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# Factorization Algebras and Geometry
# Special Session A5

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The space of conformal blocks is an object that describes the collection of all correlation function in a two-dimensional conformal field theory. Mathematically, it is a vector space computed from the input of a pointed curve $X$ (the "space-time"), a vertex algebra $V$ (which encodes the chiral conformal field theory), a collection $V$-representations, and possibly a principal $G$-bundle $P$ on $X$ (the gauge bundle), where $G$ is an algebraic group. As the curve $X$ varies in $\mathcal{M}_{g,n}$, and the bundle $P$ in $\mathrm{Bun}_{G,X}$—the moduli space of principal $G$-bundles on $X$—spaces of conformal blocks give rise to interesting sheaves equipped with projectively flat connection (the Knizhnik–Zamolodchikov connection).

A particularly beautiful instance of this construction happens in the case of the Wess–Zumino–Witten model of conformal field theory. Here, the representation-theoretic input (the vertex algebra and its modules) correspond to an affine Kac-Moody algebra at positive integer level, and the space of conformal blocks may be identified with spaces of non-abelian theta functions on $\mathrm{Bun}_{G,X}$ and its parabolic cousins. A rich combinatorial structure emerges which allows for the computation of the dimensions of spaces of conformal blocks through the celebrated Verlinde formula.

In the last 2-3 decades, new rigorous geometric approaches to conformal field theory have emerged, allowing the conformal block construction to be re-cast and generalized in several directions. First, in the theory of chiral algebras, developed by Beilinson and Drinfeld, conformal blocks appear as the *zeroth chiral homology*—one of a (potentially infinite) tower of cohomology groups. It is an interesting problem, both purely mathematical and in physical contexts, to interpret these 'higher' conformal blocks. Similarly, in the formalism of factorization algebras developed by Costello and Gwilliam, they appear as the *factorization homology.* This framework also allows one to treat higher-dimensional examples through the use of derived geometry, and applies to more general classes of quantum field theories.

The proposed special session "Factorization Algebras and Geometry" aims to bring together experts in the theory of these generalized conformal blocks with a view towards their geometric properties, and to explore how the rich interplay of representation theory, moduli spaces, and combinatorics, may be productively generalized in these settings.

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