our techniques to study a wider class of statistical mechanics models at criticality such as (self-)interacting diffusions in random environment.

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Limit laws in metric measure spaces

Maria Gordina
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We consider Dirichlet boundary problems in metric measure spaces. Results include properties of the spectrum, regularity and L^p-estimates of eigenfunctions, as well as irreducibility of the corresponding stochastic processes. A number of examples will be given including both local and non-local Dirichlet forms, hypoelliptic diffusions and stochastic processes on fractals, and applications to limit laws such as small deviations and large time behavior of the heat content.

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Regularization in Kraichnan's Passive Scalar Model

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The advection of a passive scalar by a random velocity field can induce the dissipation of multiscale norms even in the absence of diffusion, and they can be used to gauge mixing and regularizing properties of stochastic transport. The advection by a d-dimensional Gaussian vector field with a power-law covariance spectrum (Kraichnan's model) satisfies quantitative estimates for the evolution of negative Sobolev norms of passive scalars, which imply that generalized solution taking values in Sobolev spaces of negative order immediately become weak solutions of positive regularity. Joint work with Lucio Galeati and Mario Maurelli.

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How does the supercritical GMC converge?

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TBA

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The Allen-Cahn equation with weakly critical initial datum

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We study the Allen-Cahn equation in dimension 2 with white noise initial datum, motivated by the study of the generic evolution of phase fields. In a weak coupling regime, where the nonlinearity is damped in relation to the smoothing of the initial condition, we prove Gaussian fluctuations. The effective variance that appears can be described as the solution to an ODE. Our proof builds on a Wild expansion of the solution, which is controlled through precise combinatorial estimates. Joint works with Simon Gabriel, Martin Hairer, Khoa Lê and Nikos Zygouras.

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Enhanced dissipation, and residual diffusivity

Gautam Iyer
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In many situations, the combined effect of advection and diffusion causes enhances dissipation. I will talk about this in two contexts: The first is for a random class of flows (ala Pierrehumbert) for which we show that the system dissipates energy on time scales of order $O(|\log\kappa|)$, where κ is the molecular diffusivity. The second is in a discrete time setting where show that the effective diffusivity does not vanish with the molecular diffusivity. This is joint work with SJ Son and W. Cooperman and J. Nolen.

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Fluctuations of Stochastic Heat Equation and KPZ equation

Xue-Mei Li
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In this talk, we explore the stochastic heat equation and the KPZ equation, each influenced by space time Gaussian noiise with long-range spatial dependence. These equations produce solutions that admit a stationary field. Our focus is on the fluctuation problem associated with diffusively scaled solutions from their average. While the behavior of compactly supported correlations—typically known to dissipate at large scales—is well-documented, our research shifts to examining long-range dependent noise with an asymptotic profile, inspired by empirical data and physical considerations. We investigate whether this dependence is maintained in the large-scale scaling limit. Our findings not only confirm its persistence but also reveal a key difference: the exponent in the power decay of the correlation rate plays a role akin to that of dimension in compactly supported scenarios. Furthermore, we demonstrate that the fluctuations of the appropriately scaled solutions from their mean converge weakly to the solution of a stochastic heat equation with additive noise, where the spatial correlation function is governed by the Riesz potential. In addressing the KPZ equation, we confront the challenges posed by the singular Cole-Hopf transformations. This research highlights the significance of long-range dependencies and their role in modeling more complex noise inputs in physical and mathematical models.

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Cascade equation for Stefan problem as a mean field game

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The solutions to Stefan problem with Gibbs-Thomson law (i.e., with surface tension effect) are well known to exhibit singularities which, in particular, lead to jumps of the associated free boundary along the time variable. The correct times, directions and sizes of such jumps are only well understood under the assumption of radial symmetry, under which the free boundary is a sphere with varying radius. The characterization of such jumps in a general multidimensional setting has remained an open question until recently. In our ongoing work with M. Shkolnikov and Y. Guo, we have derived a separate (hyperbolic) partial differential equation — referred to as the cascade equation — whose solutions describe the jumps of the solutions to the Stefan problem without any symmetry assumptions. It turns out that a solution of the cascade equation corresponds to a maximal element of the set of all equilibria in a family of (first-order local) mean field games. In this talk, I will present and justify the cascade equation, will show its connection to the mean field games, and will prove the existence of a solution to the cascade equation. If time permits, I will also show how these results can be used to construct a solution to the Stefan problem itself.

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Set theory and applications
Special Session B22

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Dima Sinapova
Rutgers University, USA

Recent advances in set theory have contributed to the foundations of mathematics, and have created new methods to analyze infinite mathematical objects. The two main topics of the session are:

(1) Infinite combinatorics;
(2) Descriptive set theory.

It is scheduled on July 25-26.

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Topological groups without unitary representations and submeasures

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We give new examples of topological groups that do not have non-trivial continuous unitary representations, the so-called exotic groups. We prove that all groups of the form $L^0(\phi, G)$ where ϕ is a pathological submeasure and G is a topological group, are exotic. This result extends, with a different proof, a theorem of Herer and Christensen on exoticness of $L^0(\phi, \mathbb{R})$ for ϕ pathological. In our arguments, we introduce the escape property, a geometric condition on a topological group, inspired by the solution to Hilbert's fifth problem and satisfied by all locally compact groups, all non-archimedean groups, and all Banach–Lie groups.

This is joint work with F. Martin Schneider.

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On the κ^+-Borel hierarchy of subsets of the generalized Baire space

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Descriptive set theory (DST) focuses on the study of definable subsets, particularly Borel subsets, of Polish spaces, which are separable, completely metrizable spaces. Polish spaces are ubiquitous in mathematics, and studying their Borel subsets has deepened our understanding of various phenomena in different fields.

Generalized Descriptive Set Theory (GDST) extends classical descriptive set theory by replacing countable settings with uncountable ones. A significant part of the literature in GDST focuses on the generalized Baire space ${}^{\kappa}\kappa$ and its κ^+-Borel subsets. Recently, a class of Polish-like spaces was introduced to extend the role of Polish spaces to uncountable settings.

In this talk, I will present joint work with Nick Chapman, Luca Motto Ros, and Beatrice Pitton. Our project aims to extend current knowledge of the generalized Baire space ${}^{\kappa}\kappa$ to this wider class of Polish-like spaces. We address both basic questions about the κ^+-Borel hierarchy, aimed at setting a good foundation for the theory, and more complex ones, providing a comprehensive overview of its structure in general.

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Consistency Strength of Generalized Perfect Set Property

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The study of non-separable completely metrizable spaces, like for example λ^ω when λ has cofinality ω, shows that they have many similarities with their separable counterparts, the Polish spaces, and in fact it is possible to build a sensible descriptive set theory on them, and to define an analogue of the Perfect Set Property. It is therefore of interest to understand which sets enjoy such property. As in the separable case, the answer depends on large cardinals. We will show some upper bound consistency result, using Prikry-like forcings, and some lower bound consistency result, that uses what can be considered the first steps in generalized inner model descriptive set theory.

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Commutativity of Cofinal types

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The Tukey order finds its origins in the concept of Moore-Smith convergence in topology, and is especially important when restricted to ultrafilters with reverse inclusion. The Tukey order on ultrafilters over ω was studied intensively by many, but still contains several fundamental unresolved problems. I will present a recently discovered connection to a parallel study at the realm of measurable cardinals, and explain how different the Tukey order is at that levels when compared to the situation on ω. In the second part of the talk, I will demonstrate how ideas and intuition from ultrafilters over measurable cardinals led to new results at the level of ω and present an essentially new method of constructing Tukey-top ultrafilters using Diamond-like Principles on ω.

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The Borel complexity of proper homotopy equivalence of graphs

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In this talk, we will discuss the relation of proper homotopy equivalence (PHE) of infinite, finite-valence graphs (where we impose a natural Polish topology on the space). We study the Borel complexity of PHE, as well as the homeomorphism relation of infinite type surfaces. This is joint work with Hannah Hoganson.

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Topological applications of sparse families

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Let κ be a cardinal. A family of sets $\mathcal{F}$ is called *κ-sparse* if every κ-sized subfamily of $\mathcal{F}$ has an uncountable union. An $\aleph_1$-sparse family in $[\aleph_n]^\omega$, which is in addition cofinal with respect to containment, exists in ZFC for every $n < \omega$. However, the existence of a cofinal $\aleph_1$-sparse family in $([\aleph_\omega]^\omega, \subseteq)$ is independent of ZFC. Indeed, these families exist in the constructible universe, but they are killed by the Chang Conjecture variant $(\aleph_{\omega+1}, \aleph_\omega) \twoheadrightarrow (\aleph_1, \aleph_0)$. On the other hand, an $\aleph_4$-sparse cofinal family in $([\aleph_\omega]^\omega, \subseteq)$ exists in ZFC.

Sparse cofinal families are the combinatorial skeleton beneath seemingly different problems in various areas of mathematics. We will review some topological applications of them, in particular to Noetherian cardinal invariants and Corson compacta, from our past work with various coauthors.

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Boolean valued semantics for infinitary logics

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It is well known that the completeness theorem for $L_{\omega_1\omega}$ fails with respect to Tarski semantics. Mansfield showed that it holds for $L_{\infty\infty}$ if one replaces Tarski semantics with Boolean valued semantics. We use forcing to improve his result in order to obtain a stronger form of Boolean completeness (but only for $L_{\infty\omega}$). Leveraging on our completeness result, we establish the Craig interpolation property and a strong version of the omitting types theorem for $L_{\infty\omega}$ with respect to Boolean valued semantics.

Time permitting we also relate this work to Asperó and Schindler's proof of the forcibility of Woodin's axiom $(*)$ over models of ZFC and large cardinals.

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The SLO principle for Borel subsets of the generalized Cantor space

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The Wadge hierarchy establishes a hierarchy of complexity through the comparison of sets via continuous reductions. The Semi-Linear Ordering principle (SLO) asserts that, for any two subsets A and B of a space X, either A can be continuously reduced to B or the complement of B can be continuously reduced to A. While classical descriptive set theory primarily focuses on studying subsets of the space of all countable binary sequences, generalized descriptive set theory aims at developing a higher analogue in which ω is replaced with an uncountable cardinal κ satisfying the condition $2^{<\kappa} = \kappa$. Motivated by understanding the Wadge structure for (various classes of) generalized Borel sets, in this talk we will first discuss the consistency of the failure of the SLO principle for $\mathbf{\Sigma}^0_2(\kappa^+)$ sets and then, starting from the bottom of the Wadge hierarchy, we will analyse the validity of the semi-linear ordering principle as we ascend through the difference hierarchy.

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Groups without the Generic Point Property

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We show that various groups of homeomorphisms of a strongly locally homogeneous Peano continuum which is not the circle, the sphere, or the real projective plane do not have the generic point property, and it particular have non-metrizable universal minimal flow. This class of spaces includes all closed manifolds of dimension at least three, for which the result was already known, all closed surfaces which are not the sphere or the real projective plane, as well as homogeneous Peano continua such as the Menger curve. This is a joint work with Gianluca Basso and Alessandro Codenotti.

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Forcing applied to Ramsey theory of Fraïssé structures

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Forcing has been a central tool for unlocking the area of Ramsey theory on Fraïssé structures which have forbidden substructures. The first instance of this appeared in the speaker's work [5] for colorings of finite triangle-free graphs inside the Henson graph. These ideas were quickly extended in [6] and [11]. Subsequent work has expanded applications of forcing to prove Ramsey theorems for a large collection of Fraïssé structures, as exposited in [7]. We will survey the current state of the art in this arena, pointing out where new work has been able to achieve some of the same Ramsey theorems by purely combinatorial methods, starting with [10], as well as instances where the forcing point of view is still optimal.

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Wadge hierarchy on ordinal numbers

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As a contribution to the study of the Wadge hierarchy on general topological spaces, some results and open problems are presented concerning ordinals endowed with their order topology.

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Developments in Namba Forcing

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One way to study the properties of the infinite cardinals is to examine the extent to which they can be changed by forcing. In 1969 and 1970, Bukovský and Namba independently showed that $\aleph_2$ can be forced to be an ordinal of cofinality $\aleph_0$ without collapsing $\aleph_1$. The forcings they used and their variants are now known as Namba forcing. This talk will follow the recent result of the speaker that it is consistent modulo the existence of an inaccessible cardinal that classical Namba forcing for $\aleph_2$ has the weak ω_1-approximation property, answering a question of Cox and Krueger. There are a number of variations of this argument that have implications for weak guessing models, the study of $\aleph_{\omega+1}$, and the minimality of collapsing extensions.

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Dynamics of compressible Euler equations and complex flows
Special Session B23

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The topic of this special session is the analysis of partial differential equations (PDE's) arising in physics, which are nonlinear hyperbolic or systems that combine hyperbolic with parabolic features. Such equations are ubiquitous in multiple domains of applied sciences, ranging from high-speed flows in fluids, to flows of complex systems, to plasma physics, and astrophysics. The aim is to bring together people that work on compressible Euler equations (and related subjects) with specialists who work on flows of complex and multiscale systems and to stimulate an exchange between these subjects. We will focus on theory, numerics, as well as modeling and applications, and their interplay. We hope to explore new directions, and to stimulate new collaborations.

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An initial-boundary value problem for system of conservation laws with unknown boundary

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In this talk, we study a hyperbolic system of balance laws in one dimension on a domain with a boundary. The shape of the boundary is not prescribed a priori, and it is an unknown of the problem itself.

Under appropriate conditions at the boundary and on the source term, we show the existence of a non-characteristic boundary and of a weak entropy solution of small total variation, defined on the resulting domain, that is global in time. We show an application to an inverse problem motivated by the study of planar steady supersonic flow past a wedge.

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BV weak solutions with bounded support and long-time behavior to an Euler-type flocking model

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Recent results on the global in time existence of weak solutions with bounded support to a hydrodynamic model of flocking-type in a one-space dimension and the long time behavior would be presented. An appropriate notion of entropy weak solutions with bounded support is given to capture the behavior of solutions with initial data that has finite total mass confined in a bounded interval and initial density uniformly positive therein without any restrictions on the size of the total variation of the initial data. We show global in time existence of entropy weak solutions with concentration along the interfaces that separate the vacuum with the non-vacuum regions, for any initial data of bounded variation having the structure above. The analysis relies on the front tracking algorithm, a detailed study of the decay of the wave fronts and the influence of the shock discontinuities along the interfaces. In addition, we capture the time-asymptotic limit for such solutions unconditionally, showing the asymptotic decay towards flocking profiles without any further restrictions on the data.

