AMS-UMI International
Joint Meeting 2024
Palermo, July 23–26, 2024

Mean field games and related topics
Special Session A1

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Mean field games are a recent and active area of research which presents new challenging mathematical questions, from analytical, probabilistic, and numerical sides. They originated from the seminal works of Lasry and Lions, and, simultaneously, Huang, Malhamé and Caines, in 2006.

This class of models is very popular not only in the mathematical community, but also in economics, engineering, life and social sciences, since they are widely used for applications; to mention a few, they include models of oil production, volatility formation, crowd motion, energy consumption, social networks, bitcoin mining.

Mean field games represent limit models for symmetric non-cooperative N-player games, as the number of players tends to infinity. On the other hand, mean field control problems, also called optimal control of McKean-Vlasov dynamics, represent limit models for cooperative N-player games. Systems with a large number of interacting agents, both cooperative or non-cooperative, arise in several physical, social and economic models, but finding explicitly the equilibria, or even numerically, is typically an intractable problem when N is large, due to the curse of dimensionality. For this reason, limit models have been introduced: letting $N = \infty$, instead of finite, may restore some tractability of the model and thus permits to understand the qualitative behavior of the multitude of players. One of the main object of study in mean field games is nowadays the master equation, which is a PDE stated on the infinite dimensional Wasserstein space of probability measures. The master equation encapsulates all the features of the mean field game and represents the value of the game for any initial distribution. Its counterpart for mean field control problems is a Hamilton-Jacobi-Bellman equation in the Wasserstein space.

The aim of this special session is to gather together the big community of researchers working on mean field games in Italy and in the United States. Possible topics of the presentations will be the following: existence and uniqueness of equilibria, under various forms of dynamics and costs (optimal stopping, correlated equilibria, jumps, ...); well-posedness of the master equation; convergence of the N-player game towards a suitable mean field game; asymptotics: fluctuations, large deviations, long time behavior; numerical methods, also based on machine learning techniques; applications to economics and finance, with a particular focus on energy transition.

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Deterministic ergodic Mean Field Games with congestion

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I consider deterministic Mean Field Games (MFG) with a cost functional continuous with respect to the distribution of the agents and satisfying a gap condition at infinity, and compare them with the static MFG with such a cost.

Under the coercivity condition on the Hamiltonian

$$H(x,p,m) - H(x,0,m) \geq a_o |p|^\beta\,, \quad a_o > 0,\, \beta > 1\,,$$

I show how to build a solution of the ergodic MFG system of 1st order PDEs from any solution of the static MFG with cost $F(x,m) := H(x,0,m)$. This leads to new existence results under general assumptions, in particular for non-separable Hamiltonians. The motivating examples for such Hamiltonians are some models of congestion in crowd dynamics with non-local dependence on the crowd distribution, such as

$$H(x,p,m) = \frac{|p|^\beta}{(k * m(x) + \sigma)^\alpha} - F(x,m)\,, \quad \beta > 1, \sigma \geq 0, \alpha > 0.$$

Next I prove that the measure component of any solution to the ergodic MFG must solve the associated static MFG, under the following assumption on the Hamiltonian

$$H(x,p,m) - H(x,0,m) \leq a_1\, p \cdot H_p(x,p,m)\,, \quad a_1 > 0\,.$$

Such necessary condition for the solvability of the ergodic MFG implies new uniqueness results and, in some cases with multiple solutions, the characterization of all of them.

Some of these results were proved earlier for the special case $H(x,p,m) = |p|^2 - F(x,m)$ in the joint work with Hicham Kouhkouh [1], where also the asymptotics for large time of finite horizon MFG were analysed.

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Convergence analysis of controlled particle systems arising in deep learning: from finite to infinite sample size

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In this talk we will consider a class of neural SDEs and discuss the limiting behavior of the associated sampled optimal control problems as the sample size grows to infinity. The neural SDEs with N samples can be linked to an N-particle system with centralized control. We analyze the Hamilton–Jacobi–Bellman equation corresponding to the N-particle system and establish regularity results which are uniform in N. The uniform regularity estimates are obtained by the stochastic maximum principle and the analysis of a backward stochastic Riccati equation. Using these uniform regularity results, we show the convergence of the minima of objective functionals and optimal parameters of the neural SDEs as the sample size $N \to +\infty$. The limiting objects can be identified with suitable functions defined on the Wasserstein space of Borel probability measures. Furthermore, quantitative algebraic convergence rates will also be discussed. The talk will be based on a joint work with H. Liao, C. Mou and C. Zhou.

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Linear quadratic Mean Field Games in Hilbert spaces and some applications

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We study a class of linear quadratic Mean Field Games (MFG) in infinite dimension, where the state variable lives in a Hilbert space. Our motivations are problems where the state equation is a PDE or a delay equation which can be written as an ODE in a suitable Hilbert space. As a starting point, we study the case, considered in most finite dimensional contributions on the topic, where the dependence on the distribution enters just in the objective functional through the mean. This feature allows, similarly to the finite dimensional case, to reduce the usual mean field game system to a Riccati equation and a forward-backward coupled system of abstract evolution equations. Such system is completely new in infinite dimension and no results have been proved on it so far. We show existence and uniqueness of solutions for such system, applying a delicate approximation procedure.

Moreover, we study the well-posedness of the *Master equation* associated to the above mentioned MFG system and, with the help of it, we aim at proving convergence of the discrete model to the continuous one (work in progress).

We apply the results to a production output planning problem with delay in the control variable.

Keywords
Mean Field Games, infinite dimensional linear-quadratic control, delay equations, Master equation, convergence

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Fourier Galerkin approximation of mean field control problems

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The goal of this talk is to introduce a finite dimensional approximation of the solution to a mean field optimal control problem set on the d-dimensional torus, without relying on particle-based methods. Our approximation is obtained by means of a Fourier-Galerkin method, the main principle of which is to truncate the Fourier expansion of probability measures on the torus. However, this operation has the main feature not to leave the space of probability measures invariant, which drawback is know as *Gibbs*' phenomenon.

First, we manage to prove that, for initial conditions in the 'interior' of the space of probability measures and for sufficiently large levels of truncation, the Fourier-Galerkin method actually induces a new finite dimensional control problem whose trajectories take values in the space of probability measures with a finite number of Fourier coefficients. Subsequently, our main result asserts that, whenever the cost functionals are smooth and convex, the optimal control, trajectory, and value function from the approximating problem converge to their counterparts in the original mean field control problem. Noticeably, we show that our method yields a polynomial convergence rate directly proportional to the data's regularity. This convergence rate is faster than the one achieved by the usual particles approach, offering a more efficient alternative. Furthermore, our technique also provide an explicit method for constructing an approximate optimal control along with its corresponding trajectory. This talk is based on a joint work with François Delarue.

[1]The authors acknowledge the financial support of the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation programme (AdG ELISA project, Grant agreement No. 101054746).
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Deep Backward and Galerkin Methods for the Finite State Master Equation

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This paper proposes and analyzes two neural network methods to solve the master equation for finite-state mean field games (MFGs). Solving MFGs provides approximate Nash equilibria for stochastic, differential games with finite but large populations of agents. The master equation is a partial differential equation (PDE) whose solution characterizes MFG equilibria for any possible initial distribution. The first method we propose relies on backward induction in a time component while the second method directly tackles the PDE without discretizing time. For both approaches, we prove two types of results: there exist neural networks that make the algorithms' loss functions arbitrarily small and conversely, if the losses are small, then the neural networks are good approximations of the master equation's solution. We conclude the paper with numerical experiments on benchmark problems from the literature up to dimension 15, and a comparison with solutions computed by a classical method for fixed initial distributions.

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The coupling method in (McKean-Vlasov) stochastic control

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The coupling method is a popular and far-reaching technique to quantify the speed of convergence to equilibrium of Markov process. The basic principle is that of constructing on a single probability space two realisations of the same Markov dynamics with different initial conditions and in showing that the law of the two processes get exponentially close in time. The aim of this talk is to promote the idea that one can construct contractive couplings between controlled processes as well, which imply exponential turnpike estimates for the optimal controls and processes, as well as estimates on the derivative of the value function that are uniform in time. In particular, a controlled version of coupling by reflection will be discussed in some detail.

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Pasting of discrete time mean field equilibria and Donsker-type results for mean field games

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In this talk we discuss mean field games in discrete time on general probability spaces. Using dynamic programming and a forward-backward algorithm, we will construct mean field equilibria of multi period models as concatenation of equilibria of one-step games. We will also present results on convergence of discrete time games to continuous time counterparts akin to Donsker's invariance principle. The talk is based on a joint work with J. Dianetti, M. Nendel and S. Wang.

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Long Range Games

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I will introduce a general extension of Mean Field Games that includes Kac-Potential interactions and Graphon interaction. In the context of finite state space we show that the basic properties of mean-field games continue to hold: existence of Nash equilibria, uniqueness under monotonicity, approximation of the N-player game by the limit independent games.

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Self-organizing equilibria and their local stability in a Kuramoto mean field game

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Recently a Mean Field Game version of the classical Kuramoto model has been proposed in [1], describing synchronization phenomena in a large population of rational interacting oscillators. In this talk, I will discuss existence and uniqueness (up to phase transition) of the incoherent equilibrium and the self-organizing equilibrium, given that the interaction parameter is sufficiently large. Furthermore, I will also present some local stability properties of the self-organizing equilibrium with respect to dynamic equilibria in a long time regime.

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Stationary Mean-field Games of Singular Control

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In this talk I will present recent and ongoing results on existence, uniqueness, and characterization of equilibria for mean-field games with singular controls. This class of problems finds natural applications in Economics and Finance, such as in investment problems in oligopolies. In those games, the representative agent employs a bounded-variation control in order to maximize an ergodic profit functional depending on a long-time average of the controlled state-process. Several variants of the considered games will be presented, which will differ with respect to the dimension of the state-process and the optimality criterion employed.

[1]Funded by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation)- Project-ID 317210226- SFB 1283.
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Asymptotic behavior mean field games: coercive and non coercive case

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In this talk we will revisit the recent results on the description of the asymptotic behavior of some deterministic mean field game models, namely: the classical system, the system arising from the control of acceleration and the case nonholonomic dynamics. We will discuss about the results and about the questions that are still open. Finally, we will see how the ergodic mean field game system associated with a calculus of variation problem captures the behavior of Nash equilibria.

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Mean field games on homogeneous Lie groups

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We study short-time existence of classical solutions to mean field games systems defined on homogeneous Lie groups.

More precisely, we consider an homogeneous Lie group, endowed with a family of dilations, which can be identified with $\mathbb{R}^d$. Let $\{X_1, \ldots, X_m\}$ (with $m < d$) be a family of vector fields which satisfies the Hörmander condition; in particular, together with their commutators, these vector fields generate the Lie group. We consider second-order mean field games systems where the differential operators are given in terms of these vector fields and where the couplings are strongly regularizing. In our model, each agent can move only along the directions generated by $X_1, \ldots, X_m$ but it can still reach every point due to the Hörmander condition.

In order to obtain existence of solution to these mean field games, we first study existence and uniqueness of the subelliptic Fokker-Planck equation and separately of the Hamilton-Jacobi equation.

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Second order PDEs on the Wasserstein space

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We prove a comparison result for viscosity solutions of second order parabolic partial differential equations in the Wasserstein space. The comparison is valid for semisolutions that are Lipschitz continuous in the measure in a Fourier-Wasserstein metric and uniformly continuous in time. The class of equations we consider is motivated by Mckean-Vlasov control problems with common noise and filtering problems. We also mention applications for prediction problems with expert advice. The proof of comparison relies on a novel version of Ishii's lemma, which is tailor-made for the class of equations we consider.

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Knot Theory and Applications
Special Session A2

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Knot theory is a very fertile branch of low-dimensional topology with ramifications in many other areas of mathematics and natural sciences. The goal of this special session is to enable an exchange of methods and ideas as well as exploration of fundamental research problems in the fields of knot theory and low-dimensional topology, from theory to applications in mathematics and science, and to provide high quality interactions across fields. The talks will touch upon the themes below:

- algebraic, categorical and combinatorial invariants in knot theory
- interplay between links and braids
- representations of knot theory in 3-manifolds
- use of knot theory in biological and biochemical models.

The session is scheduled on July 23-24.

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Mock Alexander Polynomials

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This talk will discuss joint work with Neslihan Gügümcü. We study generalizations of the Formal Knot Theory (Kauffman 1981) state summation to general diagrammatic situations where the number of active regions in the diagram is equal to the number of crossings in the diagram. These situations include classical knot and link diagrams with non-adjacent starred regions (regions where the state sum is not evaluated and where the diagrams cannot move), knotoids, linkoids, knots and link diagrams in a torus and other configurations. The generalizations have many properties quite different from the classical Alexander polynomial and can often detect different species of chirality. Many examples will be given in the talk and we will discuss problems related to generalizing the Fox-Milnor Theorem and relationships with generalizations of the Dehn Presentation of the classical knot group.

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Hyperbolicity of Staked Links

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A staked link is obtained as a link diagram with finitely many points chosen in the complementary regions such that strands in the link diagram cannot pass over them. This is equivalent to taking a link in a handlebody. In this talk, we report on joint work as in the references. We focus on hyperbolicity of the complement of the link in the handlebody. In particular, we prove that every link can be staked to be hyperbolic, thereby implying every link in S^3 has a minimal staked hyperbolic volume. We give a characterization of hyperbolicity when the projection is alternating. We further discuss a version of composition and give a variety of results for specific classes of staked links, including volume bounds.

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From the Kauffman bracket polynomial to cubic skein modules and beyond

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The study of linear and quadratic skein modules over the last thirty-seven years has lead to a very rich skein theory that is connected to many disciplines of mathematics and physics, such as algebraic geometry, hyperbolic geometry, Topological Quantum Field Theories (TQFT), and statistical mechanics. There is, however, another class of skein modules with more parameters than the linear and quadratic cases which, save for a few exceptions, have been largely neglected until now. The cubic skein module is the first object in this class which awaits exploration. This is a joint work with Mathathoners VIII.

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Non-triviality of welded knots and the ribbon torus knots

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In this paper we study welded knots and their invariants. We focus on generating examples of non-trivial knotted ribbon tori as the tube of welded knots that are obtained from classical knot diagrams by welding some of the crossings. Non-triviality is shown by determining the fundamental group of the concerned welded knot. Sample examples under consideration are the standard diagrams of the family of $(2, q)$ torus knots and the twist knots. Standard diagrams of knots from Rolfsen's tables with 6 crossings are also discussed which are not in the family of torus and twist knots.

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The Roger-Yang Skein Algebra of the n-punctured torus

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Skein modules were introduced by Przytycki in [2] with the goal of building an algebraic topology based on knots, generalising the skein theory for links in S^3 to arbitrary 3-manifolds. Turaev introduced the Conway and Kauffman skein modules independently in [3]. The Kauffman bracket skein module, which serves as the generalisation of the Kauffman bracket polynomial, has a natural algebra structure given by the stacking of skeins (elements in the skein module) when the 3-manifold is a thickened surface. This algebra, known as the Kauffman bracket skein algebra, has deep connections with algebraic and hyperbolic geometry. The Roger-Yang skein algebra is a generalisation of the Kauffman bracket skein algebra for punctured surfaces, allowing framed arcs between punctures [4]. It is known that this new algebra is related to the decorated Teichmüller space introduced by Penner [1]. Further study of the Roger-Yang skein algebra will help better understand the connections between hyperbolic geometry and quantum topology. In this talk, we give a presentation of the Roger-Yang skein algebra of the n-punctured torus, $n > 0$.

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[1]The speaker acknowledges the support of Dr. Max Rössler, the Walter Haefner Foundation, and the ETH Zürich Foundation.
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Relations between knot invariants

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Ernst and Sumners' theorem [2], affirming that knots constitute a form of big data, coupled with the comprehensive knot tabulation by Burton, Hoste, Thistlethwaite, and Weeks, along with numerous computations of knot invariants, establishes the groundwork for employing big data methodologies such as machine learning in knot theory [1, 3, 4, 5]. Utilizing dimension reduction and machine learning methods, such as Ball Mapper and other techniques inspired by the topological data analysis (TDA), yields valuable insights into the statistical characteristics of knots, offers compelling means to visually represent and analyze relations between knot invariants [2, 6]. Invariants in focus include the Jones, Alexander, HOMFLYPT and Khovanov polynomials and numerical invariants such as signature and Rasmussen s-invariant.

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Dynamics and mechanics of knotted DNA and RNAs

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I will report on a series of theoretical and computational studies of DNA and RNAs that present knots and other forms of structural entanglement [1]. I will first consider model bacterial DNAs that are both knotted and supercoiled, and discuss how the simultaneous presence of knots and supercoiling creates long-lived multi-strand interlockings that might may be relevant for the simplifying action of topoisomerases. I next consider how entangled nucleic acids behave when driven through narrow pores [2-4], a setting that models translocation through the lumen of enzymes, and discuss the biological implication for a certain class of viral RNAs [4].

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Identifying knots and theta-curves via neural networks

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We show that Long Short-Term Memory (LSTM) based Neural Networks can be applied to successfully predict the knot type and theta-curve type of entangled curves in 3D space. The model is 99% successful at detecting the knot type of open polymeric chains resembling real proteins. In the case of native protein structures predicted by AlphaFold 2, the model can distinguish between trivial and non-trivial knot type with an accuracy of 93% [1].

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Vortex knot cascade by geodesic flows in a knot polynomial space

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It is well known that topologically complex tangles of superfluid vortex knots and links tend to decay through a topological cascade producing a system of unlinked vortex loops [1, 2]. Here we propose a geometric interpretation of this unlinking process by geodesic flows in an abstract knot polynomial space [3]. By taking advantage of the re-formulation of the Jones polynomial in terms of helicity contributions [4] we introduce a discrete metric space, whose points represent different knot types, and show that optimal unlinking paths in this space can describe some fundamental features of the topological decaying process observed in superfluid simulations.

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Everything AlphaFold tells us about protein knots

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I will report on a series of numerical and experimental studies to demonstrate a new type of knots [1,2] and lassos discovered in proteins based on AI methods (data are deposited in AlphaKnot [3] and AlphaLasso [4] databases). Next, I will concentrate on proteins with non-twist type of knot [5] and discuss how such proteins can fold with just one threading across the topological barrier.

Finally, I will present a cross-proteome-wide big data analysis of topologically knotted protein structures based on the AlphaFold and ESMFold predictions that essentially cover >90% of known genomic open reading frames. Using this approach, we found that there are approx. 700,000 protein structures. Next, I will show that the 3_1 knot type is the most prevalent in all proteomes, and the knotted proteins account for 0.4% of all tested proteomes regardless of their evolutionary an d habitat differences. Thus, although knotted proteins make up only approx. 0.4% of the proteome, are ubiquitous throughout all kingdoms of life. Finally I will show that all organisms contain at least one knotted protein [6].

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On the Jones polynomial of quasi-alternating links

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We use graph theoretical approach to extend the result of Thistlethwaite in [3] on the structure of the Jones polynomial of alternating links to the wider class of quasi-alternating links defined for the first time in [2]. In particular, we prove that the Jones polynomial of any prime quasi-alternating link that is not a $(2, n)$-torus link has no gap and this gives an affirmative answer to [1, Conjecture 2.3]. As an application, we show that the differential grading of the Khovanov homology of any prime quasi-alternating link that is not a $(2, n)$-torus link has no gap. Also, we show that the determinant is an upper bound for the breadth of the Jones polynomial for any quasi-alternating link. Finally, we prove that the Jones polynomial of any non-prime quasi-alternating link L has more than one gap if and only if L is a connected sum of Hopf links.

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Congruence subgroups of braid groups and crystallographic quotients

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This talk delves into the relationship between two families of groups, respectively subgroups and quotients of classical braid groups: congruence subgroups of braid groups and crystallographic braid groups, respectively introduced Arnol'd and Tits. We recall and introduce some elements belonging to congruence braid groups and we establish some (iso)-morphisms between crystallographic braid groups and corresponding quotients of congruence braid groups. Finally, we study the lower central series of congruence braid groups related to the braid group B_3, showing in particular that corresponding quotients are all *almost* crystallographic. Joint work with C. Damiani, O. Ocampo, and C. Stylianakis.

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From Kirby diagrams to triangulations of PL 4-manifolds

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Kirby diagrams, i.e. links in the 3-sphere equipped with integers associated to some components, are a classical representation method for compact PL 4-manifolds encoding their handle decompositions.

On the other hand, regular edge-colored graphs have proved to be, in many ways, an useful combinatorial tool to encode triangulations and so represent compact PL manifolds of any dimension.

We present an algorithm which, given a suitable Kirby diagram (L, d) of a compact PL 4-manifold M, produces an edge-colored graph $\Gamma(L, d)$ representing M and directly "drawn" over a planar diagram of the link ([2]).

As a consequence, the combinatorial structure of $\Gamma(L, d)$ allows to obtain upper bounds for the value of some invariants of M ([2], [3]). In particular, for any closed orientable PL 4-manifold M, an estimation can be given, in terms of combinatorial properties of L, for the *trisection genus* of M, an invariant defined within the theory of *trisections*, which were introduced in [4] as a generalization to dimension 4 of the classical concept of Heegaard splitting of a 3-manifold, and are currently intensively studied.

Moreover, the presented algorithm turns out to be an useful tool in the study of triangulations of exotic pairs of 4-manifolds; in fact, most examples of such pairs are known only through their representation by Kirby diagrams, while explicit triangulations can be rarely found in literature.

Actually, as shown in [1], the implementation of our algorithm in the *Regina* software package produced several examples of triangulations of exotic pairs as well as the first known triangulations of some Akbulut's corks, thus opening the possibility to gain an insight into their structural features.

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A New Combinatorial Invariant of Doubly Periodic Tangles

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Doubly periodic tangles, or *DP tangles*, are complex entangled structures consisting of curves embedded in the thickened plane $\mathbb{E}^2 \times I$, so can be defined as lifts of links in the thickened torus, $T^2 \times I$. They serve as a significant framework for analyzing and understanding the topological properties of interwoven filament systems across micro-, meso- and macro-scales, including, but not limited to, polymer melts, fabric-like structures, molecular chemistry, and cosmic filaments. The topological classification of DP tangles is at least as hard a problem as the full classification of knots and links in the three-space and is approached by constructing topological invariants. To reduce the complexity of this problem, the idea is to to consider the quotient of a DP diagram under a periodic lattice, namely a link diagram in the (flat) torus T^2 that we call *(flat) motif*. This approach leads to a diagrammatic theory of the topological equivalence of DP tangles, which has been established in [1] on the level of motifs, and that generalizes works initiated by Grishanov et al. related to textiles ([4, 5]).

In this talk we introduce new topological invariants of DP tangles. We will, in particular, present the notion of *axis-motif*, that is a set of arcs in the flat torus which can be viewed as a blueprint of a DP tangle capturing the different directions along which its components are organized. This will lead to the definition of the *directional type* of the DP tangle, which constitutes a topological invariant of DP tangles ([2]). We will then introduce the concept of *density* of a motif τ, defined in terms of the total number of arcs of the axis-motif of τ, which gives rise to a new invariant called *density of the DP tangle* τ_∞, defined as the minimal density over all axis-motifs of τ_∞. However, we will note that this topological invariant is not strong enough to distinguish two DP tangles of different directional types. Thus, by using the fact that the set of arcs of an axis-motif of a motif τ can be partitioned into a specific triple of integers, that we call *arc-triple of* τ, we will present a stronger invariant of DP tangles, called *minimal arc-triple*. This notion leads to a characterization of the directional type of a DP tangle by its minimal arc-triple. All the above invariants of DP tangles are measures that naturally inform on their topological complexity, they refer to global topological properties of theirs, and they add to the list of the existing invariants. In the end of this talk we will present examples of computations of our invariants on several DP tangles, comparing them with some known numerical invariants.

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The Ideal Theory and Arithmetic of Rings, Monoids, and Semigroups. Special Session A3

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During the last 20 years, the theory involving the structure of the arithmetic and ideal theory of various algebraic structures has been a popular topic and taken several important steps forward. Many applications of this theory, with particular attention to the multiplicative monoids of integral domains and their combinatorial or numerical applications to ring theory, have appeared throughout the mathematical literature. It is the aim of this session to review recent developments in this area by bringing together researchers from different areas of algebra under the umbrella of commutative monoids, semigroups, and rings. Topics to be covered include multiplicative ideal theory and general ideal systems, arithmetic in Krull and Prüfer monoids, commutative monoid rings, integer-valued polynomials, numerical monoids and congruence monoids, direct sum decompositions of modules, and various aspects of non-unique factorization.

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Square-difference factor absorbing ideals of a commutative ring

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Let R be a commutative ring with $1 \neq 0$. A proper ideal I of R is a *square-difference factor absorbing ideal* (sdf-absorbing ideal) of R if whenever $a^2 - b^2 \in I$ for $0 \neq a, b \in R$, then $a + b \in I$ or $a - b \in I$. In this paper, we introduce and investigate sdf-absorbing ideals.

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Star operations related to Polynomial closure

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Let E be a subset of K. A polynomial $f(X) \in K[X]$ is D-*integer-valued* over E if $f(E) \subseteq D$, and the set $\mathrm{Int}(E, D)$ of such polynomials is a subring of $K[X]$. The *polynomial closure of* E *in* D is the largest subset F of K such that $\mathrm{Int}(E, D) = \mathrm{Int}(F, D)$ ([1]).

Such a closure has been studied in several contexts from a topological point of view: for example, if D is a valuation domain, the polynomial closure is a topology if and only if D has dimension 1 (see [1, Theorem 5.3]) and [4, Theorem 2.7]); when D is also a rank-one discrete valuation domain (DVR) this topology coincides with the v-adic topology ([1, Proposition 4.5]).

The polynomial closure can also be studied as a star-operation (see [2]).

Generalizing the results obtained in [3] for some classes of Prüfer domains, we show that the polynomial closure and the v-operation coincide if D is an integrally closed domains or if D has residue characteristic 0.

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Multiplicative lattices, their primes and spectral spaces

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Prime ideals play a crucial role in multiplicative ideal theory, being a key tool for investigating ideal-theoretic properties of commutative rings. Moreover prime spectra are central in algebraic geometry, since they are foundation of scheme theory. However many other algebraic structures admit a prime spectrum: non-commutative rings, monoids, groups, etc. The aim of this talk is to introduce and study a natural framework where to study spectra, that of multiplicative lattices. We will focus on some ideas developed in [1] and [2].

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The isomorphism problem for ideal class monoids of numerical semigroups

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A *numerical semigroup* S is a submonoid of $(\mathbb{N},+)$ such that $\mathbb{N}\setminus S$ has finitely many elements, where $\mathbb{N}$ denotes the set of non-negative integers. The set $\mathbb{N}\setminus S$ is know as the set of gaps of S.

A set of integers I is a *(relative) ideal* of S if $I+S\subseteq I$ and I has a minimum (having a minimum is equivalent to $a+I\subseteq S$ for some integer a). On the set of ideals of S, we define the following relation: $I\sim J$ if there exists an integer z such that $I=z+J$. The set of ideals modulo this equivalence relation is known as the *ideal class monoid* of S, denoted $\mathcal{C}\ell(S)$. Addition of two classes $[I]$ and $[J]$ is defined in the natural way: $[I]+[J]=[I+J]$. The ideal class monoid of a numerical semigroup was introduced in [1], where some basic properties and bounds for its cardinality where given.

There is a one to one correspondence between classes of ideals in $\mathcal{C}\ell(S)$ and normalized ideals of S, that is, ideals of S whose minimum is zero: in each class $[I]$ we choose $-\min(I)+I\in[I]$. Denote by $\mathfrak{I}_0(S)$ the set of normalized ideals of S, which is a submonoid of the set of relative ideals of S. The above correspondence is indeed a monoid isomorphism between $\mathcal{C}\ell(S)$ and $\mathfrak{I}_0(S)$. By using this correspondence, in [2], we gave new bounds for the cardinality of the ideal class monoid of a numerical semigroup with the help of Apéry sets and Kunz coordinates. We also studied several properties and notable elements of the posets $(\mathfrak{I}_0(S),\subseteq)$ and $(\mathfrak{I}_0(S),\preceq)$ (where $I\preceq J$ if there exists $K\in\mathfrak{I}_0(S)$ such that $I+K=J$) that reflected attributes and invariants of the semigroup S.

In this talk, we prove that if S and T are numerical semigroups such that there is a poset isomorphism from $(\mathfrak{I}_0(S),\subseteq)$ to $(\mathfrak{I}_0(T),\subseteq)$, then S and T must be equal. To achieve this result, we first prove that the poset of gaps of a numerical semigroup S, with respect to the order induced by S ($a\leq_S b$ if $b\in a+S$), completely determines S. The poset $(\mathbb{N}\setminus S,\leq_S)$ is isomorphic to the poset of ideals $\mathfrak{P}_0(S)=\{\{0,g\}+S : g\in\mathbb{N}\setminus S\}$ (under inclusion). We then show that any isomorphism from $(\mathfrak{I}_0(S),\subseteq)$ to $(\mathfrak{I}_0(T),\subseteq)$ restricts to an isomorphism between $(\mathfrak{P}_0(S),\subseteq)$ and $(\mathfrak{P}_0(T),\subseteq)$.

We will also prove that if S and T are numerical semigroups whose ideal class monoids are isomorphic, then S and T must be equal. To this end, we will use the fact that quarks in $(\mathfrak{I}_0(S),+)$ are unitary extensions of S (oversemigroups of S that differ in one element with S), and that a numerical semigroup is completely determined by its unitary extensions. Induction on the genus (number of gaps) of S and T, which is proven to be the same, is the last ingredient needed to finish the proof.

We will finish the talk with some questions regarding the poset $(\mathfrak{I}_0(S),\preceq)$.

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Norms and elasticities in rings of algebraic integers

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One of the central notions in the study of factorization in monoids and domains is the notion of elasticity. If we let X denote a monoid or domain, and $x \in X$ can be factored as

$$x = \pi_1 \pi_2 \cdots \pi_n$$

with each $\pi_i \in X$ an irreducible, then we say that x has a factorization of length n. We now define the elasticity of x as

$$\rho(x) = \sup\{\frac{n}{m} | \text{where } x \text{ has (irreducible) factorizations of lengths } n \text{ and } m\}.$$

The elasticity of X can be obtained by taking the supremum over the elasticities of all of the atomic elements in X.

It is well-known (see [3] and [4]) that if R denotes a ring of algebraic integers, then the elasticity of R is given by

$$\rho(R) = \begin{cases} \frac{D(\mathrm{Cl}(R))}{2}, \text{ if } R \text{ is not a UFD} \\ 1, \text{ if } R \text{ is a UFD} \end{cases}$$

where $D(\mathrm{Cl}(R))$ is the Davenport constant of the class group of R.

If R is a ring of algebraic integers, the set of integral norms of R forms an atomic monoid that has been shown, in many cases, to effectively mirror the factorization structure of the parent ring R. For instance, if R is a ring of integers with quotient field F, and F is Galois over $\mathbb{Q}$, then R is a UFD if and only if $N(R)$, the set of integral norms of R, is a UFM (see [1]). An analogous result also holds for HFDs ([2]).

In this talk, we will discuss a concept that generalizes the notion of the Davenport constant for a finite abelian group. We then show that this is the "right" notion needed to talk about the relationship between the elasticity of the set of norms and its parent ring in the Galois case. Special attention will then be devoted to the quadratic case where some strong results may be obtained.

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The Ideal Theory and Arithmetic of Rings, Monoids, and Semigroups

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A 1927 conjecture of Emil Artin predicts that for any integer g, not -1 and not a square, there are infinitely many primes p for which the multiplicative group mod p is generated by g. While Artin's conjecture remains unsolved, it has generated quite a lot of good mathematics. I will discuss applications of some of this theory to the elasticity of orders in quadratic fields. As one example, the Generalized Riemann Hypothesis implies that infinitely many orders inside $\mathbb{Q}(\sqrt{2})$ are half-factorial, confirming a conjecture of Coykendall.

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Monoid Algebras and Weak Notions of ACCP

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For a submonoid M of a torsion-free abelian group and a commutative ring R, let $R[M]$ denote the monoid algebra of M over R. In this talk, we will discuss some recent progress on weaker notions of the ACCP property (i.e., every ascending chain of principal ideals stabilizes). In particular, we will discuss the ascent of such properties from the pair (M, R) to the monoid algebra $R[M]$. We will focus on the special case where R (and so $R[M]$) is an integral domain.

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Algebraic Properties of Subsemigroups and Semigroup Ideals of Factorial Monoids

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Consider a factorial monoid $F = F^\times \times \mathcal{F}(P)$, where P is a set of prime elements and $F^\times$ are the units of F. We will discuss the general algebraic and arithmetic properties of subsemigroups H of F, determining when they are Krull, root closed, bounded factorization, etc. Attention will be paid to subtle technical details that arise when adapting concepts from the factorization theory of monoids to the setting of semigroups. Additionally, the role of the units $F^\times$, particularly the set $F^\times \cap H$, will be discussed in detail. We then will specialize to semigroup ideals, namely nonempty subsets H of F satisfying $HF = H$. Here, many of the technical concerns involving units are quickly resolved and we obtain a much clearer picture of the algebra, which is often wild from a factorization-theoretic perspective. In particular, we will exhibit a large class of semigroup ideals which exhibit pathological factorization properties in the "furcus" family of conditions (bifurcus, m-furcus, and multifurcus).

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Skew Mal'cev-Neumann Series Ring Over a Dedekind Domain

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There are only a few known constructions of Dedekind prime rings. Our goal is to give a new one. Let D be a commutative Dedekind domain. We study the ring of skew Mal'cev-Neumann series,

$$R = D((G; \alpha)),$$

for an ordered group G and a homomorphism $\alpha : G \to \mathrm{Aut}(D)$. We show that R is a (simple) noncommutative Dedekind domain under some assumptions on α. Furthermore, the canonical morphism from the ideal class group of D to the group of stable isomorphism classes of R-ideals is surjective. This gives us a way to construct new examples of noncommutative Dedekind domains with some control over their simple modules.

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Egyptian integral domains and the ring of reciprocals

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An Egyptian fraction

$$q = \frac{1}{a_1} + \ldots + \frac{1}{a_n}$$

is a representation of a rational number q as a sum of distinct unit fractions. It is well-known since ancient time (the first known proofs goes back to Fibonacci) that any rational number q can be represented as an Egyptian fraction.

In this talk we start by discussing the existence of Egytpian fractions in arbitrary integral domains. We say that an element x of an integral domain D is *Egyptian* if there exist distinct elements $d_1, \ldots, d_n$ such that

$$x = \frac{1}{d_1} + \ldots + \frac{1}{d_n},$$

and that D is an *Egyptian domain* if all its elements are Egyptian. The ring of integers $\mathbb{Z}$ is clearly an Egyptian domain. We show that also the rings with nonzero Jacobson radical are Egyptian while polynomial rings over any ring are not Egyptian.

For this study, it is useful to consider the ring of reciprocals $R(D)$, defined as the subring of the quotient field of D generated by all fractions $\frac{1}{d}$ for nonzero $d \in D$. It can be observed that a domain D is Egyptian if and only if $R(D)$ coincides with the quotient field of D.

In the second part of the talk we focus on the description of the ring $R(D)$ in the case D is a polynomial ring over a field in a finite number of variables.

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Asymptotic behaviour of the v-number of homogeneous ideals

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Let $S = K[x_1, \dots, x_n]$ be the standard graded polynomial with coefficients over a field K, and let $I \subset S$ be a homogeneous ideal. The v-number of I is defined as the minimum degree of an homogeneous polynomial $f \in S$ such that $(I : f) \in \text{Ass}(I)$ is an associated prime of I. This invariant was introduced in relation to minimum distance functions and Reed-Muller type codes. In the present talk, we show the following

Theorem 1. [3, Theorem 1.1], [5, Theorems 3.1 and 4.1] *Let $I \subset S$ be a graded ideal. Then, for all $k \gg 0$, $\text{v}(I^k)$ is a linear function of the form $\alpha(I)k + b$ where*

$$\lim_{k\to\infty} \frac{\text{v}(I^k)}{k} \;=\; \alpha(I)$$

is the initial degree of I and $b \in \mathbb{Z}$. Here $\alpha(I)$ is the minimum degree of an homogeneous element belonging to I.

We then survey the recent numerous studies on this and related topics, like monomial ideals, integer programming and graded filtrations, and then we discuss some open questions.

