AMS-UMI International
Joint Meeting 2024
Palermo, July 23–26, 2024

Automorphic forms, Galois representations, and L-functions
Special Session B19

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This session is scheduled on July 25-26. The focus of the proposed scientific session is to present recent developments in a wide research area of modern number theory which sits in the wide framework of the Langlands program. Automorphic forms arise naturally in many different settings of number theory; under the deep web of conjectures that form the Langlands program, they should be related to Galois representations, and discovering properties on one of these gives us information about the other. The natural tool to connect them are complex (and p-adic) L-functions and their relations with Selmer groups, conjecturally described in great generality by Bloch–Kato (in their equivariant Tamagawa number conjecture) as a wide generalization of the Birch and Swinnerton-Dyer Conjecture for elliptic curves, a Milllennium Problem. One of the tools to attack the Bloch–Kato conjecture is the theory of Euler systems and their relation with complex (and p-adic) L-functions. In recent years, there have been many important advances in this area, most notably:

- New constructions of Euler systems via algebraic cycles, which allow one to study automorphic forms and Galois representations for motives of algebraic groups possibly different from GL_2;
- Variation of automorphic forms in families, especially using higher degree coherent cohomology which provides new ways to study Galois representations and their L-functions by deforming them p-adically.

These new developments have exciting applications on many outstanding open conjectures in number theory among which the Bloch–Kato conjectures, as already mentioned, the Iwasawa main conjecture, the study of (completed) cohomology of Shimura varieties.

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Schedule and Abstracts

July 25, 2024

11:30–12:15 p-Adic variantion of de Rham modular sheaves and applications
Adrian Iovita (Concordia University, CANADA and Università degli Studi di Padova, ITALY)

Abstract. One of the main topics in this session is the theme of the variation of automorphic forms in p-adic families, especially using higher degree coherent cohomology.

In this talk I will present yet another way of studying Galois representations, L-functions and p-adic L-functions attached to automorphic eigenforms by deforming de Rham cohomology modular sheaves. These are large Banach-sheaves on appropriate strict neighbourhoods of certain components of the ordinary locus in Shimura varieties, with increasing filtrations and integrable connections. By studying the action of the Hecke operators on the de Rham cohomology of these sheaves with connections, one obtains very interesting applications.

These ideas lead so far to new ways of defining triple product p-adic L-functions for automorphic forms of finite slope, instead of ordinary, and studying Katz-type p-adic L-functions in cases in which p is not split in the CM field.

12:30–12:50 Balanced triple product p-adic L-functions and classical weight one forms
Luca Dall'Ava (University of Milan, ITALY)

Abstract. The aim of this talk is to introduce a new balanced triple product p-adic L-function and discuss its application to the equivariant Birch & Swinnerton-Dyer conjecture. We state a conjecture in a rank-1 situation analogous to the Elliptic–Stark conjecture formulated by Darmon–Lauder–Rotger in rank-2 and prove it in the CM case; this work fits in the general framework studied by Darmon–Lauder–Rotger and Andreatta–Bertolini–Seveso–Venerucci. Time permitting, we will explain briefly the intriguing technical difficulties behind the construction of this new p-adic L-function whose main feature is to allow classical weight one modular forms in the chosen families. That is joint work with Aleksander Horawa.

14:30–14:50 Kolyvagin's conjecture and Perrin-Riou's main conjecture for modular forms, part I
Stefano Vigni (Università di Genova, ITALY)

Abstract. In this talk, I will state and (time permitting) sketch a proof of an analogue for p-adic Galois representations attached to a higher (even) weight newform f of Kolyvagin's conjecture on the p-indivisibility of derived Heegner points on elliptic curves, where p is a prime number that is ordinary for f. Our strategy, which is inspired by work of W. Zhang in weight 2, builds crucially on results of H. Wang on the indivisibility of Heegner cycles over Shimura curves. In her talk, Maria Rosaria Pati will explain how our work on Kolyvagin's conjecture can be combined with other ingredients to yield a proof of the counterpart for f of Perrin-Riou's Heegner point main conjecture for elliptic curves ("Heegner cycle main conjecture" for f).

15:00–15:20 Kolyvagin's conjecture and Perrin-Riou's main conjecture for modular forms, part II
Maria Rosaria Pati (Université de Caen Normandie, FRANCE)

Abstract. In this talk, I will state and sketch a proof of the counterpart for a higher (even) weight newform f of Perrin-Riou's Heegner point main conjecture for elliptic curves "Heegner cycle main conjecture" for f). Our strategy of proof builds on, among other ingredients, our work on Kolyvagin's conjecture for f that was described by Stefano Vigni in his talk. This is joint work with Matteo Longo, Stefano Vigni and Haining Wang.