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A non linear and non local hyperbolic–parabolic system inspired by biology

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Motivated by the description of general predator–prey dynamics and with the specific aim of controlling parasites' propagation in pest control problems, we propose a model consisting of a non linear and non local hyperbolic equation together with a parabolic one. The unknown functions are the densities of the two competing populations. Prey/parasites are assumed to diffuse, while the movement of predators is directed towards the regions where the concentration of prey is greater. This situation is modelled by a non linear and non local function of the prey's density. The two PDEs are further coupled through the source terms, that comprise Lotka–Volterra type interactions and also the presence of external space- and time-dependent controls.

We present analytical results on the well posedness of this class of systems, both on the whole space $\mathbb{R}^n$ and on a bounded domain. The presentation is concluded by some numerical simulations, to illustrate the qualitative properties of the solutions.

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Stability of a numerical method for a transport equation, with the help of noise

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The term "regularization by noise" refers to the phenomenon where the presence of noise may lead to better well-posedness of a differential equation, such as the delay of blowup, higher regularity, or existence/uniqueness for problems whose deterministic counterpart is ill-posed.

As a concrete example, I will present a recent paper where we analyse a numerical method for a stochastic transport equation. The velocity field is assumed to be of low regularity (essentially, $V \in L^\infty(\mathbb{R}^d; \mathbb{R}^d)$ with $\mathrm{div} V \in L^p(\mathbb{R}^d)$ for $p > d$) — lower than what is required in the deterministic theory. The stochastic term is of the transport type, with a spatially heterogeneous noise coefficient. We prove that the method satisfies an energy bound, and hence converges weakly to the exact solution.

This is joint work with Kenneth Karlsen and Peter Pang (both University of Oslo).

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Analysis of traveling waves for Navier–Stokes–Korteweg type models

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We study existence and spectral stability of traveling waves for Navier–Stokes–Korteweg type models, underlying in particular the interplay between capillarity and viscosity effects. Motivated by the theory of superfluidity and the mathematical modeling of semiconductor devices, we first consider the compressible Euler equations with linear and nonlinear viscosity and where the dispersive term is originated by the quantum effects described through the Bohm potential. The existence of traveling waves (or *viscous dispersive shocks*) is proved for appropriate end states defining Lax shocks for the underlying Euler system, and for arbitrary shock amplitude. The results are obtained by taking advantage of suitable Lyapunov functions and, in the case of large shocks, the framework of existence includes also the case of oscillatory profiles, that is when the effects of the dispersion play a significant role. The model with nonlinear viscosity is formulated in terms of density and velocity and the existence is proved without restrictions for the viscosity and dispersion parameters. The spectral stability of these profiles is also analyzed: we prove stability of the essential spectrum of the linearized operator, provided the end states are subsonic or sonic, and, using the Evans function, we derive estimates for the modulus of possible unstable eigenvalues. The latter allows us to investigate numerically the behavior of the Evans function in sufficiently large region of the unstable half–plane, providing numerical evidence for point spectral stability of arbitrary large, possibly non–monotone profiles. The same results are presently under investigation for the classical Navier–Stokes–Korteweg model, with physical viscosity and general Korteweg terms.

This is a joint project with P. Marcati (GSSI), D. Zhelyazov (U. Surrey), R. Folino and R. Plaza (UNAM).

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Non-uniqueness and energy dissipation for 2D Euler equations with vorticity in Hardy spaces

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We show by convex integration that uniqueness of solutions to the 2D incompressible Euler equations fails in the class of admissible (i.e. energy dissipating), compactly supported, $L^\infty_t L^2_x$ velocity fields having vorticity in the real Hardy space $H^p(\mathbb{R}^2)$, for any $p \in (0, 1)$.

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Error analysis of the θ-method for ODEs with irregular vector fields

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The goal of this talk is to investigate the convergence of numerical methods applied to ordinary differential equations with vector fields in the DiPerna-Lions class. We will prove logarithmic rates of convergence (in the mesh-size) of the approximating flows constructed via the so-called θ-method towards the unique regular Lagrangian flow of the given vector field. I will also discuss the applications to the linear transport equation. This is the first result concerning a posteriori error estimate for implicit schemes applied to ordinary differential equations with Sobolev vector fields.

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On a stochastic fluid-structure interaction problem with compressible fluid and elastic shell

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In this talk, we will present our recent results on a non-linearly coupled stochastic fluid-structure interaction problem that involves an isentropic compressible fluid interacting with a thin elastic membrane. The elastodynamics problem is modeled by linear shell equations determining the displacement of a 3D time-dependent domain that contains a fluid whose flow is described by the Navier-Stokes equations. The noise is applied both to the fluid equations as a volumetric body force, and to the structure as an external forcing to the deformable fluid boundary. We will discuss the challenges arising due to the random motion of the time-dependent fluid domain and present our recent result that provides the existence of weak martingale solutions. We will briefly describe our approach to constructing these solutions which involves a splitting scheme, penalization of boundary behavior and domain extension, construction of an artificial structure variable and a stopping time argument.

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Existence of weak solutions to the kinetic Cucker-Smale equations with local alignment and stochastic kinetic transport

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We present a result on existence of weak martingale solutions for a stochastic kinetic Cucker-Smale equation with local alignment and stochastic kinetic transport. The Cucker-Smale equations, which were introduced in [1] and [2], model the collective dynamics and behavior of agents, such as flocks of birds, schools of fish, and herds, where the velocity of each agent evolves in time and is influenced by the velocities and the spatial locations of other nearby agents in the system.

While it is possible to model each agent in the flock or herd individually, when the number of agents is large, it is more tractable to instead provide a kinetic description of the collective dynamics. This involves introducing a function $f(t, x, v)$ for $t \in [0, T]$, $x \in \mathbb{R}^d$, and $v \in \mathbb{R}^d$, which heuristically is a density function describing the relative quantity of agents having position $x \in \mathbb{R}^d$ and velocity $v \in \mathbb{R}^d$ at a given time t. To model the ways in which differences in velocities and spatial positions between agents influence the evolution of $f(t, x, v)$, one can use an interaction kernel to describe pairwise interactions between agents, and a local alignment term, which describes the tendency of agents to align in direction with agents sharing the same position. It is well-known that weak (function-valued) solutions exist to the deterministic Cucker-Smale kinetic model with an interaction term and local alignment, due to work in [3].

The goal of the present work is to extend the well-posedness result for the deterministic kinetic Cucker-Smale dynamics in [3] to the stochastic regime, where the kinetic Cucker-Smale dynamics are perturbed by stochastic kinetic transport. The stochastic kinetic transport represents the effect of random noise, such as random wind or currents, that globally and randomly perturbs the dynamics of the entire system. Such kinetic equations with stochastic kinetic transport have been of recent interest, for example in the case of the Boltzmann equations [4]. We show existence of weak martingale solutions to the kinetic Cucker-Smale equations with stochastic kinetic transport by using approximation parameters to approximate the local velocity and by constructing appropriate approximate solutions. We then establish uniform bounds on the approximate solutions by deriving suitable a priori estimates. Finally, to complete the passage to the limit in the approximate solutions, we develop new stochastic velocity averaging results in the spirit of [4] to pass to the limit in time-dependent and spatially-dependent quantities, such as the density and the momentum, and then we establish tightness of laws of the approximate solutions in order to use the Skorokhod representation theorem.

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Existence of weak solutions for the unsteady $p(\cdot,\cdot)$-Navier–Stokes equations via a fully-discrete approximation.

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We establish the well-posedness, stability, and (weak) convergence of a fully-discrete approximation of the unsteady $p(\cdot,\cdot)$-Navier–Stokes equations employing an implicit Euler step in time and a discretely inf-sup-stable finite element approximation in space. The result, beside the interest for the numerical approximation, gives an alternative way to prove existence of weak solutions. Moreover, numerical experiments that supplement the theory are presented.

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On Bayesian data assimilation for PDEs with ill-posed forward problems

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We consider Bayesian data assimilation for time-evolution PDEs, for which the underlying forward problem may be very unstable or ill-posed. We formulate assumptions on the forward solution operator of such PDEs under which stability of the posterior measure with respect to perturbations of the noisy measurements can be proved. We also provide quantitative estimates on the convergence of approximate Bayesian filtering distributions computed from numerical approximations. For the Navier-Stokes equations, our results imply uniform stability of the filtering problem even at arbitrarily small viscosity, when the underlying forward problem may become ill-posed, as well as the compactness of numerical approximations in a suitable metric on time-parametrized probability measures.

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Asymptotic limits for unipolar and bipolar Euler-type systems

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In this talk, we present some recent developments in the study of asymptotic limits for Euler-type systems via the relative energy method. The first limit that we consider is the high-friction limit. In [1], one establishes this limit for a bipolar Euler-Poisson system towards a bipolar drift-diffusion system, whereas in [3] this limit yields a diffusion-aggregation equation from an Euler-Riesz system. These results extend the ones obtained previously in [4] and [5], and are compared regarding the admissible range of the adiabatic exponents. The high-friction limit connects hyperbolic systems with parabolic equations.

Furthermore, we also consider the zero-electron-mass and quasi-neutral limits for a bipolar Euler-Poisson system [2]. In the former limit, an adiabatic electron system is obtained, while in the combined regime one arrives at an Euler system. In this case, these limiting procedures yield hyperbolic systems from hyperbolic equations.

The technical challenges in these studies are addressed either via the theory of Riesz potentials and related Hardy-Littlewood-Sobolev inequalities, or via standard elliptic regularity theory together with Sobolev embeddings.

The results presented were obtained in collaboration with A. E. Tzavaras, J. A. Carrillo, and Y. P. Choi.

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Global regularity for the one-dimensional stochastic Quantum-Navier-Stokes equations

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In this talk I will present some new results concerning the analysis of the stochastically forced 1D Quantum-Navier-Stokes equations. In particular for $x \in \mathbb{T}$, the 1-dimensional flat torus, and $t \in [0,T]$, the system under studying is the following:

$$
\begin{cases} \mathrm{d}\rho + \partial_x(\rho u)\mathrm{d}t = 0 \\ \mathrm{d}(\rho u) + [\partial_x(\rho u^2 + p(\rho))]\mathrm{d}t = [\partial_x(\mu(\rho)\partial_x u) + \rho\partial_x\left(\dfrac{\partial_{xx}\sqrt{\rho}}{\sqrt{\rho}}\right)]\mathrm{d}t + \mathbb{G}(\rho,\rho u)\mathrm{d}W. \end{cases} \tag{1}
$$

The unknowns $\rho > 0$ and $u \in \mathbb{R}$ denote the density and the velocity of the fluid, while

$$
p(\rho) = \rho^\gamma, \quad \gamma > 1, \quad \mu(\rho) = \rho^\alpha, \quad \alpha \geq 0,
$$

represent the isoentropic pressure and the viscosity coefficient. The stochastic forcing term $\mathbb{G}(\rho,u)\mathrm{d}W$ is a multiplicative noise defined on a stochastic basis with a complete right-continuous filtration $(\Omega, \mathfrak{F}, (\mathfrak{F}_t)_{t\geq 0}, \mathbb{P})$ together with a cylindrical $(\mathfrak{F}_t)$-Wiener process $W(t)$. The related stochastic integral is understood in the Itô sense.
Our analysis is focused on the existence of solutions to (**??**) which are strong both in PDEs and probability sense. In particular, we prove the local well-posedness of the problem up to a maximal stopping time τ which depends on the $W^{2,\infty}$ norm of the solution (ρ, u) and we derive some a priori estimates in the case of the viscosity exponent $\alpha \in [0, \frac{1}{2}]$, which allow us to extend the local strong solution to a global one by controlling the arising of vacuum states of the density. The analysis is performed for a wide class of density dependent viscosity coefficients and as a byproduct of our results we also get the global well-posedness for the deterministic case.

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Density of wild data for the Euler system of dynamics

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We adapt Glimm's approximation method to the framework of convex integration to show density of wild data for the (complete) Euler system of gas dynamics. The desired infinite family of entropy admissible solutions emanating from the same initial data is obtained via convex integration of suitable Riemann problems pasted with local smooth solutions. In addition, the wild data belong to the BV class. The results presented are based on a joint work with Eduard Feireisl [**?**].

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Recent developments for the one-dimensional compressible Euler system with local and non-local interactions and dissipation terms.

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In this talk I will summarise our recent findings on the existence of regular and weak solutions for the compressible Euler equations with nonlocal interaction terms including attraction-repulsion and alignment [1]. In particular, I will present the relative entropy structure based on the two-velocity reformulation of the system [2], the viscous approximation [4], and the long-time behaviour of solutions [3].

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Vanishing physical viscosity solutions of characteristic initial-boundary value problems for systems of conservation laws.

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Consider the viscous approximation of a nonlinear system of conservation laws in one space variable

$$\mathbf{g}(\mathbf{v}^\varepsilon)_t + \mathbf{f}(\mathbf{v}^\varepsilon)_x = \varepsilon\Big(\mathbf{D}(\mathbf{v}^\varepsilon)\mathbf{v}^\varepsilon_x\Big)_x,$$

where $\mathbf{g}, \mathbf{f} : \mathbb{R}^N \to \mathbb{R}^N$ are given functions, $\mathbf{D}$ is also a given function attaining values in the space of $N \times N$ matrices, and the unknown $\mathbf{v}^\varepsilon$ attains values in $\mathbb{R}^N$. The archetypical examples are the compressible Navier-Stokes equations and the viscous MagnetoHydroDynamics (MHD) equations that formally boil down, in the inviscid $\varepsilon \to 0^+$ limit, to the compressible Euler and inviscid MHD equations, respectively.