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Differences in sets of lengths for monoids of plus-minus weighted zero-sum sequences

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Let $(G, +)$ be a finite abelian group. A (finite) sequence of elements $g_1 \dots g_k$ is called a zero-sum sequence if $g_1 + \cdots + g_k = 0$; in the current context sequences that just differ in the ordering of the terms are considered as equal. The set of all zero-sum sequences over G, denoted $\mathcal{B}(G)$, forms a submonoind of the monoid of all sequences over G, denoted $\mathcal{F}(G)$. It is a Krull monoid and there are numerous results on its arithmetic, in particular as results on its arithmetic translate directly into results on the arithmetic of rings of algebraic integers.

More recently the following related monoid has been investigated (see the references). A sequence is called a plus-minus weighted zero-sum sequences if $\epsilon_1 g_1 + \cdots + \epsilon_k g_k = 0$ for some choice of 'weights' $\epsilon_i \in \{+1, -1\}$. The set of all these sequences is denote $\mathcal{B}_{\pm}(G)$; it is also a submonoid of $\mathcal{F}(G)$. For most G, it is not a Krull monoid yet still a C-monoid. Moreover, its arithmetic is related to the arithmetic of normsets for rings of integers of quadratic number fields.

When investigating the arithmetic of monoids, sets of lengths are an important notion. For an element a of a multiplicative monoid H, one says that a has a factorization of lengths l if there are irreducible elements $a_1, \dots, a_l$ such that $a = a_1 \dots a_l$. One denotes by $\mathsf{L}(a)$ the set of all l that are a lengths of a.

Since these monoids, $\mathcal{B}_{\pm}(G)$, are finitely generated monoids the Structure Theorem of Sets of Lengths holds (see for example [1, 4] for details), that is, all its sets of lengths are Almost Arithmetical Multiprogressions with a global bound M and a difference from a certain finite set of differences $\Delta^*(\mathcal{B}_{\pm}(G))$.

The goal of this talk is to present results on the sets $\Delta^*(\mathcal{B}_{\pm}(G))$, in particular its maximum. In case the order of G is odd, we obtain fairly precise results, in particular the maximum is equal to $\exp(G) - 2$. For groups of even order, the situation is much less clear and $\Delta^*(\mathcal{B}_{\pm}(G))$ can contain considerably larger elements.

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Palermo, July 23–26, 2024

On an Inverse Zero-sum Question for Rank Two Groups

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For an abelian group G, the Davenport Constant of G is the smallest integer ℓ such that any sequence of ℓ terms from G must contain a non-trivial zero-sum subsequence. If $k \geq 0$ is an integer, one can refine this question by asking how long a sequence of terms from G must be to guarantee a nontrivial zero-sum of length at most $\mathsf{D}(G) - k$, so with k less terms than that guaranteed by definition of the Davenport constant. Let $\mathsf{s}_{\leq \mathsf{D}(G)-k}(G)$ denote this minimal ℓ such that any sequence of ℓ terms from G must contain a nontrivial zero-sum subsequence of length at most $\mathsf{D}(G) - k$. Wang and Zhao determined its precise value for rank two abelian groups $G = C_m \oplus C_n$ with $m \mid n$, showing

$$\mathsf{s}_{\leq \mathsf{D}(G)-k}(G) = m + n - 1 + k \quad \text{ for } k \in [0, m-1]$$

(the constant is infinite for larger values of k). At the extremal values $k = 0$ and $k = m$, this recovers well-known exact formulas for the Davenport Constant and the η Constant of rank two abelian groups (the latter asking for a nontrivial zero-sum subsequence of length at most the exponent of G). It remained an open question to characterize the structure of all extremal sequences of length $m + n - 2 + k$ failing to contain a nontrivial zero-sum of length at most $\mathsf{D}(G) - k = m + n - 1 - k$. Prior work completely reduced the general characterization problem to that for rank two p-groups of the form $C_p \oplus C_p$, with partial progress resolving the case of small $k \leq \frac{2p+1}{3}$. Here, we resolve all open cases, showing that a sequence of $2p - 2 + k$ terms avoiding any nontrivial zero-sum with length at most $2p - 1 - k$, where $k \in [2, p-2]$, must consist of three distinct terms e_1, e_2 and $e_1 + e_2$, with e_1 and e_2 repeated $p - 1$ times and $e_1 + e_2$ repeated k times, for some basis (e_1, e_2) of G. Hence all extremal sequences have the form $e_1^{p-1} \cdot e_2^{p-1} \cdot (e_1 + e_2)^k$.

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On the arithmetic of the monoid of plus-minus weighted zero-sum sequences

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Let G and G_1 be abelian groups and let $\mathcal{F}(G)$ be the free abelian monoid with basis G. By $\mathcal{B}_{\pm}(G) = \{\prod_{i=1}^{n} g_i \mid n \in \mathbb{N}_0, (g_i)_{i=1}^{n} \in G^n \text{ and } \sum_{i=1}^{n} \varepsilon_i g_i = 0 \text{ for some } (\varepsilon_i)_{i=1}^{n} \in \{-1,1\}^n\} \subseteq \mathcal{F}(G)$, we denote the monoid of plus-minus weighted zero-sum sequences.

In this talk, we discuss and describe when $\mathcal{B}_{\pm}(G)$ satisfies various well-known properties, like being a Mori monoid, a C-monoid or a finitely generated monoid. As a byproduct, we rediscover and strengthen some of the results of [1] and [2].

We will also put our focus on two problems that were profoundly studied for many other types of monoids. These problems are the isomorphism problem and the characterization problem.

- **The isomorphism problem:** *Let $\mathcal{B}_{\pm}(G)$ and $\mathcal{B}_{\pm}(G_1)$ be isomorphic (as monoids). Determine when G and G_1 are isomorphic (as groups).*
- **The characterization problem:** *Suppose that the systems of sets of lengths of $\mathcal{B}_{\pm}(G)$ and $\mathcal{B}_{\pm}(G_1)$ coincide. Describe when G and G_1 are isomorphic (as groups).*

The main results of our talk provide partial solutions to each of these problems.

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The stable rank of a ring of integer-valued polynomials

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The stable rank of a ring, introduced by Bass and simplified by Vaserstein, is an important invariant in algebraic K-theory, but, we think, underappreciated in commutative ring theory.

Stable rank is comparable to Krull dimension: By a result of Heitmann $s.r.(R) \leq \dim(R) + 1$, for Prüfer rings. (But s.r. is not really a measure of dimension, since $s.r.(R) = 1$ for every local ring.)

We will show $s.r.(R) = 2$ for some 2-dimensional Prüfer rings, namely, rings of integer-valued polynomials over rings of integers in number fields (other than imaginary quadratic) and discuss possible generalizations.

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Nontriviality of rings of integral-valued polynomials

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In this work we consider rings of integral-valued polynomials over subsets S of the ring of all algebraic integers $\overline{\mathbb{Z}}$, defined as $\mathrm{Int}_{\mathbb{Q}}(S, \overline{\mathbb{Z}}) = \{f \in \mathbb{Q}[X] \mid f(S) \subseteq \overline{\mathbb{Z}}\}$. The family of these rings has been introduced by Alan Loper and Nicholas J. Werner in 2012 to obtain an example of a Prüfer domain strictly contained between $\mathbb{Z}[X]$ and the classical ring of integer-valued polynomials $\mathrm{Int}(\mathbb{Z})$. We correct here a minor wrong claim of that paper, namely, that for any $S \subseteq \overline{\mathbb{Z}}$, the ring $\mathrm{Int}_{\mathbb{Q}}(S, \overline{\mathbb{Z}})$ is always nontrivial, i.e., it strictly contains $\mathbb{Z}[X]$. For example, if S comprises all the roots of unity $\xi_n, n \in \mathbb{N}$ it is not difficult to show that $\mathrm{Int}_{\mathbb{Q}}(S, \overline{\mathbb{Z}}) = \mathbb{Z}[X]$ using the fact that the ring of integers of $\mathbb{Q}(\xi_n)$ is monogenic. We completely characterize those subsets S of $\overline{\mathbb{Z}}$ for which $\mathrm{Int}_{\mathbb{Q}}(S, \overline{\mathbb{Z}})$ is nontrivial in terms of pseudo-divergent sequences and pseudo-stationary sequences contained in S with respect to some fixed extension of the p-adic valuation to $\overline{\mathbb{Q}}$, as p runs through the set of prime integers. These sequences have been introduced by Chabert in 2010 in order to study the polynomial closure of subsets of rank one valuation domains.

We produce several examples of subsets of $\overline{\mathbb{Z}}$ of unbounded degree for which the corresponding ring $\mathrm{Int}_{\mathbb{Q}}(S, \overline{\mathbb{Z}})$ is trivial or not. In particular, we show that the monogenicity of the ring of integers of $\mathbb{Q}(s)$, for $s \in S$, is not a necessary condition for $\mathrm{Int}_{\mathbb{Q}}(S, \overline{\mathbb{Z}})$ to be trivial.

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Integer-valued polynomials over subrings of matrix algebras

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Integer-valued polynomials with coefficients in noncommutative algebras have been studied since around 2010. As there is no substitution homomorphism in this case, it is not clear when such polynomials form a ring. In 2012, N. Werner gave a positive answer for integer-valued polynomials with matrix coefficients, by showing that $\mathrm{Int}(\mathrm{M}_n(D)) = \{f \in \mathrm{M}_n(K)[x] \mid f(\mathrm{M}_n(D)) \subseteq \mathrm{M}_n(D)\}$, where D is an integral domain with quotient field K, is a subring of $\mathrm{M}_n(K)[x]$. S. Frisch showed that this also holds for integer-valued polynomials over upper triangular matrices. These results were extended by J. Sedighi Hafshejani, A. R. Naghipour, and M. R. Rismanchian to certain kinds of block matrix algebras. We study subalgebras of $\mathrm{M}_n(D)$ of the following form: let $\precsim$ be a preorder on $\{1, \dots, n\}$ and define $\mathrm{M}_{\precsim}(D) = \{(a_{ij}) \in \mathrm{M}_n(D) \mid a_{ij} \neq 0 \Rightarrow i \precsim j\}$. In other words, the ring $\mathrm{M}_{\precsim}(D)$ consists of those matrices whose coefficients in certain positions (determined by $\precsim$) are zero, while the others range freely over D. We show that the set of integer-valued polynomials $\mathrm{Int}(\mathrm{M}_{\precsim}(D)) = \{f \in \mathrm{M}_{\precsim}(K)[x] \mid f(\mathrm{M}_{\precsim}(D)) \subseteq \mathrm{M}_{\precsim}(D)\}$ is a ring and give a description in terms of polynomials with coefficients in K.

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[1]The speaker is supported by the Austrian Science Fund (FWF): P 35788
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Palermo, July 23–26, 2024

Integer-Valued Polynomials for Semidomains

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A semidomain is a subsemiring of an integral domain. One might think of a semidomain as an integral domain in which the condition of having additive inverses is relaxed. In our discussion, we generalize the notion of integer-valued polynomials for the class of semidomains. This talk is based on joint works with Scott T. Chapman and Nathan Kaplan.

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Bounds for syzigies of monomial curves

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Despite the simplicity of the objects involved, this question is still not well understood.

A numerical monoid $\Gamma \subseteq \mathbb{N}$ is a cofinite additive submonoid of $\mathbb{N}$. Every numerical monoid has a unique minimal set of generators $g_0 < g_1 < \cdots < g_e \in \Gamma$, and we write $\Gamma = \langle g_0, g_1, \ldots, g_e \rangle$ to denote this fact. The cardinality $e+1$ of the minimal set of generators is called the embedding dimension of Γ, and it is denoted by $edim(\Gamma)$. The monoid morphism $\varphi : \mathbb{N}^{e+1} \to \mathbb{N}$ defined by $\varphi(a_0, a_1, \ldots, a_e) = \sum_{i=0}^{e} a_i g_i$ yields an isomorphism $\Gamma \cong \mathbb{N}^{e+1}/\ker(\varphi)$, where $\ker(\varphi) = \{(\mathbf{a}, \mathbf{b}) \in \mathbb{N}^{e+1} \times \mathbb{N}^{e+1} \mid \varphi(\mathbf{a}) = \varphi(\mathbf{b})\}$ is the kernel congruence of φ. A *minimal presentation* of Γ is a minimal set of generators of the congruence $\ker(\varphi)$. Minimal presentations are not unique, but their cardinality depends only on Γ. This cardinality is denoted by $\rho(\Gamma)$, and it is referred to as the number of minimal relations of Γ.

The main question we consider in this talk is: how many minimal relations can a numerical monoid have? The picture for lower bounds is quite clear, since it is known that $\rho(\Gamma) \geq e$, with equality attained for *complete intersection* semigroups. On the other hand, finding sharp upper bounds for the number of minimal relations of a numerical monoid is a surprising complex problem.

Let K be a field and $P = K[[x_0, x_1, \ldots, x_e]]$.

The semigroup ring of Γ is $R_\Lambda = K[[t^\gamma : \gamma \in \Gamma]] = K[[t^m, t^{g_1}, \ldots, t^{g_\nu}]] \subseteq \; [[t]]$.

We have $R_\Lambda = P/I_\Lambda$, where I_Λ is the toric ideal of the semigroup Γ

$$I_\Lambda = \left(x_0^{\alpha_0} \cdots x_\nu^{\alpha_e} \surd x_0^{\beta_0} \cdots x_\nu^{\beta_e} \mid \sum_{i=0}^{\nu} \alpha_i g_i = \sum_{i=0}^{\nu} \beta_i g_i \right).$$

The number of minimal relations $\rho(\Gamma)$ is actually the number of minimal generators of I_Λ, and thus it is possible to bound the number of relations in this way by using techniques coming from commutative algebra.

In this talk, we provide an useful connection between the world of semigroup rings and toric ideals and that of monomial ideals, by associating to I_Λ a monomial ideal J_Λ having larger Betti numbers; using this approach, we can thus bound $\rho(\Gamma)$ by translating the original problem in the context of monomial ideals, granting us access to some classical results in this context. We provide a general bound for the number of relations, and for the Betti numbers of I_Λ at large, in function of e and g_0, as well as finding some cases in which this bound is sharp. Furthermore, we use this approach to study a question of Herzog and Stamate, asking for a bound for $\rho(\Gamma)$ in function of $g_e \surd g_0$.

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On factorization invariants of ideal extensions of free commutative monoids

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Let $\mathbb{N}$ be the set of non negative integers and I be a non-empty subset of $\mathbb{N}$. We denote by $\mathbb{N}^{(I)}$ the additive monoid consisting of all sequences $\mathbf{n} = (n_i)_{i\in I}$ such that $n_i = 0$ but for finitely many $i \in I$. If S is a submonoid of $\mathbb{N}^{(I)}$, the set of gaps of S is defined as $\mathcal{H}(S) = \mathbb{N}^{(I)} \setminus S$. An element $\mathbf{a}$ of S is an *atom* if whenever $\mathbf{a} = \mathbf{b} + \mathbf{c}$ for some $\mathbf{b}, \mathbf{c} \in S$, then either $\mathbf{b} = \mathbf{0}$ or $\mathbf{c} = \mathbf{0}$. The set of atoms of S is denoted by $\mathcal{A}(S)$. S is called an *ideal extension* of $\mathbb{N}^{(I)}$, if $S^* = S \setminus \{\mathbf{0}\}$ is an ideal of $\mathbb{N}^{(I)}$, that is, $S^* + \mathbb{N}^{(I)} \subseteq S^*$. Moreover, for a submonoid S of $\mathbb{N}^{(I)}$, we say that S is a *gap absorbing monoid* if

(1) $2\mathcal{H}(S) \subseteq \mathcal{H}(S) \cup \mathcal{A}(S) \cup 2\mathcal{A}(S)$, and
(2) $\mathcal{H}(S) + \mathcal{A}(S) \subseteq \mathcal{A}(S) \cup 2\mathcal{A}(S)$.

In this work, we study some invariants of Factorization Theory of these kind of monoids. In particular, we provide some properties about Betti Elements, Delta set, Catenary degree and Omega primality (see [2] and [3] for the definition of these invariants). For instance, for the Delta set $\Delta(S)$, a conjecture by N. Baeth in [1,Conjecture 4.16] states that $\Delta(S) = \{1\}$ holds for every complement finite ideal S in $\mathbb{N}^d$, with d any positive integer. We provide an affirmative answer to this question in the case $S \subseteq \mathbb{N}^{(I)}$ is a gap absorbing monoid. This work is a continuation of [1], and is dedicated to its author.

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Discrete valuation overrings of a Noetherian ring

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Let R be a Noetherian ring and P be a regular prime ideal of R. Then there is a rank one discrete valuation overring of R with (regular) maximal ideal M, so that $M \cap R = P$. This is a natural generalization of Chevalley's result that every local Noetherian domain is dominated by a rank one discrete valuation overring.

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Recent advances in numerics for deterministic and stochastic dynamical systems
Special Session A4

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The special session is focused on presenting recent advances in numerical modeling for evolutionary problems (such as Ordinary and Partial Differential Equations, Stochastic Differential Equations). The focus of each contribution is conveyed on numerical integrators, the analysis of their relevant properties, also towards possibile applications. Accuracy, stability and structure-preserving issues are considered, together with their role in selected applications. Theoretical issues are also supported by a proper selection of numerical experiments.

The session is scheduled on July 23–24.

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Transient dynamics under structured perturbations: a bridge between unstructured to structured pseudospectra

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> The figure shows that the unstructured spectral value set can be a misleading indicator of the robustness of stability. D. Hinrichsen and A. J. Pritchard (2005), p. 532

> By contrast, the eigenvalues that arise from structured perturbations do not bear as close a relation to the resolvent norm and may not provide much information about matrix behavior. L. N. Trefethen and M. Embree (2005), p. 458

The structured ε-stability radius is introduced as a quantity to assess the robustness of transient bounds of solutions to linear differential equations under structured perturbations of the matrix. This applies to general linear structures such as complex or real matrices with a given sparsity pattern or with restricted range and corange, or special classes such as Toeplitz matrices. The notion conceptually combines unstructured and structured pseudospectra, allowing for the use of resolvent bounds as with unstructured pseudospectra and for structured perturbations as with structured pseudospectra, which has important applications.

We propose and study an algorithm for computing the structured ε-stability radius, which solves eigenvalue optimization problems via suitably discretized rank-1 matrix differential equations that originate from a gradient system. The proposed algorithm has essentially the same computational cost as the known rank-1 algorithms for computing unstructured and structured stability radii. Numerical experiments illustrate the behavior of the algorithm.

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Steady-state density preserving method for a class of second-order stochastic differential equations

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We devise a method for the long-time integration of a class of damped second-order stochastic systems. The introduced numerical scheme has the advantage of being completely explicit for general nonlinear systems while, in contrast with other commonly used integrators, is able to compute the evolution of the system with high numerical stability and precision in very large time intervals. Notably, the method has the important property of preserving, for all values of the stepsize, the steady-state probability density function of any linear system with a stationary distribution. Numerical simulations are presented to illustrate the practical performance of the introduced method.

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Nonstandard numerical integration of local and nonlocal differential models

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Reliable numerical simulations of real-life applications should be based on methods able to catch the main features of the phenomenon under consideration. This means that the numerical method must preserve properties such as positivity, boundedness, monotonicity, asymptotic values, conservation laws of the exact solution.

In this talk, we focus on differential models of physical or biological phenomena, where the modelled quantities are, e.g., chemical concentrations, population sizes or the temperature, thus these should necessarily be positive. General-purpose numerical methods are not usually derived to satisfy this property, thus they may produce nonphysical solutions, at least on coarse grids. Recently, some numerical schemes have been proposed which compute positive solution by design, as the nonstandard finite difference schemes, introduced in [4].

The first part of the talk regards time-fractional reaction-advection-diffusion problems with positive solution. We apply a standard space discretization by finite difference schemes, L1 or Grünwald-Letnikov methods for the approximation of the fractional derivative in time; then we suitably modify each scheme using a nonstandard time integration, to obtain positive solutions. We analyse the stability and the convergence of the proposed schemes. We compare these methods with their standard counterpart on some significant test examples.

The second part of the talk is concerned with an age-group SIR (Susceptible-Infected-Recovered) model, which is composed by a nonlinear system of ordinary differential equations. We prove the conservation of the total population, the positivity of the analytical solution, and derive the final size of the epidemic. For the numerical approximation, we consider standard and nonstandard finite difference schemes, and a Modified Patankar-Runge-Kutta (MPRK) method. We prove that the standard finite difference scheme preserves the positivity only for a small stepsize, while the nonstandard one and the MPRK method are unconditionally positive. A model for the diffusion of information in social networks is considered for application of the presented results on real data.

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Stabilization of synchronous solutions in networks

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We consider a network of identical agents, coupled through linear antisymmetric nearest neighbor coupling. The single agent dynamic has an attractor and we are interested in stabilizing the corresponding synchronous solution in the network. In this talk: (i) We show how and when it is possible to choose an appropriate coupling of the agents so that the synchronous solution is stable, and (ii) we show that this guarantee of stability comes without having to let the coupling strength be too large. Our construction is based on the Master Stability Function and on solving a suitable inverse eigenvalue problem for the coupling matrix. Numerical implementations will be presented.

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Structure-preserving numerics: yesterday, today and tomorrow

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This talk aims to outline some recent advances on structure-preserving numerical methods for deterministic and stochastic differential equations, highlighting the basic principles of the so-called geometric numerical integration by its history. After providing a glance to deterministic structure-preservation, the talk moves towards the direction of stochastic geometric numerical integration, according to the following two directions:

- *track 1: geometric numerical integration of stochastic Hamiltonian problems.* For these problems, two different scenarios are visible: if the noise is driven in the Ito sense, the expected Hamiltonian function exhibits a linear drift in time; in the Stratonovich case, the Hamiltonian is pathwise preserved. In both case, the talk aims to highlight the attitude of selected numerical methods in preserving the aforementioned behaviors. A long term investigation via backward error analysis is also presented;
- *track 2: structure-preserving numerics of stochastic PDEs.* In this case, the attention is focused on the stochastic Korteweg-de Vries equation, characterized by certain invariance laws under the exact dynamics. Our goal is to analyze whether they can also be reproduced along the numerical dynamics provided by stochastic θ-methods for the time integration of the spatially discretized system.

For all tracks, numerical evidence supporting the theoretical inspection will be provided. The investigation of above tracks is based on the joint research in collaboration with Chuchu Chen (Chinese Academy of Sciences), David Cohen (Chalmers University of Technology & University of Gothenburg), Stefano Di Giovacchino and Annika Lang (Chalmers University of Technology & University of Gothenburg).

The talk also aims to address tentative future directions for geometric numerical integration and a vision of potentially new fields of inspection where structure preservation can be directly involved.

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Derivation of new linearly implicit methods for stiff PDEs models

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Partial Differential Equations (PDEs) constitute one of the most common tools for modeling several phenomena. This talk focuses on the efficient numerical solution of PDEs models coming from real applications, such as corrosion of materials, sustainability (Maldon et al., Entropy, 2020), vegetation (Eigentler et al., Bull. Math. Biol., 2019). The numerical treatment of these models is not trivial, since a related spatial discretization, performed e.g. via finite differences, finite elements, spectral methods, often leads to large and highly stiff Initial Value Problems (IVPs). These two issues are difficult to treat simultaneously: in fact, the stiffness could force the use of very dense temporal grids, which however would make the computational cost of the method very high, due to the non-negligible size of the problem.

In this talk, we propose new efficient linearly implicit numerical methods for solving large and highly stiff problems. They are derived by stabilizing explicit numerical schemes through the so-called TASE (Time-Accurate and highly-Stable Explicit) preconditioners [1]. In particular, to improve the classical TASE-RK methods [2], starting from them we derive two new classes of numerical schemes: TASE-peer methods [3], which can be parallelized; AMF TASE-W methods [5], which have better stability properties than TASE-RK, and require the solution of a lower number of linear systems per integration step. We finally show the derivation of adapted discretizations for two reaction-diffusion PDEs models: one for the corrosion of metallic materials [6], and one for vegetation growth in the African Savannah [4]. Numerical results testify the efficiency of the proposed methods.

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A spectral method for dispersive solutions of the nonlocal Sine-Gordon equation

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Moved by the pressing need for rigorous and reliable numerical tools for the analysis of peridynamic materials, the authors propose a model able to capture the dispersive features of nonlocal soliton-like solutions obtained by a peridynamic formulation of the Sine-Gordon equation. The analysis of the Cauchy problem associated to the peridynamic Sine-Gordon equation with local Neumann boundary condition is performed in this work through a spectral method on Chebyshev polynomials nodes joined with the Störmer-Verlet scheme for the time evolution. The choice for using the spectral method resides in the resulting reachable numerical accuracy, while, indeed, Chebyshev polynomials allow straightforward implementation of local boundary conditions. Several numerical experiments are proposed for thoroughly describe the ability of such scheme. Specifically, dispersive effects as well as the ability of preserving the internal energy of the specific peridynamic kernel are demonstrated.

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Optimization of eigenvalues based on dynamical low-rank

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The use of techniques based on dynamical low-rank approximation in eigenvalue problems, when the underlying solution has a low-rank structure, has proven to be very effective, both in the case of the rightmost eigenpairs of linear operators and in the computation of the effective eigenvalue for the neutron transport equation. In this talk, we focus on the latter problem. In particular, we consider a previously introduced special low-rank inverse power iteration, which is very efficient in lowering memory requirements, and provide suitable rank adaptations. We focus on a combination of the aforementioned method with techniques to optimise quantities of interest in order to obtain specific values of the effective eigenvalue. This is a joint work with L. Einkemmer, J. Kusch and R. McClarren.

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An IVP solver for Filippov systems on co-dimension 2 manifolds and regularizations for the moments vector field

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We present a numerical solver for the integration of systems with discontinuous right-hand side, enabling sliding dynamics on co-dimensional 1 manifolds and 2. It is based on an adaptive Runge Kutta-type integrator, coupled with event detection for switch between different regimes, and on the method of moments to define the slip. The latter method can automatically detect co-dimensional 2 tangential exits. We further monitor other general exit points that may occur. Indeed, a co-dimension 2 discontinuity manifold can be attractive in finite time by partial sliding or spiraling.

Moreover, we introduce a novel regularization for discontinuous vector fields that is able to overcome issues of the bilinear regularization near first order tangential exit points and that is conjectured to converge to the moments vector field on the discontinuity manifold.

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Numerical preservation of stochastic dissipativity

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Standard numerical analysis for stochastic differential equations has a clear understanding of stability in the linear case or when the drift coefficient satisfies a one-sided Lipschitz condition and the diffusion term is globally Lipschitz. By looking at many applications, it is obvious that we need a deeper mathematical and numerical insight into stability of problems with non-global Lipschitz coefficients.

This talk is aimed to analyze nonlinear stability properties of θ-methods for stochastic differential equations under non-global Lipschitz conditions on the coefficients. In particular, the concept of exponential mean-square contractivity is introduced for the exact dynamics; additionally, stepsize restrictions in order to inherit the contractive behaviour over the discretized dynamics are also given. A selection of numerical tests confirming the theoretical expectations is also presented.

Moreover, we will briefly tackle some future frontiers concerning numerical dissipativity for stochastic partial differential equations.

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Joint Meeting 2024
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Adaptive Tau-Leaping Strategies for Efficient Kinetic Monte Carlo Simulations in Spatially Non-Uniform Systems

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In this presentation, we address the computational challenges encountered by traditional Kinetic Monte Carlo (KMC) approaches, particularly in systems with diverse timescales and spatial non-uniformity. Our work focuses on enhancing the efficiency of KMC time integration, with a specific emphasis on lattice structures in surface chemistry applications. We introduce two novel adaptive tau-leaping methods and their associated time integration strategies, inspired by the "n-fold" direct KMC approach. These methods enable simultaneous execution of multiple reactions, advancing time using adaptively-selected coarse increments, thereby significantly improving computational efficiency. Through numerical experiments, we demonstrate the efficacy of our proposed methods in comparison to existing approaches, using a catalytic surface kinetics application involving ammonia decomposition as a case study. Our findings underscore the promising potential of these strategies to streamline KMC simulations in spatially heterogeneous kinetic systems, paving the way for broader applications in complex reaction environments.

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Factorization Algebras and Geometry Special Session A5

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Brian Williams
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The space of conformal blocks is an object that describes the collection of all correlation function in a two-dimensional conformal field theory. Mathematically, it is a vector space computed from the input of a pointed curve X (the "space-time"), a vertex algebra V (which encodes the chiral conformal field theory), a collection V-representations, and possibly a principal G-bundle P on X (the gauge bundle), where G is an algebraic group. As the curve X varies in $\mathcal{M}_{g,n}$, and the bundle P in $\mathrm{Bun}_{G,X}$—the moduli space of principal G-bundles on X—spaces of conformal blocks give rise to interesting sheaves equipped with projectively flat connection (the Knizhnik–Zamolodchikov connection).

A particularly beautiful instance of this construction happens in the case of the Wess–Zumino–Witten model of conformal field theory. Here, the representation-theoretic input (the vertex algebra and its modules) correspond to an affine Kac-Moody algebra at positive integer level, and the space of conformal blocks may be identified with spaces of non-abelian theta functions on $\mathrm{Bun}_{G,X}$ and its parabolic cousins. A rich combinatorial structure emerges which allows for the computation of the dimensions of spaces of conformal blocks through the celebrated Verlinde formula.

In the last 2-3 decades, new rigorous geometric approaches to conformal field theory have emerged, allowing the conformal block construction to be re-cast and generalized in several directions. First, in the theory of chiral algebras, developed by Beilinson and Drinfeld, conformal blocks appear as the *zeroth chiral homology*—one of a (potentially infinite) tower of cohomology groups. It is an interesting problem, both purely mathematical and in physical contexts, to interpret these 'higher' conformal blocks. Similarly, in the formalism of factorization algebras developed by Costello and Gwilliam, they appear as the *factorization homology*. This framework also allows one to treat higher-dimensional examples through the use of derived geometry, and applies to more general classes of quantum field theories.

The proposed special session "Factorization Algebras and Geometry" aims to bring together experts in the theory of these generalized conformal blocks with a view towards their geometric properties, and to explore how the rich interplay of representation theory, moduli spaces, and combinatorics, may be productively generalized in these settings.

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Topological modular forms and conformal field theories

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We survey some applications of the theory of topological modular forms to the study of quantum field theories, especially conformal and topological ones, in two and three dimensions.

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The modular functor perspective on spaces of conformal blocks

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Modular functors are consistent systems of projective mapping class group representations, see e.g. [2] for an introduction. In particular, to each compact oriented surface equipped with labels for its boundary components, one associates a vector space with a projective representation of the mapping class group of that surface. This vector space is called the *space of conformal blocks* for that surface. In my talk, I will describe modular functors using modular operads. Moreover, I will explain how factorization homology can be used to classify modular functors. The talk is on joint work with Adrien Brochier [3], and is partially also based on joint work with Lukas Müller [4] [5].

In more detail, we will identify genus zero modular functors with values in a symmetric monoidal bicategory $\mathcal{S}$ with cyclic $\mathcal{S}$-valued algebras over the framed little disks operad fE_2. These are characterized in [4], and they always extend uniquely to an *ansular functor*, i.e. a modular algebra over the modular operad of handlebodies [5].

One has the following weak uniqueness result for extensions from genus zero modular functors to modular functors defined at all genera:

Theorem 1 ([3])**.** *The space of extensions of a genus zero modular functor to a modular functor is contractible if it is non-empty.*

This is a far-reaching generalization of [1].

Those cyclic fE_2-algebras that turn out to extend to higher genus are exactly the ones for which the so-called *skein modules*, that in this context will be defined using factorization homology, associated to the different handlebodies with the same boundary surface are isomorphic. We call these cyclic fE_2-algebras *connected*. We then arrive at:

Theorem 2 ([3])**.** *The moduli space of $\mathcal{S}$-valued modular functors is equivalent to the 2-groupoid of connected cyclic $\mathcal{S}$-valued fE_2-algebras.*

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Palermo, July 23–26, 2024

Higher rank series invariants for plumbed 3-manifolds

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The Witten-Reshetikhin-Turaev (WRT) invariants provide a powerful framework for constructing a family of invariants for framed links and 3-manifolds starting from the data of a modular tensor category and surgery presentations for the 3-manifolds. An ongoing pursuit in quantum topology revolves around the categorification of these invariants. Recent progress has been made in this direction, particularly through a physical definition of a new series invariant for a class of plumbed 3-manifolds. These invariants exhibit a convergence towards the WRT invariants in their limits. In this talk, I will present a refinement of such series invariants and show how one can obtain infinitely many new series invariants starting from the data of a root lattice of rank at least 2.

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Vertex algebras from divisors on Calabi-Yau threefolds

Dylan Butson
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I will explain two conjecturally equivalent constructions of vertex algebras associated to divisors S on certain toric Calabi-Yau threefolds Y, and some partial results towards the proof of their equivalence. One construction is geometric, as a convolution algebra acting on the homology of certain moduli spaces of coherent sheaves supported on the divisor, following the proof of the AGT conjecture by Schimann - Vasserot and its generalization to divisors in $\mathbb{C}^3$ by Rapcak-Soibelman-Yang-Zhao. The other is algebraic, as the kernel of screening operators on lattice vertex algebras determined by the GKM graph of Y and a Jordan-Holder filtration of the structure sheaf of S. This provides a correspondence between the enumerative geometry of coherent sheaves on Calabi-Yau threefolds and the representation theory of W-algebras and ane Yangian-type quantum groups.

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Vertex algebras from divisors on Calabi-Yau threefolds

Surya Raghavendran
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In seminal work, Grojnowski-Nakajima constructed an action of the Heisenberg algebra on equivariant cohomology of Hilbert schemes. I will describe two holomorphic factorization algebras in three complex dimensions that furnish higher dimensional uplifts of the Heisenberg and Virasoro vertex algebras respectively. Conjecturally, mode algebras of these factorization algebras act on coherent cohomology of moduli of twisted Higgs sheaves on surfaces, and in a particular example, the action admits a cohomological deformation to the one studied by Grojnowski-Nakajima. I will describe motivation and evidence for this conjecture, rooted in a new mathematical understanding of a nebulous superconformal eld theory in six dimensions.

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Raviolo vertex algebras

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Surya Raghavendran
Department of Mathematics, Yale University
Brian R. Williams
Department of Mathematics and Statistics, Boston University

I will describe recent work with B. Williams about an algebraic structure modeling the local observables in mixed holomorphic-topological quantum field theories in three dimensions. The resulting algebraic structure is directly analogous to a vertex algebra, but where holomorphic functions on a punctured complex curve are replaced by (derived) functions on a punctured 3-manifold that are constant along the leaves of chosen transverse holomorphic foliation. Our construction provides a geometric interpretation of a vertex-algebraic structure shown by Oh and Yagi to describe local operators in holomorphic-topological quantum field theories.

Theorem 1 (NG, BW). *Let V be a vector space. The structure of a raviolo vertex algebra on V is equivalent to that of a 1-shifted Poisson vertex algebra.*

Due to their formal similarities, many vertex-algebraic constructions pass over to the setting of raviolo vertex algebras. For example, there are raviolo analogues of free-field algebras, affine Kac-Moody algebras, and Virasoro and superconformal algebras. Time permitting, I will describe one such construction appearing in recent work with S. Raghavendran and B. Williams whereby we realize.

Theorem 2 (NG, SR, BW). *To any $\mathcal{N} = 2$ superconformal raviolo vertex algebra V there are two canonically defined 2-shifted Poisson schemes $\mathcal{M}_H[V]$ and $\mathcal{M}_C[V]$ called its Higgs and Coulomb branches.*

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Raviolo vertex algebras and conformal blocks

Charles Young [1]
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One very promising approach to higher conformal blocks is through factorization algebras in their various incarnations, smooth and algebraic.

In this talk, though, I'll discuss an attempt to do something more naive and direct: namely to generalize the usual definition of rational conformal blocks/coinvariants on the Riemann sphere by simply replacing the various algebras that appear – for example, the algebra of regular functions on the configuration space of marked points – by their derived analogs in higher settings.

The case of raviolo vertex algebras – in the sense recently introduced by Garner and Williams – is an attractive arena in which to try this, because it is a comparatively mild generalization of the usual situation in complex dimension one.

I'll introduce a notion of configuration space in the raviolo setting, and construct a model in dg algebras of the derived global sections of its structure sheaf, by using the Thom-Sullivan functor. Using that, I'll define a space of coinvariants in the raviolo setting, and finally show that the state-field map of raviolo vertex algebras emerges in the limit in which marked points collide.

