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Recent Trends in Stochastic Analysis
Special Session B21

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Stochastic analysis methods and robust techniques have recently led to a variety of advances in the theory of (stochastic) partial differential equations (PDEs). The spectrum of applications is very wide and includes stochastic homogeneization, fluid mechanics, mathematical finance, statistical mechanics and the stochastic quantization of quantum field theories. The goal of the session is to bring together mathematicians who have applied stochastic analysis to (stochastic) PDEs of many different kinds, in particular singular stochastic PDEs such as KPZ and Φ^4, Fisher-KPP PDE, Navier-Stokes PDE, and Stefan problems among others. The organizers are confident that such an exchange of ideas will lead to a number of further advances across different areas of (stochastic) PDEs.

For more information visit
`umi.dm.unibo.it/jm-umi-ams/special-sessions/special-sessions-on-25-26-july-2024`.

Schedule and Abstracts

July 25, 2024

11:30–12:10 Hyperbolic Anderson model with Lévy white noise: fluctuations of the spatial average
Raluca Balan (University of Ottawa, CANADA)

Abstract. In this talk, we study the hyperbolic Anderson model in dimension 1 driven by a space-time Lévy white noise with finite variance. Motivated by recent active research on limit theorems for SPDEs driven by Gaussian noise, we present the first study in this Lévy noise setting. The goal of the talk is to show that, with appropriate normalization and centering, the spatial average of the solution converges in distribution to the standard normal distribution, and to estimate the speed of this convergence in the Wasserstein (or Kolmogorov) distance. This talk is based on joint work with Guangqu Zheng (University of Liverpool).

12:20–13:00 The Allen-Cahn equation with weakly critical initial datum
Tommaso Rosati (University of Warwick, UK)

Abstract. We study the Allen-Cahn equation in dimension 2 with white noise initial datum, motivated by the study of the generic evolution of phase fields. In a weak coupling regime, where the nonlinearity is damped in relation to the smoothing of the initial condition, we prove Gaussian fluctuations. The effective variance that appears can be described as the solution to an ODE. Our proof builds on a Wild expansion of the solution, which is controlled through precise combinatorial estimates. Joint works with Simon Gabriel, Martin Hairer, Khoa Lê and Nikos Zygouras.

14:30–15:10 Limit laws in metric measure spaces
Maria Gordina (University of Connecticut, USA)

Abstract. We consider Dirichlet boundary problems in metric measure spaces. Results include properties of the spectrum, regularity and L^p-estimates of eigenfunctions, as well as irreducibility of the corresponding stochastic processes. A number of examples will be given including both local and non-local Dirichlet forms, hypoelliptic diffusions and stochastic processes on fractals, and applications to limit laws such as small deviations and large time behavior of the heat content.

15:20–16:00 Regularization in Kraichnan's Passive Scalar Model

Francesco Grotto (Università di Pisa, ITALY)

Abstract. The advection of a passive scalar by a random velocity field can induce the dissipation of multiscale norms even in the absence of diffusion, and they can be used to gauge mixing and regularizing properties of stochastic transport. The advection by a d-dimensional Gaussian vector field with a power-law covariance spectrum (Kraichnan's model) satisfies quantitative estimates for the evolution of negative Sobolev norms of passive scalars, which imply that generalized solution taking values in Sobolev spaces of negative order immediately become weak solutions of positive regularity. Joint work with Lucio Galeati and Mario Maurelli.

17:00–17:40 How does the supercritical GMC converge?
Martin Hairer (École Polytechnique Fédérale de Lausanne, SWITZERLAND)

Abstract. TBA

July 26, 2024

11:30–12:10 Enhanced dissipation, and residual diffusivity
Gautam Iyer (Carnegie Mellon University, USA)

Abstract. In many situations, the combined effect of advection and diffusion causes enhances dissipation. I will talk about this in two contexts: The first is for a random class of flows (ala Pierrehumbert) for which we show that the system dissipates energy on time scales of order $O(|\log \kappa|)$, where κ is the molecular diffusivity. The second is in a discrete time setting where show that the effective diffusivity does not vanish with the molecular diffusivity. This is joint work with SJ Son and W. Cooperman and J. Nolen.

12:20–13:00 Fluctuations of Stochastic Heat Equation and KPZ equation
Xue-Mei Li (École Polytechnique Fédérale de Lausanne, SWITZERLAND)

Abstract. In this talk, we explore the stochastic heat equation and the KPZ equation, each influenced by space time Gaussian noiise with long-range spatial dependence. These equations produce solutions that admit a stationary field. Our focus is on the fluctuation problem associated with diffusively scaled solutions from their average. While the behavior of compactly supported correlations—typically known to dissipate at large scales—is well-documented, our research shifts to examining long-range dependent noise with an asymptotic profile, inspired by empirical data and physical considerations. We investigate whether this dependence is maintained in the large-scale scaling limit. Our findings not only confirm its persistence but also reveal a key difference: the exponent in the power decay of the correlation rate plays a role akin to that of dimension in compactly supported scenarios. Furthermore, we demonstrate that the fluctuations of the appropriately scaled solutions from their mean converge weakly to the solution of a stochastic heat equation with additive noise, where the spatial correlation function is governed by the Riesz potential. In addressing the KPZ equation, we confront the challenges posed by the singular Cole-Hopf transformations. This research highlights the significance of long-range dependencies and their role in modeling more complex noise inputs in physical and mathematical models.

14:30–15:10 Cascade equation for Stefan problem as a mean field game
Sergey Nadtochiy (Illinois Institute of Technology, USA)

Abstract. The solutions to Stefan problem with Gibbs-Thomson law (i.e., with surface tension effect) are well known to exhibit singularities which, in particular, lead to jumps of the associated free boundary along the time variable. The correct times, directions and sizes of such jumps are only well understood under the assumption of radial symmetry, under which the free boundary is a sphere with varying radius. The characterization of such jumps in a general multidimensional setting has remained an open question until recently. In our ongoing work with M. Shkolnikov and Y. Guo, we have derived a separate (hyperbolic) partial differential equation — referred to as the cascade equation — whose solutions describe the jumps of the solutions to the Stefan problem without any symmetry assumptions. It turns out that a solution of the cascade equation corresponds to a maximal element of the set of all equilibria in a family of (first-order local) mean field games. In this talk, I will present and justify the cascade equation, will show its connection to the mean field games, and will prove the existence of a solution to the cascade equation. If time permits, I will also show how these results can be used to construct a solution to the Stefan problem itself.

15:20–16:00 Weak coupling scaling of critical SPDEs
Giuseppe Cannizzaro (University of Warwick, UK)

Abstract. The study of stochastic PDEs has known tremendous advances in recent years and, thanks to Hairer's theory of regularity structures and Gubinelli and Perkowski's paracontrolled approach, (local) existence and uniqueness of solutions of subcritical SPDEs is by now well-understood. The goal of this talk is to move beyond the aforementioned theories and present novel tools to derive the scaling limit (in the so-called weak coupling scaling) for some stationary SPDEs at the critical dimension. Our techniques are inspired by the resolvent method developed by Landim, Olla, Yau, Varadhan, and many others, in the context of particle systems in the supercritical dimension. Time allowing, we will explain how it is possible to use our techniques to study a wider class of statistical mechanics models at criticality such as (self-)interacting diffusions in random environment.

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