In the case of initial-boundary value problem, it is known that, owing to boundary layers phenomena, different viscous mechanism (that is, different choices of the matrix $\mathbf{D}$) yield different solutions in the vanishing viscosity limit. In my talk I will consider the initial boundary-value problem for the conservation law

$$\mathbf{g}(\mathbf{v}^\varepsilon)_t + \mathbf{f}(\mathbf{v}^\varepsilon)_x = \mathbf{0}$$

and encode information on the underlying viscous mechanism in the formulation of the boundary condition. I will then discuss a new wave front-tracking algorithm providing, for small total variation data, global-in-time existence of admissible solutions of the initial-boundary value problem. The hypotheses cover the most interesting, albeit technically demanding, cases, that is physical (mixed hyperbolic-parabolic) viscosity and characteristic boundary. In particular, the result applies to the inviscid limit of the Navier-Stokes and viscous MHD equations, written in both Eulerian and Lagrangian variables.

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Renormalized solutions for the Maxwell-Stefan system

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The Maxwell-Stefan system is commonly used to describe diffusion processes of multi-component systems. In this talk (i) we show how a multicomponent Euler system emerges in the high-friction limit via a process of velocity alignment, and how the Maxwell-Stefan system may be viewed as the zero-mean-motion approximation of the former. (ii) Extend the notion of renormalized solutions to the Maxwell-Stefan system and use it in conjunction with symmetrized relative entropy to obtain uniqueness results.

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Post-quantum Group-based Cryptography
Special Session B24

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Maria Tota
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This session is scheduled on July 25-26. We bring cryptographers, group theorists, quantum computational theorists to discuss various aspects of post-quantum group-based cryptography. Group-based cryptography is a relatively new family of classes of post-quantum primitives with great potential. Due to the diversity of backgrounds of the speakers, we are aiming to create interdisciplinary areas for collaborations. The topics of talks could generate new mathematical questions for group theorists as well as for cryptographers striking applying new mathematical objects and problems for the cryptographic schemes. This meeting is designed to exchange the new results in this direction and well as creating national and international collaborations among the speakers and attendees.

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Automaton group-based cryptography

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In cryptography most famous protocols (RSA, Diffie-Hellman, and elliptic curve methods) depend on the structure of commutative groups and they are related to the difficulty to solve integers factorization and discrete logarithms. In 1994 Shor provided a quantum algorithm that solves these problems in polynomial time. As a consequence, researchers are now interested in finding alternative methods in cryptography that are secure in a post-quantum world. Some of the candidates have been known for years, while others are still emerging. Group theory, and in particular non-abelian groups, offers a rich supply of complex and varied problems for cryptography.

In this talk, we present an overview of the current state-of-the-art in post-quantum group-based cryptography. We will describe in particular the class of automaton groups as suitable platfrom for cryptography, and we present new results and some open problems.

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Hash functions with special linear groups

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Ramón Flores
Universidad de Sevilla

Delaram Kahrobaei
City University of New York

Thomas Koberda
University of Virginia

In this talk new families of Tillich-Zémor hash functions are defined, using higher dimensional special linear groups over finite fields as platforms. We will show that the Cayley graphs of these groups combine fast mixing properties and high girth, which together give rise to good preimage and collision resistance of the corresponding hash functions.

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Secret sharing schemes using representation theory of finite p-groups

Keivan Mallahi-Karai

June 20, 2024

In this talk, I will describe a new approach toward constructing secret sharing schemes that is based on the representation theory of finite p-groups. I will explain connections to the Kirillov's orbit method, and a number of other works that aim at characterizing minimal faithful representations of p-groups. This is based on a joint work with Delaram Kahrobaei.

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A Guide to the Design of Digital Signatures based on Cryptographic Group Actions

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Edoardo Persichetti
Florida Atlantic University, Sapienza University of Rome

Federico Pintore
Department of Mathematics - University of Trento

Paolo Santini
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Krijn Reijnders
Radboud University Nijmegen

Recent years have witnessed a revival of cryptography based on group actions, mainly due to its role in post-quantum cryptography, i.e. the branch of cryptography whose goal is designing cryptosystems believed to be secure even in the presence of quantum attackers. For instance, several works have proposed digital signature schemes based on group actions, as well as a variety of techniques aimed at improving their performance and efficiency. Most of these techniques can be explained as transforming one Sigma protocol into another, while essentially preserving security. In this talk, we present a unified taxonomy of such techniques. In particular, we describe all techniques in a single fashion, show how they impact the performance of the resulting protocols and analyse how different techniques can be combined for optimal performance. This analysis is meant to provide a flexible tool which is easy to adapt and employ in the design of future schemes.

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On the security of some group-based key exchange protocols

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In this talk we will discuss the security of some group-based key exchange protocols with emphasis on the semantic security and its connection with well-known group-theoretic decidability problems. We will make considerations on how this analysis can affect the choice of platform groups, presenting concrete examples.

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Quantum-Safe Data Aggregation Solution for Smart Meter Networks

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Smart meters have replaced traditional electricity meters in recent years, marking a significant advancement in energy management. Italy is the nation in Europe that first expressed interest in first-generation smart meters and is currently replacing them with second-generation ones. These digital devices offer several advantages, enabling daily consumption monitoring and support for real consumption-based billing. By continuously monitoring, it is possible to identify consumption peaks and enhance network stability at different levels of aggregation. In this context, it is crucial to have a cryptographic scheme that preserves consumers' privacy. This talk aims to describe a protocol that enables an unreliable data aggregator to compute the sum of several plaintexts working only with the corresponding ciphertexts. Our proposal, unlike other solutions to this problem, does not require communication between smart meters and is quantum-safe.

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Navigating in graphs of elliptic curves

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Many past and current cryptographic challenges are based on the difficulty of efficiently navigating in graphs whose vertices represent elliptic curves from certain families. A relevant instance is given by isogeny graphs between supersingular elliptic curves, which have been proposed as valid postquantum candidates. Although some of these protocols did not pass the test of time, others are still alive and keep motivating the fundamental research of these curves. In this talk, such graphs and their difficulty assumptions will be reviewed. Furthermore, other families of graphs naturally arising from elliptic curves over finite fields will be presented, highlighting differences and possible connections. It will be noted how this topic is linked to several distinct fields, highlighting the group actions underlying the considered graphs. It is based on two ongoing works, joint with Eleni Agathocleous, Antoine Joux, Marzio Mula and Federico Pintore.

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Automorphisms of bibraces and their applications to symmetric-key cryptography

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In a XOR-based alternating block cipher the plaintext is masked by a sequence of layers each performing distinct actions: a highly nonlinear permutation, a linear transformation, and the bitwise key addition. When assessing resistance against *classical* differential attacks (where differences are computed with respect to XOR), the cryptanalysts must only take into account differential probabilities introduced by the nonlinear layer, this being the only one whose differential transitions are not deterministic. The temptation of computing differentials with respect to another difference operation runs into the difficulty of understanding how differentials propagate through the XOR-affine levels of the cipher. In this talk we introduce a special family of *braces* that enable the derivation of a set of differences whose interaction with *every* layer of an XOR-based alternating block cipher can be understood. We show that such braces can be described also in terms of alternating binary algebras of nilpotency class two. Additionally, we present a method to compute the automorphism group of these structures through an equivalence between bilinear maps. By doing so, we characterise the XOR-linear permutations for which the differential transitions with respect to the new difference are deterministic, facilitating an alternative differential attack.

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Applications of varieties over finite fields to the differential uniformity of polynomials

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Algebraic curves over finite fields hold significance not only in theory but also establish profound links with various branches of mathematics and combinatorics. They serve as crucial instruments in tackling topics such as APN functions, planar functions, APcN functions, and APN permutations. In this presentation, I'll delve into the applications of algebraic curves in exploring the aforementioned concepts.

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Cryptanalysis of Xifrat1-Sign.I quasigroup-based digital signature scheme

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We propose a new cryptanalysis of the `Xifrat1-Sign.I DDS` digital signature scheme proposed by Jianfang "Danny" Niu in 2023 as a candidate to NIST's post-quantum Cryptography Standardization project [1]. The scheme is based on a quasigroup Q of order 16 satisfying restrictive commutativity property, stating that $(ab)(cd) = (ac)(bd)$ for all elements $a, b, c, d \in Q$. The scheme, using multiplication in the quasigroup and several rounds of mixing, constructs a 768-bit shared secret key. We study properties of the underlying quasigroup and expose a vulnerability that allows us to recover the key from public data within seconds in all tested cases. Another attack on the scheme was proposed in July 2023 by Lorenz Panny using the linearization. Our attack takes much less time to recover the secret key. The implementation of the attack in GAP and statistical analysis of the scheme are given in [2].

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A new primitive for multivariate schemes arising from Boolean functions theory

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Multivariate public-key cryptography is one of the families of post-quantum cryptography. Most of the schemes belonging to this family can be seen as applying an affine transformation to a quadratic map from $\mathbb{F}_q^n$ to $\mathbb{F}_q^m$, where n, m are positive integers, q is a power of a prime and $\mathbb{F}_q$ is a finite field with q elements. In this talk, we will consider a more general notion of equivalence relation, the CCZ-transformation, borrowed from the area of cryptographically relevant Boolean functions. To allow a generic CCZ-transformation, we make use of the *twisting*, introduced by Canteaut and Perrin in [1].

Definition 1. Two functions $F, G : \mathbb{F}_q^n \to \mathbb{F}_q^m$ are equivalent via t-twist, for $0 \le t \le \min(n, m)$, if $F(x, y) = (T(x, y), U(x, y)) = (T_y(x), U_x(y))$ and $G(x, y) = (T_y^{-1}(x), U_{T_y^{-1}(x)}(y))$, where $T : \mathbb{F}_q^t \times \mathbb{F}_q^{n-t} \to \mathbb{F}_q^t$, $U : \mathbb{F}_q^t \times \mathbb{F}_q^{n-t} \to \mathbb{F}_q^{m-t}$, and $T_y(x)$ is a bijection for any fixed $y \in \mathbb{F}_q^{n-t}$.

Using the twisting, we define the following pair of public and secret keys.
For $F : \mathbb{F}_q^n \to \mathbb{F}_q^m$ a t-twistable function, we consider the equivalent function via t-twist G. For random A_1, A_2 affine bijections of $\mathbb{F}_q^m$ and $\mathbb{F}_q^n$ respectively, we set $G_{pub} = A_1 \circ G \circ A_2$. Then, the public key is $\mathtt{pk} = G_{pub}$ and the secret key is the tuple $\mathtt{sk} = \langle F, A_1, A_2 \rangle$.

We present an encryption scheme and a signature scheme using the above cryptographic primitive. We provide further specifications on the map F, hence on the choice of quadratic T and U, and we call it the UOV-CCZ scheme. Focusing on the latter, we study its structure and its properties. Finally, we study whether and how some known attacks to multivariate cryptographic schemes can be applied to our proposal. For example, the *linearization attack*, proposed by Patarin in [2], works only under some restriction on the choice of parameters.

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Security Analysis of ZKPoK based on MQ problem

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In this talk I will investigate the security of a new Zero-Knowledge (ZK) Proof-of Knowledge (PoK) scheme based on Multivariate Quadratic (MQ) problem, presented in [1, 2]. The security of this new scheme is related to a new hard problem: the so-called $\mathtt{DiffMQ}_{\mathrm{H}}$.

I will present a new efficient probabilistic algorithm for solving the $\mathtt{DiffMQ}_{\mathrm{H}}$. The algorithm solves $\mathtt{DiffMQ}_{\mathrm{H}}$ in polynomial time if $m - n \in O(1)$. For more details see [3].

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Conjugacy Search Problem in Contracting Self-similar Groups

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We propose self-similar contracting groups as a new platform for cryptographic schemes based on simultaneous conjugacy search problem (SCSP). The class of these groups contains extraordinary examples like Grigorchuk group, which is known to be non-linear, and groups in this class admit a natural normal form based on the notion of a portrait. While for some groups in the class the conjugacy search problem has been studied, there are many groups for which no such algorithms are known. We discuss benefits and drawbacks of using these groups in cryptography and provide computational analysis of variants of the length based attack on SCSP for some groups in the class, including Grigorchuk group.

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Some group-theoretical results on block ciphers in a long-key scenario

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Most modern block ciphers belong to two families of symmetric cryptosystems, i.e. Substitution-Permutation Networks (SPN) and Feistel Networks. Typically, in both cases, each encryption function is a composition of key-dependent permutations of the plaintext space, called *round functions*, designed in a such way to provide both *confusion* and *diffusion*. Confusion is provided applying public non-linear vectorial Boolean functions, called S-boxes, whereas diffusion is obtained by means of public linear maps, called diffusion layers. The private component of the cipher, i.e. the *key*, is derived from the user-provided information by means of a public procedure known as *key-schedule*. When the round functions are made in such a way the confusion and diffusion layers are followed by the XOR-addition with the so-called *round-key*, where the round-key is every possible vector in the message space, the cipher is a *long-key cipher*.

Since the seventies, many researchers have studied the relationship between some algebraic properties of the confusion/diffusion layers and some algebraic weaknesses of the corresponding ciphers, using a permutation-group-theoretical approach. In 1975, Coppersmith and Grossman considered a set of permutations which can be used to define a block cipher and, by studying the permutation group that they generate, they linked some properties of this group and the security of the corresponding cipher. From this work a new branch of research was born, which focuses on group-theoretical properties that can be exploited to attack encryption methods. Kaliski, Rivest and Sherman proved that if the permutation group generated by the encryption functions of a cipher is too small, then the cipher is vulnerable to birthday-paradox attacks. Calderini, Civino and Sala proved that if such group is isomorphic to a subgroup of the affine group of the plaintext space, induced by a sum different to the classical bitwise XOR, then it is possible to embed a dangerous trapdoor on it. More relevant, Paterson built a DES-like cipher whose encryption functions generate an imprimitive group and showed how the knowledge of this trapdoor can be turned into an efficient attack to the cipher. For this reason, showing that the group generated by the encryption functions of a given cipher is primitive and not of affine type became a relevant branch of research. In 2016, Bannier, Bodin and Filiol generalized the imprimitive attack shown by Paterson by means of a trapdoor which consists in mapping a partition of the plaintext space into a (different) partition of the ciphertext space. The authors also proved that only *linear* partitions can propagate round-by-round in a long-key SPN.

In this talk, first we present some conditions ensuring that linear partitions cannot propagate in a long-key SPN [**??**]. Then, we study some properties of the linear-partition propagation under the action of a long-key Feistel Network. In particular, we will see that also in a Feistel-Network-like long-key framework, if the cipher allows partition propagation, then the partitions are linear ones. Moreover, we provide a partial generalisation in the Feistel Network case [**??**] of the results given in the SPN case.