This talk is based in part on joint work with Luigi Alfonsi and Hyungrok Kim.

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First chiral homology on $\mathbb{P}^1$ and V-module extensions

Jethro van Ekeren, Juan Guzman
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In this talk I will present the construction of an isomorphism between equivalence classes of extensions $0 \to A \to B \to C \to 0$ of V-modules, where V is a vertex operator algebra, and the dual space of the first chiral homology group of V over $\mathbb{P}^1$ with coefficients A and C inserted at ∞ and 0 respectively.

This work is part of an ongoing project on chiral homology at IMPA, together with Reimundo Heluani and Thadeu Henrique Cardoso.

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Hodge filtrations in chiral homology

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Many of the deepest results in the representation theory of vertex algebras revolve around the structure of spaces of conformal blocks (i.e., of chiral homology in degree zero) at the boundary of the moduli space of elliptic curves. In this talk I will present some results in this direction on chiral homology in higher degree, deduced from analysis of a sort of Hodge filtration on the associated chiral chain complex. This is joint work with R. Heluani.

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Factorization algebras in the geometric Langlands correspondence

Nick Rozenblyum
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The geometric Langlands correspondence is an equivalence of appropriate categories of sheaves on moduli spaces associated to a smooth proper algebraic curve and a reductive group G. The theory of factorization algebras gives a local-to-global approach to studying these categories. I will give an overview of the proof of the geometric Langlands correspondence focusing on the local-to-global aspects which are of independent interest. This is joint work with Arinkin, Beraldo, Chen, Færgeman, Gaitsgory, Lin, and Raskin.

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Geometric variational models with nonlocal energies
Special Session A6

Sara Daneri
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Ihsan Topaloglu
Virginia Commonwealth University, USA

Nonlocal geometric variational models have attracted significant attention in recent years. Nonlocal interactions are ubiquitous in physical models, from material science to chemistry and biology. The pertinence of nonlocal geometric models is that they naturally introduce length scales, which then are used to investigate the geometry of microstructures in macroscopic domains. The study of such models is a very active research direction, aiming at understanding e.g.: collective behavior in biological systems; the geometry of phase transitions/singularities of steady states for a large class of PDEs; the mechanisms behind pattern formation in self-organizing systems induced by competing short-range attractive/long-range repulsive interactions. The aim of this session is to bring together leading experts from US, Italy and Europe working on complementary and interconnected problems in the field, so as to favour fruitful interactions and collaborations.

For more information visit `https://sites.google.com/view/ams-umi-geom-var/`.

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Equilibrium shapes of liquid drops with discrete charges

Cyrill Muratov, *Matteo Novaga*
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Philip Zaleski
Department of Mathematical Sciences, New Jersey Institute of Technology

In this talk I will present our treatment of a geometric variational problem arising from modeling the equilibrium shapes of liquid drops whose energy presents a competition of surface tension with the repulsive Coulombic energy of a fixed number of point charges inside the drop. The continuum analog of this problem in which the liquid is treated as a perfect conductor is known to be variationally ill-posed, hence the discrete nature of the charges preserved in our model presents a non-trivial regularization whose properties are far from obvious. In our model, we make a simplification of no dielectric contrast between the liquid and its surroundings, which nevertheless is an appropriate assumption for charged drops of liquid helium that are used in applications to quantum chemistry. For large numbers of charges, we identify a sharp charge threshold as the volume of the drop goes to infinity jointly with the number of charges. This threshold separates the regime of existence of minimizers from that of non-existence and turns out to be considerably lower than the one predicted by Rayleigh for continuum charge distributions, and below the threshold the minimizer looks like a small perturbation of a ball with charges distributed approximately uniformly over the drop surface. Above the threshold, on the other hand, it is always convenient to evaporate a single charge from the drop and move it to infinity to lower energy.

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Energy driven pattern formation for local/non-local systems: the isotropic and crystalline case

Eris Runa
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In this talk I will discuss symmetry breaking and pattern formation for a family of functionals with local/non-local interactions in general dimension. Such functionals arise naturally in applications and contain a local term penalizing interfaces, and a non-local term favouring oscillations. In most physical situations such functionals have either isotropic or crystalline symmetries. The equilibrium between these two terms will result in the emergence of pattern formation. In particular we will show that the minimizers are periodic stripes.

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The elastica functional as the critical Gamma-limit of the screened Gamow model

Cyrill Muratov, Matteo Novaga
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Theresa Simon
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I will consider the large mass limit of a nonlocal isoperimetric problem in two dimensions with screened Coulomb repulsion. In this regime, the competition between perimeter and the repulsion simplifies to leading order by the nonlocal interaction localizing on the boundary of the sets. For an appropriate choice of screening constant, the surface area is exactly cancelled, requiring an analysis of the next order contribution. It turns out that then the nature of the problem changes from length minimization to curvature minimization: I will prove that the Gamma limit is given by (the relaxation of) the elastica functional, i.e., the integral over the squared curvature over the boundary.

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Nonlocal isoperimetric problems: lamellar pattern, lens cluster, and a new partitioning problem

Lia Bronsard
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We first present a nonlocal isoperimetric problem for three interacting phase domains, related to the Nakazawa-Ohta ternary inhibitory system which describes domain morphologies in a triblock copolymer. We consider global minimizers on the two-dimensional torus, in the droplet regime where some of the species have vanishingly small mass but the interaction strength is correspondingly large. In this limit there is splitting of the masses, and each vanishing component rescales to a minimizer of an isoperimetric problem for clusters in 2D. These results have led to a new type of partitioning problem that I will also describe. These represent work with S. Alama, X. Lu, C. Wang, S. Vriend and M. Novack.

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An Infinite Double Bubble Theorem

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Michael Novack
Department of Mathematical Sciences, Carnegie Mellon University

The classical double bubble theorem characterizes the minimizing partitions of $\mathbb{R}^n$ into three chambers, two of which have prescribed finite volume. In this talk we will discuss a variant of the double bubble theorem in which two of the chambers have infinite volume. Such a configuration is an example of a *(1,2)-cluster*, or a partition of $\mathbb{R}^n$ into three chambers, two of which have infinite volume and only one of which has finite volume. For $(1,2)$ clusters, the analogue of the double bubble is the *lens cluster*, and we show that it is minimizing. Furthermore, it is is unique in $\mathbb{R}^n$ for $n \leq 7$, with the same uniqueness holding in $\mathbb{R}^n$ for $n \geq 8$ under a natural growth assumption.

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A max-min property of the ball

Almut Burchard
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I will present recent joint work with Davide Carazzato and Ihsan Topaloglu on maximizing a functional that involves the minimization of the Wasserstein distance between disjoint sets of equal volume. The functional appears as a repulsive interaction term in some models describing biological membranes. Using a symmetrization-by-reflection technique, we show that balls are the unique maximizers of the functional.

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Diffuse improvement of flatness in codimension two

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The Allen–Cahn energy

$$F_\varepsilon(u) = \int \left[\epsilon |du|^2 + \frac{(1-u^2)^2}{4\epsilon}\right]$$

for real-valued maps u is by now a well-understood way to approximate the area functional for hypersurfaces. Critical points of it converge to minimal hypersurfaces as we send the scaling parameter ε to zero, and the same holds for the gradient flow, which converges to the mean curvature flow for hypersurfaces. Inspired by this parallel, De Giorgi proposed a conjecture which is analogous to the Bernstein problem for minimal graphs: *given an entire critical point $u : \mathbb{R}^n \to \mathbb{R}$ in dimension $n < 9$, monotone in x_n, is it necessarily a function of just x_n, up to rotations?*

Savin solved this conjecture for minimizers, which can be seen to be equivalent to a mild additional assumption. We discuss an analogue in codimension two, for the abelian Yang–Mills–Higgs energy

$$E_\varepsilon(u, \nabla) = \int \left[|\nabla u|^2 + \frac{(1-|u|^2)^2}{4\varepsilon^2} + \varepsilon^2 |F_\nabla|^2\right],$$

which is known to approximate area in codimension two; here u is a complex-valued map and $\nabla = d - i\alpha$ a connection on the trivial complex line bundle, with curvature F_∇. The result is based on an improvement of flatness in the style of Allard and is partly inspired by an alternative proof of Savin's theorem by Wang. It also uses recent stability results in dimension two by Halavati.

This is joint work with Guido De Philippis (NYU Courant) and Aria Halavati (NYU Courant).

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Topological spin textures stabilized by long range dipolar interaction in ferromagnetic thin films and their applications

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Topological spin textures are potential bit-encoding states for various information technology applications including high-density/high-speed memory and unconventional computing such as neuromorphic, probabilistic and reservoir computing. As a consequence, these systems have been widely studied experimentally and theoretically in recent years. Nevertheless, despite numerous attempts, a satisfactory theoretical description of these objects is still lacking today due to the highly non-trivial character of the magnetostatic interaction that plays a major role in determining the nature of magnetization patterns in ferromagnets. In this talk we will present a pedagogical overview of magnetization patterns existing in the case of ferromagnetic thin films. We will build a phase diagrams to locate magnetic patterns (uniform, stripe, helicoid, bubble, skyrmion...) as a function of the ferromagnetic layer thickness, applied magnetic field and other physical parameters. We will start with simplified "toy" models enabling physical insight of what a ferromagnet is and the origin of hysteresis (Stoner-Wohlfarth, thin wall models...). We will continue with results involving rigorous mathematical analysis [1-4]. This talk will be illustrated with experimental and numerical observations. We will finally give a concrete example of a promising applications using topological spin textures for ultra-fast/ultra-low power physical random number generation with various applications from probabilistic AI to Monte Carlo based intensive calculations in finance.

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Internal structure of the 2-pi wall

Hans Knüpfer
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The 2 pi wall is a type of transition layer which appears in thin ferromagnetic films. We discuss existence and structure of the 2 pi wall from the variational point of view. This is joint work with A. Capella and C. Muratov

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On the stability of the ball for attractive-repulsive energies

Marco Bonacini
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Riccardo Cristoferi
Department of Mathematics, IMAPP, Radboud University
Ihsan Topaloglu
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In this talk I will consider a class of attractive-repulsive energies, given by the sum of two nonlocal interactions with power-law kernels, defined over sets with fixed measure. It has recently been proved by R. Frank and E. Lieb that the ball is the unique (up to translation) global minimizer for sufficiently large mass. After a review of the literature on this minimization problem, I will focus on the issue of the stability of the ball, in the sense of the positivity of the second variation of the energy with respect to smooth perturbations of the boundary of the ball. I will present a characterization of the range of masses for which the second variation is positive definite (large masses) or negative definite (small masses). Moreover, I will discuss the connection between the stability of the ball and its local/global minimality. This talk is based on a work in collaboration with Riccardo Cristoferi and Ihsan Topaloglu.

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Minimizers of Energies with Repulsive-Attractive Power Law Interactions

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Cameron Davies
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Rupert Frank
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We will discuss the minimization of energies with certain repulsive-attractive potentials

$$I_{\alpha,\beta}(\mu) = \int_{\mathbb{R}^d} \int_{\mathbb{R}^d} \Big(\frac{\|x-y\|^\alpha}{\alpha} - \frac{\|x-y\|^\beta}{\beta} \Big) d\mu(x) d\mu(y).$$

The equilibrium measure of such an energy arises naturally as a steady state of an aggregation model where particles repel one another at short range, but are attracted to each other when far apart. We are particularly interesting in how the choices of β and α, i.e. the strength of repulsion at short range and strength of attraction at long range, respectively, affect the dimension of the support of the minimizing measure, and will provide new results for when the support of the equilibrium is a sphere or a ball.

[1]Acknowledgments: this research was supported by the NSF Mathematical Sciences Postdoctoral Research Fellowship, Grant Number 2202877
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Generative Adversarial Networks: A study of the Dynamics

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Generative Adversarial Networks (GANs) was one of the first Machine Learning algorithms to be able to generate remarkably realistic synthetic images. In this presentation, we delve into the mechanics of the GAN algorithm and its profound relationship with optimal transport theory. Through a detailed exploration, we illuminate how GAN approximates a system of PDE, particularly evident in shallow network architectures. Furthermore, we investigate the phenomenon of mode collapse, a well-known pathological behavior in GANs, and elucidate its connection to the underlying PDE framework through an illustrative example.

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Graphs associated with groups: advances and applications Special Session A7

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This special session aims to bring together mathematicians interested in exploring the synergy between graphs and group theory. Graph theory and group theory are two fundamental branches of mathematics with a wide range of applications in various fields. In particular, the situation becomes quite interesting when working in the intersection of these two areas, where a variety of ideas belonging to the first theory can be applied to the other. In recent years, the application of graphs to the study of groups and their properties has gained significant attention, as documented by several inspiring papers in literature. The session will provide a platform to discuss recent developments, share research findings, and foster collaboration in this exciting interdisciplinary field. In fact we seeks to promote the exchange of ideas, techniques, and applications of graph theory in the context of group theory by facilitating discussions on recent advancements, open problems and potential directions for future research. Participants will gain insights into how graph theory can be used to solve problems in group theory and vice versa.

For more information visit `https://sites.google.com/unisa.it/umi-ams-a7/home`.

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The total graph of a gain graph

Matteo Cavaleri, Alfredo Donno [1], *Stefano Spessato*
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The *total graph* $T(\Gamma)$ of a graph $\Gamma = (V_\Gamma, E_\Gamma)$ was introduced by M. Behzad in his doctoral thesis in 1965. It is the graph with vertex set $V_\Gamma \cup E_\Gamma$, whose adjacencies are such that:

- $u \sim v$ in $T(\Gamma)$ when $u, v \in V_\Gamma$ and $u \sim v$ in Γ;
- $v \sim e$ in $T(\Gamma)$ when $v \in V_\Gamma$ is an endpoint of $e \in E_\Gamma$;
- $e \sim f$ in $T(\Gamma)$ when $e, f \in E_\Gamma$ are incident in Γ.

In particular, the graph Γ and its line graph are two subgraphs of $T(\Gamma)$. The spectrum of the total graph of a regular graph was investigated by Cvetković in 1973.

Recently, two possible definitions were introduced for the total graph of a signed graph: a signed graph (Γ, σ) consists of an underlying graph Γ, endowed with a signature function σ associating a sign ± 1 with each edge.

A further generalization of a signed graph is given by the notion of *gain graph*: a gain graph over a group G is a pair (Γ, ψ) consisting of an underlying graph Γ, endowed with a gain function ψ associating an element of a group G with every oriented edge, in such a way that the inverse element is associated with the opposite orientation of that edge.

We present here a definition of total graph for a gain graph by using G-phases, which can be regarded as the generalization of the notion of incidence matrix. Our construction is well defined, in the sense that gain graphs that are switching equivalent have switching equivalent total graphs; moreover, it recovers the existing notions of total graph for signed graphs.

By defining a suitable action of the gain group G on the set of G-phases, we are able to describe the sets of G-phases inducing the same switching equivalence class of gain functions on Γ or, equivalently, on its total graph.

Finally, we determine the spectrum of the total graph of a gain graph (Γ, ψ) whose underlying graph is regular: we express its eigenvalues in terms of the eigenvalues of (Γ, ψ), where the notion of spectrum is referred to a unitary representation of the gain group G.

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[1]Supported by the "National Group for Algebraic and Geometric Structures, and their Applications"(GNSAGA - INdAM)
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Groups whose common divisor graph on p-regular classes is a forest

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Let G be a finite p-separable group, where p is a prime. In the last decades, some authors have investigated how the features of conjugacy classes of non-central p-regular elements of G incluence its local structure. In this framework, the common divisor graph built on the mentioned set of classes has proven to be a very useful tool to capture certain arithmetical properties of them, which constrain the structure of the p-complements of G. The aim of this talk is to present new progress in this research area. More concretely, we will classify the structure of the p-complements of G when the common divisor graph on conjugacy classes of non-central p-regular elements is a forest. In particular, we will provide further evidence for some open problems regarding this graph.

[1]This research is supported by Ayuda a Primeros Proyectos de Investigación (PAID-06-23) from Vicerrectorado de Investigación de la Universitat Politècnica de València (UPV), and by Proyecto CIAICO/2021/163 from Generalitat Valenciana (Spain).
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Groups and Graphs: recent results

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Let G be a finite group. An element g of G is called *vanishing element* if there exists an irreducible character χ of G such that $\chi(g)$ =0; in this case, we say that the conjugacy class of g is a vanishing conjugacy class. In this talk we discuss some arithmetical properties concerning the sizes of the vanishing conjugacy classes. This context is neatly portrayed by the *prime graph* of G for class sizes.

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The powerful class of groups

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The notion of powerful class of finite p-groups was coined by Avinoam Mann in 2011. In this talk we focus on pro-p groups of finite powerful class. These groups are p-adic analytic, and the torsion elements always form a subgroup. It is shown that there are only finitely many finite p-groups of fixed coclass and powerful class.

We also sketch how the above results provide an upper bound for the p-length of a finite p-solvable group in terns of the powerful class of its Sylow p-subgroup.

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Connectedness of the rank graph and properties of almost simple groups

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The generating graph of a group G is the graph on the elements of G in which x and y are joined if $G = \langle x, y \rangle$; it encodes how generating pairs are spread within the elements of the group. This graph has been studied by several authors and a lot is known about it. It has been conjectured for a long time that the graph induced by its non-isolated vertices is connected for every finite group. We will talk about this conjecture: we present a reduction to groups without abelian normal subgroups and the proof of a weaker version of this conjecture in the setting of the rank graph. This graph has vertex set the elements of the groups and x and y are joined if they belong to a generating set of minimal cardinality. When this cardinality (which we denote by $d(G)$) is 2, the rank graph coincides with the generating graph, so the former can be viewed as a generalisation of the latter to groups which are not necessarily 2-generated. In a paper in preparation and in [Chap. 4, 2], jointly with A. Lucchini, we prove the following.

Theorem 1. *The graph induced by the non-isolated vertices of the rank graph is connected for every finite group G such that $d(G) \geq 3$.*

Statements about connectedness of these kind of graphs are usually related to the structure of almost simple groups. We will present a fundamental ingredient of the proof of the theorem above, which is in fact a general result about almost simple groups, proved jointly with M. Costantini and A. Lucchini in [1].

Theorem 2. *Let G a finite almost simple group with socle G_0. If G/G_0 is abelian, then G contains an abelian subgroup A such that $G = AG_0$.*

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Graphs defined on groups: some interactions

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My topic is not Cayley graphs, but graphs with vertex set the group, which capture some of the group structure. The oldest of these is the *commuting graph*, in which two elements are joined if they commute. Used by Brauer and Fowler in the seminal 1955 paper showing that there are only finitely many finite simple groups with a given involution centraliser, this was perhaps the first step to the Classification of the Finite Simple Groups. Other graphs defined since include the power graph (two elements joined if one is a power of the other), the enhanced power graph, nilpotency graph, and solubility graph (two elements joined if the group they generate is cyclic, nilpotent or soluble respectively), together with various contractions of these. Other graphs depend on generation properties: for example, in the generating graph, two elements are adjacent if they generate the group.

I will focus on several types of result linking these graphs to the groups:

- The graphs may be used to prove a group-theoretic result, as in the Brauer–Fowler theorem. I will give a new result of this type.
- Equality of two of the graphs often define interesting classes of groups, such as EPPO groups, Dedekind groups, and 2-Engel groups.
- Finally, it can happen that one of these graphs contains an interesting graph (such as one with large girth) buried within it.

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On the diameter of graphs associated with groups

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The association of graphs to groups goes back to the 19th century when Cayley in [2] introduced a graph that encodes the abstract structure of a group. Later other graphs have been considered. More precisely, given a group property $\mathcal{P}$ and a group G one can consider the graph whose set of vertices is G and two vertices x and y are adjacent if and only if the subgroup generated by x and y has the property $\mathcal{P}$. In particular if the property $\mathcal{P}$ denotes solubility or commutativity, then the resulting graphs are called the solubility graph and the commutativity graph of the group G, respectively.

In discussing the connection and the diameter of such graphs, it is customary to exclude the unit of the group and sometimes all universal vertices. For instance, some results related to connectivity can be found in [1] and [3].

This talk aims to provide an overview of problems concerning the diameter of graphs associated with groups, with particular attention to the solubility graph and the commutativity graph.

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On the Gowers trick for classical simple groups

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Let A, B, C be subsets of a finite group G. Let $\mathrm{Prob}(A, B, C)$ be the probability that if a and b are uniformly and randomly chosen elements from A and B respectively, then $ab \in C$. Let k be the minimal degree of a non-trivial complex irreducible character of G.

Gowers [1] proved the following beautiful theorem.

Theorem 1 (Gowers)**.** *If $\eta > 0$ is such that $|A||B||C| > |G|^3/\eta^2 k$, then*

$$(1-\eta)\frac{|C|}{|G|} < \mathrm{Prob}(A, B, C) < (1+\eta)\frac{|C|}{|G|}.$$

In this talk we will consider the special case when G is a classical simple group and when A, B, C are special kinds of sets. We will require at least two of these sets to be normal. (Recall that a subset of a group G is normal if it is invariant under conjugation by every element of G.) This is joint work [2] with Francesco Fumagalli.

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[1] The first author was partially supported by the Italian INdAM-GNSAGA.

[2] The second author thanks the hospitality of the Department of Mathematics at the University of Florence. He was also supported by the National Research, Development and Innovation Office (NKFIH) Grant No. K138596, No. K132951 and Grant No. K138828.

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Critical groups

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The concept of critical class arises naturally in the reconstruction of the directed power graph of a finite group G by its undirected counterpart ([1], [2]). The presence in G of such classes makes that reconstruction for G a much more difficult task, with respect to groups having no critical class. That fact suggests that those $x \in G$ such that their equivalence class $[x]_{\mathtt{N}}$, with respect to the closed twin relation in the power graph, is critical could have important impact on properties and structure of G. We call such elements the critical elements of G and study their properties. We then consider the extreme case of groups $G \neq 1$ having all their nontrivial elements critical and call such groups *critical groups*. We characterize the critical groups, giving a complete description of their structure.

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Vertex-transitive graphs and derangements

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A derangement is a permutation without any fixed points. An outstanding property of derangements in a finite transitive permutation group is their abundance. What can we say about the portion of derangements of a group acting transitively on the vertices of a graph? In the present talk, we will introduce a lower bound on the portion of derangements depending only on the valency of the graph, and we will discuss its asymptotic behaviour compared to other general lower bounds.

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Representations of Finite Groups on Posets, Lattices, and Distributive Lattices

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The objective of this presentation is to highlight recent advancements in refining results initially proposed by Babai regarding representations of finite groups as automorphism groups of posets, lattices, and distributive lattices. These refinements employ an indirect method, utilizing the concept of a Haar graph.

Let R be a group, and let S be a subset of R. The Haar graph $\mathrm{Haar}(R,S)$ of R with connection set S is defined as a graph with a vertex set of $R \times \{-1,1\}$, where two distinct vertices $(x,-1)$ and $(y,1)$ are deemed adjacent if and only if $yx^{-1} \in S$. The automorphism group of $\mathrm{Haar}(R,S)$ contains a subgroup isomorphic to R. When the automorphism group of $\mathrm{Haar}(R,S)$ equals R, the Haar graph $\mathrm{Haar}(R,S)$ is termed a Haar graphical representation of the group R.

During this seminar, we delve into the methodology employed to classify finite groups admitting Haar graphical representations. Subsequently, we demonstrate how this classification facilitates enhancements to a 1980 result by Babai regarding group representations on posets and distributive lattices, thereby achieving optimal outcomes in this domain.

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Configurations in projective spaces and related research in commutative algebra and algebraic geometry
Special Session A8

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Elena Guardo
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The study of subvariety arrangements in projective spaces is a well-established and highly fruitful area of research. This field has produced numerous profound discoveries that have had a significant impact across various branches of mathematics.

In recent years, there has been a growing interest in studying configurations of points, lines, conics, and hyperplanes. These configurations are not only intriguing in their own right but also serve as powerful tools for uncovering unexpected properties within Commutative Algebra and Algebraic Geometry.

Representative examples of the importance of subvariety arrangements in Commutative Algebra and Algebraic Geometry include the use of grids, root systems, and Kochen-Specker Sets (including the Penrose dodecahedron) to analyze properties related to projections (such as Geproci sets, Weddle varieties). Several configurations give examples of unexpected curves, cones, and hypersurfaces. Contact star configurations were used to give a geometrical interpretation of some variety defined by Hadamard products. Additionally, arrangements like the Fermat arrangement, Star and Steiner configurations, and various others have played a crucial role in investigating containment problems, with a focus on the Waldschmidt constant, resurgence, and the conjectures of Demailly and Chudnovsky. Such research is often connected with the study of monomial ideals and, in particular, ideals defined over graphs and hypergraphs.

Our central aim is to stimulate open dialogue among researchers, promoting the sharing of progress and novel ideas. Through this, we want to encourage experts from different mathematical areas to collaborate on problems collectively. We hope to establish an enduring connection among researchers, fostering a spirit of collaboration that persists even after the session ends.

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Adding generators to monomial ideals

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In this talk we will show how the concept of an "elementary collapse" from discrete homotopy theory can be used to enlarge a given monomial ideal, while preserving most of its homological properties.

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"To infinity... and beyond!" with initial degrees of configurations of points

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In [1] and [2] the authors study the Waldschmidt constant

$$\widehat{\alpha}(I_{\mathbb{X}}) = \lim_{t \to \infty} \frac{\alpha(I_{\mathbb{X}}{}^{(t)})}{t}$$

of the ideal $I_{\mathbb{X}}$ defining k-configurations and standard k-configurations (which are special k-configurations of points whose coordinates are integer values). A k-configuration of points in $\mathbb{P}^2$ is a finite set $\mathbb{X}$ of points in $\mathbb{P}^2$ decomposable as $\mathbb{X} = \bigcup_{i=1}^{s} \mathbb{X}_i$ supported on suitable lines L_i $(1 < i \leqslant s)$ such that L_i contains $\mathbb{X}_i$ for each $i = 1, \ldots, s$ but does not contain any point of $\mathbb{X}_j$ for all $j < i$.

Here, we present some recent advances on the Waldschmidt constant of configurations of points in $\mathbb{P}^2$ supported on lines but that are not standard k-configurations, called partial intersections. It is a work currently in progress based on a joint project arised during the WICA II workshop (CIRM - Trento).

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Lefschetz properties of squarefree monomial ideals via Rees algebras

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The theory of Rees algebras of monomial ideals has been extensively studied, and as a consequence, many (sometimes partial) equivalences between algebraic properties of monomial ideals, and combinatorial properties of simplicial complexes and hypergraphs are known. In this talk, we will see how this theory can be used to find interesting examples in the theory of Lefschetz properties. We will also explore the consequences of known results from Lefschetz properties to the Rees algebras of squarefree monomial ideals. Applications include a connection between symbolic powers and f-vectors of simplicial complexes, and the positivity of mixed multiplicities of some families of squarefree monomial ideals.

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Generalized Weddle loci, tensors, and the geometry of projections

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I will introduce the notion of generalized Weddle loci (see [3]), which are important in the study of configurations of points in projective spaces and in computer vision, and I will discuss some recent application of the loci to the description of special configurations, projections, and spaces of tensors. I will mainly focus on a work in progress related the use of generalized Weddle loci for determining the structure of geproci sets (see [1]), the rank of linear systems of quadrics (see [2]), and the structure of spaces of partially symmetric tensors.

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Towards the classification of geproci sets

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A geproci set of points is a set whose general projection to a hiperplane is a complete intersection. We give examples of geproci sets in characteristic zero and present the so-called standard construction of nongrid geproci sets. Moreover we discuss recent results in the classification of geproci sets on skew lines having two transversals.

The talk is based on a joint project with Luca Chiantini, Pietro De Poi, Giuseppe Favacchio, Brian Harbourne, Giovanna Ilardi, Juan Migliore, Tomasz Szemberg and Justyna Szpond.

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The geproci property in positive characteristic

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The geproci property is a recent development in the world of geometry. We call a set of points $Z \subseteq \mathbb{P}^3_k$ an (a, b)-geproci set (for GEneral PROjection is a Complete Intersection) if its projection from a general point P to a plane is a complete intersection of curves of degrees $a \leq b$. Nondegenerate examples known as grids have been known since 2011. Nondegenerate nongrids were first described in 2018, working in characteristic 0. Almost all of these new examples are of a special kind called half grids.

In this paper, based partly on the author's thesis, we use a feature of geometry in positive characteristic to give new methods of producing geproci half grids and non-half grids.

Theorem 1. *Let $\mathbb{F}_q$ be the field of size q, where q is some power of a prime. Then $Z = \mathbb{P}^3_{\mathbb{F}_q} \subseteq \mathbb{P}^3_{\overline{\mathbb{F}}_q}$ is a $(q + 1, q^2 + 1)$-geproci half grid.*

Theorem 2. *The complement $Z \subseteq \mathbb{P}^3_{\mathbb{F}_q}$ of a maximal partial spread of deficiency d is a nontrivial $\{q + 1, d\}$-geproci set. Furthermore, when $d > q + 1$, Z is also not a half grid.*

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Construction of free and nearly free curves

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We construct new free or nearly free curves starting from a curve C by adding inflectional tangents, or lines passing through the singularities of C, or lines in the tangent cone of some singularities of the curve. We give new results and construct many examples (joint work with A. Dimca, P. Pokora and G. Sticlaru).

Then we construct new families of free and nearly free curves starting from a plane cubic curve C and adding some of its hyperosculating conics, that is conics with the property that the intersection index of such a conic with a given cubic is exactly 6.

We get new insight into the geometry of the 27 hyperosculating conics of the Fermat cubic curve.

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Gotzmann's persistence theorem for products of projective spaces

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Gotzmann's regularity and persistence theorems provide tools which allow us to find explicit equations for the Hilbert scheme $\mathrm{Hilb}_P(\mathbb{P}^n)$. A natural next step is to generalise these results to the multigraded Hilbert scheme $\mathrm{Hilb}_P(X)$ of a smooth projective toric variety X. In 2003 Maclagan and Smith ([1],[2]) generalise Gotzmann's regularity theorem to this case. We present new persistence type results for the product of two projective spaces, and time permitting discuss how these may be applied to a more general smooth projective toric variety.

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Configurations with triple points

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The talk concerns configurations with the maximal number of triple points. We show a construction of an infinite series of line arrangements (in characteristic two) with only triple intersection points. We will also present some results on the existence and non-existence of line arrangements (in various characteristics) with up to 19 lines maximizing the number of triple points.

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Zero-dimensional schemes via Hadamard products

Cristiano Bocci
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In this talk I will show how to use Hadamard products of projective varieties to construct set of points which are star configurations, fat grids or Gorenstein with given h $\surd$vector.

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On rational complete intersections

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The known results about the rationality vs irrationality of Fano complete intersections $X^n \subset \mathbb{P}^{n+c}$ of dimension $n = 3, 4, 5$ and fixed type $(d_1, \ldots, d_c)$ suggest an uniform approach to treat several open cases: index one; index two; quartic fourfolds and fivefolds; etc). From one hand one would like to decide the rationality/irrationality of every element in the numerous cases where the stable irrationality of the very general element is known (e.g. quartic fourfolds and fivefolds, quintic fivefolds, etc); from the other hand one hopes to put some further light on several longstanding conjectures (e.g. the irrationality of the very general cubic fourfold).

[1]The author has been partially supported by the PRIN 2020 *Squarefree Gröbner degenerations, special varieties and related topics*, by the PRIN 2022 *Birational geometry of moduli spaces and special varieties* and is member of the G.N.S.A.G.A. of INDAM
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Complete intersections on Veronese surfaces

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In "Commentationes Geometricae" Euler asked when a set of points in the plane is the intersection of two curves, that is, using the modern terminology, when a set of points in the plane is a complete intersection. In the same period, Cramer asked similar questions so that this type of questions is presently known as the Cramer-Euler problem. In this talk, we consider a generalization of the Cramer-Euler problem: characterize the possible complete intersections lying on a Veronese surface, and more generally on a Veronese variety. The main result describes all possible reduced complete intersections on Veronese surfaces. More precisely, we prove the following theorem:

Theorem 1. *If $\mathbb{X} \subseteq V_{2,d} \subseteq \mathbb{P}^{N_{2,d}}$ is a reduced complete intersection of type $(a_1, \dots, a_r)$, with $a_1 \leq \cdots \leq a_r$, then one of the following holds:*

(1) $(d, r, (a_1, a_2, \dots, a_r)) = (2, 4, (1, 1, 1, 2))$, *that is, $\mathbb{X}$ is a conic lying on $V_{2,2}$;*
(2) $(d, r, (a_1, a_2, \dots, a_r)) = (2, 5, (1, 1, 1, 2, a_5))$, *any $a_5 \in \mathbb{N}$, that is, $\mathbb{X}$ is a set of $2a_5$ complete intersection points of a conic lying on $V_{2,2}$ and a hypersurface of degree a_5;*
(3) $(d, r, (a_1, a_2, \dots, a_r)) = (d, N_{2,d}, (1, 1, \dots, 1))$ *for any $d \geq 2$, that is, $\mathbb{X}$ is a reduced point;*
(4) $(d, r, (a_1, a_2, \dots, a_r)) = (d, N_{2,d}, (1, 1, \dots, 1, 2))$ *for any $d \geq 2$, that is, $\mathbb{X}$ is a set of two reduced points.*

We formulate a conjecture for the general case of complete intersection subvarieties of any dimension, and we prove it in the case of the quadratic Veronese threefold. Our main tool is an effective characterization of all possible Hilbert functions of reduced subvarieties of Veronese surfaces.

Conjecture 1. Let $\mathbb{X} \subseteq V_{n,d} \subseteq \mathbb{P}^{N_{n,d}}$ be a reduced subvariety with $d > 1$. Then $\mathbb{X}$ is a complete intersection of type $(a_1, \dots, a_r)$, with $a_1 \leq \cdots \leq a_r$ if and only if

- $r = N_{n,d}$, $a_1 = \ldots = a_{N_{n,d}} = 1$, any n, d, that is $\mathbb{X}$ is a reduced point;
- $r = N_{n,d}$, $a_1 = \ldots = a_{N_{n,d}-1} = 1$, $a_{N_{n,d}} = 2$, any n, d, that is $\mathbb{X}$ is a set of two reduced points;
- $r = N_{n,d}$, $a_1 = \ldots = a_{N_{n,d}-2} = 1$, $a_{N_{n,d}-1} = 2$, $a_{N_{n,d}} = b$, any n, $d = 2$, any $a \geq 2$, that is $\mathbb{X} = \mathcal{C} \cap H_b$ for $\mathcal{C} \subseteq V_{n,2}$ a conic and H_b a degree b hypersurface;
- $r = N_{n,d} - 1$, $a_1 = \ldots = a_{N_{n,d}-2} = 1, a_{N_{n,d}-1} = 2$, $d = 2$, any n, that is $\mathbb{X}$ is a conic.

[1]The author is supported by the '0-Dimensional Schemes, Tensor Theory, and Applications' project 2022E2Z4AK – funded by European Union – Next Generation EU within the PRIN 2022 program (D.D. 104 - 02/02/2022 Ministero dell'Università e della Ricerca).
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Toric Ideals of Graphs and the Fundamental Group

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This talk will discuss a surprising connection between combinatorial commutative algebra and algebraic topology. Specifically, for a connected biparite finite simple graph G, we will look at the relationship between the toric ideal associated with G, I_G, and the fundamental group of G, $\pi_1(G)$. We will examine how properties of the fundamental group $\pi_1(G)$ allow us to obtain important algebraic information about the toric ideal I_G.

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Simplices osculating to rational normal curves

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L. Schaffler
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In this talk we discuss one of the many variations of the following question: does there exist a rational normal curve in $\mathbb{P}^d$ passing through n given points? For $n = d + 3$ points in general linear position an affirmative answer is given by the well known Castelnuovo's Lemma. While, for any collection of $n \geq d + 4$ points, the paper [1] provides explicit equations that always give an answer to the question.

We will consider special sets of points, namely the vertices of two simplices whose faces osculate a given rational normal curve. For example, one can consider $d = 2, n = 6$ that is the vertices of two triangles osculating to a conic. And also, $d = 3, n = 8$, that is two tetrahedrons osculating to a twisted cubic curve. This particular instance was considered and answered by von Staudt in 1856, by Hurwitz in 1882, and by White in 1921. In particular, White also claimed to have an argument to prove the general case, but a proof was not provided.