15:30–15:50 The Gross–Kohnen–Zagier theorem via p-adic uniformization
Marti Roset Julia (McGill, CANADA)

Abstract. Let S be a set of rational places of odd cardinality containing infinity and a rational prime p. We can associate to S a Shimura curve X defined over $\mathbb{Q}$. The Gross–Kohnen–Zagier theorem states that certain generating series of Heegner points of X are modular forms of weight $3/2$ valued in the Jacobian of X. We will state this theorem and outline a new approach to prove it using the theory of p-adic uniformization and p-adic families of modular forms of half-integral weight. This is joint work with Lea Beneish, Henri Darmon and Lennart Gehrmann.

16:00–16:20 Equidistribution of CM points on Shimura curves and ternary quadratic forms
Francesco Maria Saettone (Ben-Gurion University of the Negev, ISRAEL)

Abstract. Equidistribution of "special" points is a theme of both analytic and geometric interest in number theory. In particular, this seminar plans to deal with the case of CM points, generalizing the results of [1] to quaternionic Shimura curves over a totally real number field.

The first part will be devoted to a short geometric description of the aforementioned curves and in particular of their special fiber. Subsequently, I plan to describe an equidistribution result of reduction of Galois orbits of CM points in the special fiber of Shimura curves associated both to an unramified and ramified quaternion algebra.

The first results boil down to a correspondence between CM points and primitive representations of certain ternary quadratic forms by the means of optimal embeddings. We then conclude by exploiting subconvexity bounds on the Fourier-Whittaker coefficients of the automorphic theta series (attached to our ternary forms) to obtain the desired equidistribution.

References

[1] Jetchev, Dimitar; Kane, Ben. *Equidistribution of Heegner points and ternary quadratic forms.* Math. Ann. 350 (2011), no. 3, 501-532.

17:00–17:20 Modularity of mod p Tate–Shafarevich classes
Matteo Tamiozzo (Université Sorbonne Paris Nord, FRANCE)

Abstract. Let $E/\mathbf{Q}$ be an elliptic curve; modularity of E translates geometrically into the existence of a surjection from the Jacobian of a modular curve to E. A similar modularity property is expected for classes in $Ш(E/\mathbf{Q})$. More precisely, these cohomology classes correspond to curves of genus one with a point over every completion of $\mathbf{Q}$, whose Jacobian is isomorphic to E. Jetchev and Stein conjectured in [2] that such curves can be realized inside (quotients of) the Jacobians of modular curves over $\mathbf{Q}$. After introducing the conjecture, I will present the following result for p-torsion Tate–Shafarevich classes, whose proof builds on the techniques introduced in [1].

Theorem 1. *([3], Theorem 1.3.1) Assume that E has squarefree conductor and does not have complex multiplication. Let $p > 3$ be a prime of good ordinary reduction. Assume that $\bar{\rho}$: $\mathrm{Gal}(\bar{\mathbf{Q}}/\mathbf{Q}) \to \mathrm{Aut}_{\mathbf{F}_p}(E[p])$ is surjective and ramified at every prime factor of the conductor congruent to ± 1 modulo p. Then every class in $Ш(E/\mathbf{Q})[p]$ is modular.*

References

[1] M. Bertolini, H. Darmon, *Iwasawa's main conjecture for elliptic curves over anticyclotonic $\mathbf{Z}_p$-extensions*, Ann. Math. (2005)

[2] D. Jetchev, W. Stein, *Visibility of the Shafarevich-Tate group at higher level*, Doc. Math. (2007)

[3] M. Tamiozzo, *Congruences of modular forms and modularity of Tate-Shafarevich classes*, preprint

17:30–17:50 Hodge polygons of formal $\mathcal{O}_D$-modules and the p-adic half space
Andrea Marrama (Università degli Studi di Padova, ITALY)

Abstract. Let p be a prime number. The p-adic analogue of the complex upper half plane, together with its tower of étale coverings, has long played a prominent role in the p-adic local Langlands program, especially through its cohomology, but also as a uniformising space of p-adic models of certain Shimura curves. Thanks to Drinfeld, this space and its higher dimensional versions admit a moduli interpretation, namely in terms of "formal $\mathcal{O}_D$-modules". In this talk, I will introduce some combinatorial invariants of the latter objects, which help describing the geometry of the p-adic half space and, possibly, of more general moduli of formal groups.