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Using regular subgroups in block cipher cryptanalysis

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In the context of iterated block ciphers classical cryptanalysis techniques and their generalizations exploit linear relations between the inputs (plaintexts) and outputs (ciphertexts) of an encryption function. Usually the plaintext and ciphertext space is given by an n dimensional vector space over the binary field $\mathbb{F}_2$, $V = \mathbb{F}_2^n$, and the operation which is exploited in the linear relations is the XOR sum $(+)$.

Alternative operations $\circ$ can be defined over the space V in order to obtain that $(V, \circ)$ is still a vector space. Such operations come from the elementary abelian regular subgroups of $\mathrm{Sym}(V)$, that are the conjugates to the usual translation group $T_+(V)$. In this talk, we will focus on the properties of the elementary abelian regular subgroups contained in the general affine group. We will give a characterizations of such groups and we will discuss how they can be exploited in order to implement possible attacks to block ciphers.

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Subsets of groups in public-key cryptography

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Subsets of groups defined by language-theoretic conditions (rational, algebraic, context-free,...) are an interesting object of study in group theory and algorithmic problems when defined on subsets are in general harder than when defined on finitely generated subgroups.

In this talk, we suggest the usage of algebraic subsets instead of subgroups in public-key cryptography, playing the role of subgroups in some classical key exchange protocols. We also introduce new group theoretic problems arising from this work.

This is joint work with António Malheiro and the results can be found in the preprint [1].

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Exploring problems and platforms in group-based cryptography

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Group-based cryptography is mainly concerned with the study of cryptographic protocols which rely on problems within nonabelian Group Theory. These problems typically manifest as search variants stemming from decision-type problems.

Once a cryptographic system is established, the primary challenge lies in identifying an appropriate platform, whether it is a singular group or a class of groups, endowed with favorable algorithmic properties. Indeed, this platform should render the underlying problems sufficiently complex to effectively withstand attacks.

This presentation aims to investigate various decision problems in Group Theory from two distinct perspectives. Firstly, we will explore their application in proposed cryptographic systems. Subsequently, we will embark on a comprehensive discussion regarding the advantages and disadvantages of various suggested platforms.

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Analysis, control and inverse problems in climate sciences
Special Session B25

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The ongoing climate emergence and its consequences have emphasized the crucial role of a deep mathematical research on such topic. The fight against climate change, the study and possibly prediction of extreme events, the analysis of potential effects on the enviroment have nowadays attracted significant attention of scientists from a wide range of disciplines. In particular, the investigation of differential equations which describe such phenomena is certainly of fundamental importance: from the definition and analysis of reliable climate models, the study of the dependence on significant parameters, and finally the employment of such models to infer social and enviromental impacts of climate change.

The scope of this special session is to gather experts on the aforementioned subjects in order to present and discuss new mathematical developments on such topics. Our aim is to give an interdisciplinary overview of the problems connected to climate change and the related techniques arising in mathematical and numerical analysis, stochastic calculus, dynamical systems, ODEs and PDEs analysis.

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Nonautonomous Dynamical Systems for Climate Change & Climate Variability: An Application to a Simple Ocean Model

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The theory of nonautonomous and random dynamical systems (NDSs and RDSs) provides an appropriate mathematical setting for the study of time-dependent forcing, both natural and anthropogenic, upon a climate system characterized by intrinsic variability [1]. In this theory, the forward attractors of autonomous dynamical systems are replaced by pullback and random attractors (PBAs and RAs) and classical bifurcations by "tipping points." Over the last decade-and-a-half, these relatively novel concepts have been applied to a number of simple climate models, atmospheric, oceanic and coupled [2].

Important insights into the study of PBAs and RAs arising from climate dynamics have been provided by novel tools from algebraic topology [3,4]. These tools have led to the introduction and analysis of topological tipping points and we present them here as applied to a simple model of the wind-driven double-gyre ocean circulation [5].

The model is a low-order approximation of a spectral quasigeostrophic model for the subtropical and subpolar gyres of the North Atlantic or North Pacific ocean basin, subject to time varying zonal winds [6]. The recent tools from algebraic topology applied to it are Branched Manifold Analysis through Homologies (BraMAH) and the Templex, which combines the complex underlying BraMAH with a directed graph that captures the flow in the dynamical system's phase space [4].

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Typicality of extreme events in the climate system – A large deviation theory perspective –

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In this presentation I will shortly present the main tools we use to analyse and predict various types of weather and climate extreme events. I will then focus on persistent climate extremes, like heatwaves and cold spells, and discuss the new insights we gain when analysing them through the lens of large deviations theory. This framework has given rise to the concept of "typicality" concerning certain weather and climate extremes, indicating their property to exhibit similarities in spatial patterns, temporal evolution, and underlying physical processes, with this resemblance intensifying as events become more extreme. Recent research confirms that highly intense heatwaves, characterized by prolonged local temperature anomalies, consistently coincide with specific large-scale circulation patterns. This suggests that there is a typical way the atmosphere realises an extreme temperature anomaly.

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From Micro to Macro in Modeling Sea Ice

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Earth's sea ice covers form a key component of the climate system. Their precipitous declines impact the polar marine environment and its ecosystems, with ripple effects felt far beyond the polar regions. As a material sea ice exhibits composite structure on many length scales. A principal challenge is how to use microstructural information to find effective properties on larger scales relevant to climate and ecological models. From microscale brine inclusions in sea ice to the Arctic ice pack itself, and from algae living inside the brine inclusions to polar bears roaming the ice surface, we'll tour recent advances in modeling sea ice and the ecosystems it hosts. Areas of mathematics we'll visit include percolation, fractal geometry, random matrix theory, advection diffusion, inverse problems, topological data analysis, and uncertainty quantification.

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Analysis of a two-layer energy balance model: long time behaviour and greenhouse effect (Part 1)

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We study a two-layer energy balance model, that allows for vertical exchanges between a surface layer and the atmosphere. The evolution equations of the surface temperature and the atmospheric temperature are coupled by the emission of infrared radiation by one level, that emission being partly captured by the other layer, and the effect of all non-radiative vertical exchanges of energy. Therefore, an essential parameter is the absorptivity of the atmosphere, denoted ε_a. The value of ε_a depends critically on greenhouse gases: increasing concentrations of CO_2 and CH_4 lead to a more opaque atmosphere with higher values of ϵ_a.

First we study the associated ODE model, and we prove that global existence of solutions of the system holds if and only if $\varepsilon_a \in (0, 2)$, and blow up in finite time occurs if $\varepsilon_a > 2$. (Note that the physical range of values for ε_a is $(0, 1]$). Next, we explain the long time dynamics for $\varepsilon_a \in (0, 2)$, and we prove that all solutions converge to some equilibrium point. We also present some numerical results showing the classical S-shaped structure of the equilibrium points. Finally, motivated by the physical context, we study the dependence of the equilibrium points with respect to the involved parameters, and we prove in particular that the surface temperature increases monotonically with respect to ε_a. This is the key mathematical manifestation of the greenhouse effect for this toy model.

Finally, we turn to the PDE model, and we prove global existence and convergence results, a key tool being the existence of comparison principles (and the informations obtained on the ODE model). We also provide some properties of the stationary solutions.

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Analysis of a two-layer energy balance model: long time behaviour and greenhouse effect (Part 2)

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We study a two-layer energy balance model, that allows for vertical exchanges between a surface layer and the atmosphere. The evolution equations of the surface temperature and the atmospheric temperature are coupled by the emission of infrared radiation by one level, that emission being partly captured by the other layer, and the effect of all non-radiative vertical exchanges of energy. Therefore, an essential parameter is the absorptivity of the atmosphere, denoted ε_a. The value of ε_a depends critically on greenhouse gases: increasing concentrations of CO_2 and CH_4 lead to a more opaque atmosphere with higher values of ϵ_a.

First we study the associated ODE model, and we prove that global existence of solutions of the system holds if and only if $\varepsilon_a \in (0,2)$, and blow up in finite time occurs if $\varepsilon_a > 2$. (Note that the physical range of values for ε_a is $(0,1]$). Next, we explain the long time dynamics for $\varepsilon_a \in (0,2)$, and we prove that all solutions converge to some equilibrium point. We also present some numerical results showing the classical S-shaped structure of the equilibrium points. Finally, motivated by the physical context, we study the dependence of the equilibrium points with respect to the involved parameters, and we prove in particular that the surface temperature increases monotonically with respect to ε_a. This is the key mathematical manifestation of the greenhouse effect for this toy model.

Finally, we turn to the PDE model, and we prove global existence and convergence results, a key tool being the existence of comparison principles (and the informations obtained on the ODE model). We also provide some properties of the stationary solutions.

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On response theory for climate models

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A methodology to establish response theory for a class of nonlinear stochastic partial differential equations has been provided in [1]. Specifically, it is shown that for a certain class of observables, the averages of those observables against the stationary measure of the SPDE are differentiable (linear response) or, under weaker conditions, locally Hölder continuous (fractional response) as functions of a deterministic additive forcing.

The physical motivation for studying the response to perturbations in the forcings for models in geophysical fluid dynamics comes from climate change and relate to the question as to whether long term statistical properties of the dynamics derived under current conditions will be valid under different forcing scenarios.

In this talk we look at the the key features of the methodology for stochastic PDEs in [1], with a particular interest on its application to quasi-geostrophic models for the ocean and energy balance models for the global temperature.

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Linear response and control of the statistical properties of dynamics

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The concept of linear response of a dynamical system describes how the statistical properties of the system change when a certain "infinitesimal" perturbation is applied to the system. This concept has important relations with the study of climate models and the climate change ([5]).

One inverse problem associated to linear response is the one of finding the best infinitesimal perturbation in order to modify the statistical properties of the system in some wanted direction. This inverse problem can be formalized in different ways (see e.g. [1],[2],[3],[5]]). The existence of an optimal solution has been proved in some classes of deterministic and random systems and numerical methods for its approximation have been shown. In the talk we will briefly discuss the mathematical structure of the problem and review some recent result on the control of the statistical properties of random dynamical systems and expanding maps.

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The Geometry of Time-Dependent Spherical Random Fields

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Space-time random fields defined over the two-dimensional unit sphere $\mathbb{S}^2$ find a wide set of applications in Climate Sciences. Moreover, the investigation of the behavior for geometric functionals of random fields on manifolds has drawn recently considerable attention. In this talk, we consider fluctuations over time for the excursion area and the level curves' length of time-dependent Gaussian spherical random fields. We focus on both long and short memory assumptions, presenting a different behavior starting from the analysis of the Wiener chaos decompositions of the geometric functionals of interest. In particular, in the long memory case, we show that the fluctuations are dominated by a single chaotic component, while in the short memory case, we show that all chaoses contribute in the limit, no variance cancellation occurs and a Central Limit Theorem can be established by Fourth-Moment Theorems and a Breuer-Major argument. The talk is based on the two articles [1, 2].

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Topological analysis of weather extremes

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Volumes in a dynamical system's phase space can stretch, compress, fold or tear; the combination of these processes gives rise to a structure in its space. The topology of this structure is the signature of the mechanisms that act to shape the system's flow in this phase space [1]. Our topological approach uses the novel concept of templex [2], which consists of finding a topological representation for the branched manifold approximated by a cell complex and a directed graph that prescribes the allowed connections between the highest dimensional cells of the complex as a function of the flow. The templex properties include the non-equivalent ways of circulating along the complex, which are essential to provide a complete description of the system's dynamics. The templex provides the key features of the topological structure underlying a dynamical system.

Weather extremes can be classified by two scalars: the instantaneous dimension and the persistence of a state of a dynamical system [3,4]. The computation of these asymptotic scalars for weather observables provides information on the rarity, and persistence of specific states. In this work, we present a first attempt to relate the instantaneous dimension and other local metrics to the topological properties of the templex of the system under study.

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Dynamical units for attractors with high-order symmetries extracted from templex

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Chaotic attractors produced by three-dimensional systems are well characterized by a template that can be viewed as a knot-holder: consequently, the approach was mainly based on knot theory [1,2,3]. Unfortunately, in higher dimensional space, all knot are trivial and this theory can no longer be used. Recently, we introduced a new concept, templex, based on the construction of a complex of cells from a trajectory visiting the chaotic attractor [4,5]. The key novelty was i) to orientate the cell according to the flow, ii) to associate with such a complex the flow which governs the evolution of the trajectory, thus leading to a directed graph which synthetizes most of the topological properties of the attractor. Since the complex can be constructed in any dimension, a templex can be obtained for high-dimensional attractor. By reducing the complex to a *minimal* form, the corresponding reduced digraph allows to extract some dynamical units which are of two types, oscillating (O) and switching (S), and which structure the attractor. Thus, the attractor is viewed as a combination of units based on which a taxonomy of chaotic invariant sets, initially introduced by Otto E. Rössler [6] and recently extended [7], can be generalized. Chaotic attractors with high-order symmetries [8,9] are here investigated in terms of templexes and their structure in terms of O- and S-units are provided.

A climate attractor is treated with this procedure validated with the previous examples.

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Acoustic waves analysis in atmosphere modelling

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Fluid dynamic equations are used to model various phenomena arising from physics, engineering, astrophysics, geophysics. In particular in the case of a geophysical flow, such as the atmospheric flows one feature is that they take place at different time and length scales and it is important to understand which phenomena occur according to the use of single scales or to the interactions of them (i.e. internal gravity waves, Rossby waves, cloud formation). From a mathematical point of view, these various physical behaviors give rise to different singular limits and, consequently to a different analysis of the asymphtotics of the governing equations. In this talk we will focus on a combined low Mach number, low Rossby number limit and we will analyze a very simplified model given by a linearized continuity equation and by the classical momentum equation which include terms that take into account of gravitation and rotation and we will show, according to the values of different scales, that the asymptotic behavior of the model will be those of an incompressible fluid or of a geostrophic flow (see [1]).

The common feature of this kind of limit in the ill prepared data framework is, the presence of high frequency time oscillations along the so called acoustic waves. Those waves are supported by the gradient part of the velocity field, which, as consequence becomes infinte. In this scenario it becomes extremely important to understand under which conditions the behaviour of this waves can be controlled and their presence is harmless. In this talk we will show in which cases those waves have a dispersive behaviour and how dispersion may help in order to recover the necessary compactness of the velocity field.