Our main result in [2] is the following:

Theorem 1. *Let $d \geq 2$ and let $\mathcal{C} \subseteq \mathbb{P}^d$ be a rational normal curve. Consider distinct points $P_i, P'_i \in \mathcal{C}$ for $1 \leq i \leq d + 1$ and let π_i, respectively π'_i, be the osculating hyperplanes to $\mathcal{C}$ in P_i, respectively P'_i. Define points*

$$R_i = \mathop{}\limits_{j\neq i} \pi_j \text{ and } R'_i = \mathop{}\limits_{j\neq i} \pi'_j.$$

Then, the points $R_1, \ldots, R_{d+1}, R'_1, \ldots, R'_{d+1}$ lie on a rational normal curve.

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On the codimension of permanental varieties

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Mateusz Michałek
Universität Konstanz

Emanuele Ventura
Politecnico di Torino

We study *permanental varieties*, i.e. varieties defined by the vanishing of permanents of fixed size of a generic matrix. Permanents and their varieties play an important, and sometimes poorly understood, role in combinatorics. However, there are essentially no geometric results about them in the literature, in very sharp contrast to the well-behaved and ubiquitous case of determinants and minors. Motivated by the study of the singular locus of the permanental hypersurface, we focus on the codimension of these varieties. We introduce a $\mathbb{C}^*$-action on matrices and prove a number of results. In particular, we improve a lower bound on the codimension of the aforementioned singular locus established by von zur Gathen in 1987. This is joint work with Boralevi, Carlini and Michałek.

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Functional inequalities and PDEs Special Session A9

Filomena Feo
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Emanuel Indrei
Sam Houston State University, USA

Elisabetta Tornatore
University of Palermo, ITALY

This session deals with the most recent results in PDEs, functional inequalities and their applications to PDEs. We will mainly focus on existence, regularity, qualitative properties of solutions to PDEs, and optimal functional inequalities. In this framework, inequalities play a crucial role in establishing uniform bounds, global existence results and large-time behavior, decay rates, and existence/uniqueness of blow-up solutions to various classes of differential equations. Furthermore, functional inequalities are also intrinsically fundamental in many sub-fields of analysis. The invited speakers are actively involved in this research area with distinguished contributions. They will discuss their latest achievements and future developments. The session aims to bring together experts in different fields to encourage fruitful discussions, develop new ideas, and promote knowledge exchange between participants.

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The LGC method: Recent progress on several problems in harmonic analysis

Victor Lie
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In this talk we touch upon three problems:

- (joint with my postdoc Bingyang Hu) the boundedness of the trilinear Hilbert transform along the moment curve:

$$T_C(f_1, f_2, f_3)(x) := \text{p.v.} \int_{\mathbb{R}} f_1(x-t) f_2(x+t^2) f_3(x+t^3) \frac{dt}{t}, \quad x \in \mathbb{R}\,.$$

- (joint with C. Benea and F. Bernicot) the boundedness of the hybrid trilinear Hilbert transform:

$$T_H(f_1, f_2, f_3)(x) := \text{p.v.} \int_{\mathbb{R}} f_1(x-t) f_2(x+t) f_3(x+t^3) \frac{dt}{t}, \quad x \in \mathbb{R}\,.$$

- (joint with my graduate student Martin Hsu) the boundedness of the 2D non-resonant Carleson–Radon transform:

$$CR(f)(x,y) := \sup_{a \in \mathbb{R}} \left| \text{p.v.} \int_{\mathbb{R}} f(x-t, y-t^2) \, \frac{e^{a\,i\,t^3}}{t} \, dt \right|, \quad (x,y) \in \mathbb{R}^2\,.$$

The leit-motif in the successful approach of all of the above problems is the appeal to the LGC methodology–introduced by the speaker and further developed in various collaborative works–that involves several key ingredients, including the following interdependent elements:

- a sparse-unform decomposition of the input function(s) adapted to an appropriate time-frequency foliation of the phase-space,
- a structural analysis of suitable maximal “joint Fourier coefficients", and
- a level set analysis with respect to the time-frequency correlation set.

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The optimal Leray-Trudinger inequality

Giovanni Pisante, Georgios Psaradakis, Giuseppina di Blasio
Department of Mathematics and Physics, University of Campania

In this talk, we will present some recent results on Leray-Trudinger type inequalities that are closely related to Trudinger-Moser and Hardy inequalities. The plan of the talk is to present the origin and the history of the problem and to present an optimal analogous of the Trudinger's inequality. The presentation is based on joint works with G. di Blasio and G. Psaradakis.

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Classification of singular solutions in the half-space

Luigi Montoro
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We provide a classification result for positive solutions to $\sqrt{\Delta} u = 1/u^{\gamma}$ in the half space, under zero Dirichlet boundary condition. We refer to [1].

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Mathematical models for chemotaxis system and their applications in medicine

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We are interested in qualitative properties as blow-up phenomena, decay in time, boundedness, global existence to solutions of some classes of parabolic systems. In particular we consider a chemotaxis system in a bounded and smooth domain with no-flux boundary condition. In multicellular organisms, the chemotaxis of cell populations plays a crucial role throughout the life cycle:

- in the formation of the embryo and during embryonic development for the cell positioning;
- In the adult, the immune cell migration to sites of inflammation and fibroblasts into wounded regions to initiate healing.

There are same mechanisms during cancer growth, allowing tumour cells to invade the surrounding environment or stimulate new blood vessel growth (angiogenesis).
This talk is connected to a joint research with S. Vernier-Piro and T. Yokota.

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Liouville theorems for geometric PDEs

Alberto Roncoroni
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In this talk I will present Liouville-type theorems for two critical PDEs of geometric and variational nature. The first one is the critical p √Laplace equation in $\mathbb{R}^n$ (and, possibly, in Riemannian manifolds), while the second one is the CR Yamabe equation in the Heisenberg group $\mathbb{H}^n$. It is well-known that both equations have a geometric and variational origin; indeed the critical p √Laplace equation is related to the Sobolev inequality and, for $p = 2$, to the Yamabe problem while the CR Yamabe equation is related to the Folland-Stein inequality and to the CR-Yamabe problem.

Liouville theorems regarding the classification of entire and positive solutions to these equations have been obtained in [4, 5] and in [3] under a finite energy assumption and remove this hypothesis is still an open and challenging problem. In the talk I will present two recent Liouville theorems in which we remove the finite energy assumption for the critical p √Laplace equation in $\mathbb{R}^n$, with $n = 2$ or $n = 3$ provided $3/2 < p < 2$, and for the CR Yamabe equation in $\mathbb{H}^1$.

The first result has been obtained in collaboration with G. Catino and D.D. Monticelli and is contained in the paper [2], while the second one has been obtained with G. Catino, Y.Y. Li and D.D. Monticelli and is contained in [1].

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Growth of Sobolev norms for completely resonant quantum harmonic oscillator

Beatrice Langella, Alberto Maspero, Maria Teresa Rotolo
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We consider the linear time-dependent quantum harmonic Schrödinger equation in $\mathbb{R}^2$:

$$i\partial_t u = \frac{1}{2}(-\partial_{x_1}^2 - \partial_{x_2}^2 + x_1^2 + x_2^2)u + V(t,x,D)u, \qquad x \in \mathbb{R}^2,$$

where $V(t,x,D)$ is a self-adjoint pseudodifferential operator of degree zero, 2π periodic in time. We prove that under suitable conditions on the symbol of the potential $V(t,x,D)$ there exist solutions of the equation that exhibit unbounded growth in time of their positive Sobolev norms, and we show that the class of symbols satisfying such condition is generic in the Fréchet space of classical time-dependent symbols of order zero.

The strategy to prove growth of positive Sobolev norms mainly consists of two parts: in the first we use a pseudodifferential normal form to extract an effective resonant equation governing the dynamics, and in the second we prove that a dispersive energy estimate holds for the effective equation and use it to find a solution with growing positive norms.
The main difficulty lies in the proof of the energy estimate, that in turn relies on Mourre's theory of positive commutators and on the existence of a conjugate operator for the effective equation. We give generic conditions on the classical Hamiltonian dynamics of the principal symbol of the perturbed operator that turn out to be sufficient to prove a Mourre estimate for the perturbed operator.
The main novelty of the work is the proof of the generic existence of a conjugate operator for an equation with spatial variable in dimension two. To this aim we adapt a geometric construction done by Colin de Verdière in [1] to our context.
In this talk I will try to present the general problem and the key results needed for this proof. This is a joint work with B. Langella and A. Maspero.

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A p-Laplacian problem in $\mathbb{R}^N$ with singular, convective, critical reaction

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Laura Baldelli
IMAG, Departamento de Análisis Matemático, Universidad de Granada

The talk is devoted to the problem

$$\begin{cases} -\Delta_p u = \lambda w(x) f(u, \nabla u) + u^{p^*-1} & \text{in } \mathbb{R}^N, \\ \quad u > 0 & \text{in } \mathbb{R}^N, \\ \; u(x) \to 0 & \text{as } |x| \to +\infty, \end{cases}$$

where $N \geq 2$, $1 < p < N$, and $\lambda > 0$. The nonlinear term $f : (0, +\infty) \times \mathbb{R}^N \to (0, +\infty)$ is a continuous function which is singular in the first variable and p-sublinear with respect to the second one. The weight $w : \mathbb{R}^N \to (0, +\infty)$ satisfies suitable summability and decay conditions.

The problem exhibits several features:

- the perturbation f is singular, i.e., it blows up when the solution vanishes;
- f encompasses also convection terms, that is, depending on the gradient of the solution;
- the 'dominating' reaction term has critical growth;
- the setting is the whole $\mathbb{R}^N$;
- pointwise decay (at infinity) of the solutions is required.

We will present an existence result that combines variational methods, truncation techniques, and concentration compactness arguments, together with set-valued analysis and fixed point theory. In addition, De Giorgi's technique, a priori gradient estimates, and nonlinear regularity theory will be employed to ensure local $C^{1,\alpha}$ regularity of solutions, as well as their pointwise decay at infinity.

The result is new even in the non-singular case, also for the Laplacian.

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Boundedness results for the p-Laplacian on noncompact Riemannian manifolds

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We present some boundedness results for the p-Laplace operator in domains with finite volume, on noncompact Riemannian manifolds.
Specifically, we deal with two different eigenvalue problems for the p-Laplacian: the first has Neumann boundary conditions, the second is a Steklov problem.
We consider also the boundedness of the solutions to the Schrödinger equation, under Neumann boundary conditions, with non-necessarily bounded potential.
The assumptions ensuring L^q or L^∞ bounds for the solutions to the equations are offered either in terms of the isoperimetric function or of the isocapacitary function of the domain.
We also present some examples that show the optimality of our assumptions.

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A stability result for the first Robin-Neumann eigenvalue: a double perturbation approach

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Gianpaolo Piscitelli [1]
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Let $\Omega = \Omega_0 \setminus \overline{\Theta} \subset \mathbb{R}^n$, $n \geq 2$, where Ω_0 and Θ are two open, bounded and convex sets such that $\overline{\Theta} \subset \Omega_0$ and let $\beta < 0$ be a given parameter. We consider the eigenvalue problem for the Laplace operator associated to Ω, with Robin boundary condition on $\partial\Omega_0$ and Neumann boundary condition on $\partial\Theta$. In [2] it is proved that the spherical shell is the only maximizer for the first Robin-Neumann eigenvalue in the class of domains Ω with fixed outer perimeter and volume.

We establish a quantitative version of the afore-mentioned isoperimetric inequality; the main novelty consists in the introduction of a new type of hybrid asymmetry, that turns out to be the suitable one to treat the different conditions on the outer and internal boundary. Up to our knowledge, in this context, this is the first stability result in which *both* the outer and the inner boundary are perturbed.

This talk is based on a joint work with Gloria Paoli (Friedrich-Alexander-Universität Erlangen-Nürnberg) and Simone Cito (Università del Salento).

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The Well-posedness of Cylindrical Jets with Surface Tension

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In 1879 Rayleigh studied the stability of infinite cylindrical jets, inspired by the experiments of Plateau. The principal question that Rayleigh asked is: under what circumstances the jet is stable, for small displacements. In this talk I will discuss the short time stability for the initial condition belonging to some Sobolev space, and the initial jet boundary being uniformly bounded away from the axis of symmetry. This is proved by the method of paradifferential calculus and paralinearization. The salient feature of these results is that no smallness assumption is imposed on the initial condition. The results are taken from a joint paper with Dr Yucong Huang.

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On the approximation of optimal thin layers in thermal insulation

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We are interested in the thermal insulation of a body Ω surrounded by a bulk layer Σ of insulating material. We consider a Robin boundary condition on the boundary of Σ that does not touch Ω; this corresponds to a model of heat transfer between the insulated body and the environment determined by convection. We will address the problem of proving the existence of an optimal Σ, and we will study a way to approximate the problem via first-order asymptotic development by Γ-convergence.

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On the singular planar Plateau problem

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The classical Plateau problem asks which surface in three-dimensional space spans the least area among all the surfaces with boundary given by an assigned curve S. This problem has many variants and generalizations, along with (partial) answers, and has inspired numerous new ideas and techniques. In this talk, we will briefly introduce the problem in both its classical and modern contexts, and then we will focus on a specific vectorial (planar) type of the Plateau problem. Given a curve S in the plane, we can ask which diffeomorphism T of the disk D maps the boundary of D to S and spans the least area, computed as the integral of the Jacobian of T, among competitors with the same boundary condition. For simply connected curves, the answer is provided by the Riemann map, and the minimal area achieved is the Lebesgue measure of the region enclosed by S. For more complex curves, possibly self-intersecting, new analysis is required. I will present a recent result in this sense, obtained in collaboration with Prof. Riccardo Scala from the University of Siena, where the value of the minimum area is computed with an explicit formula that depends on the topology of S.

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On the Shape of Small Liquid Drops Minimizing Nonlocal Energies

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We study the equilibrium shape of liquid drops minimizing the fractional perimeter under the action of a potential energy. We prove, with a quantitative estimate, that the small volume minimizers are convex and uniformly close to a ball. This is a joint work with Matteo Novaga (Pisa) and Fumihiko Onoue (Munchën).

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Stability for the logarithmic Sobolev inequality

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Jean Dolbeault
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Nikita Simonov
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This talk is devoted to stability results for the Gaussian logarithmic Sobolev inequality, with explicit stability constants.

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Higher-order singular perturbation models for phase transitions

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Variational models of phase transitions take into account double-well energies singularly perturbed by gradient terms, such as the Cahn-Hilliard free energy. The derivation by Γ-convergence of a sharp-interface limit for such energy is a classical result by Modica and Mortola. We consider a singular perturbation of a double-well energy by derivatives of order k, and show that we still can describe the limit as in the case $k = 1$ with a suitable interfacial energy density, in accord with the case $k = 1$ and with the case $k = 2$ previously analyzed by Fonseca and Mantegazza. The main issue is the derivation of an optimal-profile problem on the real line describing the interfacial energy density, which must be conveniently approximated by minimum problems on finite intervals with homogeneous condition on the derivatives at the endpoints up to order $k-1$. To that end a careful study must be carried on of sets where sequences of functions with equibounded energy are "close to the wells" and have "small derivatives", in terms of interpolation inequalities and energy estimates.

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Nonlinear patterns and waves in biological systems
Special Session A10

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As widely observed in different fields, self-organisation in several fields of applications often leads to the emergence of patterned and travelling wave solutions. A few examples include the formation of spots and travelling vegetation bands in water-limited regions [1,2], the firing patterns of neurons [4], and the emergence of inflammatory aggregates related to immune responses [3]. These relevant phenomena are mostly modelled from the mathematical viewpoint via reaction/advection/cross-diffusion equations. As these models aim to incorporate relevant real-life features, such as complex spatial domains and space/time-dependent parameters, their complexity has steadily increased in the last few years. In turn, an increasing effort of the scientific community has been devoted to advancing the analytical and numerical tools to analyse the emerging solutions and predict their long-time evolution.

The aim of this Special Session is to provide an overview of some of the most recent developments in the investigation of patterned and travelling wave solutions arising in a wide variety of mathematical models. Bringing together experts from the US and the European communities will allow the interactions and the exchange between different techniques and foster new collaborations.

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Coherent structures in an IntraGuild predation reaction-diffusion model with anti-predator behavior

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This study introduces the following cross-diffusion model defined on the spatial domain $\Omega \subset \mathbb{R}$:

$$(1) \quad \begin{cases} \dfrac{\partial N_1}{\partial \tau} = D_1 \Delta N_1 + N_1 \left(B - a_{11} N_1 - a_{12} N_2 - a_{13} N_3\right), & \text{on } \mathbb{R}_+ \times \Omega, \\[2ex] \dfrac{\partial N_2}{\partial \tau} = D_2 \Delta N_2 + D \nabla \cdot \left(N_2 \nabla N_3\right) + N_2 \left(-M_2 + a_{21} N_1 - a_{23} N_3\right), & \text{on } \mathbb{R}_+ \times \Omega, \\[2ex] \dfrac{\partial N_3}{\partial \tau} = D_3 \Delta N_3 + N_3 \left(-M_3 + a_{31} N_1 + a_{32} N_2\right), & \text{on } \mathbb{R}_+ \times \Omega, \end{cases}$$

where $N_1(\tau, X), N_2(\tau, X), N_3(\tau, X)$, represent the densities of a basal resource, an intermediate consumer (the IGPrey) and an omnivorous predator (the IGPredator), respectively. The model describes the dynamics of intraguild predation communities, where predators and prey also compete for shared resources within an ecological guild. It incorporates IntraGuild Prey exhibiting anti-predator behavior, dispersing along local gradients in predator density, while local dynamics are described by the Lotka-Volterra functional form [1].

Beginning with an overview of existing results concerning the existence and stability of the steady state of homogeneous coexistence, we demonstrate that the local dynamics support the bistability of the spatially homogeneous equilibrium with oscillations due to a subcritical Hopf bifurcation.

We prove that the predator avoidance strategy described by cross-diffusion is crucial for pattern formation in the reaction-diffusion system and characterize the cross-diffusion-driven Turing bifurcation. Using the formalism of amplitude equations, we derive the asymptotic profiles of the stationary solutions, revealing that anti-predator behavior can account for segregation patterns between IntraGuild Prey and IntraGuild Predator observed in field studies. Through a combination of analytical and numerical tools, we demonstrate that the predator avoidance strategy serves as a mechanism that stabilizes coexistence states in Intraguild Predation communities beyond the conditions imposed by the corresponding spatially homogeneous model.

We shall also prove that the system is able to reproduce spatially non-homogeneous periodic-in time species distribution, in the neighborhood of the codimension-2 Turing-Hopf bifurcation point, where the stationary and temporal instability coexist.

In the latter part of the presentation, we address the issue of localized pattern. We demonstrate the existence of stationary spikes, namely far-from-equilibrium highly localized patterns that are singularly perturbed solutions of the model system. Using asymptotic techniques, we shall derive explicit conditions for the existence of the spike patterns and construct their asymptotic profile.

Finally, we identify multiple bifurcating branches of localized states organized in a characteristic snakes-and-ladders structure, termed *homoclinic snaking*. Adopting the spatial dynamics description, we show that the origin of the spatially localized patterns is due to a reversible $1:1$ resonance (Hamilton-Hopf bifurcation), whose normal form gives the explicit form of the small-amplitude branch of homoclinic solutions.

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Emergence of Vascular Networks

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The emergence of vascular networks is a long-standing problem which has been the subject of intense research in the past decades. One of the main reasons being the widespread applications that it has in tissue regeneration, wound healing, cancer treatment, etc. The mechanisms involved in the formation of vascular networks are complex and despite the vast amount of research devoted to it, there are still many mechanisms involved which are poorly understood. Our aim is to bring insight into the study of vascular networks by defining heuristic rules, as simple as possible, and to simulate them numerically to test their relevance in the vascularization process. We will present a hybrid agent-based/continuum model that couples blood flow, oxygen flow, capillary network dynamics, and tissue dynamics. And we will show simulations that demonstrate the ability of our model to capture the main features of vascular networks. This is joint work with P. Degond, B. Aymard, L. Castella, A. Lorsignol, P. Kennel, F. Plouraboue, and D. Peurichard.

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Competing effects in fourth-order aggregation-diffusion PDEs: a variational approach

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The talk concerns the analysis of fourth-order aggregation-diffusion equations using an optimal transport approach. These models have been recently obtained as approximation of nonlocal systems of PDEs describing cell-cell adhesion. This is a crucial mechanisms regulating collective cell migration during tissue development, homeostasis and repair, allowing cell populations to self-organise and form and maintain complex tissue shapes. In a recent work, we use the 2-Wasserstein gradient flow structure of such equations to give sharp conditions for global in time existence of weak solutions, in any dimension and for general initial data. The energy involved presents two competing effects: the Dirichlet energy and the power-law internal energy. Homogeneity of the functionals reveals critical regimes that we analyse. In addition, we study a system of two Cahn-Hilliard-type equations exhibiting an analogous gradient flow structure.

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The role of boundary constraints in simulating biological systems with nonlocal dispersal

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Population and vegetation models often use nonlocal forms of dispersal to describe the spread of individuals and plants. When these long-range effects are modeled by spatially extended convolution kernels, the mathematical analysis of solutions can be simplified by posing the relevant equations on unbounded domains. However, in order to numerically validate these results, these same equations then need to be restricted to bounded sets. Thus, it becomes important to understand what effects, if any, do the different boundary constraints have on the solution. To address this question we present a quadrature method valid for convolution kernels with finite second moments. This scheme is designed to approximate at the same time the convolution operator together with the prescribed nonlocal boundary constraints, which can be Dirichlet, Neumann, or what we refer to as free boundary constraints. We then apply this scheme to study pulse solutions of an abstract 1-d nonlocal Gray-Scott model as a case study. We consider convolution kernels with exponential and with algebraic decay.

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A nodal ghost method based on variational formulation and regular square grid for elliptic problems on arbitrary domains and applications to biological network formation

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This talk focuses on the numerical solution of elliptic partial differential equations (PDEs), specifically addressing the challenges arising from irregular domains. Both finite element method (FEM) and finite difference method (FDM) face difficulties in dealing with arbitrary domains. We introduce a novel nodal symmetric ghost method based on a variational formulation approach, which combines the advantages of FEM and FDM. The method employs bilinear finite elements on a structured mesh and provides a detailed implementation description. The convergence rates are validated with many numerical experiments, in both one and two space dimensions. At the end, we show an application to biological network formation in a leaf-shaped domain, computing the solution of a reaction-diffusion equation for the conductivity tensor, coupled with a Poisson equation for the pressure of the fluid.

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Stopping waves: Geometric analysis of coupled bursters

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Bursting is a type of electrical activity seen in many neurons and endocrine cells where episodes of action potential firing are interspersed by silent phases. In [1] we investigate partial synchrony and wave propagation in a population of square-wave bursters. In particular, by using a prototypical polynomial bursting model and slow/fast bifurcation analysis, we study why electrically coupled model bursters typically synchronize very easily, as reflected in the tendency for simulated excitation waves to propagate far into the region of silent cells when an excitation gradient is imposed. Such simulation are inspired by, but do not reproduce, experimentally observed Ca^{2+} waves in pancreatic islets exposed to a glucose gradient. Our analyses indicate a possible modification of the model so that the excitation waves stop at the border between "active" and "silent" cells. We verify this property by simulations using such a modified model for a chain, and for a cubic cluster, of coupled cells. Furthermore, we show how our one- and two-parameter bifurcation analyses allow us to predict where the simulated waves stop, for both the original model and the modified version.

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Pattern formation and stability for a kinetic model of ants

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We present an interacting particle system to model the behaviour of collectives of ants. The particles are modelled as Active Brownian Particles interacting only via chemical pheromones. The particles also have an antenna by which they sense the pheromones. The particles form typical Keller–Segel collapse clusters or, depending on the length of the antenna, travelling clusters. We study the formal mean field limit PDE model to substantiate these particle behaviours. We show analytical and numerical results for the PDE. We begin by demonstrating analytical well-posedness and uniform boundedness globally in time. Using a convergent scheme we are also able to obtain the linear instability curve in the parameter space. In the linearly unstable regime finite volume simulations demonstrate that there is pattern formation reflecting either the Keller–Segel collapse or lane formation, depending on the length of the antenna and the interaction strength. The patterning is also explained by the shape of the growing eigenfunctions associated to the linear theory. We also show there is a region of bistability where Keller–Segel collapse and lane formation compete.

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Topics on the complex spatio-temporal dynamics of spatially heterogeneous biological systems.

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From the development of plants and animals to large-scale vegetation organization, biological systems are often characterized by the emergence of patterns in heterogeneous environments. In particular, vegetation and animal population distributions adjust to varying levels of precipitation, sun, and soil quality, and generate patterns according to both nonlinear interactions with each other, but also their fitness at different locations in space. In embryonic development, particularly in the development of brain areas, ample experimental evidence established the importance of morphogen gradients interaction with nonlinear competitions between gene expression These motivate the fine understanding of pattern formation in spatially heterogeneous domains. In his seminal paper [1], Alan Turing immediately made the observation that: *"most of an organism, most of the time, is developing from one pattern to another, rather than from homogeneity into a pattern"*. Despite this importance, little remains known mathematically about pattern formation in heterogeneous environments. Indeed, most mathematical works on pattern formation deal with homogeneous systems or include gradients of parameters crossing bistable regimes, and we are still lacking a systematic understanding of the phenomenon of crossing transiently regions of parameters associated with multiple bifurcations. Recent investigations of these problems uncovered an astounding complexity of phenomena.

In this talk, I will review recent works and ongoing explorations around the problem of pattern formation in heterogeneous environments and will outline a few open problems. In particular, I will present some recent techniques developed to account for the remarkable reliability of brain area formation and will argue that homeoprotein diffusion, a phenomenon associated with extermely local diffusion of transcription factors, could in fact have far-reaching consequences on the formation and stabilization of sharp boundaries between developing brain region [2]. Mathematically, I will show that this phenomenon could be related to the behavior of general reaction-diffusion systems in competition with external cues. In these systems, typically exhibiting a bistable regime between two states, I will show that small diffusion leads to the emergence of two domains with a sharp transition between them using asymptotic methods involving Wentzel–Kramers–Brillouin expansions (or Hopf-Cole transforms) and viscosity solutions [2]. Contrasting with this paradigm, new data showed that genetic mutations can shatter the transition into multiple ectopic domains. I will show that these can be related to the crossing of multiple Turing instabilities and that the shape of the gradient will strongly impact the presence and nature of the patterns observed [3]. More generally, complex, possibly chaotic phenomena arise at the crossing of branches of periodic orbits that I will also review in this talk [4].

These topics cover collaborations with C. Quiñinao, A. Prochiantz, S-J Chou's laboratory, D. Patterson, S. Levin and others.

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Noise-induced patterns in biological systems

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Patterns across various scales in time and space are pervasive in nature. However, for many of these systems, deterministic descriptions often require fine-tuning of parameters to facilitate pattern formation, a process at odds with their observed natural robustness. Moreover, intrinsic stochasticity, such as fluctuations in small copy numbers of interacting basic variables, can significantly impact these systems. This necessitates characterizing their microscopic dynamics through master equations, which provide the probability of observing a system in a given state at a given time. The application of van Kampen's system size expansion to approximate analytically solutions of these master equations reveals how demographic noise emerges perturbatively and how intrinsic stochasticity can self-consistently amplify, yielding nearly regular oscillations in space and time.

In this talk, we will explore two natural systems where demographic noise plays a crucial role, demanding an inherently stochastic, individual-based description. Firstly, we will examine a stochastic one-dimensional model of coupled clock cores and their phosphorylation states in *Anabaena*, a multicellular filamentous cyanobacterium [1]. This model demonstrates that demographic noise can instigate stochastic oscillations, termed quasi-cycles, beyond the region where deterministic limit cycles with circadian periods occur. Additionally, it reproduces the observed spatio-temporal coherence along filaments, indicating that noise can paradoxically organize into regular quasi-periodic orbits, fostering macroscopic order from microscopic disorder. Secondly, we will investigate the development of trichome patterns on the epidermis of wild-type *Arabidopsis thaliana* leaves [2]. The statistical characterization of these patterns reveals power spectra with fat tails, a signature indicative of noise-driven stochastic Turing patterns, absent in patterns driven by deterministic instabilities. We will present a theoretical model incorporating demographic noise arising from birth-death processes of genetic regulators, which we analyze both analytically and through stochastic simulations. This model successfully captures the observed experimental features of trichome patterns.

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Harmonic Analysis and Geometric Measure Theory Special Session A11

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Abstract. This section aims at exploring the recent progress in and interplay between a few of the most prominent families of problems in Harmonic Analysis, such as

- Fourier restriction inequalities, decoupling and local smoothing estimates;
- problems related to the set and maximal Kakeya conjecture;
- singular integrals in the Euclidean, doubling, and non-doubling setting, and their application to Geometric Measure theory;
- Continuous and discrete Radon transforms, oscillatory integrals related to Carleson's theorem, and discrete operators in Harmonic Analysis.

These circles of problems, and the researchers working on them, all rely on diverse technical tools from a variety of fields of contemporary mathematics such as modern Analysis, Algebraic Geometry, Combinatorics, and Number Theory. In contrast with the increasingly specialized nature of their problems, researchers coming from different subfields share the same language of classical Harmonic Analysis and Geometric Measure Theory. This special session aims at highlighting both shared aspects, and at fostering interaction between researchers from each subfield. The invited speakers draw from both pools of world-leaders in the field and of innovative, accomplished and diverse earlier-career mathematicians. The Italian and American Harmonic Analysis communities will both be well represented, with additional emphasis on geographic diversity

For more information visit

`sites.google.com/view/fdiplinio/home/umi-ams-special-session`

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Singular integrals along variable codimension one subspaces

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The talk concerns maximal operators on $\mathbb{R}^n$ defined by taking arbitrary rotations of tensor products of an $(n - 1)$-dimensional Hörmander–Mihlin multiplier with the identity in one coordinate. These maximal operators are connected to differentiation problems and maximally modulated singular integrals such as Sjölin's generalization of Carleson's maximal operator.

As the main result, we prove a weak-type $L^2(\mathbb{R}^n)$ estimate on band-limited functions. As corollaries, we obtain a sharp $L^2(\mathbb{R}^n)$ estimate for the maximal operator restricted to a finite set of rotations in terms of the cardinality of the finite set and a version of the Carleson–Sjölin theorem. Our approach relies on higher dimensional time-frequency analysis elaborated according to the directional nature of the operator under study.

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Localised variants of multilinear restriction

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The multilinear Fourier Restriction estimates for hypersurfaces of Bennett–Carbery–Tao have played a central role in many recent developments in Euclidean Harmonic Analysis. Recently, Bejenaru introduced some variants in which a gain in the estimates is possible if the functions are Fourier localised to a neighbourhood of a submanifold of the hypersurfaces involved. In this talk, we present an alternative approach to that of Bejenaru for obtaining such refined estimates which, in particular, connects them to the theory of Kakeya–Brascamp–Lieb inequalities. Furthermore, we will present generalisations of Bejenaru's estimates under finer localisation conditions on the functions.

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Analysis on trees with nondoubling flow measures

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We consider infinite nonhomogeneous trees endowed with nondoubling flow measures of exponential growth. In this setting we develop a Calderón–Zygmund theory, we construct a family of dyadic sets, and we define BMO and Hardy spaces, proving a number of desired results extending the corresponding theory as known in the classical setting. This construction was inspired by a paper by Hebisch and Steger on homogeneous trees equipped with the canonical flow.

Moreover, given a nonhomogeneous tree T equipped with a flow measure m and a flow Laplacian $\mathcal{L}$ which is self-adjoint on $L^2(m)$, we prove weighted L^1-estimates for the heat kernel of $\mathcal{L}$ and its gradient. Such estimates combined with the Calderón–Zygmund theory adapted to the space (T, m) can be applied to study the boundedness properties of the Riesz transform associated with $\mathcal{L}$ and of spectral multipliers of $\mathcal{L}$.

This is a joint work with Matteo Levi, Alessio Martini, Federico Santagati and Anita Tabacco.

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Multi-parameter potential theory: results and problems

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Multi-parameter potential theory might be seen as the tensor-product version of classi- cal potential theory. It naturally arises, for instance, in the study of function spaces on poly-discs. Despite this short description, progress in the area has been slow and many basic questions remain unanswered. I will present a (biased) survey of old and recent results, and some problems which are central to have further progress. Some of the recent results were obtained by I. Holmes, P. Mozolyako, K.M. Perfekt, G. Sarfatti, A. Volberg, and myself.

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Oscillatory integrals over locally compact fields: A unified theory (part I)

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Here we consider oscillatory integrals defined over general locally compact fields $\mathbb{K}$. When $\mathbb{K} = \mathbb{R}$ is the real field, oscillatory integrals are a basic object of study in harmonic analysis. On the other hand, complete exponential sums or character sums can be realised as oscillatory integrals over the p-adic field $\mathbb{K} = \mathbb{Q}_p$. These are basic objects in number theory. In both cases, for real oscillatory integrals and exponential sums, there is an extensive literature giving sharp bounds for these oscillating entities.

In this talk, we discuss a unified theory for oscillatory integrals defined over any locally compact field. This is joint work with Gian Maria Dall'Ara.

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Oscillatory integrals over locally compact fields: A unified theory (part II)

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Jim Wright
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Here we consider oscillatory integrals defined over general locally compact fields $\mathbb{K}$. When $\mathbb{K} = \mathbb{R}$ is the real field, oscillatory integrals are a basic object of study in harmonic analysis. On the other hand, complete exponential sums or character sums can be realised as oscillatory integrals over the p-adic field $\mathbb{K} = \mathbb{Q}_p$. These are basic objects in number theory. In both cases, for real oscillatory integrals and exponential sums, there is an extensive literature giving sharp bounds for these oscillating entities.

In this talk, we discuss a unified theory for oscillatory integrals defined over any locally compact field. This is joint work with Jim Wright.

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Quantitative norm convergence of triple ergodic averages for commuting transformations

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We discuss a quantitative result on norm convergence of triple ergodic averages with respect to three general commuting transformations. For these averages we prove an r-variation estimate, $r > 4$, in the norm. We approach the problem via real harmonic analysis, using the recently developed techniques for bounding singular Brascamp-Lieb forms. It remains an open problem whether the analogous norm-variation estimates hold for all $r \geq 2$ as in the cases of one or two commuting transformations, or whether such estimates hold for any $r < \infty$ for more than three commuting transformations. This is joint work with Lenka Slavíková and Christoph Thiele.

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The oscillatory bi-est operator

Cristina Benea
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Abstract: The oscillatory bi-est operator is yet another example of a modulation-invariant multi-linear operator which presents certain curvature features, since it consists of the classical bi-est operator and carries an oscillatory factor in the form of a complex exponential. This is a joint work with I. Oliveira, and builds up on related works with F.Bernicot, V. Lie, M. Vitturi.

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On the curved trilinear Hilbert transform

Bingyang Hu
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The goal of this talk is to discuss the L^p boundedness of the trilinear Hilbert transform along the moment curve. The main difficulty in approaching this problem(compared to the classical approach to the bilinear Hilbert transform) is the lack of absolute summability after we apply the time-frequency discretization (which is known as the LGC methodology introduced by V. Lie in 2019).

To overcome such a difficulty, we develop a new, versatile approach – referred to as *Rank-II LGC* (which is also motivated by the study of the non-resonant bilinear Hilbert-Carleson operator by C. Benea, F. Bernicot, V. Lie, and V. Vitturi in 2022), whose control is achieved via the following interdependent elements:

1). a sparse-uniform decomposition of the input functions adapted to an appropriate time-frequency foliation of the phase-space;

2). a structural analysis of suitable maximal "joint Fourier coefficients";

3). A level set analysis with respect to the time-frequency correlation set.

This is a joint work with my postdoc advisor Victor Lie from Purdue.

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Fitting Smooth Functions to Data

Charles Fefferman
Princeton University, USA

The talk will discuss the following problems, which my friends and I have studied over the last 20 years or so:

Let E be an arbitrary subset of $\mathbb{R}^n$. How can we decide whether a given function $f : E \to \mathbb{R}$ extends to a C^m function on $\mathbb{R}^n$ for fixed m? If such an extension F exists, then how small can we take its C^m norm? What can we say about F and its derivatives at points in or near E? Can we take F to depend linearly on f for fixed E and m?

Suppose E is finite. Can we compute an F with C^m norm of smallest possible order of magnitude? How many computer operations does it take? What if we demand merely that F agree with f on E to a given tolerance, rather than demanding that $F = f$ on E? What if we are allowed to delete a few points of E as "outliers"? Which points should we delete?