18:00–18:20 On p-adic uniformization of abelian varieties and p-divisible groups
Jackson S. Morrow (University of North Texas, USA)

Abstract. Given an abelian variety over a finite extension K of $\mathbb{Q}_p$, Fontaine constructed an integration map from the Tate module of A to its Lie algebra. This map gives the splitting of the Hodge–Tate short exact sequence. In recent work with Iovita and Zaharescu, we extended this integration map to the $\overline{K}$-points of the perfectoid universal cover of A, and used this result to give a uniformization of the points of the underlying p-divisible group. Later in joint work with Howe and Wear, we gave a different perspective on this uniformization using 1-motives and extended the uniformization to certain kinds of p-divisible groups. In this talk, I will explain the constructions of each of these results and state some follow up questions concerning these uniformizations.

July 26, 2024

11:30–12:15 Weil height and modular Galois representations
Lea Terracini (Università di Torino, ITALY)

Abstract. An algebraic field is said to have the *Bogomolov property*, (property (B) for short), if the Weil height is uniformly bounded below outside torsion points. It is well known that property (B)

holds for number fields and for potentially abelian extensions; moreover a theorem by Habegger proves property (B) for the field generated by the torsion points of an elliptic curve defined over $\mathbb{Q}$. Together with F. Amoroso we generalized this result, establishing property (B) for the extension cut out by the Galois representation associated to a modular form, assuming the existence of a strong supersingular prime and the fullness of the image of the residual representation; moreover we conjectured that property (B) holds unconditionally for modular Galois representations.
In my talk, I will explain the main idea underlying the proof, and discuss some aspects of particular relevance in the theory of automorphic forms.

12:30–12:50 The algebra $\mathbb{Z}_\ell[[\mathbb{Z}_p^d]]$ and applications to Iwasawa theory
Ignazio Longhi (Università degli Studi di Torino, ITALY)

Abstract. Let ℓ and p be distinct primes, and let Γ be an abelian pro-p-group. I will discuss a structure theorem for the algebra $\Lambda := \mathbb{Z}_\ell[[\Gamma]]$ and its consequences on the structure of Λ-modules. In particular, if $\Gamma \simeq \mathbb{Z}_p^d$ is the Galois group of an extension K/k, with k a global field, one obtains explicit formulae for the orders and ℓ-ranks of certain Iwasawa modules (namely ℓ-class groups and ℓ-Selmer groups) associated with the finite subextensions of K. In the case of ℓ-class groups, this provides different proofs and generalizations of results of Washington and Sinnott.

14:30–14:50 Drinfeld quasi-modular forms of higher level
Maria Valentino (University of Calabria, ITALY)

Abstract. When working with global function fields of characteristic $p > 0$ complex valued (Jacquet–Langlands) automorphic forms are only half of the story, the other part being Drinfeld modular forms. As in the classical characteristic zero setting the space of Drinfeld forms is not stable under differentiation; this is why it is interesting to introduce Drinfeld quasi-modular forms, which indeed behaves well under the action of differential operators. In this talk we will present some new and recent advances on Drinfeld quasi-modular forms for congruence subgroups.

15:00–15:20 Dihedral long root A-packets of p-adic G_2 via theta correspondence
Raúl Alonso[1] (University of California, Santa Barbara, USA)

Abstract.

Let G be a split exceptional group of type G_2 over a number field F. According to Arthur's conjectures, the square-integrable automorphic representations of G can be classified into near-equivalence classes, known as global A-packets, indexed by certain homomorphisms known as global A-parameters. For each place v of F, the representations of $G(F_v)$ which can appear as a local component of a representation in a given global A-packet form the corresponding local A-packet.

We will focus on the A-packets attached to a specific type of non-tempered A-parameters of the group G, called dihedral long root A-parameters. These A-parameters can be seen to factor through A-parameters of the group PU_3. Thus, we propose a construction of the non-archimedean local A-packets of G attached to a dihedral long-root A-parameter as theta lifts of the corresponding local A-packets of PU_3, relying on the exceptional theta correspondence between $\mathrm{PU}_3 \rtimes \mathbb{Z}/2\mathbb{Z}$ and G studied in [2].

This talk is based on the joint work [1].

References

[1] R. Alonso, Q. He, M. Ray, M. Roset, *Dihedral long root A-packets of p-adic G_2 via theta correspondence*, preprint.

[2] P. Bakić, G. Savin, *Howe duality for a quasi-split exceptional dual pair*, Math. Ann. **389** (2024), 325–364.

[1]Participation in the conference partially supported by Journal of Number Theory.

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