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A quasi-geostrophic coupled ocean atmosphere model

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Ocean atmosphere coupling and thermodynamical effects are essential to understand low frequency variability on annual and decadal scale. We study well-posedness of a system of PDEs describing the atmosphere by two quasi-geostrophic layers coupled to one further quasi-geostrophic deep ocean layer. Furthermore, there are two equations describing the development of the atmosphere and ocean temperature. More specifically, we consider the model which Vannitsem et al. in [1] used, which is based on previous models by Charney and Strauss '80, Reinhold and Pierrehumbert '82; Pierini '11. These models have been studied intensively numerically, but not analytically. We will describe the first steps in this direction: existence of weak and strong solutions, weak-uniqueness, existence and finite-dimensionality of the global attractor, determining modes. An interesting asymmetry in the unknowns with respect to regularity, dimension of attractor and determining modes emerges.

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Dynamical systems approaches to the study of climate extremes with applications to recent events

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Between 1980 and now, climate and extreme weather events have caused, in Europe, up to 520 billion euros in economic losses, according to a 2022 report released by the European Environment Agency. Many weather extreme events such as heatwaves, droughts, tropical cyclones and cold spells will change in frequency and/or intensity due to the ongoing anthropogenic climate change, driving our planet to unprecedented threats. From a mathematical point of view, such extreme events are difficult to characterize because they are rare and they do occur at specific spatiotemporal scales of the dynamics, not necessarily the largest or the smallest of the climate system. Their study requires a dynamical systems framework capable of tracking their probability of occurrence, predictability and persistence. In this talk I will describe how dynamical systems theory helps in building numerical and theoretical tools for weather extreme events, using recurrences of patterns termed analogues. These tools can be used to study whether climate change do already affect the dynamics of extremes.

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The interaction of physics and mathematics: modelling, solving and simulating large-scale flows

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This talk focuses on one particular model, the semi-geostrophic equations . First proposed in 1948 by meteorologist Arnt Eliassen, the semi-geostrophic equations constitute a model for the formation of fronts in the atmosphere. While these equations have been well studied by the meteorological community, fundamental problems concerning their mathematical analysis remain open. In this talk, I will present a novel constructive proof of the existence of *geostrophic* Lagrangian solutions using semi-discrete optimal transport techniques, for both incompressible and compressible flows. This proof gives rise to a very effective numerical method, in the full 3D setting. It might also offer a pathway to settle the existence and uniqueness of Eulerian solutions, thus addressing a major theoretical open problem.

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Reconstruction of forcings in models of geophysical fluid dynamics

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Data assimilation is a term from the geosciences which refers to estimating the current state of a model (of geophysical fluid dynamics) from current and past observations. In this contribution, we consider data assimilation schemes that not only estimate the underlying states of a dynamical system but simultaneously reconstruct unknown components of the dynamics.

We focus on simple 2D-Navier Stokes and Transport-Diffusion equations (for instance for atmospheric aerosols or tracer gases) and reconstruct forcings or surfacd fluxes, along with the underlying dynamical states. Tracer gases and aerosols play an important role in the dynamics of the atmosphere; tracer gases such as ozone, methane, or CO_2 for instance impact the radiative transfer and are thus linked to the energy budget of the planet ("greenhouse effect"), while aerosols (especially in the lower troposphere) are common pollutants with strong and potentially adverse effects on the environment, human activity, and health.

We discuss two algorithms that both apply in the context of the 2D-Navier Stokes as well as the transport–diffusion equations. The discussed algorithms are very simple and not optimised for performance, but they permit a fully rigorous mathematical analysis. Insights from this analysis can thus inform the design of more complex (and better performing) approaches.

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Background states of the atmosphere: analysis and computation using Optimal Transport

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The decomposition of an atmospheric flow into a background state and perturbations to it plays a central role in several areas of atmospheric science. This talk will describe how the theory of Optimal Transport (OT) can be used to uniquely define and reliably compute such a background state. Specifically, we consider energy minimisers over the class of *Modified Lagrangian Mean* (MLM) states proposed in [1]. Translated in to the language of OT, such states can be viewed as minimisers of a suitable *optimal transport cost*, over a class of *source measures*, each one defined by a free surface. Using this viewpoint, we prove the existence and uniqueness of such states. In fact, optimality conditions imply that this free surface optimisation problem reduces to a standard OT problem, whose solution implicitly defines the energy minimising free surface. Thanks to recent advances in numerical OT, the computation of energy minimising MLM states is therefore numerically tractable.

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Filtering Dynamical Systems Using Observations of Statistics

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We consider the problem of filtering dynamical systems, possibly stochastic, using observations of statistics [1]. Thus, the computational task is to estimate a time-evolving density $\rho(v,t)$ given noisy observations of the true density $\rho^{\dagger}$; this contrasts with the standard filtering problem based on observations of the state v. The task is naturally formulated as an infinite-dimensional filtering problem in the space of densities ρ. However, for the purposes of tractability, we seek algorithms in state space; specifically, we introduce a mean-field state-space model, and using interacting particle system approximations to this model, we propose an ensemble method. We refer to the resulting methodology as the ensemble Fokker–Planck filter (EnFPF).

Under certain restrictive assumptions, we show that the EnFPF approximates the Kalman–Bucy filter for the Fokker–Planck equation, which is the exact solution to the infinite-dimensional filtering problem. Furthermore, our numerical experiments show that the methodology is useful beyond this restrictive setting. Specifically, the experiments show that the EnFPF is able to correct ensemble statistics, to accelerate convergence to the invariant density for autonomous systems, and to accelerate convergence to time-dependent invariant densities for non-autonomous systems. We discuss possible applications of the EnFPF to climate ensembles and to turbulence modeling.

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Galerkin Approximations of Nonlinear DDEs: Bifurcation Analysis and Noise-Driven Chaos

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Delay differential equations (DDEs) are widely used in many applied fields to account for delayed responses of the modeled systems to internal/external factors. In contrast to ODEs, the phase space associated even with a scalar DDE is infinite-dimensional. Oftentimes, it is desirable to have low-dimensional ODE systems to approximate the DDE dynamics. In this talk, we will discuss a recently developed Galerkin scheme for general nonlinear DDEs. The main ingredient is a type of polynomials that are orthogonal under an inner product with a point mass. The associated Galerkin scheme enjoys some nice properties that help reduce the derivation of the convergence results to basic functional analysis exercises. Analytic formulas are available within this approach, which help simplify the numerical treatment.

We will also discuss further dimension reduction using the center manifold technique for DDE bifurcation analysis. The developed framework leads to a rigorous and computationally efficient way to approximate the Stuart-Landau normal forms for the considered DDEs. We will show how insights gained from such normal forms can help design stochastic perturbations to further enrich the time variability of the otherwise periodic DDE dynamics. The approach will be illustrated on a cloud-rain DDE model in the context of Hopf bifurcations and noise-driven chaos, and also on a delay model of the El Niño-Southern Oscillation (ENSO) in the context of saddle-node bifurcation of periodic orbits and noise induced decadal variability.

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A pollution model on Network

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We study the joint determination of optimal investment and optimal depollution in a spatiotemporal framework where pollution is transboundary and spatial component is described by a network structure. Pollution is controlled at a global level by a central planner. The problem is solved explicitly and the optimal investment and depollution are found. In conclusione, some investigations on the impact of heterogeneity on the optimal path are performed.

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Analysis and control for degenerate evolution equations with applications to climate sciences

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In this talk we study the Energy Balance Climate Models (EBCM), in particular we consider the Budyko-Sellers model, that is a particular kind of EBCM.
We talk about some results concerning the approximate multiplicative controllability of degenerate reaction-diffusion equations with applications to the 1-D Budyko-Sellers model.
We also introduce a new version of the Budyko-Sellers model using time-fractional degenerate heat equations to simulate anomalous heat diffusion in several situations.

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Recent Advances in Biological, Epidemiological and Social Dynamics
Special Session B26

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Innovative mathematical models for biological, medical, epidemiological, and social dynamics are now more crucial than ever. In recent years, such models have found wide application in, for example, the study of infectious diseases (COVID-19 in particular), the behavior of large crowds, online dissemination of misinformation, and the development of new immunotherapies. This Special Session aims to collect the most recent efforts of the mathematical community to produce reliable models spanning a wide range of topics relevant to the biological and social sciences. We will explore diverse classes of models: including, but not limited to, models based on ordinary and partial differential equations, networks, kinetic theory, and agent-based approaches, as well as modern data-driven techniques. Aspects relating to phenomenological modeling, mathematical analysis, model calibration and validation, as well as applications will be considered. The ultimate goal of the Special Session is to foster the exchange of ideas among researchers with diverse mathematical expertise, working in a broad range of application areas. The Special Session also promotes the activities of the UMI Group Modellistica Socio-Epidemiologica (*Socio-Epidemiological Modeling Group*).

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Epidemic models incorporating the role of individuals' viral load

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In classical epidemic models, a neglected aspect is the heterogeneity of disease transmission and progression linked to the viral load of each infectious individual. Here, we attempt to investigate the interplay between the evolution of individuals' viral load and the epidemic dynamics from a theoretical point of view. In the framework of multi–agent systems [3], we propose a particle stochastic model describing the infection transmission through interactions among agents and the individual physiological course of the disease. Agents have a double microscopic state: a discrete label, that denotes the epidemiological compartment to which they belong, and a microscopic trait, representing a normalized measure of their viral load. Specifically, we consider Susceptible–Infected–Removed–like dynamics where the disease transmission rate [2] or the isolation rate [1] of infectious individuals may depend on their viral load. We derive kinetic evolution equations for the distribution functions of the viral load of the individuals in each compartment, whence, via suitable upscaling procedures, we obtain a macroscopic model for the densities and viral load momentum. We perform then a qualitative analysis of the ensuing macroscopic model, and we present numerical tests in the case of both constant and viral load–dependent model parameters.

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Recent advances on behavioural integral epidemic models with information index

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The information index has been introduced and developed by P. Manfredi, A. d'Onofrio and coauthors in a series of papers starting from 2007 [1,4,5]. It is a distributed delay which describes the opinion-driven human behavioural changes [6,7]. The information index can be considered as an extension of the concept of prevalence-dependent contact rate due to V. Capasso in the seventies [3]. In this talk, we discuss integral models describing the epidemic propagation of an infectious disease. The models are behavioural in the sense that the constitutive law for the Force of Infection includes an information index. We show some results obtained via qualitative analysis and numerical simulations. In particular, we show that when the memory of the past values of the infection is exponentially fading, the stability of an endemic state is guaranteed. Through numerical simulations, we show that self-sustained oscillations may arise when the memory is more focused in the disease's past history [2]. This research is in collaboration with E. Messina, C. Panico (University of Naples Federico II) and A. Vecchio (IAC-CNR).

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Advanced compartmental models for pandemic management

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The COVID-19 pandemic highlighted the need to quickly respond, via public policy, to the onset of an infectious disease breakout. Deciding the type and level of interventions a population must consider to mitigate risk and keep the disease under control could mean saving thousands of lives. Here we provide a framework for capturing population heterogeneity whose consideration may be crucial when developing a mitigation strategy based on non-pharmaceutical interventions. Precisely, we use age-stratified data to segment our population into groups with unique interactions. Our model will be fit to census data for the state of New Jersey (but could be easily adapted to other states and countries), thus we consider 7 different groups corresponding to age brackets of census data denoted by a subscript, e.g. S_j, $j = 1, \dots, 7$. The interactions among different age groups will be encoded by a matrix $L = (l_{k,j})$, where $l_{k,j}$ quantifies the interactions that group k is having with group j. Let us denote by $N_j = S_j + E_j + I_j + R_j$ the total population in the j-th age group (which is constant), and set $\widetilde{N}_j = \sum_{k=1}^{h} l_{k,j}\, N_k$, which represents the total number of individuals the j-th susceptibles are interacting with. Then we set:

$$
\begin{cases}
\dot{S}_j = -u\, \dfrac{\beta S_j}{\widetilde{N}_j} \displaystyle\sum_{k=1}^{h} l_{k,j}\, I_k \\
\dot{E}_j = u\, \dfrac{\beta S_j}{\widetilde{N}_j} \displaystyle\sum_{k=1}^{h} l_{k,j}\, I_k - \delta E_j \\
\dot{I}_j = \delta E_j - \gamma I_j \\
\dot{R}_j = \gamma I_j
\end{cases}
\tag{1}
$$

for $j = 1, \dots, h$, where $1 - u$ is the lockdown rate, β the infection rate, L the interaction matrix, δ the latent period, and γ the recovery rate. We sub-divide the interaction matrix $l = (l_{k,j})$ into a well-known socially driven subset of uniquely interacting groups. We define a set of l^i to be n interaction matrices where $l^i_{k,j}$ encodes some portion of the interaction between groups k and j due to the i-th subset of interactions. It holds that $\sum_{i=1}^{n} l^i_{k,j} = l_{k,j}$ for every k, j and thus $l = \sum_{i=1}^{n} l^i$. Each new interaction matrix brings the ability to encode the regulation of that subgroup of interactions, and we will denote by u_i the "amount of allowed interactions" such that the severity of lockdown for each group of interactions is $1 - u_i$. Therefore our final model reads

$$
\begin{cases}
\dot{S}_j = -\beta \dfrac{S_j}{\widetilde{N}_j} \left(u_1 \displaystyle\sum_{k=1}^{h} l^1_{k,j}\; I_k + u_2 \sum_{k=1}^{h} l^2_{k,j}\; I_k + \dots + u_n \sum_{k=1}^{h} l^n_{k,j}\; I_k \right) \\
\dot{E}_j = \beta \dfrac{S_j}{\widetilde{N}_j} \left(u_1 \displaystyle\sum_{k=1}^{h} l^1_{k,j}\; I_k + u_2 \sum_{k=1}^{h} l^2_{k,j}\; I_k + \dots + u_n \sum_{k=1}^{h} l^n_{k,j}\; I_k \right) - \delta E_j \\
\dot{I}_j = \delta E_j - \gamma I_j \\
\dot{R}_j = \gamma I_j
\end{cases}
\tag{2}
$$

with $j = 1, \dots, h$ and other parameters set as in (1).