What if F is required to satisfy constraints, e.g. $F \geq 0$ or F convex? What if the C^m norm is replaced by a Sobolev norm?

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Hardy spaces and dilations on homogeneous groups

Tommaso Bruno
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On a homogeneous Lie group, I will discuss a characterization of the one-parameter groups of dilations whose associated Hardy spaces in the sense of Folland and Stein are the same. The talk will be based on a joint work with J. T. van Velthoven [BvV].

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Uniform bounds for trilinear Fourier multiplier forms

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Let $d \geq 1$. Given three test functions f_1, f_2 and f_3 on $\mathbb{R}^d$ and a symbol function $m : \mathbb{R}^{3d} \to \mathbb{C}$, consider the trilinear form

$$\Lambda(f_1, f_2, f_3) = \iiint_{\xi_1+\xi_2+\xi_3=0} \widehat{f_1}(\xi_1)\widehat{f_2}(\xi_2)\widehat{f_3}(\xi_3)m(\xi_1, \xi_2, \xi_3)\, d\xi_1 d\xi_2 d\xi_3.$$

Assume the symbol function to be smooth except for a singularity at a d-dimensional linear subspace $\Gamma \subset \mathbb{R}^{3d}$, which is the image of $\{(\xi, \xi, \xi) : \xi \in \mathbb{R}^d\}$ under a map $L = L_1 \oplus L_2 \oplus L_3$ whose blocks L_i satisfy $K^{-1}|x|^2 \leq L_i x \cdot x \leq K|x|^2$ for some K and all $i \in \{1, 2, 3\}$ and all $x \in \mathbb{R}^d$. Under the natural derivative bounds on m (relative to the above mentioned singularity), we show that the form is bounded on $L^{p_1}(\mathbb{R}^d) \times L^{p_2}(\mathbb{R}^d) \times L^{p_3}(\mathbb{R}^d)$ provided that the exponents are in the local $L^2(\mathbb{R}^d)$-range and satisfy the natural scaling conditions

$$\frac{1}{p_1} + \frac{1}{p_2} + \frac{1}{p_3} = 1, \quad 2 < p_1, p_2, p_3 < \infty.$$

The bound on the form depends on the matrices L_1, L_2 and L_3 only through the parameter K (which is invariant under scalings). In particular, the result recovers the uniform bounds in local L^2-range for the bilinear Hilbert transform and extends them to the two dimensional bilinear singular integrals such as the bilinear variant of the Beurling transform. This is based on joint work with Marco Fraccaroli and Christoph Thiele.

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Endpoint estimates for Fourier multipliers with Zygmund Singularities

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We consider Fourier multipliers on the real line with frequency singularities along thin sets: the prototypical examples are given by Hřmander-Mihlin multipliers which are singular along higher-order lacunary sets and corresponding square functions, and by Marcinkiewicz-type multipliers. For these operators we prove the sharp endpoint mapping properties, generalizing previous results of Tao and Wright. Underlying the proofs of such results is a family of generalized Zygmund-type inequalities which are related to the (dual version of the) Chang-Wilson-Wolff inequality. A further and more sophisticated approach will allow us to develop the theory for general sets of singularities that satisfy suitable Zygmund-type assumptions, leading to the full range of (weighted) Lp-estimates, sparse bounds, and a characterization of the Littlewood-Paley property.

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On the non-resonant Carleson–Radon transform

Martin Hsu
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In this talk, we discuss the L^p boundedness of the 2D non-resonant Carleson–Radon transform:

$$CR(f)(x,y) := \text{p.v.} \int_{\mathbb{R}} f(x-t, y-t^2)\, \frac{e^{i\, a(x,y)\, t^3}}{t}\, dt\,, \quad (x,y) \in \mathbb{R}^2\,, \tag{1}$$

where $a(x,y)$ is an arbitrary real measurable function. We prove the following:

Theorem 1. *For $p \in (1,\infty)$, there's a constant C_p independent of $a(x,y)$ such that*

$$\|CR(f)\|_{L^p} \leq C_p \|f\|_{L^p}\,.$$

Previously known results addressed only the restricted situation with $a(x,y) = a(x)$ or $a(x,y) = a(y)$. Our theorem is the first instance treating (**??**) in full generality.

Our approach relies on the LGC method and involves the following key elements:

- a sparse-uniform dichotomy of the input function adapted to appropriate time-frequency foliation of the phase-space;
- a joint structural analysis of the linearizing stopping-time function $a(x,y)$ in relation to the Gabor coefficients of the input;
- a level set analysis on the time-frequency correlation set.

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On integer distance sets

Marina Iliopoulou
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An integer distance set is a set in the Euclidean plane with the property that all pairwise distances between its points are integers. In this talk we will explain that any integer distance set lies on a single line or circle, apart from perhaps a small number of its points. This helps us address some questions by Erdős on the size of integer distance sets. For instance, we deduce that integer distance sets in general position are very sparse. The main idea of the proof is transforming the points in an integer distance set into lattice points on some high-degree variety, and then using existing results from number theory to control the number of these lattice points. This is joint work with Rachel Greenfeld and Sarah Peluse.

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Discrepancy on the sphere with respect to caps of fixed radius

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A classical result of J. Beck guarantees that the optimal discrepancy of an N-point set in $\mathbb{S}^d$ over all spherical caps with unrestricted radii is of the order $N^{-\frac{1}{2}-\frac{1}{2d}}$. However, it was completely unclear what happens if one fixes the radius. We provide a partial answer to this question by describing a set of radii for which the above bound continues to hold for spherical caps of a fixed radius. To this end we introduce gegenbadly approximable numbers (an analog of badly approximable numbers in diophantine approximations) for which the values of Gegenbauer polynomials stay away from zero in a certain quantitative sense. We also discuss other versions of this question in various settings (e.g. balls in the unit cube or torus), an interesting role played by the case $d \equiv 1 \mod 4$, as well as the so-called 'freak theorem' about continuous functions which have mean zero over all spherical caps of a given radius. The talk is based on joint work with M. Mastrianni and S. Steinerberger.

[1]Acknowledgements: The speaker was supported by the NSF grant DMS-2054606
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The oscillatory bi-est operator

Alexander Volberg
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Abstract: Suppose you wish to find a $2^n \times 2^n$ matrix by asking this matrix question that it honestly answers. For example you can ask question "What is your $(1, 1)$ element?" Obviously you will need exponentially many questions like that. But if one knows some information on Fourier side one can ask only log n questions if they are carefully randomly chosen. Of course one pays the price: first of all one would find the matrix only with high confidence (high probability bigger than $1 - \delta$), secondly the error ϵ. I will explain how this can be done using harmonic analysis and probability. The main ingredient is dimension free Remez inequality.

The classical Remez inequality bounds the supremum of a bounded-degree polynomial on an interval X by its supremum on any subset $Y \subset X$ of positive Lebesgue measure. There are many multivariate generalizations of the Remez inequality, but they have constants that depend strongly on dimension. Here we show that a broad class of domains X and test sets Y—termed *norm designs*—enjoy dimension-free Remez-type estimates.

Theorem 1. *Let $n \geq 1$, $\eta > 0$ and $K \geq 2$. Consider $Y = \prod_{j=1}^n Z_j$ for sets $Z_1, Z_2, \ldots, Z_n \subset \mathbf{D}$ such that for all $1 \leq j \leq n$ we have $|Z_j| = K$ and*

(1) $$\min_{z \neq z' \in Z_j} |z - z'| \geq \eta\,.$$

Then for any analytic polynomial $f : \mathbf{D}^n \to \mathbb{C}$ of degree d and individual degree $K - 1$,

(2) $$\|f\|_{\mathbf{D}^n} \leq C(d, K)\|f\|_Y\,.$$

Here constant $C(d, K) = C(d, K, \eta) = C(K, \eta)^d$, and $C(K, \eta) > 0$ depends only on K and η, and not on dimension n.

Moreover if all $Z_j = \Omega_K := \{e^{2\pi \mathbf{i} k/K} : k = 0, 1, \ldots, K - 1\}$ then $C(d, K) \leq \big(\mathcal{O}(\log K)\big)^{2d}$.

This is the joint work with Lars Becker, Ohad Klein, Joseph Slote, and Haonan Zhang.

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On Implicitly Oscillatory Multilinear Integrals

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For $1 \leq j \leq N$ let $\varphi_j : B \to \mathbb{R}^1$ be C^1 surjections, and let $B \subset \mathbb{R}^2$ be an open ball. Consider scalar-valued multilinear forms

$$T(f_1, \ldots, f_N) = \int_{\mathbb{R}^2} \prod_{j=1}^{N} (f_j \circ \varphi_j)\, \eta$$

where η is a smooth compactly supported cutoff function. We aim to prove *a priori* multilinear smoothing inequalities of the form

$$|T(f_1, \ldots, f_N)| \leq C \prod_j \|f_j\|_{W^{p,s}}$$

where $p < \infty$, $W^{p,s}$ is the Sobolev space of functions with s derivatives in L^p, and crucially, s is strictly negative. The emphasis is on the existence of p, s, not on optimal thresholds.

Such an inequality broadly asserts that if at least one factor has Fourier transform supported at high frequencies, then $|T(f_1, \ldots, f_N)|$ is small — hence the term "implicitly oscillatory".

The first result of this type was due to Bourgain [?] for a particular example with $N = 3$. A compactness theorem, without any quantitative inequality, was proved by Joly, Métivier, and Rauch [?], again for $N = 3$.

We state two results under a mild transversality hypothesis on the mappings φ_j. Firstly, for $N = 4$ and C^ω mappings φ_j, the inequality holds if and only if there does not exist a resonance — that is, a nonconstant C^ω solution $(g_1, \ldots, g_4)$ of $\sum_j (g_j \circ \varphi_j) \equiv 0$ in some connected open subset of $\mathbb{R}^2$.

Secondly, for $N = 3$, the inequality holds for $C^{1,\alpha}$ mappings for any $\alpha > 0$, under a certain nonlinearizability hypothesis on the 3-web in $\mathbb{R}^2$ defined by $(\varphi_j : 1 \leq j \leq 3)$. This hypothesis is a version of nonvanishing curvature of the web, but requires only C^1 regularity, whereas the classical definition of 3-web curvature involves third order derivatives.

Certain sublevel set inequalities are a key element for the proofs of both results. Other elements include local Fourier expansion aka microlocal analysis, stationary phase, and decomposition into structured and pseudorandom summands in the spirit of much work in additive combinatorics. Due to the time constraint, little can be said about the proofs in this talk.

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A numerical method for the solution of boundary value problems on convex planar domains

J. Hulse, S. Llewellyn Smith and Elena Luca, Loredana Lanzani
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The Unified Transform Method (UTM) was pioneered in the early 1990s by A. S. Fokas and I. M. Gèlfand in their study of the numerical solution of boundary value problems for elliptic PDEs and for a large class of nonlinear PDEs. The UTM provides a connection between the Fourier Transform method for linear PDEs (FT) and its nonlinear counterpart, namely the Inverse Spectral method - also known as Non Linear Fourier Transform method (NLFT).

At the heart of the matter is a new derivation of the FT method for linear equations in one and two (space) variables that follows the same conceptual steps needed to implement the NLFT method for a class of nonlinear evolution equations, thus pointing to a unified approach to the numerical solution of linear and nonlinear PDEs.

From the very beginning, the UTM has attracted a great deal of interest in the applied mathematics community. A multitude of versions of the original method have since been developed, each dealing with a specific family of equations. Here we focus on a 2003 result of A.S. Fokas and A.A. Kapaev pertaining to the study of boundary value problems for the Laplacian on convex polygons: their original approach relied on a variety of tools (spectral analysis of a parameter-dependent ODE; Riemann-Hilbert techniques, etc.) but it was later observed by D. Crowdy that the method can be recast within a complex function-theoretic framework which, in turn, expands the applicability to so-called circular domains (domains bounded by arcs of circles, with line segments being a special case).

We extend the original approach of Fokas and Kapaev for polygons, to arbitrary convex domains. It turns out that ellipses (which are not circular in the sense of Crowdy) are of particular relevance in applications to engineering because the most popular heat exchangers (namely the shell-and-tube exchangers) have elliptical cross section.

This is joint work with J. Hulse (Syracuse University), S. Llewellyn Smith (UCSD & Scripps Institute of Oceanography) and Elena Luca (The Cyprus Institute).

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TBA

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Topological and Variational Methods for Differential and Difference Equations Special Session A12

John R. Graef
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Luisa Malaguti
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The investigation of ordinary differential equations and systems maintains a constant interest in the mathematical literature. This has never been more apparent than in recent years with the exploding interest in the fractional calculus and fractional differential equations. The great attention towards these equations and systems is mainly due to their role as models in many areas of science and technology such as in diffusion processes, celestial mechanics, flows in porous media, population dynamics, epidemic models, and more recently in social science areas. These equations and systems are also used to obtain solutions to diffusion and transport equations that are often in good agreement with experimental data. In some cases, the underlying process is discretized, which justifies the study of the associated discrete models.

New requests by the scientific community and the development of new mathematical tools serve to motivate interesting recent achievements in several directions.

The main mathematical tools in these studies are topological or variational methods. In some cases, these techniques can be successfully combined with comparison-type techniques such as the upper and lower solution method.

The aim of this special session is to advance knowledge and interest in the study of all these types of problems by inviting researchers to participate and share their current research endeavors.

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Uniqueness of Positive Solutions for a Class of Nonlinear Elliptic Equations with Robin Boundary Conditions

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Ratnasingham Shivaji
Department of Mathematics and Statistics, University of North Carolina at Greensboro

Xiao Wang
Jiangsu University

We prove uniqueness of positive solutions to the BVP

$$\begin{cases} -\Delta u = \lambda f(u) & \text{in } \Omega, \\ \frac{\partial u}{\partial n} + bu = 0 & \text{on } \partial\Omega, \end{cases}$$

when the parameter λ is large independent of $b \in (0, \infty)$. Here Ω is a bounded domain in $\mathbb{R}^n$ with smooth boundary $\partial\Omega$, $f : [0, \infty) \to [0, \infty)$ is continuous, concave for u large, and sublinear at ∞.

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Existence results for discrete and differential nonlinear problems

Giuseppina D'Aguì [1]
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In this talk some results on the existence of two positive solutions to boundary value problems for difference equations and ordinary differential equations are presented. In particular the existence of at least two solutions is obtained by requiring suitable behaviour at infinity and at zero of the primitive of the nonlinear term. Our main tool is a critical point theorem obtained by appropriately combining the powerful classical Ambrosetti-Rabinowitz theorem with a recent non-zero local minimum theorem.

[1]The results presented are part of the research carried out within the project: PNRR-MAD-2022-12376692-PNRR-Missione 6 - Componente 2 Investimento 2.1 Valorizzazione e Potenziamento della Ricerca Biomedica del SSN

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Existence and approximation of a solution for a two point nonlinear Dirichlet problem

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The existence of at least one positive solution to a second-order nonlinear two-point boundary value problem, is established. Combining difference methods with variational or topological methods, we get a solution as the limit of an appropriate sequence of piecewise linear interpolations. Furthermore, a priori bounds on the infinite norm of a solution and its derivatives are pointed out. Some examples are also discussed to illustrate our results.

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[1]Research project of MIUR (Italian Ministry of Education, University and Research) Prin 2022 "Nonlinear differential problems with applications to real phenomena"(Grant Number: 2022ZXZTN2)
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Periodic solutions in special relativity with singular potentials

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According to special relativity theory, the law of motion $x(t) \in \mathbb{R}^3$ of a particle of mass m and charge q obeys to the Lorentz force equation

$$\frac{d}{dt}\left(\frac{m\dot{x}}{\sqrt{1-|\dot{x}|^2/c^2}}\right) = q[E(t,x) + \dot{x} \times B(t,x)]$$

where c is the speed of light while $E(t,x)$ and $B(t,x)$ are the electric and magnetic fields, respectively, at time t and position x. That equation is formally the Euler-Lagrange equation of the action functional

$$I(x) = \int_0^T mc^2\left(1 - \sqrt{1-|\dot{x}(t)|^2/c^2}\right)dt + \int_0^T q\left[-V(t,x(t)) + A(t,x(t))\cdot \dot{x}(t)\right]dt$$

where V is the electric potential and A is the magnetic vector potential, in such a way that

$$E(t,x) = -\nabla_x V(t,x) - \partial_t A(t,x) \quad \text{and} \quad B(t,x) = \nabla_x \times A(t,x).$$

For instance, when E and B are generated by charged particles that move along T-periodic orbits in space, the potentials V and A are called Liénard-Wiechert potentials and are T-periodic w.r.t. time and singular w.r.t. space. Moreover, the first term of the functional I is not globally differentiable but is convex and satisfies Szulkin's structural assumptions [5].

We use non-smooth critical point theory to prove the existence of infinitely many T-periodic solutions of the differential equation under physically reasonable conditions on V and A: for instance the cases of Liénard-Wiechert potentials [3], of the N-center problem [3] and of the perturbed Kepler problem [2] are included.

We use an adaptation of the non-smooth min-max principle [4,Theorem 3.1] to singular potentials together with arguments developed in [1] and Lusternik-Schnirelman category theory.

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Multiple solutions for Kirchhoff double phase problems

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Presentation concerns the nonlocal Dirichlet equation of double phase type with the right hand side function of superlinear and subcritical growth. The existence of two constant sign solutions (one positive, the other one negative) and of a sign-changing solution which has exactly two nodal domains and which turns out to be the least energy sign-changing solution is presented. The proof is based on variational tools in combination with the quantitative deformation lemma and the Poincare-Miranda existence theorem.

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Nearly-circular periodic solutions of a perturbed relativistic Kepler problem: the fixed-period and the fixed-energy problems

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The motion of a relativistic particle in a Kepler potential can be described by the equation

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{mx}{\sqrt{1-|x|^2/c^2}}\right) = -\alpha\,\frac{x}{|x|^3}, \quad x \in \mathbb{R}^2 \setminus \{0\},$$

where $m > 0$ is the mass of the particle, c is the speed of light, and $\alpha > 0$ is a constant. Firstly, we illustrate the Hamiltonian formulation of the problem and we focus our attention on the description of the periodic solutions. Secondly, we deal with the perturbed equation

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{mx}{\sqrt{1-|x|^2/c^2}}\right) = -\alpha\,\frac{x}{|x|^3} + \varepsilon\,\nabla_x U(t,x), \quad x \in \mathbb{R}^2 \setminus \{0\},$$

where $\varepsilon \in \mathbb{R}$. We provide existence of periodic solutions bifurcating, for ε small enough, from the set of circular solutions of the unperturbed system ($\varepsilon = 0$). Both the case of the fixed-period problem (assuming that U is T-periodic in time) and the case of the fixed-energy problem (assuming that U is independent of time) are considered. The same results are also valid in the three-dimensional space. The talk is based on papers written in collaboration with Alberto Boscaggin, Walter Dambrosio, and Duccio Papini.

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[1]The talk is based on papers written under the auspices of the Gruppo Nazionale per l'Analisi Matematica, la Probabilità e le loro Applicazioni (GNAMPA) of the Istituto Nazionale di Alta Matematica (INdAM).
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Bifurcation of periodic solutions for problems of relativistic mechanics

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We discuss bifurcation of periodic solutions for some problems of relativistic mechanics, such as the relativistic Kepler problem in special relativity and the Schwarzschild problem of general relativity. The main tool is the classical Poincaré-Birkhoff fixed point theorem, together with its higher-dimensional generalizations.

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Rich dynamics in a model related to suspension bridges

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We consider a degenerate plate model for suspension bridge-type structures, encompassing a coupled dynamics involving longitudinal and torsional oscillations. In presence of a time-periodic external force triggering a single longitudinal mode and satisfying suitable assumptions, we show the occurrence of rich and complex dynamics for the corresponding longitudinal oscillation. The goal is achieved by applying a rigorous analytical approach based on the theory of linked twist maps.

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Birkhoff-Kellogg type results with applications

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We present some classical and recent results of Birkhoff-Kellogg type in cones. We illustrate their applicability in the context of ordinary, functional and partial differential equations subject to local, nonlocal and functional boundary conditions.

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Existence of solutions for some boundary value problems associated with singular equations with Φ-Laplacian operators

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Alessandro Calamai
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We will review some recent results about the existence of solutions for different boundary value problems (BVPs) associated with second- or third-order singular equations involving the Φ-Laplacian operators.

In the recent paper [1] we investigate the following BVP

$$(\mathrm{P}) \begin{cases} (\Phi(k(t)u''(t)))' = f(t, u(t), u'(t), u''(t)), \text{ a.e. on } [0, T], \\ \\ u(0) = a, \ u'(0) = b, \ u'(T) = c \end{cases}$$

where f is a Carathédory function, $\Phi : \mathbb{R} \to \mathbb{R}$ is a strictly increasing homeomorphism, and the nonnegative function k may vanish on a set of measure zero.

Under mild assumptions, including a weak form of a Nagumo–Winter growth condition, we prove the existence of solutions of problem (P) in the Sobolev space $W^{2,p}([0, T])$. Our approach is based on fixed point techniques suitably combined to the method of upper and lower solutions.

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Multiplicity of periodic solutions for nearly resonant Hamiltonian systems

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Let us first consider a planar Hamiltonian system

$$Ju = \nabla_u \mathcal{H}(t,u)\,, \tag{1}$$

having a twist dynamics. More precisely, we assume $\mathcal{H}(t,u)$ to be continuous, T-periodic in t, continuously differentiable in $u=(q,p)$, and 2π-periodic with respect to q, and that there exist $a<b$ be such that all solutions $u=(q,p)$ of the system, starting with $p(0)\in[a,b]$, are defined on $[0,T]$ and are such that

$$p(0)=a \implies q(T)-q(0)<0\,, \qquad p(0)=b \implies q(T)-q(0)>0\,.$$

Then, the celebrated Poincaré–Birkhoff Theorem ensures the existence of at least two geometrically distinct T-periodic solutions $u=(q,p)$, with $p(0)\in\,]a,b[\,$. This multiplicity result was proved in [3] and extended to systems in $\mathbb{R}^{2M}$, providing in that case the existence of $M+1$ periodic solutions.

Let us now consider another Hamiltonian system

$$Jz = \nabla_z H(t,z)\,, \tag{2}$$

which has a completely different dynamics. We assume the function H to be continuous, T-periodic in t, twice continuously differentiable in z, with

$$A \le H_z''(t,z) \le B\,, \quad \text{for every } (t,z)\in[0,T]\times\mathbb{R}^{2N},$$

for some symmetric matrices A,B which satisfy the nonresonance condition

$$\bigcup_{\lambda\in[0,1]} \sigma((1-\lambda)JA+\lambda JB)\setminus \frac{2\pi}{T}i\mathbb{Z}=\emptyset\,.$$

Then it was proved in [1] that system (2) has a unique T-periodic solution.

In this talk I will present the results obtained in [2] for a Hamiltonian system whose Hamiltonian function has a twisting part and a nonresonant part. We consider a system of the form

$$\begin{cases} Ju = \nabla_u\mathcal{H}(t,u)+\nabla_u P(t,u,z)\\ Jz=\nabla_z H(t,z)+\nabla_z P(t,u,z)\,,\end{cases}$$

where $\nabla_u P(t,u,z)$ and $\nabla_z P(t,u,z)$ are bounded, and prove some multiplicity results for the associated T-periodic problem. We begin by assuming a nonresonance condition on the function $H(t,z)$ as the one described above, and then analyse a possible approach to resonance, together with some kind of Landesman–Lazer condition. We propose a new version of this condition, and we also treat the so-called *double resonance* situation.

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On semilinear elliptic systems with superlinear boundary conditions

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We investigate a system of elliptic equations characterized by superlinear and subcritical boundary conditions with a bifurcation parameter. Furthermore, under additional conditions on the nonlinearities near zero, we discuss the existence of a global, connected branch of positive solutions bifurcating from the line of trivial solutions, with a unique bifurcation point from infinity when the bifurcation parameter is zero. We employ bifurcation theory, degree theory, and sub- and super-solution method to obtain our results.

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On a class of Initial–Boundary Value Problems for renewal equations

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In biological or epidemiological models, different species are typically described through their densities $u_1, u_2, \ldots, u_k$ and, in general, each u_h depends on time $t \in \mathbb{R}_+$, on age $a \in \mathbb{R}_+$, on a spatial coordinate in $\mathbb{R}^2$ or $\mathbb{R}^3$ and possibly also on some other structural variables. In order to provide a unified treatment of these models, we propose an Initial–Boundary Value Problem (IBVP) for a system of balance laws, considering quite general interaction terms, possibly non linear and/or non local. In particular, we remark that in the present framework diffusion is lacking, thus any movement or evolution described by the IBVP propagates with a finite speed.

We prove well posedness of the solutions, thus local existence, uniqueness and continuous dependence on the initial datum, then we provide conditions ensuring global in time existence. Moreover, we show the stability of the solutions with respect to the functions and the parameters appearing in the IBVP.

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Existence of nonnegative mild solutions of stochastic evolution inclusions with application to climate change models

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We provide sufficient conditions for the existence of mild solutions to stochastic differential inclusions in infinite dimensional Hilbert spaces driven by a cylindrical Wiener process. Inspired by [5] we develop an approximation procedure based on the weak topology. By applying this method, we can accomplish the dual objective of proving the existence of a solution over the entire real half-line while relaxing the commonly assumed hypotheses of Lipschitz continuity and compactness found in the literature on the subject. Moreover, the problem of the non negativity of the solution is addressed. Namely, assuming an additional sign condition that involves both the deterministic and stochastic nonlinear terms as in [4], we can ensure the non-negativity of the solution starting from a non-negative initial datum. These differential inclusions find applications in climate model studies. Indeed to encompass the broadest possible range in climate change models, it is essential to consider non-deterministic differential equations. For example, cyclones can be treated as a rapidly varying component and represented as a white-noise process. In this context, nonlinear stochastic parabolic differential equations were initially proposed by North and Cahalan [1] to explore non-deterministic variability in energy balance climate frameworks. Climate models based on differential inclusions are characterized by a deterministic part, a stochastic part, or both, described by set-valued maps. Considering set-valued maps allows us to include in the model cases where the exact value of the empirical evidence cannot be calculated, but is known with a certain degree of uncertainty, see e.g. [2, 3]. The talk is based on two joint papers, one with Alessandra Cretarola and Lucia Angelini and the second one with Alessandra Cretarola and Lorenzo Guida, University of Perugia.

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A non-linear eigenvalue problem for the critical p-Laplace equation in the ball

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In this talk we show that the number of radial positive solutions of the following critical problem

$$\begin{cases} \Delta_p u(x) + \lambda K(|x|)\, u(x)\, |u(x)|^{q-2} = 0\,, & \\ u(x) > 0 & |x| < 1 \\ u(x) = 0 & |x| = 1. \end{cases}$$

undergoes a bifurcation phenomenon; here $q = \frac{np}{n-p}$, $p > 1$ and $x \in \mathbb{R}^n$. When $K(r)$ is steep enough at $r = 0$ the problem admits (at least) one solution for any $\lambda > 0$, while if $K(r)$ is too flat at $r = 0$ then it admits no solutions for λ small and two solutions for λ large.

The existence of the second solution is new even in the classical Laplace case. The proofs use Fowler transformation and dynamical systems tools: in fact if $1 < p \leq 2$ we can rely on standard tools of invariant manifold theory, while if $p > 2$ we need to develop some ad-hoc box argument to overcome some lack of regularities issues.

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Periodic solutions of differential equations with oscillating constraints

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This talk is devoted to the investigation of the topological structure of the set of harmonic solutions to a class of implicit ordinary differential equations subjected to periodic perturbations.

In order to that, we study preliminarly the harmonic solutions of periodically perturbed differential equations *in normal form* subjected to a possibly time-dependent constraint; in particular, a so-called *Differential-Algebraic equation* of an appropriate type. Using a combination of techniques from the theory of Topological Degree and Differential-Algebraic Equations, we obtain, for these systems, a degree theoretic condition ensuring a "global branching" result for the nontrivial periodic solutions.

The latter result actually turns out to be more general than the one sought by the former, original, problem. In fact the investigation of the set of harmonic solutions of our implicit differential equations boils down to the the same problem for Differential-Algebraic equations of the above mentioned type, by the means of the introduction of an extra variable.

[1]Work partially supported by INDAM-GNAMPA
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Multiple solutions for a nonlocal Dirichlet problem driven by the p-Laplacian

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The aim of the talk is to study the following problem

$$\begin{cases} -a\left(\int_\Omega u^q dx\right)\Delta_p u = \beta(x)f(t) & \text{in } \Omega, \\ u > 0 & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases} \tag{1}$$

where Ω is a bounded smooth domain in $\mathbb{R}^N$, N is a positive integer, $q \geqslant 1$, $1 < p < +\infty$, while $a \in C([0,\infty))$ is a changing sign function, $\beta \in L^\infty(\Omega)$ and f is suitable continuous function.

As pointed out for example in [2], nonlocal problems having a structure as (1) can be considered for describing biological models of the population diffusion.

The case when the reaction term does not depend on the vectorial variable has been studied first in [3, 4], when $p = 2$, and then, in [1], in the more general case when $1 < p < \infty$.

Here, following [3] and [1], we will assume that a admits a finite number of "positive bumps"; moreover, f is a positive and continuous function in a right neighbourhood of zero, satisfying a monotonicity condition.

The multiplicity of solutions will be achieved by using an auxiliary problem and combining truncation techniques, variational methods and a well known formula, due to Diaz-Saa, with the fixed point theory.

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Hyperbolic PDEs: Analytical Techniques and Applications

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Topic: The techniques of hyperbolic partial differential equations, in particular (of systems of) conservation laws, are the main topic of the special session "Hyperbolic PDEs: Analytical Techniques and Applications". The analytical results presented comprise also the case of mixed systems, in which conservation or balance laws are coupled also with ordinary differential equations, or equations of other types. The motivations for the tackled problems typically originate in very specific problems, suggested by a wide variety of applications such as fluid dynamics, epidemiology or biology and, mostly, the modeling of vehicular traffic flows.

Structure: This session is scheduled on July 23-24. It consists of 10 talks (each 45 minutes long followed by a 15-minute break).

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Going Forward and Backward in Time in Conservation Laws and Hamilton-Jacobi Equations.

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In the scalar one dimensional case, consider a conservation law and a Hamilton - Jacobi equation, i.e.,

$$\partial_t u + \partial_x f(x,u) = 0 \qquad \text{and} \qquad \partial_t U + f(x, \partial_x U) = 0\,.$$

The evident correspondence between the two equations is in deep contrast with the differences between their classical definitions of solutions. Indeed, Kružkov definition of entropy solution to a conservation law and Crandall - Lions definition of viscosity solution to a Hamilton - Jacobi equation appear as completely unrelated.

This talk presents first a framework where the well posedness of both equations can be proved and the correspondence between their solutions can be rigorously established. Then, given a time $T > 0$ and profiles $w \in \mathbf{L}^\infty(\mathbb{R};\mathbb{R})$ or $W \in \mathbf{Lip}(\mathbb{R};\mathbb{R})$, we characterize the two sets of initial data that, for the two equations, evolve into w or W at time T.

The explicit presence of the space variable in the flux or Hamiltonian f significantly enriches the theory, as shown by an explicit example. Applications to vehicular traffic management and data reconstruction are also considered.

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[1] Acknowledgments to the PRIN 2022 project *Modeling, Control and Games through Partial Differential Equations* (D53D23005620006), funded by the European Union - Next Generation EU.
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A nonlocal version of Aw-Rascle-Zhang system

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In this talk, we consider a macroscopic model for traffic flow. The model stems from the classical Aw-Rascle-Zhang system (ARZ) in which the pressure, describing the anticipation term along a trajectory, is replaced by a nonlocal version of it. The resulting system is an Euler-alignment pressureless system with a non-symmetric interaction kernel.

We tackle the study of weak solutions for this system through a sticky-particle approach, the difficulty being in the occurrence of non-conservative terms due to the singularity of the interaction kernel.

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Nonlocal Multi-D Systems of Hyperbolic Equations

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We consider the following multi-D non linear system of hyperbolic equations

$$\begin{cases} \partial_t \rho_i + \nabla \cdot (\rho_i \, V_i(\nabla \rho * \eta)) = 0 & (t,x) \in \mathbb{R}_+ \times \mathbb{R}^n \\ \rho_i(0,x) = \rho_{o,i}(x) & x \in \mathbb{R}^n \end{cases} \qquad i \in \{1, \ldots, m\} \tag{1}$$

with non-local terms in the flux functions. Here $t \in \mathbb{R}_+$ is the time, $x \in \mathbb{R}^n$ is the space variable, $\rho = (\rho_1, \ldots, \rho_m)$ denotes the vector of time dependent densities of m populations defined on the whole space $\mathbb{R}^n$, $(\rho_{o,1}, \ldots, \rho_{o,m})$ are the initial conditions, and V_i, for $i \in \{1, \ldots, m\}$, represents the velocity function for the i-th population. We assume that each V_i depends on the $n \times m$ matrix $(\nabla \rho * \eta)$, where the ji entry is

$$(\nabla \rho(t) * \eta(x))_{ji} = \partial_{x_j}(\rho_i(t) * \eta)(x) = \partial_{x_j} \int_{\mathbb{R}^n} \rho_i(t, x-y)\eta(y) dy,$$

for $j \in \{1, \ldots, n\}$ and $i \in \{1, \ldots, m\}$ and $\eta : \mathbb{R}^n \to \mathbb{R}$ is a smooth kernel.

The model (1) is of macroscopic type and is able to describe different behaviors typically emerging in population dynamics. Indeed different shapes of the kernel function η and of the velocity vectors V_i may result for example in an aggregation phenomenon, with the possible formation of clusters (or opinions), or in a segregation of the various populations. An important role is played by the support of the kernel function, which correspond to the visual range of the individuals. In the talk we present several numerical integrations showing various behaviors of (1), together with its well posedness and some analytic qualitative properties.

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Stabilization of evolution systems

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In this talk, we will discuss recent progresses in stabilization of PDE systems. We will begin by introducing a recent approach –known as Fredholm backstepping (or F-equivalence)– which has significantly improved in the last two years and has demonstrated a surprisingly remarkable efficacy [1, 2, 3]. Instead of attempting directly to solve the problem, the principle of the F-equivalence is to allow the PDE system to be invertibly transformed into a simpler PDE system whose stability is already established.

Next, we will discuss practical applications: stabilizing hyperbolic PDEs to control navigable rivers and traffic flows. They are particular examples of what is called *density-velocity* systems, which consist in two equations: a conservation of mass and a balance of momentum or energy [5]. We will present some results obtained in the last few years and show how abstract mathematical concepts, like entropic solutions, can have tangible impacts in real-world scenarios [4].

Finally, if time allows, we will dedicate a few minutes to explore how AI models can be trained to learn a mathematical intuition, in particular in stabilization of evolution systems.

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Smoothing effect and particle approximation for a nonlocal conservation law

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Conservation laws with nonlocality in the flux appear in many applied contexts such as traffic flow, sedimentation processes, supply chain modelling, and crowd movements. In this talk we consider the following version of a one-dimensional, nonlocal scalar conservation law (extensively studied in the literature recently, see for example the papers below)

$$\partial_t \rho + \partial_x(\rho W[\rho]) = 0 \tag{1}$$

where $W : \mathcal{P}(\mathbb{R}) \to L^\infty(\mathbb{R})$ is a nonlocal operator defined by

$$W[\rho](x) = v(V * \rho)(x)$$

with $\mathcal{P}(\mathbb{R})$ being the space of probability measures on $\mathbb{R}$. We assume that $V \in L^1 \setminus L^\infty(\mathbb{R};\, [0, +\infty))$ satisfies $\mathrm{supp}(V) \subset (\ \ \infty, 0]$, $V \in \mathrm{Lip}((\ \ \infty, 0])$, V left continuous at zero with $V(0^-) > 0$. The velocity map $v : [0, +\infty) \to \mathbb{R}$ is supposed to be Lipschitz continuous with $v'(\rho) \leq \ \ b$ (almost everywhere) for some $b > 0$ for all $\rho \geq 0$.

We prove that the Cauchy problem for (1) is well posed for initial data $\rho_0 \in \mathcal{P}(\mathbb{R})$, more precisely in the 2-Wasserstein space. Moreover, we prove that all solutions satisfy an instantaneous smoothing effect from $\mathcal{P}(\mathbb{R})$ to $L^\infty(\mathbb{R})$. Finally, we provide a deterministic particle approximation of (1) in terms of solutions to a follow-the-leader type ODE system.

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A 2×2 system of conservation laws with discontinuous flux and traffic applications

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We consider a Follow-the-Leader-type microscopic system and prove the rigorous micro-macro convergence in the many particle limit in presence of vacuum to weak solutions of a second order system that includes the hydrodynamic traffic flow model introduced in [3] with space discontinuous flux, that here we call CGST model, and the classical ARZ model in [1]-[4] . The general system reads

$$\begin{cases} \partial_t \rho + \partial_x \left(c\,V(h)\,\rho\right) = 0, \\ \partial_t \left(\rho\,(h + p(\rho))\right) + \partial_x \left(c\,V(h)\,\rho\,(h + p(\rho))\right) = 0. \end{cases}$$

It finds its relevance in the modeling of vehicular traffic. In this context, $t \geq 0$ denotes the time, $x \in \mathbb{R}$ the space, $\rho = \rho(t, x) \geq 0$ the density, $p = p(\rho) \geq 0$ is the pressure function, $c = c(x) > 0$ is a discontinuous function that mimics the road capacity, and $V \geq 0$ represents the speed law. The interpretation of $h = h(t, x) \geq 0$ depends on the specific model under consideration. For instance, the function h represents the mean headway in the model proposed in [3] and the velocity in the ARZ model, see [1]-[4]. We complement our result with numerical simulations of the particle method compared with some macroscopic approximate solutions obtained with the Lax-Friedrichs scheme.

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Unique Solutions to Hyperbolic Conservation Laws with a Strictly Convex Entropy

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Consider a strictly hyperbolic $n \times n$ system of conservation laws, where each characteristic field is either genuinely nonlinear or linearly degenerate. In this standard setting, it is well known that there exists a Lipschitz semigroup of weak solutions, defined on a domain of functions with small total variation. If the system admits a strictly convex entropy, we give a short proof that every entropy weak solution taking values within the domain of the semigroup coincides with a semigroup trajectory [2]. The result shows that the assumptions of "Tame Variation" or "Tame Oscillation" (see [1] and references therein), previously used to achieve uniqueness, can be removed in the presence of a strictly convex entropy.

Combined with a compactness argument, the result yields a uniform convergence rate, $\rho(\varepsilon) \to 0$ as $\varepsilon \to 0$, for a very wide class of approximation algorithms. Some partial estimates on $\rho(\varepsilon)$ are given.

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A strongly coupled PDE-ODE model in the presence of discontinuity in the flux

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In this work, we introduce a system of PDE-ODEs with discontinuity in the flux function with application to traffic flow:

$$\begin{cases} \partial_t \rho + \partial_x [f(\gamma, \rho)] = 0 & , x \in \mathbb{R},\, t \in \mathbb{R}_+ \\ \rho(0, x) = \rho_\circ(x) \\ f(\gamma, \rho) \;\surd\; \dot{y}\rho \leq F(y) \\ \dot{y} = w(y, \rho) \end{cases} .$$

where ρ denotes the traffic density, f is the flux function and γ captures a variable speed limit. Strongly coupled PDE-ODE, models have been exploited in various works to present the effect of controllers in regulating the behavior of traffic flow, among other applications. In such settings, the mass behavior of traffic is modeled through PDEs and the dynamics of controllers are modeled by a system of ODEs. The interaction of ODEs with other fronts can create a so called non-classical shock that introduces some complexities in proving the well-posedness of the problem. The discontinuity in the flux of conservation laws can arise from application point of view; e.g., changes in maximum speed in traffic flow applications. Such discontinuities cause an oscillatory behavior in the system and consequently lack of uniform bounded variation of the solutions which will restrict using the conventional tools to establish the compactness theorem.

We use the wavefront tracking method to show the existence of solution. In doing so, first we define a Riemann solution that explains both the non-classical shock and the jump in the flux. As the system lacks bounded variation, we prove the convergence of the approximate solutions using a homeomorphism. We will investigate different cases that can arise as the result of interaction of the controllers' trajectory with other waves under this setting and will discuss the potential complexities. Eventually, using the properties of the solutions in the homeomorphic space, we will show the compactness theorem and subsequently the existence of the solution.

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Scalar Approach to ARZ-Type Systems of Conservation Laws

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We are interested in 2×2 systems of conservation laws of special structure, including generalized Aw-Rascle and Zhang (GARZ) models for road traffic. The simplest representative is the Keytz-Kranzer system, where one equation is nonlinear and not coupled to the other, and the second equation is a linear transport which coefficients depend on the solution of the first equation.

In GARZ systems, the coupling is stronger, they do not have the "triangular" structure of Keytz-Kranzer. The systems we consider take the form

$$\begin{cases} \partial_t \rho + \partial_x \left(f(w,\rho) \right) = 0 \\ \partial_t \left(w\rho \right) + \partial_x \left(w f(w,\rho) \right) = 0. \end{cases}$$

We claim that it makes sense to address these systems *via* a kind of splitting approach. Indeed, in 2008, Panov proposed a robust framework for solving linear transport equations with divergence free coefficients. Our idea is to use this framework for the second equation of GARZ systems, and to exploit the scalar discontinuous flux framework for the first equation of the system.

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Hybrid and multiscale models for vehicular traffic

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Classical models for vehicular traffic include the celebrated Lighthill-Whitham-Richards one:

$$\rho_t + f(\rho)_x = 0 \tag{1}$$

where $\rho \in [0, \rho_{max}]$ is the car density, the flux verifies $f(\rho) = \rho\, v(\rho)$, and $v(\rho)$ is the average velocity. These models proved to be successful in traffic modeling and monitoring. New technologies advances allow the development of new control mechanism, in particular via autonomous vehicles. To model this Delle Monache and Goatin proposed the following PDE=ODE coupled system:

$$\begin{cases} \rho_t + f(\rho)_x = 0 \\ \dot{y}(t) = \min\{\omega(t), v(\rho(t, y(t)+))\} \\ f(\rho(t, y(t))) \surd \dot{y}(t)\rho(t, y(t)) \leq F_\alpha(\dot{y}(t)) \end{cases} \tag{2}$$

where $f(\rho) = \rho\, v(\rho)$, the control $\omega \in [0, U]$ indicates a speed chosen by the bottleneck, say an autonomous vehicle or a vehicle with autonomous cruise control, $v(\rho(t, y(t)+)) = \lim_{h \quad 0, h \geq 0} v(\rho(t, y(t)+h))$, and F_α is given by:

$$F_\alpha(\dot{y}(t)) := \max_\rho (\alpha f(\rho/\alpha) \surd \rho\dot{y}(t)), \qquad \alpha \in]0, 1[. \tag{3}$$

In this talk we report various results on the well-posedness of such system, and its extension to include a two equation PDE traffic model and a multilane setting.

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Computability Theory
Special Session A14

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Computability is a fundamental notion of mathematics. Interest in effectiveness is already apparent in the famous Hilbert problems, in particular, the second and tenth, and in early 20th century work of Dehn, initiating the study of word problems in group theory. The formal definition of computability was given by Turing, Gödel, and others in the 1930s. Problems are classified according to various logical hierarchies, giving precise complexity measurements, which closely relate computational and definitional complexity.

Since its inception, and perhaps especially so in the past 30–40 years, computability theory has seen many fascinating and dramatic developments, growing to encompass many new subfields. These include:

- Algorithmic randomness: The study of randomness for individual objects such as reals.
- Computable model theory: The study of computational aspects of mathematical structures.
- Reverse mathematics: The search for optimal axioms to prove mathematical theorems.
- Reducibility and degree structures: The study of the Turing degrees, and more generally of various notions of comparison of computability-theoretic strength.

Likewise, the subject has found links to other mathematical disciplines inside and outside of logic. This includes, for instance, work on enumeration degrees that has revealed deep and surprising relations to general topology, and work on algorithmic randomness that is closely tied to symbolic dynamics and geometric measure theory. Inside logic there are relations to model theory, set theory, effective descriptive set theory, and proof theory.

In some of these cases the bridges to seemingly distant mathematical fields have yielded completely new proofs or even solutions of open problems in the respective fields. In others, previously disparate areas have found common tools and questions, resulting in what are now essentially merged fields. An example of the latter is reverse mathematics and computable analysis, which have become deeply intertwined through new developments over the past decade.

The special session will cover the majority of areas of modern computability theory, including all those mentioned above.

For more information visit `www.computability.org/umi-ams`.

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Minimal covers in the Weihrauch degrees

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Weihrauch reducibility is a notion of reducibility that calibrates the uniform computational strength of computational problems. Despite its growing popularity, the structure of Weihrauch degrees is vastly unexplored, as most of the efforts up to this date have concentrated on the classification of the Weihrauch degrees of specific problems.

In this talk, after a short introduction to the topic we will explore some recent developments on the structure of Weihrauch degrees. Recall that, given a partial order $(P, \leq)$, we say b is a minimal cover of a if $a < b$ and there is no c such that $a < c < b$. In other words, b is a minimal cover of a if the interval (a, b) is empty. We say that b is a strong minimal cover of a if $c < b$ implies $c \leq a$.

We present a complete characterization of minimal covers and strong minimal covers in the Weihrauch degrees. Let $\{p\}^+ = \{\langle e, q\rangle : \Phi_e(p) = q \text{ and } q \not\leq_{\mathrm{T}} p\}$.

Theorem 1. *Let f and h be partial multi-valued functions on Baire space. The following are equivalent:*

(1) *f is a minimal cover of h in the Weihrauch degrees.*
(2) $f \equiv_{\mathrm{W}} h \sqcup \mathrm{id}_{\{p\}}$ *for some p with* $\mathrm{dom}(h) \not\leq_{\mathrm{M}} \{p\}$ *and* $\mathrm{dom}(h) \leq_{\mathrm{M}} \{p\}^+$.

Theorem 2. *Let f and h be partial multi-valued functions on Baire space. The following are equivalent:*

(1) *f is a strong minimal cover of h in the Weihrauch degrees.*
(2) *There is $p \in \mathbb{N}^{\mathbb{N}}$ such that $f \equiv_{\mathrm{W}} \mathrm{id}_{\{p\}}$ and $h \equiv_{\mathrm{W}} \mathrm{id}_{\{p\}^+}$.*

The previous two results have interesting consequences: they imply that the degree of id is first-order definable in $(\mathcal{W}, \leq_{\mathrm{W}})$ and that the first-order theory of the Weihrauch degrees (below id) is recursively isomorphic to the third-order theory of true arithmetic.

This is joint work with Steffen Lempp, Joe Miller, Arno Pauly, and Mariya Soskova.

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Computability theory and existentially closed groups

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Existentially closed groups were introduced in 1951 in analogue with algebraically closed fields. Since then, they have been further studied by Neumann, Macintyre, and Ziegler, who elucidated deep connections with model theory and computability theory. We review some of the literature on existentially closed groups and present new results that further refine these connections.

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Characterizing learnability for families of structures

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In this talk, we study learning of countable families of countable relational structures. The framework we use combines ideas from computable structure theory and inductive inference, and it was defined in a series of papers by Bazhenov, Fokina, Kötzing, and San Mauro ([2],[3]). The framework models the scenario in which, given a family of structures $\mathfrak{K}$, a *learner* receives more and more information about the atomic diagram of a copy of some $\mathcal{A} \in \mathfrak{K}$ and, at each stage, is required to output a conjecture about the isomorphism type of such a structure.

A natural criterion to consider is **Ex**-*learning* in which we require the learner to stabilize to the correct conjecture in finitely many steps. Together with Bazhenov and San Mauro in [1] we gave a descriptive set-theoretic characterization of **Ex**-learning. Namely, we showed that a family of structures is **Ex**-learnable if and only if the corresponding isomorphism problem continuously reduces to E_0, the equivalence relation of eventual agreement on $2^{\mathbb{N}}$. Replacing E_0 with other equivalence relations, one obtains a hierarchy to rank such isomorphism problems. That is, a family of structures $\mathfrak{K}$ is *E-learnable* for an equivalence relation E if there is a continuous reduction from the isomorphism problem associated with $\mathfrak{K}$ to E.

To get a better understanding of which families of structures are E-learnable it is useful to provide a model-theoretic characterization of E-learnability. This has been done for some equivalence relations (see e.g., [2, 1]): these characterizations rely on the ordering between the structures in the family with respect to the inclusion of their Σ_2^{inf}-theories. A natural question is to ask for every n which is (if exists) the equivalence relation E^n such that a family of structures $\mathfrak{K}$ is E^n-learnable if and only if the ordering between the structures in $\mathfrak{K}$ with respect to the inclusion of their Σ_n^{inf}-theories is a partial order. It turns out that these equivalence relations exist and they are natural ones: namely, they are the (iterations of the) Friedman-Stanley jump of equality on $\mathbb{N}$ and $2^{\mathbb{N}}$.

We continue showing that other learning criteria coming from the classical setting of inductive inference of formal languages or recursive functions have nice model-theoretic characterization as well.

This talk collects joint works with Bazhenov, Jain, Marcone, San Mauro and Stephan.

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On structures with non-computable presentations

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In computable structure theory, one usually measures the complexity of a structure by identifying the structure with its atomic diagram. The structure is then computable if so is its atomic diagram. The notion naturally relativises to non-computable oracles. However, for structures without computable presentations, a finer way to measure the complexity is sometimes more natural and suitable.

In this talk we discuss different approaches to measure the complexity of non-computable structures and explain several of our recent results.

[1]"This research was funded by the Austrian Science Fund (FWF) P 36781.
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On normality, supernormality, finite state dimension, and point to set principles

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Effective and resource-bounded dimensions were defined by Lutz in [6] and [5] and have proven to be useful and meaningful for quantitative analysis in the contexts of algorithmic randomness, computational complexity and fractal geometry (see the surveys [2,7,3,10] and all the references in them).

The point-to-set principle (PSP) of J. Lutz and N. Lutz [8] fully characterizes Hausdorff and packing dimensions in terms of effective dimensions in the Euclidean space, enabling effective dimensions to be used to answer open questions about fractal geometry, with already an interesting list of geometric measure theory results (see [4,9]).

Finite state dimension [1] is the lowest level effectivization of Hausdorff dimension and is closely related to Borel normality. In this talk I will review its main properties, prove a new characterization in terms of information content approximated at a certain precision, and consider new generalizations of normality. I will then prove a finite-state dimension point to set principle [11].

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[1] Research supported in part by Spanish Ministry of Science and Innovation grant PID2022-138703OB-I00 and by the Science dept. of Aragon Government: Group Reference T64_20R (COSMOS research group)
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Nash-Williams theorem for sequences of finite range in ATR_0

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In [1], Crispin Nash-Williams proved that if $(Q, \leq_Q)$ is a *well quasi-order* (henceforth *wqo*), then so is the set of transfinite sequences over Q with finite range, ordered by embeddability. In this talk, we will show that this result is provable in the subsystem of second-order arithmetic ATR_0: together with previous results by Shore [2], this determines the reverse mathematical strength of Nash-Williams' result.

We will see that, interestingly, the proof relies on the notion of *better quasi-order* (henceforth *bqo*), a strengthening of the concept of wqo again due to Nash-Williams. We will present how the "degree of bqo-ness" of a quasi-order Q relates to the order-theoretic properties of $Q^{<\omega^{1+\alpha}}$, the set of sequences over Q of length less than $\omega^{1+\alpha}$, and to those of $\dot{V}_\alpha(Q)$, which corresponds (roughly) to the α-iterated powerset of Q. These observations will lead to an equivalence between $\dot{V}_\alpha(Q)$ and the *indecomposable* sequences in $Q^{<\omega^{1+\alpha}}$, and, together with a slight modification of clopen Ramsey theorem, will lead to the desired proof in ATR_0.

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From combinatorial theorems to well-ordering principles

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Many well-ordering preservation principles and theorems from Ramsey Theory are known to be equivalent to various systems of Reverse Mathematics. For example Ramsey's Theorem for triples and the well-ordering preservation principle for base-ω exponentiation are both equivalent to Arithmetical Comprehension. The resulting equivalence between the combinatorial theorem and the well-ordering principle is usually obtained indirectly.

We present some examples of direct implications from Ramsey-type and Hindman-type theorems to well-ordering principles. The proofs yield computable reductions. For example we show that the well-ordering principle for the ε-ordering reduces to Ramsey's Theorem for colorings of exactly large sets. This work is joint with Mainardi and Zdanowski.

We furthermore present some preliminary results on generalizations of the Free Set, Thin Set and Rainbow Ramsey Theorem to colorings of exactly large sets (and, more generally, to colorings of barriers) for which we obtain weak anti-basis results. This work is joint with Gjetaj and Vivi.

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Weihrauch degrees of indivisibility

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We call a structure $\mathcal{M}$ over $\mathbb{N}$ *indivisible*, if for every colouring of $\mathbb{N}$ with finitely many colours there is a monochromatic isomorphic copy of $\mathcal{M}$. A typical example of an indivisible structure is $(\mathbb{Q}, <)$. For a fixed indivisible structure $\mathcal{M}$ we can then study the computational task $\mathrm{Ind}\mathcal{M}$, which receives as input a k-colouring of $\mathbb{N}$ and which has to output a monochromatic copy of $\mathcal{M}$. This programme was recently formulated by Kenneth Gill [3, 4], who obtained results about the Weihrauch degree [1] of $\mathrm{Ind}\mathbb{Q}$ and some other structures. In a largely independent development, Dzhafarov, Solomon and Valenti [2] studied the Weihrauch degree of the tree pigeon hole principle TT^1_+, which can be seen as indivisibility of the full binary tree with relations for "is in the left subtree below" and "is in the right subtree below". It is easy to see that $\mathrm{Ind}\mathbb{Q} \equiv_{\mathrm{W}} \mathrm{TT}^1_+$.

Our contribution here is to solve some of the questions left open in these two projects. In particular, we prove that:

Theorem 1. $\mathrm{C}_{k+1} \not\leq_{\mathrm{W}} \mathrm{TT}^1_k$ *for all* $k \geq 1$.

Here C_{k+1} is *finite closed choice* with $k+1$ elements, the problem receives as input an enumeration of some (maybe none) but not all elements of $\{0, 1, \ldots, k \quad 1\}$, and the valid outputs are the elements not appearing in the input. With TT^1_k we denote the restriction of the tree pigeon hole principle to k colours. Overall the gist of the resulting picture is that the opportunities to code more discrete information into an instance of TT^1_+ (or equivalently $\mathrm{Ind}\mathbb{Q}$) beyond which colours can appear in solutions are very limited (but exist).

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What can be uniformly computed from descending sequences?

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How hard is it to find an infinite descending sequence in a countable ill-founded linear order? What about an infinite bad sequence in a countable non-well-quasi-order? We investigate these questions in the computability-theoretic framework of Weihrauch (uniform) reducibility, where the above problems are denoted DS and BS respectively. First, we will show that DS cannot compute an infinite path in a given finitely branching tree, resolving the primary open question in our previous paper [1]. Then we will show that DS cannot compute even a single element which is extendible (to an infinite bad sequence) in a given non-well-quasi-order, refuting our false claim in [1]. This talk is based on ongoing joint work with Arno Pauly and Manlio Valenti [2].

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Computability of the Whitney Extension Theorem

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A very relevant topic in computable analysis which, not surprisingly, has marked the development of the subject is the *effective* reformulation of classical mathematical theorems of the form

$$(\forall x \in X)(\exists y \in Y)A(x,y) \quad (*)$$

for X and Y suitable topological spaces. This exactly corresponds to showing that a corresponding multi-valued function, assigning to any $x \in X$ a $y \in Y$ such that $A(x,y)$, is *computable.*

A strictly related interesting aspect, also from a philosophical perspective, is to evaluate to what extent the classical proofs of those classical results have an algorithmic nature. Some of such proofs present indeed an intuitive computational flavour, but, so to say, some computable steps are hidden under the surface. Hence, in such cases, the proofs of computability of the corresponding multi-valued functions can retrace classical proofs by showing in a rigorous way the computational content that was only sketched in them. Nevertheless, removing the rind to get to the computational pith might contain non-trivial steps, and it depends first of all on the correct choice of suitable translations of the classical concepts into computational notions.

An interesting example is given by the well-known Urysohn-Tietze Extension Theorem. A computable version of this theorem was proved by Klaus Weihrauch in [4]. In order to prove this result, Weihrauch provided in fact an *effectivization* of the classical proof contained in [1].

The Urysohn-Tietze Theorem classically finds a generalization in the Whitney Extension Theorem. For the real case, this theorem states that for any given (non-empty) closed set $F \subseteq \mathbb{R}^n$ and a *jet* of order m of functions on F, there exists a total continuous function g in $C_m(\mathbb{R}^n)$ such that both g as well as its partial derivatives coincide on F with the corresponding partial functions of the jet. Here a jet is a finite sequence of continuous functions defined on F satisfying Taylor's condition, and which behave like partial derivatives of each other. A classical proof of this statement is contained in [3] and, as a preliminary result, Stein proves an extension theorem for the limit case in which the jet consists only of a single continuous function, providing then another proof for the Urysohn-Tietze Extension Theorem.

In this talk I will check the computability of the construction of Whitney's extensions through an effectivization of the proof given by Stein. A preliminary investigation of its computational content brought already in [2] to the systematic classification of different formulations of the projection point problem onto closed subsets of $\mathbb{R}^n$, and it turned out that only the problem of finding approximated projection points of an $x \in \mathbb{R}^n$ onto a closed $F \subseteq \mathbb{R}^n$ up to a given error bound ε is computable with respect to full information for closed sets (which consists of an exact open covering of the complement of the set and of a dense subset of the set itself). In fact, the effectivization of Stein's proof requires the use of approximations in several different aspects and in a way that implies a non-trivial departure from its original formulation.

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Computable Π_2 Scott sentences

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Scott [2] showed that for any countable structure $\mathcal{A}$, there is an $L_{\omega_1\omega}$-sentence φ whose countable models are exactly the isomorphic copies of $\mathcal{A}$. We call φ a *Scott sentence* for $\mathcal{A}$. Montalbán [1] showed that for a countable ordinal $\alpha \geq 1$, $\mathcal{A}$ has a $\Pi_{\alpha+1}$ Scott sentence iff the orbits of all tuples are defined by Σ_α formulas. In particular, for $\alpha = 1$, $\mathcal{A}$ has a Π_2 Scott sentence iff the orbits of all tuples are defined by existential formulas. In $L_{\omega_1\omega}$-formulas, there are countable disjunctions and conjunctions, but only finite strings of quantifiers. *Computable* infinitary formulas are formulas of $L_{\omega_1\omega}$ in which the infinite disjunctions and conjunctions are over c.e. sets. These formulas, while infinite, seem comprehensible.

Some structures, such as the ordered group of rationals and the standard model of arithmetic, have a *computable* Π_2 Scott sentence. Just from the definition, we get necessary and sufficient conditions for this. We give examples illustrating why certain other conditions, in particular, on families of formulas that define the orbits, are either not sufficient, or not necessary. For a special class of structures, in a language with just unary relation symbols, we show that $\mathcal{A}$ has a computable Π_2 Scott sentence iff there is a computable Π_2 formula $\beta(x)$ saying, in all models of the $\forall\exists$-theory of $\mathcal{A}$, that the type of x is isolated.

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A new way of classifying word problems

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The study of word problems dates back to the work of Dehn in 1911. Given a recursively presented algebra A, the word problem of A is to decide if two words in the generators of A refer to the same element. Much is known about the complexity of word problems for familiar algebraic structures: e.g., the Novikov-Boone theorem, one of the most spectacular applications of computability to general mathematics, states that the word problem for finitely presented groups is unsolvable. Yet, the computability theoretic tools commonly employed to measure the complexity of word problems (e.g., Turing or m-reducibility) are defined for sets, while it is generally acknowledged that many computational facets of word problems emerge only if one interprets them as equivalence relations.

In this work, we revisit the world of word problems, with a special focus on groups and semigroups, through the lens of the theory of equivalence relations, which has grown immensely in recent decades. To do so, we employ computable reducibility, a natural effectivization of Borel reducibility.

This talk collects joint works with Uri Andrews, Valentino Delle Rose, Meng-Che Ho, and Andrea Sorbi.

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Degrees and applications of e-pointed trees

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Enumeration pointed trees (e-pointed trees) arise in work by Montalbán [1] in computable structure theory and the study of possible degree spectra of structures. They have since also proved useful in the study of the hyperenumeration degrees.

McCarthy [2] studied e-pointed trees on Cantor space, and showed that the enumeration degrees of e-pointed trees in Cantor space are exactly cototal degrees. He further showed that this characterization remains unchanged if we introduce uniformity or allow for dead ends.

We prove that the enumeration degrees of e-pointed trees in Baire space behave differently. Allowing dead ends gives rise to a very large class of degrees, characterized in terms of hyperenumeration reducibility and containing all Π^1_1 degrees. On the other hand, the class of Baire e-pointed trees without dead ends does not even contain all arithmetic degrees. Nevertheless, it is strictly larger than the class of all cototal degrees. Sandwiched properly in between the cototal degrees and the degrees of e-pointed trees without dead ends are the introenumerable enumeration degrees.

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Palermo, July 23–26, 2024

Generic and coarse computability of Abelian p-groups

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Motivated by asymptotic density problems in combinatorial group theory, and by approximately computable sets introduced and studied by Jockusch and Schupp in computability theory, we formulated the notions of densely computable structures such as generically and coarsely computable structures. We first studied generically and coarsely computable equivalence structures and injection structures, and generically and coarsely computable isomorphisms of these structures. Returning to groups, we now investigate generically computable Abelian p-groups, as well as other variants of approximately computable Abelian p-groups. There is Khisamiev's characterization of reduced Abelian p-groups of certain length with computable isomorphic copies. We define a countable group to be generically computable if it has a computably enumerable subgroup the domain of which is an asymptotically dense set. While every countable Abelian p-group is isomorphic to a generically computable group, this is not the case with Abelian groups in general. We further consider a graded family of elementarity conditions for computably enumerable subgroups that approximate groups. We also investigate coarsely computable Abelian p-groups, which are approximated by computable groups. We show that every countable Abelian p-group is isomorphic to a coarsely computable group.

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On the complexity of some topological properties in highly computable graphs

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The problem of deciding whether a graph has an *Eulerian* path, namely a path visiting each edge of the graph exactly once, has a very long history in mathematics, dating back to the famous problem of *the seven bridges of Könisberg*, solved by Euler in 1736.

Around 200 years later, Erdős, Grünwald and Vászsonyi extended Euler's result by characterizing those graphs which admit an infinite Eulerian path. This characterization strongly relies on the *number of ends* of a graph, namely, the maximum number of infinite connected components which can be obtained by removing a finite number of edges. Indeed, only graphs with one or two ends can have Eulerian paths.

From the point of view of computability theory, their result highlights an important difference between computable graphs and *highly computable* graphs, the latter notion corresponding to computable and locally finite graphs where, in addition, one can uniformly compute the degree of each vertex. In fact, Bean showed that, while there are computable, locally finite graphs which admit an Eulerian path but no computable one, every highly computable graph admitting Eulerian paths must have a computable one.

However, deciding whether a given graph admits an Eulerian path at all is a difficult problem: as Kuske and Lohrey have shown, such problem is Π^0_2-complete even when restricting to connected, highly computable graphs. Interestingly, this turns out to be the same difficulty of simply counting the number of ends in a highly computable graph.

Motivated by these considerations, we have studied how the difficulties of these two problems precisely relate. We have found that counting the ends of the graph indeed represents the hardest task when deciding the existence of an Eulerian path. More precisely, we have shown that:

(1) deciding existence of Eulerian paths is only (2-c.e.)-complete when restricting to highly computable graphs with one end,
(2) the same problem realizes precisely the m-degrees of Δ^0_2 sets in the case of highly computable graphs with two ends.

To get these results we have conducted a detailed analysis, which we believe of independent interest, of the computational hardness of what we call the *separation problem*: to decide whether a finite set of edges separates a connected and highly computable graph into two or more infinite connected components. Two results here are particularly relevant. On the one hand, we show that any function which takes as input a highly computable graph and outputs a finite set of edges separating the graph must compute the halting problem. On the other hand, the separation problem turns out to be (non-uniformly) decidable for highly computable graphs with finitely many ends: in fact, from the number of ends of a graph and a single maximally separating set, we can compute the whole collection of separating sets for this graph.

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Revisiting the reverse mathematics of the Tietze extension theorem: preserving suprema

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The following version of the Tietze extension theorem is well-known to be provable in RCA_0 [1–2]. Let $\mathcal{C}$ be a closed subset of a complete separable metric space $\mathcal{X}$, and let $f\colon \mathcal{C} \to [-1, 1]$ be a continuous function. Then f extends to a continuous $g\colon \mathcal{X} \to [-1, 1]$, where $\forall x \in \mathcal{C}\ g(x) = f(x)$. However, in this formulation g need not preserve the supremum of f. That is, it may be that $\sup_{x\in\mathcal{X}} |g(x)| > \sup_{x\in\mathcal{C}} |f(x)|$. We show that the following version of the supremum-preserving Tietze extension theorem requires Π^1_1-CA_0. Let $\mathcal{C}$ be a closed subset of a complete separable metric space $\mathcal{X}$, and let $f\colon \mathcal{C} \to [-1, 1]$ be a continuous function. Then f extends to a continuous $g\colon \mathcal{X} \to [-1, 1]$, where $\forall x \in \mathcal{C}\ g(x) = f(x)$ and additionally $\forall x \in \mathcal{X}\ \exists y \in \mathcal{C}\ |g(x)| \leq |f(y)|$. This is an unusual example of a statement about continuous functions requiring Π^1_1-CA_0.

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Mathematics for Quantum and Statistical Physics
Special Session A15

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University of Milan, ITALY

This session is devoted to recent advances in the mathematical understanding of phenomena of interest in Statistical and Quantum Physics, and related developments in modern Probability Theory and Analysis. Topics will include Universality in Statistical Mechanics and Condensed Matter, Symmetry Breaking effects, Localization in Quantum Systems, Non-Equilibrium Phenomena, construction of Quantum Field Theory models and Interaction Effects of Quantum Particles.

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The Nonlinear Schroedinger equation with bounded initial data

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Abstract: We discuss the behavior of the NLS on R or the discrete NLS on Z with bounded initial data. Examples include quasi-periodic and random data. On Z, polynomial bounds in time are proved for all bounded initial data. In the continuum, local existence is established for smooth data. We also discuss the long time behavior of a smoothly driven anharmonic oscillator. This is joint work with B. Dodson and A. Soffer.

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Ground state properties of interacting bosons

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The interacting Bose gas is a system in quantum statistical mechanics where complex collective behavior emerges from the underlying many-body theory, posing crucial challenges to its rigorous mathematical description.

In recent years, progress has been made in understanding its ground state properties; in certain scaling limits it has been possible to prove the occurrence of the Bose-Einstain condensation phase transition [2,4] and to obtain expressions for the excitation spectrum [1], justifying the linear dispersion relation predicted by Bogoliubov. For the description of disordered materials it is crucial to understand whether similar properties are stable in presence of randomly placed impurities. We show that for a suitably rescaled interactions and in presence of Poisson distributed impurities (Kac–Luttinger model), Bose-Einstein condensation occurs into the minimizer of a Hartree-type functional [3].

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Non-perturbative behavior of interacting Bosons at intermediate densities

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Much attention has been given to systems of interacting Bosons in the dilute regime, where powerful theoretical tools such as Bogolyubov theory give detailed and accurate predictions. In this talk, I will discuss a different approach to studying the ground state of Boson systems, which Carlen, Lieb and I have recently found to be accurate at all densities. In particular, it allows us to probe the system in the intermediate density regime, which had, until now, only been accessible to costly Monte-Carlo simulations. In this talk, I will first give an overview of this Simplified approach, and will discuss evidence for non-perturbative behavior in the intermediate density regime obtained using this tool.

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Foundations of kinetic theory: recent progress and open directions

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Foundations of kinetic theory: recent progress and open directions.

We consider deterministic, time-reversible dynamics with random initial data, in a low-density scaling. Under suitable assumptions on the initial measure, a strong chaos property is propagated in time, which also encodes the transition to irreversibility. This result is complemented by large deviation estimates and by a theory of small fluctuations, allowing to establish the connection between microscopic and hydrodynamic scales, for perturbations of a global equilibrium. Many of the open problems left require a deeper understanding of the coupling mechanisms between deterministic and stochastic dynamics.

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Decay of Information for the Kac Evolution

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We consider a system of M particles in contact with a heat reservoir of $N \gg M$ particles. The evolution in the system and the reservoir, together with their interaction, are modeled via the Kac's Master Equation. We chose the initial distribution with total energy $N+M$ and show that if the reservoir is initially in equilibrium then the entropy of the system decays exponentially to a very small value. We base our proof on a similar property for the Information.

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Current fluctuations for the boundary driven zero-range process: microscopic versus macroscopic approach and a theory of non-reversible resistor-like networks

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We compute the joint large deviation rate functional in the limit of large time for the current flowing through the edges of a finite graph where a boundary driven system of stochastic particles is evolving with zero-range dynamics. This generalizes one dimensional results previously obtained with different approaches [5, 3]. We use new techniques that illuminate various connections and complementary perspectives. In particular, we use a variational approach to derive the rate functional by contraction from a level 2.5 large deviation rate functional. We perform an exact minimization and obtain finally as a rate functional a variational problem involving a superposition of cost functions for each edge. The contributions from different edges are not independend since are related by the values of a potential function on the nodes of the graph. The rate functional on the graph is a microscopic version of the continuous rate functional predicted by the macroscopic fluctuation theory [4] and we show indeed a convergence in the scaling limit. If we split the graph into two connected regions by a cutset and are interested just on the current flowing through the cutset, we show that the result is the same of an effective system composed by just one effective edge. This is what happens at macroscopic level and it is expected also for other model [1]. The characteristics of this effective edge are related to the capacities of the graph and can be obtained by a reduction of the graph by elementary transformations like in electric networks (effective components in parallel, in series and N-star to effective complete N graphs). Our reduction procedure is directly related to the reduction of the so called *trace process* and, since the dynamics is in general not reversible, it is closely related to the theory of non reversible electric networks in [2].

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Spectral properties of some quantum Kac models and entropy production bounds

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We compute the spectrum for a class of quantum Markov semigroups describing systems of N particles interacting through a binary collision mechanism. These quantum Markov semgroups are associated to a novel kind of quantum random walk on graphs, with the graph structure arising naturally in the quantization of the classical Kac model, and we show that the spectrum of the generator of the quantum Markov semigroup is closely related to the spectrum of the Laplacian on the corresponding graph. For the direct analog of the original classical Kac model, we determine the exact spectral gap for the quantum generator. We also give a new and simple method for studying the spectrum of certain graph Laplacians. We also discuss entropy production for these Master equations.

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Time-reversal symmetry and response in an external magnetic field

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Given a Hamiltonian system, we show that there are infinitely many time reversal symmetries that lead obtain to statistical mechanical results, such as the Onsager Reciprocal Relations, or the fluctuation relations, even in presence of magnetic fields or rotating refefrence frames. Consequently, the Casimir modification of such relations is not necessary. We illustrate both the classical and the non-relativistic quantum mechanical theories. In particular, we prove that the spin-field interaction does not break the time reversal invariance of the dynamics, and that it does not require additional conditions for such a symmetry to hold, compared to the spinless cases. These results are relevant for experiments such as diffusion in solutions, thermoelectricity and spin charge transport.

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Weakly and strongly coupled maps: a self-consistent transfer operator approach

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To investigate the statistical properties of dynamical systems which exhibit chaotic behaviour, researchers have developed a well-established method stemming from Ruelle's work in 1970. This method focuses on analyzing the evolution of measures (densities) rather than individual trajectories. The evolution of density is described by a linear operator known as the transfer operator, which offers significant advantages. However, many natural phenomena, particularly those involving interacting dynamical systems coupled by mean-field forces, cannot be adequately described by linear operators alone. In such cases, the evolution in the thermodynamic limit is characterized by a transfer operator that incorporates the coupling effect, known as the self-consistent transfer operator (see e.g. [1]). Building on a joint work with Stefano Galatolo (University of Pisa) and Matteo Tanzi (King's college, London), in this presentation I will discuss recent and new techniques based on self-consistent transfer operators that provide valuable statistical insights, including convergence to equilibrium, especially in scenarios involving weak or moderate coupling.