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Balance Laws in Biological Models

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Balance laws, i.e., partial differential equations of the form $\partial_t u + \nabla \cdot f(t, x, u) = g(t, x, u)$, appear in a variety of models devoted to epidemiology, structured population dynamics, cell growth, cluster formation and predator - prey or pursuit - evasion games. They may contain local and non local terms, be set in unbounded or bounded domains with different boundary conditions, and be coupled with equations of other types.

The present talk is devoted to analytic results establishing the well posedness of models based on balance laws, with numerical integrations showing the qualitative behavior of solutions in specific cases. A class of predator - prey models motivated by pest control and some structured epidemiological models will be considered in detail.

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Kinetic formulation of cross-diffusion models for predator–prey dynamics

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We introduce a mesoscopic framework for the modeling of predator–prey dynamics in a two-dimensional space, where the different species evolve via both intraspecific and interspecific interactions. We will show how to combine in a suitable way diffusion- and fast-reaction-type limits to formally derive, at a macroscopic level, reaction–cross-diffusion systems involving a Beddington–DeAngelis-like functional response.

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Modeling some aspects of stem cell therapy in cardiac tissue regeneration

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An acute myocardial infarction (MI) is followed by a multiphase reparative response, in which, in particular, the damaged tissue is replaced with a fibrotic scar composed of cardiac fibroblasts and collagens. This eventually leads to impaired cardiac function, bringing to adverse consequences such as cardiac atrophy and arrhythmogenicity

In the meantime, the damaged tissue tries to recover and the immune system responds by emitting damage signals so that new cells come to the aid. These signals are the chemokines.

Several studies show that stem cell therapies can help tissue repair, after an acute myocardial infarction and are used as an alternative to traditional pharmacological management of myocardial infarction.

Stem cells have the capacity to regenerate cardiac tissue and improve the function of the damaged heart. Thus clearly the impact of the chemokines in the attraction of these cells toward the damaged area of the myocardium becomes very important.

However several biological aspects are not yet completely understood and are still under investigation.

For example, it was not yet completely clarified if the decline in cardiac fibrosis was due to replacement of dead cardiomyocytes by the stem cells (they can divide and become specialized cells, such as cardiomyocytes), or because of the direct effects of paracrine factors released from the transplanted stem cells on the extracellular matrix and on other cells and molecules participating to the tissue repair [1].

In this talk we present two mathematical models describing two different effects of stem cells on the damaged cardiac tissue.

On the one hand we generalize a recent model [2], which describes the rebuilding of the cardiac tissue by means of the differentiation of the stem cells, which replace the dead cardiomyocytes. In the new model the chemotactic effect of the chemokines on the stem cells is taken into consideration.

On the other hand we generalize a recent mathematical model [3], based on a system of PDEs, which describes the processes happening in the damaged area and bringing to the formation of fibrotic scars. In our new model we take into consideration the paracrine effects of the stem cells, in order to reduce the fibrotic scar and restore the functionality of the heart. For the paracrine effetcs, we follow in particular the paper [4], which proposes a mathematical model, based on ODEs, related to the stem cell therapy after myocardial infarction.

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On online-offline models of spread of tension and social outbursts

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Social collective behavior, like boycotts or protests, is a global phenomenon. Lately, the online spread of information and social tensions has altered the dynamics of such events. In this talk, we will introduce models that link online and offline dynamics, exploring spreading rates of these activities. Additionally, we will delve into how the online network structure impacts offline activity dynamics.

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Condensation effects in social dynamics

Mattia Zanella [1]
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In this talk, we discuss a class of models to understand the impact of population size on opinion formation dynamics, a phenomenon usually related to group conformity. To this end, we introduce a new kinetic model in which the interaction frequency is weighted by the kinetic density. In the quasi-invariant regime, this model reduces to a Kaniadakis-Quarati-type equation with quadratic drift, originally introduced for the dynamics of bosons in a spatially homogeneous setting. From the obtained PDE for the evolution of the opinion density, we determine the regime of parameters for which a critical mass exists and triggers blow-up of the solution. Therefore, the model is capable of describing strong conformity phenomena in cases where the total density of individuals holding a given opinion exceeds a fixed critical size. In the final part, several numerical experiments demonstrate the features of the introduced class of models and the related consensus effects.

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New Advances in the Mathematical Theory of Control. Special Session B27

Monica Motta
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Peter Wolenski
Louisiana State University, USA

This session is devoted to recent advances in optimal theory, including optimality conditions and invariance properties in nonstandard problem formulations. It is scheduled on July 25-26.

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Optimal controls over sweeping processes and exponential penalty method for the truncated sweeping sets

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We consider optimal control problems (P) involving a perturbed sweeping process governed by moving, nonsmooth, prox-regular, and possibly *unbounded* sweeping sets, C(t), and having *joint* endpoint constraints. More specifically, $C(t)$ is the intersection of finitely many sub-levels of smooth functions.

When the process is coupled with a standard control system, this general model incorporates different sub-models as particular cases, including second order sweeping processes, a subclass of integro-differential sweeping processes, evolution variational inequalities (EVI), and dynamical variational inequalities (DVI). Note that the presence of the discontinuous and unbounded normal cone $N_{C(t)}$ in the sweeping process renders inapplicable all the known optimality results in optimal control over standard differential inclusions.

In addition to the discrete-time approximation, the method of continuous-time exponential penalty approximation has shown to be instrumental in deriving Pontryagin-type maximum principle for special cases of (P).

The goal of this talk is to demonstrate that when deriving the Pontryagin-type maximum principle for a strong local minimum $(\bar{x}, \bar{u})$ of (P) it is greatly beneficial to develop the exponential penalty approximation not for the sweeping set $C(t)$ but rather for $C(t) \cap \bar{B}_{\bar{\varepsilon}}(\bar{x}(t))$, that is, the truncation of $C(t)$ by a ball centered at the state of the strong local minimum and having a specific radius $\bar{\varepsilon}$ carefully chosen so that the exponential penalty method can me modified successfully. Consequently, the so-obtained Pontryagin-type principle turns out to be valid under minimal local assumptions at or around the optimal state $\bar{x}$, and furthermore, the addition of the extra constraint to $C(t)$ due to the truncation, does not require *any* extra assumption, not even on the constraint qualification.

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OPTIMAL CONTROL OF PERTURBED SWEEPING PROCESSES WITH APPLICATIONS TO GENERAL ROBOTICS MODELS

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Boris Mordukhovich
Department of Mathematics, Wayne State University

Dao Nguyen
Department of Mathematics and Statistics, San Diego State University

Trang nguyen
Department of Mathematics, Wayne State University

This talk primarily focuses on the practical applications of optimal control theory for perturbed sweeping processes within the realm of robotics dynamics. By describing these models as controlled sweeping processes with pointwise control and state constraints and by employing necessary optimality conditions for such systems, we formulate optimal control problems suitable to these models and develop numerical algorithms for their solving. Subsequently, we use the Python Dynamic Optimization library GEKKO to simulate solutions to the posed robotics problems in the case of any fixed number of robots under different initial conditions.

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Applications and simulations in optimal control

Norma Ortiz-Robinson
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In this talk I will discuss a few recent projects involving modeling of applications with optimal control and their simulation using the Python dynamic optimization library GEKKO.

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Solutions to the Hamilton-Jacobi equation for dynamic optimization problems with discontinuous time dependence

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We will provide characterizations of the value function as the unique lower semicontinuous solution, appropriately defined, to the Hamilton-Jacobi equation associated with some classes of dynamic optimization problems with discontinuous time dependence. We shall consider optimal control problems of Bolza type in which the running cost is possibly non Lipschitz continuous w.r.t. the state variable and Calculus of Variations problems in which the functional to minimize comprises an end-point cost function and an integral term involving a nonautonomous Lagrangian. We shall give also some illustrative examples.

This talk is based on recent results obtained in collaboration with J. Bernis, C. Mariconda and R. Vinter.

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An Optimal Control Approach to the Problem of the Longest Self-Supporting Structure

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Michele Palladino
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Pierangelo Marcati
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The characterization of the self-supporting slender structure with the furthest length is of interest both from a mechanical and biological point of view. Indeed, from a mechanical perspective, this classical problem was developed and studied with different methods, for example using similarity solutions and stable manifolds. However, none of them led to a complete analytical solution. On the other hand, plant structures such as tree branches or searcher shoots in climbing plants can be considered elastic cantilevered beams. In this paper, we formulate the problem as a non-convex optimisation problem with mixed state constraints. The problem is solved by analysing the corresponding relaxation. With this method, it is possible to obtain an analytical characterization of the cross-section.

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Strong local optimality in state constrained optimal control problems

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In several optimal control problems coming from realistic cases, the state of the system is subject to inequality constraints. In such case, a suitable, non trivial, extension of the celebrated Pontryagin Maximum Principle (PMP) gives a necessary condition for optimality (in the spirit of KKT conditions). However, it is well known that, except for special low-dimension cases, the PMP identifies many non-optimal trajectories as extremals: that is why the optimal control community is active in the search of additional necessary and sufficient optimality conditions.

We address the problem of strong local optimality for state constrained optimal control problems where the dynamics is affine with respect to the control:

$$\begin{aligned}&\dot{\xi}(t) = f_0(\xi(t)) + u(t) f_1(\xi(t)) \quad \text{a.e. } t \in [0, T],\\ &\xi(0) = x_0, \quad \xi(T) \in \mathcal{N}_f,\\ &c(\xi(t)) \leq 0 \quad \forall t \in [0, T], \qquad\qquad |u(t)| \leq 1 \quad \text{a.e. } t \in [0, T].\end{aligned}$$

Here the state space is a smooth manifold M, the function $c\colon M \to \mathbb{R}$ defining the state constraint is assumed to be smooth on a neighborhood of its zero-level set; the vector fields f_0, f_1 are smooth and $\mathcal{N}_f$ is a submanifold of M. The cost to be minimized can be either a Mayer cost or the minimum time to reach $\mathcal{N}_f$, i.e. we consider both the following problems

$$\text{minimize} \quad \psi(\xi(T)), \quad T > 0 \text{ fixed},$$

$$\text{minimize} \quad T, \quad T > 0 \text{ free}.$$

More precisely we aim at giving sufficient conditions for the strong local optimality of an extremal which is given by the concatenation of some internal arcs and a boundary one.

The given conditions are proven to be sufficient via Hamiltonian methods, in the case when the extremal is the concatenation of an internal bang arc, followed by a boundary arc and then by a finite number of internal bang arcs. Currently we are investigating the case when in between the internal bang arcs and the boundary arc there occurs an internal singular arc.

The provided sufficient conditions are regularity conditions on the junction points between the boundary arc and the internal ones, which allow us to use a simpler version of PMP for state-constrained problems plus conditions on the order of the boundary arc, a strengthened version of the necessary conditions along the internal arcs, and the coerciveness of a suitable quadratic form (which is finite-dimensional in the case when no internal singular arc occurs).

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Can we locally optimize a second order decrease rate of a function for nonlinear and symmetric control systems?

Mauro Costantini, Pierpaolo Soravia [1]
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When a control system has all its vector fields tangent to the level set of a given smooth function u at a point $\hat{x}$, that function can still have a negative rate of decrease with respect to the trajectories of the control system in appropriate sense. In the case when the system is symmetric and u has a decrease rate of the second order, we investigate the existence of a best possible rate in the class of piecewise smooth trajectories. The problem turns out to be purely algebraic, and depends on the eigenvalues of matrices constructed from a basis matrix whose elements are the second order Lie derivatives of u at $\hat{x}$ with respect to the vector fields of the system.

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Directionality of constrained control systems and discontinuities in clearance

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Jeremy LeCrone
Department of Mathematics and Statistics, University of Richmond

Control theory studies the evolution of dynamical systems which are actively influenced by some external agent (or controller). We will discuss control systems with kinodynamic constraints on admissible trajectories, wherein one encounters obstacles in state space which must be avoided as the system evolves. In this setting, one defines a system-dependent clearance function quantifying the shortest admissible distance to the obstacle set. We will focus on points of discontinuity in the clearance function and how these discontinuities are experienced as one traverses admissible trajectories. This investigation of discontinuities will leads us to explore a common directionality condition for velocities at a point, characterized by strict positivity of the minimal Hamiltonian.

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Envelope Generators and Clearance Irregularities in State Constrained Control Systems

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We consider nonlinear state–constrained control problems. Specifically, we investigate irregularities of the system–dependent clearance function, i.e. the value function quantifying the minimal cost to reach the constraining obstacle set (or equivalently, the target set).

In this talk, I will present a set of sufficient conditions demonstrating how local directionality in system dynamics, together with corresponding configurations for the obstacle set, guarantee the existence of clearance discontinuities both on the obstacle boundary and propagating out into free space. A selection of applications will be explored. We will discuss further questions regarding interactions between the structure of clearance discontinuities and medial axis structures, which may be associated with sets of irregularities in the gradient of clearance.

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The minimal time problem of a sweeping process with a discontinuous perturbation

Peter Wolenski, Vinicio Ríos
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Since their inception in the seventies ([2, 3]), sweeping processes have become one of the most popular categories of dynamical systems. Their adaptability to explain nonsmooth phenomena has been a source of motivation for research in areas such as nonsmooth analysis, differential inclusions, control theory, and its associated optimization paradigm, namely, optimal control theory. In this talk, we consider an important instance of the latter framework: the minimal time problem. Specifically, we discuss for the first time the minimal time function of a sweeping process with a state constraint set that is constant (commonly referred to as the *reflecting boundary problem* [5]) and prox-regular, and whose right-hand-side is being perturbed by a discontinuous multifunction satisfying the *Dissipative-Lipschitz property*. Our analysis follows the traditional route based on the invariant properties of the epigraph and hypograph of the minimal time function, which has been successful in characterizing such a function as a proximal solution of a Hamilton-Jacobi equation for different types of dynamics (e.g., Lipschitz differential inclusions [6], Dissipative-Lipschitz differential inclusions [4], and sweeping processes with Lipschitz perturbations [1]). Accordingly, we use the prox-regularity of the state constraint set in combination with the dissipative-Lipschitz condition of the perturbing multifunction to establish the first Hamiltonian characterization of strong invariance for sweeping processes with discontinuous perturbations. This result leads to an interpretation of the minimal time function through a pair of Hamilton-Jacobi inequalities, one of which exhibits a limiting component that is intrinsic to the discontinuity of the system.