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Synchronization and averaging in a simple dynamical systems with fast/slow variables

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We study a family of dynamical systems obtained by coupling a chaotic (Anosov) map on the two-dimensional torus – the chaotic variable – with the identity map on the one-dimensional torus – the neutral variable – through a dissipative interaction. We show that the two systems synchronize, in the sense that the trajectories evolve toward an attracting invariant manifold, and that the full dynamics is conjugated to its linearization around the invariant manifold. When the interaction is small, the evolution of the neutral variable is very close to the identity and hence the neutral variable appears as a slow variable with respect to the fast chaotic variable: we show that, seen on a suitably long time scale, the slow variable effectively follows the solution of a deterministic differential equation obtained by averaging over the fast variable.

Systems with fast/slow variables of the kind studied here appear in many physical problem and have been proposed as a first (rather small) step toward the construction of a kinetic limit based understanding of Fourier's Law.

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Disordered systems beyond the permutation invariance

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In this talk I will review some results obtained for the mean-field spin-glass models when the disorder is not permutation invariant. Cases include the convex and non-convex multispecies, with emphasis on the deep Boltzmann machines in the Nishimori line, and the multy-bath SK model.

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A Parisi Formula for Quantum Spin Glasses

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We establish three equivalent versions of a Parisi formula for the free energy of mean-field spin glasses in a transversal magnetic field. These results are derived from available results for classical vector spin glasses by an approximation method using the functional integral representation of the partition function. In this approach, the order parameter is a non-decreasing function with values in the non-negative, real hermitian Hilbert-Schmidt operators. For the quantum Sherrington-Kirkpatrick model, we also show that under the assumption of self-averaging of the self-overlap, the optimising Parisi order parameter is found within a two-dimensional subspace spanned by the self-overlap and the fully stationary overlap.

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Response functions of many-body condensed matter systems

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In this talk I will discuss rigorous results about charge transport in gapless interacting fermionic lattice models. I will outline a strategy that has been used in the last years to compute the response functions of a wide class of quantum systems, including one-dimensional metals and two and three-dimensional semimetals. The approach is based on: analytic continuation of real-time response functions to imaginary times; renormalization group analysis of imaginary-time correlations and resolution of the scaling limit; lattice conservation laws and Ward identities to prove universality. I will focus on the analysis of the edge response function of interacting 2d quantum Hall systems, and on the emergence of the multichannel Luttinger liquid at the macroscopic scale. In the last part of the talk, I will report about recent progress for non-translation invariant systems; in particular, I will discuss the edge transport properties of Haldane-like models in the presence of weak quasi-periodic disorder.

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Approach to equilibrium for an oscillator interacting with a harmonic thermal bath

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Federico Bonetto
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We want to understand whether a chain of harmonic oscillators can act approximately as a heat bath over a smaller system, put in contact with it, when the number of degrees of freedom of the chain is sufficiently large. As a simplified model, we consider the Hamiltonian dynamics of a single harmonic oscillator (we call it the *probe*) interacting with the chain (we call it the *bath*) through a linear coupling, and we study the dynamics of the global system over long times. The chain is made by identical particles with nearest neighbour interactions and an on–site pinning potential, while the first particle of the chain is linked with the probe by a harmonic spring.

We consider initial data that are canonically distributed, both for the probe and for the bath, but at different temperatures, and show that when the coupling is turned on, the probability distribution for the coordinates of the probe displays an approach to equilibrium. Indeed, we are able to control the decay of the correlations of the position and the momentum of the probe, and to prove that its average energy approaches thermal equilibrium with the bath. These results are valid only up to a finite time, which grows to infinity as the number of particles of the the bath tends to infinity.

We discuss the dependence of the estimates on the parameter tuning the interaction between bath and probe and how this result compares with the previous ones on the dynamics of coupled oscillators.

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The gapped phases of $O(n)$ quantum spin chains

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The ground state phase diagram of the $O(n)$ quantum spin chains with nearest neighbor interactions, for $n \geq 3$ or larger, shows two gapped phases separated by a critical point often referred to as the Reshetikhin point. One of the phases contains the $SU(n)$ invariant $P^{(0)}$ model which has been analyzed using the Temperley-Lieb algebra and, more recently, by a random loop model. These works show the ground state to be dimerized. The other phase contains a special point with exact MPS ground states that generalize the AKLT state (corresponding to the case $n = 3$). For even n, that point too is a phase with breaking of the translation invariance down to period 2. We show that it is not dimerized in the usual sense of the term and uncover other interesting new properties.

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Advances in Geometric Control Theory and Subelliptic PDEs
Special Session A16

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In the last decade, attention to sub-Riemannian geometries has rapidly spread in several directions. These are concerned with the geometrical aspects of control problems and the analytical properties of PDEs defined on such anisotropic structures. These problems arise from different natural models, such as the geometric theory of several complex variables, curvature problems, diffusion processes, human vision, and multi-agent dynamics. All of these models have in common the fact that their ellipticity directions span subspaces of dimension strictly less than the dimension of the state space, and all the remaining directions are recovered from commutators. This implies that the underlying geometric structure of the state space is of an anisotropic type, which plays a crucial role in the controllability properties of the dynamical system, as well as in the analysis and regularity of solutions to PDEs.

This special session aims to gather researchers interested in the study of geometric optimal control theory, linear/nonlinear elliptic and parabolic PDEs, and their connection represented by the sub-Riemannian setting. During the conference, we will primarily focus on the following themes: Hamilton-Jacobi equations and viscosity solutions tailored to the new geometric framework, qualitative and quantitative aspects of solutions to subelliptic PDEs such as Liouville properties, Harnack-type inequalities, and maximum principles driven by possibly degenerate elliptic and parabolic, linear and nonlinear, subelliptic operators.

We plan to divide the talks in this session into plenary communications, held by senior researchers aiming to present not only known results, but also open problems to inspire discussions and collaborations, and general communications aiming at spreading the most recent results.

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A geometric control method for bilinear quantum systems

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We consider a bilinear control systems of the type

$$\dot{x}(t) = (A + u(t)B)x(t)$$

where the state x belongs to some complex infinite dimensional Hilbert space X, the (possibly unbounded) linear operators A and B are skew-adjoint and the control u is a real valued function. Such systems arise, for instance, in quantum control with the bilinear Schrödinger equation. Our goal is to describe the *reachable set*, the set of points attainable in finite time from an initial datum using controls in a certain class. We say that the system is *controllable* if the reachable set is the whole space X.

In [1] the authors showed that if the operator B is bounded the rechable set with L^p controls, $p > 1$, is contained in a countable union of compact subsets of X. Hence, when X is infinite dimensional, this result represents an obstruction to exact controllability. We present a sufficient condition on the system for approximate controllability with piecewise constant controls [3]. On the other hand we extend, under suitable hypotheses on the commutator of the operators A and B, the definition of solution for controls in the set of Radon measures, obtaining precise *a priori* energy estimates on the solution [2] and, as a consequence, upper estimates on the reachable sets. As a particular case we have an extension to L^1 of the noncontrollability result of [1].

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On the Harnack inequality for degenerate Kolmogorov operators with non-smooth coefficients

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After recalling the classical mean value formulas for uniformly parabolic operators with smooth coefficients, we deal with strongly degenerate Kolmgorov-type operators with less regular coefficients. More precisely, following [2], we combine the Mean Value formulas established in [1] with a "descent method" due to Kuptsov to obtain formulas with improved kernels. We then exploit the mean value formulas to prove an invariant Harnack inequality and a strong maximum principle for classical solutions to a large class of subelliptic PDEs. Our analysis is carried out in the non-Euclidean framework of the associated Lie group. We point out that the proofs of our main results only rely on the classical theory developed for harmonic functions.

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Strichartz estimates in the Heisenberg group

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In this talk I will discuss Strichartz estimates on the Heisenberg group for the linear Schrödinger and wave equations involving the sublaplacian. The Schrödinger equation on the Heisenberg group is an example of a totally non-dispersive evolution equation: for this reason the classical approach that permits to obtain Strichartz estimates from dispersive estimates is not available.

Our approach, inspired by the Fourier transform restriction method initiated by Tomas and Stein, is based on Fourier restriction theorems, using the non-commutative Fourier transform on the Heisenberg group. It enables us to obtain also an anisotropic Strichartz estimate for the wave equation, for a larger range of indices than was previously known.

If time permits, I will present results on the Engel group and more general Carnot groups.

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Second order mean field games on homogeneous Lie groups

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The model of the mean field games (MFG) describes interactions among a very large number of identical agents. Evolutive second order MFGs occur when the time horizon is finite and the dynamics of the agents is stochastic. They are modelized by a system of two coupled parabolic equations: a backward in time Hamilton-Jacobi equation and a forward in time Fokker-Plank equation describing respectively the optimal cost of a generic agent and the distribution of the whole population.

We study existence of classical solutions to mean field games systems defined on homogenous Lie group. We consider an homogeneous Lie group, endowed with a family of dilations, which can be identified with $\mathbb{R}^d$. Let $\{X_1, \ldots, X_m\}$ (with $m < d$) be a family of vector fields which satisfies the Hörmander condition; in particular, together with their commutators, these vector fields generate the Lie group. We consider second order mean field games systems where the differential operators are given in terms of these vector fields and where the couplings are strongly regularizing. In our model, each agent can move only along the directions generated by $X_1, \ldots, X_m$ but it can still reach every point due to the Hörmander condition.

In order to obtain the existence of solutions to these mean field games, we first study existence and uniqueness of the subelliptic Fokker-Planck equation and separately of the subelliptic Hamilton-Jacobi equation.

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Mean value formulas for surfaces in Grushin spaces

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Department of Mathematics, University of Padova
Alessandro Socionovo
LJLL, Sorbonne Université

In this talk, we consider n-dimensional Grushin spaces, where a Riemannian metric degenerates along a line in the space, resulting in a sub-Riemannian structure. We discuss the validity of (sub-)mean value property for (sub-)harmonic functions on hypersurfaces within Grushin spaces of dimension $n > 2$. Our interest is driven by the classical counterpart: mean value formulas for harmonic functions on surfaces in the Euclidean setting are crucial for establishing the Bombieri-De Giorgi-Miranda gradient bound, which, in turn, plays a central role in the classical regularity theory. We conclude by presenting remarks and open questions about the regularity theory of minimal surfaces within this sub-Riemannian framework, which is yet to be established.

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Rectifiability in Carnot groups

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Intrinsic regular surfaces in Carnot groups play the same role as C^1 surfaces in Euclidean spaces. As in Euclidean spaces, intrinsic regular surfaces can be locally defined in different ways: e.g. as non critical level sets or as continuously intrinsic differentiable graphs. The equivalence of these natural definitions is the problem that we are studying. Precisely our aim is to generalize some results proved by Ambrosio, Serra Cassano, Vittone [1] valid in Heisenberg groups to the more general setting of Carnot groups.

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The asymptotic p-Poisson equation as $p \to \infty$ in Carnot-Carathéodory spaces

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In this talk we will deal with the asymptotic behavior of solutions to the subelliptic p-Poisson equation as $p \to +\infty$ in Carnot Carathéodory spaces. In particular, introducing a suitable notion of differentiability, we extend the celebrated result of Bhattacharya, DiBenedetto and Manfredi [BDM] and we prove that limits of such solutions solve in the sense of viscosity a hybrid first and second order PDE involving the ∞ $\sqrt{}$Laplacian and the Eikonal equation.

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Local invariants and geometry of the sub-Laplacian on H-type foliations

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Let (M, g) be a smooth, oriented, connected Riemannian manifold equipped with a Riemannian foliation with bundle-like complete metric g and totally geodesic leaves satisfying some additional symmetry conditions. The manifold is studied in the framework of sub-Riemannian geometry with bracket generating distribution transversal to the totally geodesic fibers. Equipping M with the Bott connection we find local invariants by studying the small-time asymptotics of the sub-Riemannanian heat kernel. We obtain the first three terms in the asymptotic expansion of the Popp volume for the pull-back of small sub-Riemannian balls. We address also the question of local isometry of M as a sub-Riemannian manifold and its tangent group.

This is the joint work with W. Bauer, A. Laaroussi (Leibnitz University of Hannover, Germany), S. Vega-Molino (University of Bergen, Norway)

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[1]Aknowledgement. The speaker thanks the Meltzer Foundation, University of Bergen, Norway, for sponsoring the participation in the AMS-UMI Joint International Meeting,

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Regularity theory and geometry of unbalanced optimal transport

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Using the dual formulation only, we show that the regularity of unbalanced optimal transport also called entropy-transport inherits from the regularity of standard optimal transport. We provide detailed examples of Riemannian manifolds and costs for which unbalanced optimal transport is regular. Among all entropy-transport formulations, Wasserstein-Fisher-Rao (WFR) metric, also called Hellinger-Kantorovich, stands out since it admits a dynamic formulation, which extends the Benamou-Brenier formulation of optimal transport. After demonstrating the equivalence between dynamic and static formulations on a closed Riemannian manifold, we prove a polar factorization theorem, similar to the one due to Brenier and Mc-Cann. As a byproduct, we formulate the Monge-Ampére equation associated with WFR metric, which also holds for more general costs. Last, we study the link between c-convex functions for the cost induced by the WFR metric and the cost on the cone. The main result is that the weak Ma-Trudinger-Wang condition on the cone implies the same condition on the manifold for the cost induced by WFR.

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[1] the speaker is supported by project "PDEs and optimal control methods in mean field games, population dynamics and multi-agent models", project number MUR: 2022W58BJ5 and by the MUR Excellence Department Project MatMod@TOV awarded to the Department of Mathematics, University of Rome Tor Vergata, CUP E83C23000330006

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Few counterexamples to ACF-Monotonicity formulas in Carnot groups

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In this talk, we discuss a way to obtain some counterexamples to the increasing behavior of the Alt-Caffarelli-Friedman monotonicity formula, see [1,2], in Carnot groups of step 2, see [4]. In addition, we present an overview of some statements concerning the different behavior of intrinsic harmonic functions in Carnot groups of step 2, like in the Heisenberg group $\mathbb{H}^1$, with respect to the well known monotonicity increasing behavior of the Alt-Caffarelli-Friedman functional of harmonic functions in the Euclidean setting. In particular, we introduce some results, contained in the papers, [3,4,5], obtained in collaboration with Nicolò Forcillo (Michigan State University), and then continued in [6], with Davide Giovagnoli (University of Bologna).

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Well-posedness results for Kolmogorov equations with applications to mean-field control problems for multi-agent systems

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In this talk, based on a joint project with G. Ascione, D. Castorina and F. Solombrino, we discuss some well-posedness results for Kolmogorov-Fokker-Planck equations with measurable coefficients in time and locally Hölder continuous coefficients in space with possibly unbounded drift terms and their application to the study of particle systems of the second order whose prototypical agent is driven by a McKean-Vlasov SDE, or by a Vlasov-Fokker-Planck PDE.

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Small-time controllability of bilinear PDEs with infinite-dimensional Lie bracket methods

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This talk is devoted to some recent results of global approximate controllability, in arbitrarily small times, of bilinear Schrödinger and wave equations posed on boundaryless manifolds such as tori or euclidean spaces. The analysis of the controllability properties of these PDEs is very rich, and many results of large-time controllability exist in the literature. The focus of the presentation is thus on the possibility of controlling such PDEs in arbitrarily small times. Moreover, the main ideas behind the proofs of the results will also be presented: they can be interpreted as extensions, to this infinite-dimensional setting, of classical Lie bracket methods widely used in the literature to analyse finite-dimensional nonlinear control systems.

This talk is based on the results contained in the article [1] and in a forthcoming preprint in collaboration with Karine Beauchard (ENS Rennes).

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A Mean-Field Game network model for urban planning

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We delve into a mathematical framework aimed at capturing the dynamic evolution of urban landscapes, shaped by the interplay between two entities: the workforce and business establishments. Visualizing the city as a network, where edges symbolize both residential zones and connectivity pathways, we explore how these populations vie for spatial occupancy while engaging in labor market transactions. Our formulated model is composed of a two-population Mean-Field Game system coupled with an Optimal Transport problem, showcasing the coalescence of complex dynamics. We establish the existence and uniqueness of solutions and supplement our findings with diverse numerical simulations.

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Mean field control and Mean field games for multiple species models to maintain biodiviersity

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We propose a multi-agent model, in which biodiversity is valued both through the provisioning services it enables, and through its non-use value. Thus, the interactions between local decisions and local and global consequences are intimately linked. We are interested in the roles played by global interactions on individual decisions. More specifically, we consider a multi-agent model, where individual preferences depend on the relative utility of consumption and on biodiversity. In order to use the abundance-based characterization of biodiversity, species growth is defined through a Gompertz dynamic, which allows the inclusion of a density-dependent mortality relationship. We are interested in the behaviour of the system for a large number of agents and to this end we study the problem by mean field techniques. We compare the impacts of these decisions in two different scenarios: the first, the agents are competitive and hopefully reach a Nash equilibrium; the second, they cooperate, leave the choice of the control to a social planner and reach a social optimum. The goal of the model is to take into account for the relationship between local damage to biodiversity, the local function of protection against environmental risks and the global role of prevention against global environmental disruption.

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Opinion dynamics of two populations with time-delayed coupling

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Dipartimento di Ingegneria e Scienze dell'Informazione e Matematica, Università degli Studi di L'Aquila

We study a Hegselmann-Krause type opinion formation model for a system of two populations. The two groups interact with each other via subsets of individuals, namely the leaders, and natural time delay effects are considered. By using careful estimates of the system's trajectories, we are able to prove an asymptotic convergence to consensus result. Some numerical tests illustrate the theoretical result and point out some possible applications.

This kind of model can have applications in social sciences, economics, politics, and ecology. Indeed, it is reasonable to try reaching a global consensus among individuals of different countries, or different groups of individuals in the same country, about important questions such as, e.g., ecological behaviors, climate change's reasons, appropriate strategies to reduce CO_2 emissions, etc. The proof of a consensus result for models like that can be considered as a first insight for more quantitative studies aiming to design appropriate control strategies.

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AMS-UMI International
Joint Meeting 2024
Palermo, July 23–26, 2024

Nonlinear Problems with Nonstandard Growth Conditions and Analysis on Metric Spaces
Special Session A18

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The session focuses on the study of nonlinear partial differential equations under nonstandard growth conditions which constitute an important sub-field of the calculus of variations. Variational methods are powerful tools not only in investigating existence of solutions, but also in obtaining information on the behaviour and regularity properties of minimizers and, more generally, quasiminimizers that minimize the energy functional up to a multiplicative constant.

The topics include regularity theory for a wide class of singular and degenerate elliptic and parabolic equations as well as stability properties essential in applications of PDE. As certain natural physical settings are non-smooth, by necessity the research is conducted in the general setting of (potentially) nonsmooth metric measure spaces. One of the advantages of this kind of approach is that it embraces many different spaces, as a consequence the results can be applied in manifolds, graphs, vector fields and groups, just to mention a few. We are interested both in theoretical aspects of nonlinear partial differential equations and also in their applications to the regularity theory. In particular, we present regularity questions including boundedness and Hölder continuity of solutions and higher integrability properties of the gradients of solutions.

The nonlinear partial differential equations under study are connected to many different applications, for example diffusion in highly nonhomogeneous media and the motions of multiphased fluids in porous media.

The aim of the session is bringing together leading minds in the field and early career mathematicians in a relaxed, informal atmosphere conducive of creating new scientific collaborations. We intend to make the environment more inclusive and the participants reflect the diversity of mathematicians in the field.

The Nonlinear Problems with Nonstandard Growth Conditions and Analysis on Metric Spaces Session is scheduled on July 23-24. We look forward to seeing you in Palermo!

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The Leray-Lions existence theorem under general growth conditions.

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It is an edition, under more general growth conditions, of the celebrated existence theorem of weak solutions to a class of Dirichlet problems for second order nonlinear elliptic equations under the so-called natural growth conditions, published in 1965 by Jean Leray and Jacques-Louis Lions. We describe some existence and regularity results recently obtained in collaboration with G.Cupini and E.Mascolo.

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Joint Meeting 2024
Palermo, 23–26 July 2024

Superposition of subsolutions to infinite Laplace equation in disjoint variables

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It is well known that the sum of two subharmonic functions is still subharmonic. However, in general we cannot expect the same type of results to hold for nonlinear elliptic equations. On the other hand, one may easily verify a variant of this property with disjoint variables: if $u_1(x)$ is a smooth subsolution to $\Delta u = 0$ in $\mathbb{R}^m$ and $u_2(y)$ is a smooth subsolution to $\Delta u = 0$ in $\mathbb{R}^n$, then $u_1(x) + u_2(y)$ is a subsolution of $\Delta u = 0$ in $\mathbb{R}^{m+n}$. It turns out that this new superposition property can be extended to several nonlinear operators. The purpose of this talk is to show such a result for the infinity Laplace equation. We will also discuss further generalization for infinite Laplacian associated to a frame of vector fields in $\mathbb{R}^n$. The talk is based on joint work with Qing Liu and Juan Manfredi.

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Elliptic systems and double phase functionals

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We consider the system of partial differential equations in divergence form

$$\begin{cases} -\sum_{i=1}^{n} D_i \left[A_i^1(x,u(x),Du(x))\right] &= 0, \\ -\sum_{i=1}^{n} D_i \left[A_i^2(x,u(x),Du(x))\right] &= 0, \\ \qquad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots & \\ -\sum_{i=1}^{n} D_i \left[A_i^m(x,u(x),Du(x))\right] &= 0, \end{cases}$$

where $x \in \Omega \subset \mathbb{R}^n$, Ω is bounded and open, $u : \Omega \subset \mathbb{R}^n \to \mathbb{R}^m$ and $u^1, u^2, ..., u^m$ are the components of u, so that, $u = (u^1, u^2, ..., u^m)$. We assume that u is a weak solution of the previous system, $A(x, z)$ is measurable with respect to x and continuous with respect to z.

It is well known that solutions to elliptic systems may be unbounded, [2]. Nevertheless, for some special classes of systems, it can be proved that solutions are bounded. We mention a recent result of this kind: [1]. In such a paper, we are dealing with the so-called p, q-growth conditions, [3]. Starting from [4], we discuss some examples suggested by double phase functionals.

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[1] I gladly take the opportunity to thank MUR, UNIVAQ, INdAM, GNAMPA for the support.
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On regularity for nonstandard growth functionals on metric spaces

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We present some results on a class of double phase integrals characterized by nonstandard growth conditions. The study focuses on regularity theory, specifically local and global higher integrability, for quasiminimizers of a double phase integral with (p, q)-growth. The methods are variational-based in the setting of metric measure spaces with a doubling measure and a Poincaré inequality. The main feature is an intrinsic approach to double phase Sobolev-Poincaré inequalities. The results were obtained in collaboration with Juha Kinnunen (Aalto University) and Cintia Pacchiano Camacho (Calgary University).

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Conformal transformations preserving Besov energy

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One approach to solving Dirichlet or Neumann boundary value problems on unbounded domains is by changing the metric and the measure of the space in such a way that the p–energy is preserved and the domain becomes bounded in the new metric. After the transformation, the direct methods of calculus of variation can be applied to the transformed problem. This approach has been applied in the metric measure spaces setting for uniform domains with bounded boundary by Gibara, Korte and Shanmugalingam. In this case, it is possible to use a transformation that does not change the metric or the measure on the boundary. If we want to apply this kind of approach for problems on domains with unbounded boundary, we have to use transformations that change also the boundary and therefore we have to study also what happens to the Besov spaces – i.e. the trace space of Sobolev functions. As a first step in developing methods into this directions, we will discuss in this talk what kind of transformations preserve the Besov energy.

This is based on joint work with A. And J. Björn, S. Rogovin and T. Takala.

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Dirichlet Problems on Unbounded Metric Measure Spaces

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Nageswari Shanmugalingam
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In this talk, we will discuss the Dirichlet problem on an unbounded uniform domain in a metric measure space with boundary datum belonging to an appropriate Besov class that is related to the "codimensionality" of the boundary of the domain. In the case where the boundary is itself bounded, this is accomplished via a conformal transformation that renders the domain bounded while maintaining the boundary intact, and so the known results for solving the Dirichlet problem on a bounded uniform domain can be employed. When the boundary is not bounded, this is accomplished by approximating the domain by ones with bounded boundary and applying the first case.

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Comparison of Dirichlet and Newtonian Sobolev spaces.

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We consider the following question: Suppose that (X, d, μ) is an unbounded metric measure space with $\mu(X) = \infty$. Let $D^{1,p}(X)$ be the Dirichlet space consisting of functions $f\colon X \to \mathbb{R}$ with an L^p-integrable upper gradient $\rho \in L^p(X)$, and let $N^{1,p}(X) = D^{1,p}(X) \cap L^p(X)$ be the corresponding Newtonian Sobolev space. When does one have $D^{1,p}(X) = N^{1,p}(X) + \mathbb{R}$?

We show that $D^{1,p}(X) \neq N^{1,p}(X) + \mathbb{R}$ for a relatively general class of unbounded doubling spaces X with infinite measure. We also show that for hyperbolic spaces with multiple ends, one has $D^{1,p}(X) \neq N^{1,p}(X) + \mathbb{R}$. However, in hyperbolic spaces with a single end, the question becomes more interesting; as an example of this, we discuss how in the standard n-dimensional hyperbolic space $\mathbb{H}^n$, the exponent $p = n - 1$ acts as a cut-off point for this phenomenon.

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Self-improvement of fractional Hardy inequalities in metric measure spaces

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In the setting of a compact, doubling metric measure space (X, d, μ), we say that a closed set $E \subset X$ satisfies an (α, p)-Hardy inequality for $1 < p < \infty$ and $0 < \alpha < 1$ if the following holds for all Lipschitz functions u which vanish on E:

$$\int_{X \setminus E} \frac{|u(x)|^p}{\operatorname{dist}(x,E)^{\alpha p}} d\mu(x) \leq C \int_X \int_X \frac{|u(x) - u(y)|^p}{d(x,y)^{\alpha p} \mu(B(x, d(x,y)))} d\mu(y) d\mu(x).$$

In this talk, we discuss the relationship between such fractional Hardy inequalities on X and p-Hardy inequalities in the hyperbolic filling of X. By first showing that a p-Hardy inequality implies the validity of weighted versions for a certain class of *regularizable* weights, we then use the structure of the hyperbolic filling to show self-improvement of the (α, p)-Hardy inequality in both exponent p and smoothness parameter α.

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Hölder Continuity of the Gradient of Solutions to Doubly Non-Linear Parabolic Equations

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We study the local behavior of non-negative weak solutions to the doubly non-linear parabolic equation

$$\partial_t u^q - \operatorname{div}\big(|Du|^{p-2}Du\big) = 0$$

in a space-time cylinder. Hölder estimates are established for the gradient of its weak solutions in the so-called *super-critical fast diffusion regime* $0 < p - 1 < q < \frac{N(p-1)}{(N-p)_+}$ where N is the space dimension. Moreover, decay estimates are obtained for weak solutions and their gradient in the vicinity of possible extinction time. Two main components towards these regularity estimates are a time-insensitive Harnack inequality that is particular about this regime, and Schauder estimates for the parabolic p-Laplace equation.

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[1]U.G. is a member of the Gruppo Nazionale per l'Analisi Matematica, la Probabilità e le loro Applicazioni (GNAMPA) of the Istituto Nazionale di Alta Matematica (INdAM). He was partly supported by GNAMPA-INdAM Project 2023 "Regolarità per problemi ellittici e parabolici con crescite non standard" (CUP_E53C22001930001) and by Grant 2017TEXA3H_002 "Gradient flows, Optimal Transport and Metric Measure Structures".

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A PDE-based approach to Borell-Brascamp-Lieb inequality

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In this talk, we provide a new PDE perspective for the celebrated Borell-Brascamp-Lieb inequality [2][3]. In contrast to previously known proofs involving techniques from convex analysis or optimal transport, our new proof is based on properties of diffusion equations of porous medium type, particularly a generalized concavity maximum principle and large time asymptotics. Our approach reveals a deep connection between the Borell-Brascamp-Lieb inequality and nonlinear parabolic equations. An analogous idea was introduced in [1] to prove the special case of the Prékopa-Leindler inequality by using the heat equation. Our analysis serves as a nonlinear generalization and improvement of this PDE approach. Furthermore, we recover the equality condition for the Prékopa-Leindler inequality by further exploiting additional properties of the heat equation including the eventual log-concavity and backward uniqueness of solutions.

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Joint Meeting 2024
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Existence of parabolic minimizers to the total variation flow on metric measure spaces

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In this project we discuss some fine properties and existence of the variational solutions to the Total Variation Flow. Instead of the classical Euclidean setting, we intend to work mostly in the general setting of metric measure spaces. During the past two decades, a theory of Sobolev functions and BV functions has been developed in this abstract setting. A central motivation for developing such a theory has been the desire to unify the assumptions and methods employed in various specific spaces, such as weighted Euclidean spaces, Riemannian manifolds, Heisenberg groups, graphs, etc.

The total variation flow can be understood as a process diminishing the total variation using the gradient descent method. This idea can be reformulated using parabolic minimizers, and it gives rise to a definition of variational solutions. The advantages of the approach using a minimization formulation include much better convergence and stability properties. This is a very essential advantage as the solutions naturally lie only in the space of BV functions.

We first give an existence proof for variational solutions u associated to the total variation flow. Here, the functions being considered are defined on a metric measure space (X, d, μ). For such parabolic minimizers that coincide with a time-independent Cauchy-Dirichlet datum u_0 on the parabolic boundary of a space-time-cylinder $\Omega \times (0,T)$ with $\Omega \subset X$ an open set and $T > 0$, we prove existence in the weak parabolic function space $L^1_w(0,T;\mathrm{BV}(\Omega))$. In this paper, we generalize results from a previous work by Bögelein, Duzaar and Marcellini and argue completely on a variational level. This is a joint project with Vito Buffa and Michael Collins.

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Some results about doubly nonlinear equations

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The term doubly nonlinear refers to the fact that the diffusion part depends nonlinearly both on the gradient and the solution itself. Such equations describe several physical phenomena and were introduced by Lions and Kalashnikov. These equations have an intrinsic Mathematical interest because they represent a natural bridge between the more natural generalisation of the heat equation: the p-Laplacian and the Porous Medium equations. Especially in the last few years, many papers were devoted to this topic. The idea is to give an unified approach comprehensive of the Prorus Medium and the p-Lapacian equations. The approaches are sometimes not rigorous, sometimes not with sharp assumptions or with unnecessary long proofs. In this talk I will speak about the state-of-the-art, my contribution and the open questions.

[1]We wish to thanks GNAMPA to have partially supported this mission
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Regularity and Existence Results for Doubly Nonlinear Anisotropic Diffusion Equations

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Matias Vestberg
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We present some recent results regarding the regularity and existence theory for doubly nonlinear anisotropic diffusion equations. Some questions that will be considered are the boundedness of solutions, expansion of the support, comparison principles and the existence of solutions to the Cauchy problem.

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Fine boundary continuity for degenerate double-phase diffusion

Simone Ciani
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We present a study on the boundary behavior of solutions to parabolic double-phase equations, through the celebrated Wiener's sufficiency criterion.

The analysis is conducted for cylindrical domains and the regularity up to the lateral boundary is shown in terms of either its p or q capacity, depending on whether the phase vanishes at the boundary or not. Eventually we obtain a fine boundary estimate that, when considering uniform geometric conditions as density or fatness, leads us to the boundary Hölder continuity of solutions.
In particular, the double-phase elicits new questions on the definition of an adapted capacity.

This is a joint work in collaboration with Eurica Henriques and Igor Skrypnik.

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New trends in self-similarity of groups, trees and fractals
Special Session A19

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Marialaura Noce
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Rachel Skipper
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A particularly interesting family of groups arises as transformations of discrete spaces called trees. These spaces can be thought of as graphs, i.e. a collection of vertices and edges joining them, with no cycles. If we have such a graph with infinitely many vertices, we may select a concrete vertex and look at all other vertices as descendants of this vertex. This is said to be a *rooted tree*. If each vertex has the same number of descendants we can see that the tree hanging from a specific vertex resembles the full tree. This self-similarity connects rooted trees with fractals.

The main goal of this special session is to develop and bring together experts in topics in all areas of self-similarity of groups and the spaces on which they act, including fractal and branch groups, groups acting on Cantor sets such as Thompson-like groups, tree structures related to random walks.

The study of these groups provided answer to multiple important problems in mathematics. In particular, these groups stem from the famous Burnside problem, which arose in 1902: "Can a finitely generated torsion group, i.e. in which every element has finite order, be itself infinite?". This question went unanswered for over sixty years. Then in 1964, Golod and Shafarevich constructed an infinite finitely generated group where every element has finite order. Further examples were given by Adjan and Novikov, Olshanskii, Grigorchuk, Gupta and Sidki and recently Schlage-Puchta. The constructions given by Olshanskii, Grigorchuk, Gupta and Sidki are geometric in nature, with the groups of Grigorchuk, Gupta and Sidki acting on trees. These latter examples form a more general family of the so-called Grigorchuk-Gupta-Sidki (GGS) groups, which are in turn examples of branch groups.

A *branch group* is an example of a self-similar group acting transitively on a spherically homogeneous rooted tree T and admits a structure of subnormal subgroups similar to the corresponding structure in the full automorphism group $\mathrm{Aut}(T)$ of the tree T. There are many examples of branch groups with remarkable algebraic properties, such as the first example of a finitely generated group with intermediate growth and the first example of an amenable but not elementary amenable group. They also play an important role in the classification of just infinite groups, i.e. infinite groups whose proper quotients are all finite.

Furthermore, self-similar groups have many applications to other areas of mathematics and science, such as to dynamics, probability, and cryptography, and the aim of this workshop is also to see the interdisciplinary aspects of these class of groups.

For more information visit:
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Some algebraic and combinatorial properties of tree automaton (semi-)groups

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In this talk I will introduce a class of automaton groups defined starting from graphs. This class contains important classical examples of automaton groups. I will study some algebraic, combinatorial and dynamic properties associated with such (semi-)groups and their action on rooted trees in relation to the structure of the initial graph, giving particular emphasis to the case of trees. In particular, I will focus on freeness, growth, spectral properties and some topological indices defined on the corresponding Schreier graphs.

[1]Aknowledgements The author acknowledges the GNSAGA group of INDAM
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On free groups and semigroups defined by automata

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In this talk, I will provide an overview of recent findings concerning the freeness question for automaton (semi)group, that is the problem of checking whether an automaton (semi)group generates a free (semi)group or not. It will be shown that algorithmically, this problem for automaton monoids is undecidable, solving an open problem initially posed by Grigorchuk, Nekrashevych, and Sushchanski. If time allows, I will also explore the problem of identifying sufficient conditions for an automaton group to be not free.

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Path groups

Gustavo A. Fernández-Alcober
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In this talk we introduce path groups, a generalisation of the so-called multi-EGS groups that allows for directed automorphisms along arbitrary paths of a p-adic tree. We develop the basic theory of these groups, which requires the introduction of variants of the concepts of self-similarity, fractality, or regular branchness that apply to a whole family of groups rather than to a single group. Then we calculate the Hausdorff dimension of path groups with respect to the filtration provided by the level stabilisers.

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The Hausdorff dimension of the generalized Brunner-Sidki-Vieira Groups

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Groups of automorphisms of regular rooted trees are a rich source of examples with interesting properties in group theory, and they have been used to solve very important problems, such as the General Burnside Problem[4] and the Milnor Problem[5]. Branch profinite groups are especially interesting because they are one of the two types of just infinite profinite groups. Abercrombie[1], Barnea and Shalev[2] started the study of the Hausdorff dimension on profinite groups. The Hausdorff dimension of self-similar profinite groups is still the object of several important open problems.

We present our result regarding the Hausdorff dimension of the closure of the generalized Brunner-Sidki-Vieira groups (BSV-groups for short) acting on the m-adic tree for $m \geq 2$. Which is the first examples of self-similar topologically finitely generated closed subgroups of transcendental Hausdorff dimension in the group of m-adic automorphisms. Let us briefly describe the construction of the BSV-groups. Let $\mathcal{T}$ be the m-adic tree (regular rooted tree with m descendants at every vertex), where $m \geq 2$, and note that the subtree of all descendants at an arbitrary vertex of $\mathcal{T}$ is isomorphic to $\mathcal{T}$. The automorphisms of $\mathcal{T}$ as a graph form a profinite group Aut $\mathcal{T}$ under composition. Let a and b be the two recursively defined automorphisms of $\mathcal{T}$ that rigidly permute the m subtrees hanging from the root according to the permutation $\sigma = (1\ 2\ \cdots\ m)$ and induce the following automorphisms of $\mathcal{T}$ on the m subtrees hanging from the vertices of the first level respectively:

$$(1, \dots, 1, a) \qquad \text{and} \qquad (1, \dots, 1, b^{-1}).$$

Then $H = \langle a, b \rangle$ is said to be the BSV-group.