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Consensus and control for Hegselmann-Krause type models

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We analyze the asymptotic behavior of a Hegselmann-Krause opinion formation model with delayed interactions. Under appropriate assumptions, an exponential consensus estimate is proven without any smallness restrictions on the time delay size. Since the consensus estimates are independent of the agents' number, we are able to extend the stability result to the continuum model obtained as the mean-field limit of the many-individual equation as the number of individuals goes to infinity. Some extensions to Hegselmann-Krause models on network topologies are also discussed. Finally, we illustrate a controllability result for a Hegselmann-Krause opinion formation model with leadership.

This talk is based on joint papers with Young-Pil Choi (Yonsei University, Republic of Korea), Chiara Cicolani (University of L'Aquila), Elisa Continelli (University of L'Aquila), and Alessandro Paolucci.

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A single player and a mass of agents: a pursuit-evasion-like game

Fabio Bagagiolo
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In this talk I will speak about a finite-horizon differential game of pursuit-evasion type, between a single player and a mass of agents. The player and the mass directly control their own evolution which, for the mass, is given by a first order PDE. By dynamic programming and the use of an adapted concept of non-anticipating strategies, I will derive an infinite dimensional Isaacs equation and address the problem of uniqueness of the value function as viscosity solution on a suitable invariant subset of a Hilbert space.

The novelty of the problem and of the results mainly relies on the derivation of the infinite-dimensional Isaacs equations and on its study in the Hilbert setting. A preliminary study of the controlled evolution of the mass, by the evolution of its density as time-varying state-dependent function, is also interesting. A couple of one-dimensional examples will enlighten the complexity of the game and of the model.

This research is conducted jointly with Rossana Capuani (University of Arizona, USA) and Luciano Marzufero (University of Bolzano, Italy). Some ongoing further research directions will be mentioned.

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Optimal control of moving sets

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In the classical Dido's problem we are given a set $V \subset \mathbb{R}^2$ and a number $\ell > 0$: among all curves $\gamma \subset V$ with length ℓ, we seek one which bounds a subset $\Omega \subset V$ with largest possible area. We consider a time-dependent version of this problem, modeling the optimal control of an invasive population. V is interpreted as an island, which is infested by an invasive species (for example, malaria-carrying mosquitoes). By spraying pesticides, we can progressively shrink the contaminated region $\Omega = \Omega(t)$, until it eventually becomes empty set. The main goal is to describe the optimal strategies $t \mapsto \Omega(t)$, relying on a set of necessary conditions for optimality.

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Lie brackets and state constraints

Franco Rampazzo
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The classical *inward pointing condition* (IPC) for a control system whose state x is constrained in the closure $C := \bar{\Omega}$ of an open set Ω prescribes that at each point of the boundary $x \in \partial\Omega$ the intersection between the dynamics and the interior of the tangent half-space of $\bar{\Omega}$ at x is nonempty. Under this hypothesis, for every system trajectory $x(.)$ on a time-interval $[0, T]$, possibly violating the constraint, one can construct a new system trajectory $\hat{x}(.)$ that satisfies the constraint and whose distance from $x(.)$ is bounded by a quantity proportional to the maximal deviation $d := \text{dist}(\Omega, x([0, T]))$. When (IPC) is violated, the construction of such a constrained trajectory is not possible in general. However, in this paper we prove that a "higher order" inward pointing condition involving Lie brackets of the dynamics' vector fields –together with a non-positiveness curvature-like assumption– allows for a novel construction of a constrained trajectory $\hat{x}(.)$ whose distance from the reference trajectory $x(.)$ is bounded by a quantity proportional to $\sqrt{d}$. As an application, we establish the continuity *up to the boundary* of the value function V of a connected optimal control problem, a continuity that allows to regard V as the *unique* constrained viscosity solution of the corresponding Bellman equation.

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HJ inequalities involving Lie brackets and feedback stabilizability with cost regulation

Giovanni Fusco, *Monica Motta*, *Franco Rampazzo*
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With reference to an optimal control problem where the state has to approach asymptotically a closed target while paying a non-negative integral cost, we propose a generalization of the classical dissipative relation that defines a Control Lyapunov Function to a weaker differential inequality. The latter involves both the cost and the iterated Lie brackets of the vector fields in the dynamics up to a certain degree $k \geq 1$, and we call any of its (suitably defined) solutions a *degree-k Minimum Restraint Function*. We prove that the existence of a degree-k Minimum Restraint Function allows us to build a Lie-bracket-based feedback which sample stabilizes the system to the target while *regulating* (i.e., uniformly bounding) the cost.

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Minimal Time Problems with Piecewise Constant Dynamics

Clinten Graham, Claire Pearson, and Dr. Wolenski, Claire Pearson [1]
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We study the problem of minimizing transit time in control systems characterized by region-specific dynamic constraints developed in [1]. The state space is partitioned into polytopes endowed with convex velocity sets. Within each polytope, velocities must satisfy a constant differential inclusion so that time-optimal refraction between regions follows a generalized form of Snell's Law derived via convex duality. This talk focuses on deriving geometric optimality conditions and studying classes of minimal time trajectories between adjacent regions.

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Piecewise Constant Trajectory Optimization via Computational Convex Analysis

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We discuss software approaches to computational convex analysis developed for solving minimal-time problems with piecewise constant dynamics [1]. A symbolic convex analysis toolkit in Mathematica is presented for calculating gauge and support functions of convex sets. We detail procedures for solving optimality conditions involving normal cones and subgradients. The second part of the talk is devoted to numerical solutions to the multi-region problem using a MATLAB framework. Finding exact multi-region solutions is challenging due to the combinatorial selection of interfaces. Still, this problem is convex and solvable for fixed sequences of region indices via mathematical programming. While the sequential problem appears intractable by variational techniques, our work leverages the Quickhull algorithm to numerically determine optimal sequences of region transfer. Using this method, we characterize the boundary of the reachable set and describe optimal trajectories.

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Piecewise constant control with convex bifunctions

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Convex bifunctions were introduced by Rockafellar [1, Section 29] as a means to unify generalized convex programs by distinguishing between variables and parameters. They can be viewed as a generalization of a multivalued mapping that assigns a cost to each of its elements. This talk will discuss how minimal time problems (called the Elvis problem in [2]) can be solved in two mediums, each having its own bifunction $(Fr)(\mathbf{v})$ that records the cost of using the velocity vector $\mathbf{v}$ for the time $r > 0$. The Elvis problem is the case where $(Fr)(\mathbf{v}) := r\left\{1 + \mathcal{I}_F\left(\frac{v}{r}\right)\right\}$, where $\mathcal{I}_F$ is the indicator of the available velocity set.

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Algebraic Coding Theory
Special Session B28

Matteo Bonini
Aalborg University, DENMARK

Gretchen L. Matthews
Virginia Tech, USA

Ferdinando Zullo
Università degli Studi della Campania *Luigi Vanvitelli*, ITALY

This special session will focus on the interactions between algebraic coding theory and finite geometry and algebraic geometry. These research areas developed together, each of them playing a crucial role in the advancement of the other. Renowned examples are given by the non-existence of the projective plane of order 10, proved via the study of certain error-correcting codes, the celebrated MDS conjecture. Geometric tools have been used to solve the known cases, and the use of algebraic varieties in positive characteristic to define codes which are asymptotically good.

Concretely, speakers of this special session will present recent developments on algebraic and geometric methods used in coding theory, the study of special linear codes together with their dual geometric counterparts, and the problem of designing error-correcting codes with certain incidence and geometric structures and algebraic curves or, more generally, varieties in positive characteristic.

The aim of this session is to bring together leading experts in the fields of coding theory, finite geometry and algebraic geometry in positive characteristic in order to present contemporary research directions on these topics. All levels of seniority are represented, from early-stage to more experienced researchers.

For more information visit `https://matteobonini11.wixsite.com/actumiams2024`.

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Introductory Talk

Matteo Bonini
Aalborg University, DENMARK

I will give an introductory talk for this session.

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Scattered spaces, polynomials, and MRD codes

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Linear sets find widespread use across diverse mathematical domains, such as Finite Geometry and Coding Theory. Among these, scattered linear sets hold particular significance. This presentation will delve into recent findings concerning exceptional scattered polynomials, scattered sequences, MRD codes, and their interplay with algebraic geometry over finite fields. In many instances, these results have been achieved through insightful polynomial characterizations of the underlying entities.

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On Weierstrass semigroups and good AG codes

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Ideas from algebraic geometry became useful in coding theory after Goppa's construction [3]. He had the beautiful idea of associating to a curve $\mathcal{X}$ defined over $\mathbb{F}_q$, the finite field with q elements, a code C. This code, called Algebraic-Geometric (AG) code, is constructed from two divisors D and G on $\mathcal{X}$, where one of them, say D, is the sum of n distinct $\mathbb{F}_q$-rational points of $\mathcal{X}$. It turns out that the minimum distance d of C satisfies

$$d \geq n - \deg(G).$$

This is one of the main features of Goppa's construction.

In the theory of Algebraic-Geometric codes, Weierstrass semigroups are crucial for defining bounds on the minimum distance, as well as for defining improvements on the dimension. To ensure good performance, the divisors defining such codes have to be carefully chosen, exploiting the rich combinatorial and algebraic properties of curves. We present some recent examples of the application of Weierstrass semigroups to the construction of AG codes.

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Towards the classification of maximum scattered linear sets of $\mathrm{PG}(1,q^5)$ and its implications in coding theory

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It is known that every linear set of the projective space is either a subgeometry or can be obtained as a projection of a subgeometry [5]. In [2, 7, 4], it was described how the properties of the projection vertex reflect in those of the linear set, especially with regard to those contained in $\mathrm{PG}(1,q^n)$. If $n \leq 4$, this approach led to a complete classification of them, see [1, 3].
In this talk, using various techniques borrowed from linear algebra, projective geometry, algebraic geometry over finite fields and exploiting the above mentioned approach, I will provide a classification result for scattered linear sets of maximum size in $\mathrm{PG}(1,q^5)$.
Finally, using the connections between such linear sets and maximum rank distance codes (shortly MRD codes) established in [6], I will discuss the implications of relevant results for $\mathbb{F}_{q^5}$-linear MRD codes with length 5 and minimum distance $d = 4$.

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On the decoding of Rank-metric Reed-Muller codes

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In 2021, Augot, Couvreur, Lavauzelle and Neri [1] proposed a construction of rank metric analogues of Reed–Muller codes from twisted group algebras associated to a Galois extension. However, they did not provide an efficient decoding algorithm correcting up to half the minimum distance.

In this talk, we propose two approaches addressing this issue. On one hand, for extensions with Galois group $(\mathbb{Z}/2\mathbb{Z})^n$, we identify common features shared with binary Reed–Muller codes with in particular a recursive structure in the spirit of Plotkin's $(u \mid u+v)$ construction. These observations lead to a decoding algorithm correcting up to half the minimum distance with a high probability. On the other hand, for extensions whose Galois group are a product of two cyclic groups, we propose a completely new approach based on Dickson matrices permitting to correct any error pattern up to half the minimum distance.

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The geometry of intersecting codes: bounds and constructions

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Intersecting codes are linear codes in which every pair of nonzero codewords has a non-trivially intersecting support. They are a classical subject in coding theory introduced in [4,5] and extensively explored in subsequent articles (see, e.g., [2,3,7]), primarily focusing on the binary case. In this context, such codes coincide with minimal codes, which have been intensively studied over the past 20 years. Intersecting and minimal codes find several practical applications: facilitating communication over AND channels, usage in secret sharing schemes, and connections to other structures such as frameproof codes [1] and $(2, 1)$-separating systems [6].

In this talk, we primarily delve into the geometric interpretation of intersecting codes, a subject largely unexplored until now. It is well-known that a nondegenerate linear code can be associated with a set of points (with multiplicities) in a projective space, and some coding-theoretical properties can be understood geometrically. This perspective bridges MDS codes with problems in projective geometry (the renowned MDS conjecture was initially formulated as such in [8]), covering problems with saturating sets, minimal codes with strong blocking sets, and so forth. Intersecting codes correspond to sets of points not contained within any pair of hyperplanes. We term such sets as *non-2-cohyperplanar*. This geometric interpretation of intersecting codes not only aids in visualizing fundamental properties but also paves the way for the introduction of novel constructions.

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Intersecting codes and their applications to additive combinatorics and factorization theory

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Intersecting codes are linear codes in which every pair of nonzero codewords has a non-trivially intersecting support. They are a classical subject in coding theory introduced in [4,5] and extensively explored in subsequent articles (see, e.g., [2,3,7]), primarily focusing on the binary case. In this context, such codes coincide with minimal codes, which have been intensively studied over the past 20 years. Intersecting and minimal codes find several practical applications: facilitating communication over AND channels, usage in secret sharing schemes, and connections to other structures such as frameproof codes [1] and $(2, 1)$-separating systems [6].

In this talk, building on a connection already explored in [8, 9], we explore the link between intersecting codes and the problem of the 2-wise Davenport constant: considering a sequence of elements from an elementary abelian group, it is possible to construct a matrix. Interpreting this matrix as the parity-check matrix of a code, we observe that codewords correspond to weighted zero-sum subsequences. The problem of determining the 2-wise Davenport constant is then equivalent to determining for which parameters intersecting codes exist. This allows us to deduce properties of the 2-wise Davenport constant from the properties of intersecting codes. Applying this knowledge to nonunique factorization theory in Dedekind rings, using the classical example of the ring of integers of an algebraic number field, we give some new bounds on the number of prime ideals in the factorization of the product of two irreducible elements.

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Subfield-Metric Codes

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Motivated by applications in quantum error correction, in [3] the authors introduced the subfield metric. Given positive $\lambda \in \mathbb{R}$, the λ-subfield weight wt_λ of an element x of the finite field $\mathbb{F}_{q^m}$ is 0 if $x = 0$, is 1 if $x \in \mathbb{F}_q^\times$, and takes the value λ otherwise. This weight assignment hence induces the partition $\mathbb{F}_{q^m}/\theta = \{\{0\}, \mathbb{F}_q^\times, \mathbb{F}_{q^m} \backslash \mathbb{F}_q\}$. It is extended additively to yield a weight function on $\mathbb{F}_{q^m}^n$, that is $\mathrm{wt}_\lambda(x) := \sum_{j=1}^n \mathrm{wt}_\lambda(x_j)$ for all $x = (x_1, \ldots, x_n) \in \mathbb{F}_{q^m}^n$. If $\lambda \geq 1/2$, it was shown in [3] that the induced distance function $\mathrm{d}_\lambda(x, y) := \mathrm{wt}_\lambda(x - y)$ for all $x, y \in \mathbb{F}_{q^m}^n$ is a distance function on $\mathbb{F}_{q^m}^n$. For applications in quantum error correction, the most interesting case appears to be $m = 2$ and $\lambda > 1$.

We show that the subfield metric satisfies the extension property and that this can be proved both by a direct argument and also by an application of a result of Dyshko and Wood (see [2, Theorem 5.1]).

Theorem 1. *Let C and D be a pair of $\mathbb{F}_{q^m}$-$[n, k]$ codes. Then C and D are isometric by an $\mathbb{F}_{q^m}$-linear map $\phi : C \longrightarrow D$ if and only if ϕ is the restriction to C of an $\mathbb{F}_q$-monomial transformation of $\mathbb{F}_{q^m}^n$.*

We next consider code optimality in relation to this metric. We describe a simple class of Plotkin-optimal codes. We obtain a duality result for linear codes for the subfield metric. We apply the methods of [1] to obtain explicit expressions for transform polynomials, which applied to the distance distribution of a subfield metric code, gives a linear programming bound. We demonstrate the effectiveness of this bound with some computations comparing its output to existing bounds in [3].

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How to speak tensor

Hsin-Po Wang
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Tensor is an algebraic operation that combines a set of vector spaces into a big vector space. It can be used to construct complicated codes out of trivial codes. In this talk, we will go over four creative applications: distributed storage, distributed matrix multiplication, local testability, and quantum error correction.

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Can Coding Reduce Deduplication Fragmentation?

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Data deduplication is the process of removing replicas of data chunks stored by different users on servers, and is one of the key features of modern Big Data storage devices. Despite the importance of the methods, prior theoretical works have not addressed one of the major drawbacks of deduplication: file fragmentation. Fragmentation arises when placing deduplicated data chunks of different user files in linear order in the chunk store, because neighboring chunks of the same file may be stored in sectors far apart on the store. The contributions of our work are three-fold. First, we describe a new model for file structures of the form of self-avoiding paths in graphs (such as sparse Hamiltonian path graphs, trees etc) and introduce the notation of *fragmentation level* and *jumping index*. The fragmentation level captures the worst-case "spread" of data chunks in a file when deduplicated and placed on the server. The jumping index, on the other hand, represents the worst-case number of jumps in the store that one has to make in order to retrieve a file. Second, we establish connections between the notion of the fragmentation level and bandwidth of the file graph, and introduce a new graph-theoretic problem that allows for a succinct characterization of the jumping index. Importantly, we suggest techniques for adding redundancy and coded chunks to the store that allow one to reduce the fragmentation level as well as the jumping index. The key ideas behind our approach are information-theoretic arguments regarding redundant chunk store fragmentation levels, as well as a new algorithms of *graph folding* and jumping index aggregation on trees.

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Functions-based codes: results and open problems

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One of the leading research problems in coding theory is constructing new linear codes with appropriate parameters (functional in communication systems, consumer electronics and data storage systems), determining their weight distributions and studying their dual codes. This research also pays particular attention to applications. This talk concentrates on linear codes. We aim to present results and open questions by selecting families of linear codes over finite fields with arbitrary characteristics (some of them are designed for (toward) specific intended applications). Specifically, the emphasis will be on linear codes created from functions over finite fields. We shall discuss the advantages and limitations

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New asymptotic bound for codes from hypergraphs

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Let X be a finite set and $\binom{X}{r}$ the collection of all subsets of X with r elements. An *r-uniform hypergraph* $\mathcal{H}$ with vertex set X is a subset of $\binom{X}{r}$. A *Berge cycle of length* k in a hypergraph is a sequence of k distinct vertices $v_1, \ldots, v_k$ and k distinct edges $e_1, \ldots, e_k$ such that $\{v_i, v_{i+1}\} \subseteq e_i$. A hypergraph $\mathcal{H}$ has girth at least g if $\mathcal{H}$ has no Berge cycles of length k for every $2 \leq k \leq g-1$. Denote the family of Berge cycles of length at most $g-1$ as $\mathcal{C}_{<g}$, and notice that a hypergraph has girth at least g if and only if it is $\mathcal{C}_{<g}$-free. Given a family of r-uniform hypergraphs $\mathcal{F}$, the *Turán number* for $\mathcal{F}$ is the maximum number of edges in an r-uniform N-vertex hypergraph that is $\mathcal{F}$-free, denoted $\mathrm{ex}_r(N, \mathcal{F})$. The asymptotic behavior of these numbers in general is unknown.

Suppose that H is a parity check matrix for an $[n,k,d]_q$ code, of size $t \times N$ with entries in $\mathbb{F}_q$ for some prime power q. Taking $R = \mathbb{F}_q^t$ and $S = \mathbb{F}_q$, we present the following hypergraph construction. Let $r \geq 2$ be an integer. Let $\vec{\lambda} = (\lambda_1, \lambda_2, \ldots, \lambda_r)$ be a vector whose entries are elements of S. For $R' \subset R$ and $A \subset R$, define $\mathcal{H}(A, \vec{\lambda})$ to be the r-uniform r-partite hypergraph with vertex set $\cup_{i=1}^r (R' \times \{i\})$ and edge set

$$\bigcup_{x \in R', a \in A} \{((x + \lambda_1 a, 1), ((x + \lambda_2 a, 2), (x + \lambda_3 a, 3), \ldots, (x + \lambda_r a, r))\}.$$

We show that a Berge cycle in $\mathcal{H}(A, \vec{\lambda})$ implies the existence of an equation in R whose coefficients are differences of the entries of $\vec{\lambda}$, satisfied by elements in A.

A series of code constructions by Dumer [2] establish the existence of linear codes of distance $d = 5$ and $d = 6$ with improved parameters in comparison with BCH codes.

In general we use H to denote a parity-check matrix for one of these codes. Let $A \subset \mathbb{F}_q^t$, where A is the set of columns of H. We define $\mathcal{H}_5 = \mathcal{H}(A, \vec{\lambda})$, where the elements of $\vec{\lambda}$ are a Sidon set in $\mathbb{F}_q$. Since a code of distance 6 (over a field with large enough q) can yield a hypergraph, we can apply results from hypergraphs to establish an asymptotic bound for codes.

The asymptotic sphere-packing bound for linear codes states that for any $[n,k,6]_q$ code, $k \leq n - 2\log_q(n) - O(1)$, which for a fixed small distance like $d = 6$ is the best asymptotic bound known. To obtain Theorem 2, we employ the correspondence between codes and hypergraphs described above, and use a recent result of Conlon, Fox, Sudakov, and Zhao [1]:

Theorem 1 (Cor. 1.10, Ref. 1)**.** *For $r \geq 3$, every r-uniform hypergraph on N vertices with girth 6 has $o(n^{3/2})$ edges.*

Theorem 2. *If C is a linear $[n,k,6]_q$ code, then $k \leq n - 2\log_q n - \omega(1)$, where $\omega(1)$ is a function that goes to infinity with n.*

This improves the asymptotic sphere packing bound for linear q-ary codes of distance 6.

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Union-Closed Linear Codes

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A family of subsets of a finite set is said to be *union-closed* if, for any pair of sets in the family, also their union belongs to the family. Union-closed families of sets have been investigated from a wide variety of mathematical points of view, due to the notorious long-standing conjecture which was proposed by Frankl 1979. Its statement is quite elementary:

"*For every finite union-closed family of sets with at least two elements, there exists an element that belongs to at least half of the sets in the family.*"

In this talk we study union-closed sets families arising from the theory of error-correcting codes. More precisely, we deal with union-closed sets families which coincide with families of supports of linear codes over a finite field. This linearity enriches the corresponding union-closed sets family with interesting geometric and algebraic features, which we exploit to derive results on the combinatorial side.

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Rank-Metric Lattices and Finite Geometry

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Rank-Metric Lattices (RML in short) were introduced in [1] as the q-analogue of Higher-Weight Dowling Lattices [3,4]. They are special families of geometric lattices whose elements are the $\mathbb{F}_{q^m}$-linear subspaces of $\mathbb{F}_{q^m}^n$ having a basis of vectors with rank weight bounded from above, ordered by inclusion.

In this talk, we investigate the properties of RMLs from a finite geometry perspective. We compute the Whitney numbers of the first kind for some of these lattices, providing a recursive formula. In the second part of the talk, we present asymptotic results on the density of some classes of cardinality-optimal rank-metric codes.

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Capacity of Adversarial Networks in the Multishot Regime

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Adversarial network coding studies the transmission of data over networks affected by adversarial noise. In this realm, the noise is modeled by an omniscient adversary who is restricted to corrupting a proper subset of the network edges. In [4], Ravagnani and Kschischang established a combinatorial framework for adversarial networks. The study was recently furthered by Beemer, Kilic and Ravagnani [1,2], with particular focus on the one-shot capacity: a measure of the maximum number of symbols that can be transmitted in one use of the network without errors.

In this talk, we present recent results on the capacity of networks in multiple transmission rounds. We also discuss scenarios where there is a gain in capacity in using a network multiple times for communication versus using the network once and extend bounds from the one-shot capacity regime to the multishot regime.

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Full weight spectrum cyclic subspace codes

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For a linear Hamming metric code $\mathcal{C}$ of length n over $\mathbb{F}_q$, we define the weight set as

$$w(\mathcal{C}) = \{w(c) \colon c \in \mathcal{C} \setminus \{0\}\}.$$

It is easy to see that $|w(\mathcal{C})| \leq n$. The codes attaining the equality in the above bound were called **full weight spectrum** codes; see [1].

In this talk we will focus on the analog class of codes in the framework of cyclic subspace codes. Subspace codes gained a lot of attention especially because they may be used in random linear network coding for correction of errors and erasures. Let denote by $\mathcal{G}_q(n,k)$ the Grassmaniann of all the $\mathbb{F}_q$-subspaces of $\mathbb{F}_{q^n}$ of dimension k, equipped with the subspace distance introduced in [2]. A **one-orbit cyclic subspace code** $\mathcal{C}$ is a subspace code in $\mathcal{G}_q(n,k)$ such that

$$\mathcal{C} = \{\alpha U \colon \alpha \in \mathbb{F}_{q^n} \setminus \{0\}\}$$

for some $U \in \mathcal{G}_q(n,k)$. In this framework, defined $\omega_i = |\{\alpha U \colon \alpha \in \mathbb{F}_{q^n}^*, d(U, \alpha U) = i\}|$, the weight set of the code is

$$w(\mathcal{C}) = \{\omega_i \, : \, i \in \mathbb{N} \text{ and} \, \omega_i > 0\}.$$

One can easily check that the size of such a weight set is at most k. Similarly to the Hamming metric case, we define **full weight spectrum** codes those codes $\mathcal{C} \subseteq \mathcal{G}_q(n,k)$ such that $|w(\mathcal{C})| = k$. In this talk, we will see examples and characterization results of full weight spectrum one-orbit cyclic subspace codes.

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Constant Dimension Subspace Codes in Schubert Varieties

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In [1], Ahlswede et al. came out with the idea that in network communication nodes can combine the inputs received and forward the new messages. This networking technique is known nowadays as *network coding*: transmitted data are encoded and decoded in order to increase network throughput, reduce delays and make the network robust. In the seminal paper [3], Kötter and Kschischang introduced the concept of transmitting information over a network encoded in subspaces. A **constant dimension subspace code** is a set of subspaces with fixed dimension of a given vector space over a finite field. In this context, the source sends a (basis of a) vector subspace and the receiver gathers a (basis of a) vector subspace possibly affected by noise. An important parameter to measure the error-correction capacity is the minimum distance of a (constant dimension) subspace code, which is the minimum among the subspace distances of any two distinct subspaces in the code.

In this talk, we consider constant dimension subspace codes restricted to Schubert varieties, which means that we allow as codewords only subspaces with a particular shape. We are interested in finding the largest size of a constant dimension subspace code with prescribed minimum distance value in some Schubert varieties. We will present the general problem, that turns out to have a natural description as a problem of incidence geometry. When the prescribed minimum distance is the largest possible, we provide a construction of maximum size constant dimension subspace codes which uses the notion of *linear sets* in projective geometry.

The talk is based on the recent paper [2].

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On generalized Sidon spaces

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Since [3], subspace codes have gained a strong interest due to their use for the error correction in random network coding. The first class of subspace codes was the one of cyclic subspace codes, originally introduced in [2]. We will focus on one-orbit cyclic subspace codes, i.e. given an $\mathbb{F}_q$-subspace S of $\mathbb{F}_{q^n}$, the one-orbit cyclic subspace code defined by S is

$$\mathrm{Orb}(S) = \{\alpha S \colon \alpha \in \mathbb{F}_{q^n}^*\} \subseteq \mathcal{G}_q(n,k),$$

where $\alpha S = \{\alpha s \colon s \in S\}$ and $\mathcal{G}_q(n,k)$ is the Grassmannian of k-dimensional $\mathbb{F}_q$-subspaces of $\mathbb{F}_{q^n}$. In [4] Roth, Raviv and Tamo pointed out the connection between one-orbit cyclic subspace codes and Sidon spaces. They have been introduced in [1] by Bachoc, Serra and Zémor as the q-analogue of Sidon sets, classical combinatorial objects introduced by Simon Szidon. The authors of [4] also introduced the notion of r-Sidon spaces, as an extension of Sidon spaces, which may be seen as the q-analogue of B_r-sets, a generalization of classical Sidon sets.
In this work we will investigate one-orbit cyclic subspace codes, through some properties of Sidon spaces and r-Sidon spaces, providing upper and lower bounds on the possible dimension of their *r-span* and showing explicit constructions in the case in which the upper bound is achieved.

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