Theorem 1. *Let H be the generalized Brunner-Sidki-Vieira group acting on the m-adic tree. Then the Hausdorff dimension of its closure in Γ_m is*

$$\mathrm{hdim}_{\Gamma_m}(\overline{H}) = \frac{m - \tau(m-1)\log_m 2}{m+1},$$

where the parameter τ is defined as

$$\tau := \begin{cases} 0 & \text{if } m \text{ is odd,} \\ 1 & \text{if } m \text{ is even.} \end{cases}$$

This is joint work with Jorge Fariña Asategui.

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Generalized word problem for stabilizers of bounded automata groups

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The membership subgroup problem asks whether an element in a group belongs to a fixed subgroup. This gives rise to a formal language and, in the case of stabilizers, to a graph-theoretic interpretation. After defining groups described by automata, we will study the membership languages for stabilizers in this class. In particular, we will make use of Lindenmayer's systems, which were originally introduced in biology, but recently have successfully been exploited in combinatorial group theory.

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Finitely generated Hausdorff spectra in branch groups

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Since the pioneering work of Abercrombie [1], and Barnea and Shalev [2], Hausdorff dimension and thereof, Hausdorff spectra (i.e. the set of numbers obtained as the Hausdorff dimension of closed subgroups) of countably based profinite groups have provided fruitful results. In the particular case of profinite branch groups, Benjamin Klopsch proved that the Hausdorff spectra with respect to the level stabilizers is always the full interval $[0, 1]$.

It is common to study subsets of Hausdorff spectra of profinite groups by restricting the Hausdorff dimension values to specific subgroups of the given group. For instance, the normal Hausdorff spectrum or the finitely generated Hausdorff spectrum. The first one has been extendedly studied, but very little is known about the second one. In this talk we present results related to the study of finitely generated Hausdorff spectra of (weakly) branch groups.

(Joint work with Jorge Fariña-Asategui and Oihana Garaialde-Ocaña).

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[1]The author aknowledges support from the Spanish Government, grants PID2020-117281GB-I00 and PID2019-107444GA-I00, partly from European Regional Development Fund (ERDF), and the Basque Government, grant IT1483-22.

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Palermo, July 23–26, 2024

Twin Towers of Hanoi: On the diameters of the components of the Schreier graphs

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The Hanoi Towers group H is a self-similar group acting on the rooted ternary tree. We consider the diagonal action H on pairs of vertices on the same level. It is known from the work of D'Angeli and Alfredo Donno that this action is "as transitive as it can be", given that the tree structure (and thus the length of common prefixes) is preserved.

Proposition 1. *Let Y_n be the set of pairs of vertices at level n. This set is decomposed into $n+1$ orbits $Y_{n,k}$, for $k = 0, \ldots, n$, by the diagonal action of the Hanoi Towers group, where the orbit $Y_{n,k}$ consists of all pairs (u, v) such that the longest common prefix of u and v has length k.*

The proposition above follows directly from the following two facts:
- H is a regular branch group over its commutator H' and
- H' acts level-transitively on the ternary tree.

A graph structure is induced on the set Y_n of pairs of vertices at level n by the diagonal action as follows. Two pairs of vertices are neighbors if one can be obtained from the other by the diagonal action of one of the standard generators of the Hanoi Towers group H. The obtained graph Y_n is called the level n Schreier graph of the diagonal action. The orbits $Y_{n,k}$, for $k = 0, \ldots, n$, of the diagonal action on level n are precisely the connected components of the Schreier graph Y_n. Note that $Y_{n,n}$ is isomorphic to the usual Schreier graph of the action of H on level n of the ternary tree. Denote by $D_{n,k}$ the diameter of the orbit $Y_{n,k}$. We provide the exact diameters $D_{n,k}$ for the smallest four orbits $Y_{n,k}$, with $k = n, n-1, n-2, n-3$, at any level.

Theorem 2. *The diameters of the 4 smallest orbits $Y_{n,k}$, with $k = n, n-1, n-2, n-3$, are given by*

$$D_{n,n} = 2^n - 1, \qquad \text{for } n \geq 0.$$

$$D_{n,n-1} = \begin{cases} 2, & \text{for } n = 1, \\ \frac{7}{6} \cdot 2^n - \frac{3+(-1)^n}{6}, & \text{for } n \geq 1. \end{cases}$$

$$D_{n,n-2} = \begin{cases} 6, & \text{for } n = 2, \\ \frac{13}{8} \cdot 2^n - 1, & \text{for } n \geq 3. \end{cases}$$

$$D_{n,n-3} = \begin{cases} 16, & \text{for } n = 3, \\ 32, & \text{for } n = 4, \\ \frac{65}{32} \cdot 2^n - 1 & \text{for } n \geq 5. \end{cases}$$

We also provide a method that, in principle, provides the diameter $D_{n,k}$ of any orbit at any level (modulo some finite, brute force, calculation, which depends only on the difference $n - k$, but not on n). We observe an interesting phenomenon – for a fixed level n, the largest diameter does not correspond to the largest orbit (largest connected component of the graph).

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Divergence in weakly branch groups

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The divergence of a group is a quasi-isometry invariant that measures how difficult it is to connect two points avoiding a large ball around the identity. It is easy to see that it is linear for direct products of infinite groups, and from that to deduce that it is linear for branch groups. In this talk, I will discuss divergence for weakly branch groups.

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Normal subgroups of non-torsion multi-EGS groups

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The family of multi-EGS groups form a natural generalisation of the Grigorchuk-Gupta-Sidki groups, which in turn are well-studied groups acting on rooted trees. Groups acting on rooted trees provided the first explicit examples of infinite finitely generated torsion groups, and since then have established themselves as important infinite groups, with numerous applications within group theory and beyond. Among these groups with the most interesting properties are the so-called regular branch groups. In this talk we investigate the normal subgroups in non-torsion regular branch multi-EGS groups, and we show that the congruence completion of these multi-EGS groups have bounded finite central width. Recall that the *central width* of Γ is defined as

$$w_c(\Gamma) := \sup |H : K|,$$

where H and K range over all central sections of Γ.

In particular, we prove that the profinite completion of a Fabrykowski-Gupta group has width 2, where for a pro-p group Γ the *width* of Γ is

$$w(\Gamma) := \sup_n \log_p |\gamma_n(\Gamma) : \gamma_{n+1}(\Gamma)|.$$

[1]This research was supported by the Knut and Alice Wallenberg Foundation and by the Royal Physiographic Society of Lund.
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Iterated Wreath Products in Product Action: In search of new hereditarily just infinite groups

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A just infinite group is an infinite group without infinite proper quotients. A group is said to be hereditarily just infinite (HJI) if all of its finite index subgroups are just infinite. A classical classification theorem of Grigorchuk-Wilson states that a residually finite just infinite group is either: (a) a branch group or (b) a HJI group. Branch groups have been extensively studied (e.g. Grigorchuk group), but HJI groups remain a very mysterious class. In this talk, I will report on some recent work on the search for new HJI groups. In particular, I will describe a way of embedding an infinitely iterated wreath product in product action as a subgroup of the automorphism group of a rooted tree via the "tree of partitions". This talk is based on joint work with G. Corob-Cook, G. Fernández-Alcober, M. Noce and S. Smith.

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Diagonal Actions of Groups Acting on Rooted Trees

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For a group G acting on a regular rooted d-ary tree T_d and on its boundary ∂T_d we consider the diagonal actions of G on the powers of T_d and ∂T_d. For the action of the full group $\mathrm{Aut}(T_d)$ of automorphisms of T_d we describe the ergodic decomposition of its action on $(\partial T_d)^n$ for all $n \geq 1$. To achieve it we analyze the orbits of n-tuples of elements of vertices of any fixed finite level of T_d. For a subgroup G of $\mathrm{Aut}(T_d)$ the corresponding orbits may be smaller, but sometimes they coincide with the orbits of the full group of automorphisms for all levels. In this case we say that the action of G on $\mathrm{Aut}(T_d)$ is maximally tree n-transitive. For example, maximal tree 1-transitivity is equivalent to level transitivity of the action of G on T_d. It follows from the results of [1,2] that Grigorchuk group and Basilica group act maximally tree 2-transitively on ∂T_2. We show that the action of Grigorchuk group on ∂T_2 is, in-fact, at least maximally tree 4-transitive.

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Rearrangement groups of fractals

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In 2019 J. Belk and B. Forrest introduced the family of rearrangement groups [2]. These are groups of certain "piecewise-canonical" homeomorphisms of fractals that act by permutations of the self-similar pieces that make up the fractal. Despite being countable, many of these groups seem to be dense in the group of all homeomorphisms of the fractal on which they act [3,7].

The family of rearrangement groups is a generalization of the main trio of Thompson groups F, T and V, each of which has made its appearance in many different topics. Other groups unrelated to fractals, such as certain Thompson-like groups and the Houghton groups, can also be realized in the framework of rearrangement groups.

Results about rearrangement groups include the simplicity of commutator subgroups in many examples [1,7,8], a general result about invariable generation [4], rationality of the fractal spaces on which they act [5] and a method to tackle their conjugacy problem [6]. This talk will introduce this family of groups and highlight some facts about them.

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Arithmetic and Geometric Aspects of Drinfeld Modules, Anderson Motives, and Computational Aspects
Special Session A20

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Matt Papanikolas
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This session of the 2nd Joint Meeting co-organized by the Unione Matematica Italiana (UMI) and the American Mathematical Society (AMS) will be mainly focused on the arithmetic and geometric theory of Drinfeld modules, shtukas and Anderson motives and on related topics, such as L-series, zeta, multizeta values, transcendence, Drinfeld modular varieties, Drinfeld modular forms. The session, scheduled on July 23-24, 2024, will also include computational aspects as different groups have been developing and implementing computational algorithms for Drinfeld modules.

Relevant information will be posted here:
`https://sites.google.com/uniroma1.it/drinfeld-modules/home-page`.

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The Manin-Drinfeld Property of Drinfeld Modular Varieties

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Let A be a Drinfeld coefficient ring (e.g., a polynomial ring $\mathbb{F}[T]$ over a finite field $\mathbb{F}$), and let M be the Drinfeld modular variety attached to either

(1) the linear group $GL(Y)$ of a projective A-module Y of rank $r \geq 2$, or to
(2) the $\mathfrak{n}$-th congruence subgroup of such a $GL(Y)$, where $\mathfrak{n}$ is a non-trivial ideal of A.

By work of Kapranov, Pink, Häberli, and the author, there exist natural Satake-like compactifications $\bar{M}$ of such M. The property referred to in the title is:

(MD) The divisors in $\bar{M} \setminus M$ generate a finite subgroup in the Chow group of cycles of codimension 1 of $\bar{M}$.

This had first been observed for the analogous case of classical elliptic modular curves M by Manin and Drinfeld 1973, where it means that the cusps of M generate a finite subgroup in the Jacobian $\mathrm{Jac}(\bar{M})$.

We show that the MD property holds for arbitrary Drinfeld rings A, ranks $r \geq 2$ and levels $\mathfrak{n}$, thereby generalizing earlier work in special cases ($A = \mathbb{F}[T]$ and $r = 2$, Gekeler 1984, $A = \mathbb{F}[T]$ and $r \geq 2$, Kapranov 1988, A arbitrary and $r = 2$, Gekeler 2000). The key ingredients are:

- an explicit formula in terms of partial zeta values for the vanishing orders of certain modular forms (division forms) $d_{\mathbf{u}}$ along boundary divisors of M (Gekeler 2023);
- the action of a finite abelian group $Q(M)$ both on the boundary divisors of M (where it is free and transitive) and on the parameter set $\{\mathbf{u}\}$ of the division forms $d_{\mathbf{u}}$. (In case (a), $Q(M)$ is the class group $\mathrm{Pic}(A)$ of A, while in case (b), it is an extension of $\mathrm{Pic}(A)$ by a group that depends on the rank r and the level $\mathfrak{n}$.)

Thereby, (MD) is reduced to showing the non-vanishing of certain character sums on $Q(M)$. This finally is achieved by relating character sums with values of the zeta function of A which are known to be non-zero.

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Nearly holomorphic Drinfeld modular forms for admissible coefficient rings

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Inspired by the work of Shimura, Chen and Gezmiş introduced the notion of nearly holomorphic Drinfeld modular forms for a certain admissible coefficient ring and studied their special values at CM points. In this talk, we discuss nearly holomorphic Drinfeld modular forms for a more general setting, namely for any admissible coefficient rings, as well as their special values at CM points. Moreover, motivated from the classical theory, we describe nearly holomorphic Drinfeld modular forms as global sections of a particular coherent sheaf on the compactification of the Drinfeld moduli space. This is a joint work with Sriram Chinthalagiri Venkata.

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Traces of Hecke operators on Drinfeld modular forms

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I will talk about recent work related to a trace formula for Hecke operators on Drinfeld modular forms of level 1. The formula uses Böckle-Eichler-Shimura theory [1] and point-counts on the moduli space of rank 2 Drinfeld modules. I will mention several theoretical consequences, such as a Ramanujan bound for Drinfeld modular forms and implications for the theory of oldforms and newforms. The trace formula can also be implemented algorithmically, which leads to interesting data suggesting directions for further research.

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$\mathcal{D}$-elliptic sheaves and the Hasse principle

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Let p be a rational prime, $q > 1$ a p-power integer and $F = \mathbb{F}_q(t)$. Let $d \geq 2$ be an integer and D a central division algebra over F of dimension d^2 which splits at ∞ and such that for any place x of F at which D ramifies, the invariant of D at x is $1/d$.

A $\mathcal{D}$-elliptic sheaf is a system of locally free sheaves equipped with an action of a sheafified version $\mathcal{D}$ of D. $\mathcal{D}$-elliptic sheaves are parametrized by the Drinfeld–Stuhler variety over F. When $d = 2$, it is also called the Drinfeld–Stuhler curve and it can be considered as a function field analogue of a quaternionic Shimura curve over $\mathbb{Q}$.

For quaternionic Shimura curves, in 1980s Jordan proved a criterion for the non-existence of quadratic points on them and gave an example of a quaternionic Shimura curve curve X and a quadratic field K such that X has no K-rational points but has K_v-rational points for any place v of K. This property of having local points without having global points is often called violation of the Hasse principle.

In this talk, I will explain how to generalize Jordan's criterion to Drinfeld–Stuhler varieties to obtain similar examples of quadratic extensions K/F over which the Drinfeld–Stuhler curve violates the Hasse principle.

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Uniformizing the moduli stacks of global G-shtukas

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Moduli stacks of global G-Shtukas play an important role in the global Langlands program over function fields. They were used in the proof of the function field global Langlands conjecture by Drinfeld, Lafforgue etc. In this talk, I will speak on my joint work with Urs Hartl, where we show that the moduli spaces of suitably bounded global G-Shtukas with colliding legs satisfy a p-adic uniformization isomorphism by Rapoport-Zink spaces. If time permits, I will mention some applications (e.g. to the Langlands-Rapoport Conjecture).

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Towards a Jacquet-Langlands correspondence for function field modular forms

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In this talk I will discuss a first step towards a Jacquet-Langlands correspondence for (quaternionic) Drinfeld modular forms. Using étale cohomology computations (facilitated by the theory of function field crystals) and the Eichler-Shimura isomorphism proved by Böckle, I will associate Hecke eigensystems of rank 2 Drinfeld cusp forms to Hecke eigensystems of functions on quaternionic adèlic double quotients, for a definite quaternion algebra ramified at exactly one finite place, exploiting the fact that these double quotients describe supersingular Drinfeld modules. The Drinfeld cusp forms obtained are new at the prime where the quaternion algebra is ramified, in the sense of Bandini and Valentino. Under suitable ordinariness assumptions for the special fiber of modular curves, all newforms arise in this way. The talk is based on the speaker's PhD thesis, written under the supervision of Prof. Gebhard Böckle.

[1]The speaker acknowledges support by Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) through the Collaborative Research Centre TRR 326 *Geometry and Arithmetic of Uniformized Structures*, project number 444845124, and by the Landesgraduiertenförderung at the University of Heidelberg.
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The rational torsion subgroups of the Drinfeld modular Jacobians for prime-power levels

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Fix a non-zero ideal $\mathfrak{n}$ of $\mathbb{F}_q[T]$. Let $\mathcal{T}(\mathfrak{n})$ be the rational torsion subgroup of the Drinfeld modular Jacobian $J_0(\mathfrak{n})$. A generalized Ogg's conjecture states that $\mathcal{T}(\mathfrak{n})$ coincides with the rational cuspidal divisor class group $\mathcal{C}(\mathfrak{n})$ of the Drinfeld modular curve $X_0(\mathfrak{n})$. First, we prove that for any prime-power ideal $\mathfrak{p}^r$ of $\mathbb{F}_q[T]$, the prime-to-$q(q-1)$ part of $\mathcal{T}(\mathfrak{p}^r)$ is equal to that of $\mathcal{C}(\mathfrak{p}^r)$ by studying the Hecke operators and the Eisenstein ideal of level $\mathfrak{p}^r$. Second, by relating the rational cuspidal divisors of degree 0 on $X_0(\mathfrak{p}^r)$ with Δ-quotients, where Δ is the Drinfeld discriminant function, we are able to compute explicitly the structure of $\mathcal{C}(\mathfrak{p}^r)$. As a result, the structure of the prime-to-$q(q-1)$ part of $\mathcal{T}(\mathfrak{p}^r)$ is completely determined.

Theorem 1. *Let $\mathfrak{n} = \mathfrak{p}^r \lhd \mathbb{F}_q[T]$ be a prime-power ideal with $r \geq 2$. For any prime $\ell \nmid q(q-1)$, we have*

$$\mathcal{C}(\mathfrak{p}^r)_\ell = \mathcal{T}(\mathfrak{p}^r)_\ell.$$

Theorem 2. *Let $\mathfrak{n} = \mathfrak{p}^r \lhd \mathbb{F}_q[T]$ be a prime-power ideal with $r \geq 2$. Then*

$$\mathcal{C}(\mathfrak{p}^r) \cong \left(\bigoplus_{1 \leq i \leq m} \frac{\mathbb{Z}}{|\mathfrak{p}|^{r-i} M(\mathfrak{p})\mathbb{Z}} \right) \oplus \left(\bigoplus_{m+1 \leq i \leq r-2} \frac{\mathbb{Z}}{|\mathfrak{p}|^{i} M(\mathfrak{p})\mathbb{Z}} \right) \oplus \frac{\mathbb{Z}}{M(\mathfrak{p})\mathbb{Z}} \oplus \frac{\mathbb{Z}}{N(\mathfrak{p})\mathbb{Z}},$$

where $m := \lfloor \frac{r-1}{2} \rfloor$, $M(\mathfrak{p}) := \frac{|\mathfrak{p}|^2-1}{q^2-1}$, and

$$N(\mathfrak{p}) := \begin{cases} \frac{|\mathfrak{p}|-1}{q^2-1}, & \textit{if } \deg(\mathfrak{p}) \textit{ is even.} \\ \frac{|\mathfrak{p}|-1}{q-1}, & \textit{otherwise.} \end{cases}$$

Moreover, we provide an explicit basis of $\mathcal{C}(\mathfrak{p}^r)$.

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Solving linear difference equations to check abelianess of Anderson t-modules

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Let K be perfect field containing the finite field $\mathbb{F}_q$ of q elements. Let t and τ be indeterminates and consider the skew polynomial ring

$$K[t]\{\tau\} = \left\{ \sum_{i,j=0}^{n} \alpha_{ji} t^j \tau^i \,\middle|\, n \geq 0, \alpha_{ji} \in K \right\}$$

with multiplication uniquely given by additivity and the rules

$$\begin{aligned} \tau \cdot \alpha &= \alpha^q \cdot \tau \quad \forall \alpha \in K, \\ t \cdot \alpha &= \alpha \cdot t \quad \forall \alpha \in K, \\ \tau \cdot t &= t \cdot \tau \end{aligned}$$

For an Anderson t-module over K of dimension d, its t-motive M is a left $K[t]\{\tau\}$-module which is free of dimension d as $K\{\tau\}$-module. The t-motive M and the Anderson t-module are called *abelian*, if M is also free finitely generated as $K[t]$-module.

In this talk, we explain how one can check whether a given t-motive is abelian or not by solving the linear difference equations associated to the t-motive. Using a modification of that algorithm, we also show that after scalar extension to $K(t)$, every t-motive becomes finitely generated, i.e. that $K(t) \otimes_{K[t]} \mathsf{M}$ is a finite dimensional $K(t)$-vector space.

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Wieferich primes and $\mathfrak{p}$-adic L-functions: a conjectural relation for Drinfeld modules

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Let p be an odd prime. The prime p is called *Wieferich* if the multiplicative order of 2 modulo p is the same modulo p^2. Very little is known about Wieferich primes; in fact, 1093 and 3511 are the only known ones and the question whether there are infinitely many of them remains open.

The notion of Wieferich primes can be easily generalized to a Drinfeld module E following the well-established analogy between the multiplicative group scheme and the Carlitz module. Questions related to their density appear to be as difficult as in the number field situation.

The aim of this short talk is to share the surprising observation that the condition for a prime $\mathfrak{p}$ in $\mathbb{F}[t]$ to be E*-Wieferich* is related to the $\mathfrak{p}$-adic valuation of the $\mathfrak{p}$-adic L-function of E. This generalizes the case where E is the Carlitz module, envisioned and established by Thakur [2] in 2015.

All the conjectures were stated jointly with Xavier Caruso and based on our recent work [1] where an algorithm computing L-functions of t-motives is described.

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Computation of classical and v-adic L-series of t-motives

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I will present an algorithm for computing the L-series attached to a t-motive and report on an implementation of it in SageMath (with a demo if time permits).

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Algebraic relations of special v-adic arithmetic gamma values and their period interpretations

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Let v be a finite place of $\mathbb{F}_q(\theta)$. We show that algebraic relations of arithmetic v-adic Γ-functions over $\mathbb{F}_q(\theta)$ are explained by the standard functional equations together with Thakur's analogue of the Gross–Koblitz formula. A key step is working out a formula expressing v-adic crystalline periods of Carlitz motives with complex multiplication as products of these special gamma values (a v-adic Chowla–Selberg formula). The algebraic independence of those periods in question results from determining the dimension of the motivic Galois groups, adapting and further developing existing tools.

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On generators of v-adic multiple zeta values in positive characteristic

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Let $A := \mathbb{F}_q[\theta]$ be the polynomial ring in the variable θ over a finite field of q elements, k the fraction field of A, and v a finite place of k. For each index $\mathfrak{s} \in \bigcup_{r\geq 0} \mathbb{Z}^r_{\geq 1}$, let $\zeta_A(\mathfrak{s})$ be the ∞-adic multiple zeta value (MZV) defined by Thakur, and $\zeta_A(\mathfrak{s})_v$ the v-adic MZV defined by Chang and the speaker. These values are function field analogues of the real-valued and p-adic MZV's respectively. Let $\mathcal{Z}_w$ (resp. $\mathcal{Z}_{v,w}$) be the k-vector space spanned by the ∞-adic (resp. v-adic) MZV's of weight w, and set $\mathcal{Z} := \sum_{w\geq 0} \mathcal{Z}_w$ and $\mathcal{Z}_v := \sum_{w\geq 0} \mathcal{Z}_{v,w}$. One of the central problem in this topic is to determine the structures of the k-vector spaces $\mathcal{Z}$ and $\mathcal{Z}_v$.

In the ∞-adic case, Chang showed that the natural surjection $\bigoplus_{w\geq 0} \mathcal{Z}_w \twoheadrightarrow \mathcal{Z}$ is an isomorphism. For each $\mathcal{Z}_w$, Todd and Thakur gave conjectures about its dimension and basis respectively, and Ngo Dac showed that the candidates given by Thakur indeed generate $\mathcal{Z}_w$. The following theorem gives affirmative answers of Todd's dimension conjecture and Thakur's basis conjecture:

Theorem 1 (Chang-Chen-Mishiba [2], Im-Kim-Le-Ngo Dac-Pham)**.** *Let $\mathcal{I}^{\mathrm{T}}_w$ be the set of indices $\mathfrak{s} = (s_1, \ldots, s_r)$ of weight w such that $s_1, \ldots, s_{r-1} \leq q$ and $s_r < q$. Then for each $w \geq 0$, the elements $\zeta_A(\mathfrak{s})$ $(\mathfrak{s} \in \mathcal{I}^{\mathrm{T}}_w)$ form a k-basis of $\mathcal{Z}_w$. In particular, we have $\dim_k \mathcal{Z}_w = |\mathcal{I}^{\mathrm{T}}_w|$.*

In this talk, we will discuss v-adic analogues of the above results. In the v-adic case, the result corresponding to Theorem 1 is not known. However, the following theorem holds:

Theorem 2 (Chang-Mishiba [3], Chang-Chen-Mishiba [1])**.** *$\mathcal{Z}_v$ forms a k-algebra and there exists a well-defined k-algebra homomorphism $\phi_v \colon \mathcal{Z} \twoheadrightarrow \mathcal{Z}_v$ such that $\phi_v(\zeta_A(\mathfrak{s})) = \zeta_A(\mathfrak{s})_v$.*

Theorem 2 means that the v-adic MZV's satisfy the same k-algebraic relations that their corresponding ∞-adic MZV's satisfy. Note that Thakur showed that $\mathcal{Z}_w \cdot \mathcal{Z}_{w'} \subset \mathcal{Z}_{w+w'}$. Since $\zeta_A(q-1)_v = 0$ by Goss, we have the following corollary:

Corollary 3. *For each $w \geq 0$, we have $\dim_k \mathcal{Z}_{v,w} \leq |\mathcal{I}^{\mathrm{T}}_w| - |\mathcal{I}^{\mathrm{T}}_{w-(q-1)}|$, where $\mathcal{I}^{\mathrm{T}}_n := \emptyset$ for $n < 0$.*

It is conjectured that the inequality in Corollary 3 is an equality. We will give candidate generators of the k-vector space $\mathcal{Z}_{v,w}$.

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New Families of Multiple Polylogarithm Relations over Function Fields

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We described a new family of relations between Carlitz multiple polylogarithms obtained using a new noncommutative factorization of the exponential function. We describe these relations both at the finite level, as well as in a motivic setting. Additionally, we give precise formulas for the coefficients of these relations and describe how these coefficients relate to Carlitz multiple zeta values.

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Periods of extensions of Drinfeld modules by the Carlitz module

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Drinfeld introduced 'Elliptic modules' now called Drinfeld modules, which can be regarded as a function field analogue of elliptic curves. By Anderson's work, there are higher dimensional generalizations of Drinfeld module called t-modules. As they are modules, if we consider two t-modules E and F, we can study the properties of $\mathrm{Ext}^1(E, F)$. Papanikolas and Ramachandran showed that when F is of rank 1 and dimension 1 (called the Carlitz module) and E is a Drinfeld module, $\mathrm{Ext}^1(E, F)$ has the structure of a t-module, and a special subgroup $\mathrm{Ext}^1_0(E, F)$ is isomorphic to the dual t-module associated to E. In this talk, we present recent work regarding the periods of the extensions in $\mathrm{Ext}^1_0(E, C)$ where we establish their connection with the periods and logarithms of the dual t-module associated to E. Our result generalizes the work of Chang for rank 2 Drinfeld modules.

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On extensions of Drinfeld modules by the tensor powers of the Carlitz module

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Let E be a Drinfeld module of rank $r \geq 2$ and $\mathbf{C}^{\otimes n}$ be the n-th tensor powers of the Carlitz module. By the theory of Anderson, we can associate a dual t-motive $\mathcal{M}_E$ for E and a dual t-motive $\mathcal{M}_{\mathbf{C}^{\otimes n}}$ for $\mathbf{C}^{\otimes n}$ respectively. In this talk, we aim to present some results concerning the $\mathbb{F}_q[t]$-module structure of $\mathrm{Ext}^1_{\mathfrak{F}}(\mathcal{M}_E, \mathcal{M}_{\mathbf{C}^{\otimes n}})$, where we consider the extensions of $\mathcal{M}_E$ by $\mathcal{M}_{\mathbf{C}^{\otimes n}}$ in the category of Frobenuis modules. Consequently, we prove that $\mathrm{Ext}^1_{\mathfrak{F}}(\mathcal{M}_E, \mathcal{M}_{\mathbf{C}^{\otimes n}})$ itself defines a t-module. Furthermore, we have the following short exact sequence of t-modules

$$0 \to \mathbf{C}^{\otimes n-1} \otimes \wedge^{r-1} E \to \mathrm{Ext}^1_{\mathfrak{F}}(M_E, M_{\mathbf{C}^{\otimes n}}) \to \partial\mathbf{C}^{\otimes n} \to 0,$$

where $\partial\mathbf{C}^{\otimes n}$ is the n-dimensional t-module whose $\mathbb{F}_q[t]$-module structure is determined by

$$t \cdot \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} := \begin{pmatrix} \theta & 1 & & \\ & \ddots & \ddots & \\ & & \theta & 1 \\ & & & \theta \end{pmatrix} \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}.$$

As an application of our results on $\mathrm{Ext}^1_{\mathfrak{F}}(\mathcal{M}_E, \mathcal{M}_{\mathbf{C}^{\otimes n}})$, we will present an effective construction of the rigid analytic trivialization for extensions of $\mathcal{M}_E$ by $\mathcal{M}_{\mathbf{C}^{\otimes n}}$.

[1]The travel was partially funded by NSF grant DMS 2419068 and the AMS-Simons Travel Grants.
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A duality result about special functions in Drinfeld modules of arbitrary rank

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Given a Drinfeld module ϕ over a ring A with an $\mathbb{F}_q$-rational point at ∞, Poonen proved in [1] that the kernel of the adjoint exponential $\exp_\phi^*$ is isomorphic to $\mathrm{Hom}_{\mathbb{F}_q}(\Lambda_\phi, \mathbb{F}_q)$. We make explicit the inverse of this bijection: $g \in \mathrm{Hom}_{\mathbb{F}_q}(\Lambda_\phi, \mathbb{F}_q)$ is sent to $-\sum_{\lambda \in \Lambda_\phi \setminus \{0\}} \frac{g(\lambda)}{\lambda}$.

We use this result to prove that the series $\zeta_\phi := -\sum_{\lambda \in \Lambda_\phi \setminus \{0\}} \lambda^{-1} \otimes \lambda \in \mathbb{C}_\infty \hat{\otimes} \Lambda_\phi$ is a *dual Drinfeld eigenvector*, in the sense that $(\phi_a^* \otimes 1)\zeta_\phi = (1 \otimes a)\zeta_\phi$ for all $a \in A$.

Under a functorial point of view, it turns out that ζ_ϕ is the universal object with this property; similarly, we find the universal *Drinfeld eigenvector* $\omega_\phi \in \mathbb{C}_\infty \hat{\otimes} \mathrm{Hom}_A(\Lambda_\phi, \Omega_{A/\mathbb{F}_q})$, a substitute of the special functions studied by Anglès, Ngo Dac and Tavares Ribeiro in [2]. Finally, we state that, under the $\mathbb{C}_\infty \hat{\otimes} A$-linear pairing $\_ \cdot \_$, the element $\zeta_\phi \cdot \omega_\phi \in \mathbb{C}_\infty \hat{\otimes} \Omega_{A/\mathbb{F}_q}$ is a rational differential form, partly generalizing previous rationality results by Pellarin ([3]), Green and Papanikolas ([4]), and the speaker ([5]).

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Zagier-Hoffman's conjectures in positive characteristic

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Zagier-Hoffman's conjectures in the classical setting on multiple zeta values over $\mathbb{Q}$ of Euler and Euler sums are still open. As analogues of the classical case, multiple zeta values and alternating multiple zeta values in positive characteristic were introduced by Thakur and Harada. In this talk, we determine the dimension and a basis of the span of all alternating multiple zeta values over the rational function field by finding all linear relations among them. As a consequence, we completely establish Zagier-Hoffman's conjectures in positive characteristic formulated by Todd and Thakur which predict the dimension and an explicit basis of the span of multiple zeta values of Thakur of fixed weight. In fact, our results have completed the conjectures as a corollary of the vastly generalized results which proves the conjectures for alternating MZV's [1] and cyclotomic MZV's [2].

This is a joint work with Hojin Kim, Khac Nhuan Le, Tuan Ngo Dac, and Lan Huong Pham.

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The refined class number formula for Drinfeld modules

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Taelman proved a formula for a special value of the Goss L-function attached to a Drinfeld module [1], which can be interpreted as a function field analogue of the analytic class number formula. In the same article, he stated that 'it should be possible to formulate and prove an equivariant version' of this formula.

Given a finite field $\mathbb{F}_q$, a finite Galois extension K/k of function fields over $\mathbb{F}_q$ and a Drinfeld $\mathbb{F}_q[t]$-module E defined over the ring of integers of k, in [2] we formulate and prove an equivariant refinement of Taelman's formula for $(E, K/k)$. We also derive explicit consequences for the Galois module structure of the Taelman class group of E over K.

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[1]The first author acknowledges support for this talk as part of the Grant PID2022-142024NB-I00 funded by MCIN/AEI/ 10.13039/501100011033.

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Coefficients of Drinfeld modular polynomials

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For any monic polynomial $N \in A = \mathbb{F}_q[t]$ we can define the Drinfeld modular polynomial $\Phi_N(X,Y) \in A[X,Y]$, which vanishes at pairs of j-invariants of rank 2 Drinfeld modules linked by a cyclic N-isogeny. The height of this polynomial is the largest t-degree of its coefficients, and grows rapidly with $\deg N$.

The precise asymptotic growth of this height was determined by Hsia in 1998 [2].

In this talk, we give fully explicit bounds on this height, similar to recent work in the elliptic curve case [1].

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Local Kummer theory for Drinfeld modules

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Let φ be a Drinfeld A-module of finite residual characteristic $\bar{\mathfrak{p}}$ over a local field K. We study the action of the inertia group of K on a modified adelic Tate module $T^{\circ}_{\mathrm{ad}}(\varphi)$ which differs from the usual adelic Tate module only at the $\bar{\mathfrak{p}}$-primary component. After replacing K by a finite extension we can assume that φ is the analytic quotient of a Drinfeld module ψ of good reduction by a lattice $M \subset K$. The image of inertia acting on $T^{\circ}_{\mathrm{ad}}(\varphi)$ is then naturally a subgroup of $\mathrm{Hom}_A(M, T^{\circ}_{\mathrm{ad}}(\psi))$.

This subgroup is described by a canonical local Kummer pairing that is the central subject of our study. In particular we give an effective formula for the image of inertia up to finite index, and obtain a necessary and sufficient condition for this image to be open. We also determine the image of the ramification filtration.

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On natural density of rank-2 Drinfeld modules with big Galois image

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In the groundbreaking work of Jones, he investigated the natural density of elliptic curves over $\mathbb{Q}$ with adelic Galois image that are index-2 subgroups of $\mathrm{GL}_2(\hat{\mathbb{Z}})$. Zywina later extended this result to elliptic curves over arbitrary number fields. It turns out that the natural density of elliptic curves over a number field with maximal adelic Galois image is equal to one.

As a function field analogy, Pink and Rütsche proved the open image theorem for Drinfeld modules of arbitrary rank without complex multiplication. And Zywina proved the existence of rank-2 Drinfeld modules with surjective adelic Galois representation. Hence it makes sense to study the natural density of Drinfeld modules of rank 2 with surjective adelic Galois representation. Due to Van der Heiden's result on the determinant of adelic Galois representation for Drinfeld modules, the natural density estimation can be split into two cases:

Case 1: Natural density of rank-1 Drinfeld modules with surjective adelic Galois representation.

Case 2: Natural density of rank-2 Drinfeld modules whose adelic Galois image containing $\mathrm{SL}_2(\widehat{\mathbb{F}_q[T]})$.

In this talk, we compute the natural density of rank-1 Drinfeld module over $\mathbb{F}_q[T]$ with surjective adelic Galois representation; and the natural density of rank-2 Drinfeld modules over $\mathbb{F}_q[T]$ whose $\mathfrak{l}$-adic Galois image containing the special linear subgroup for finitely many prime ideal $\mathfrak{l}$.

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Regulators of tensor/symmetric/alternating squares of Drinfeld modules of rank 2

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Regulators of Drinfeld modules or t-modules appear in the class module formulas of Demeslay, Fang, and Taelman. In this talk, we will investigate regulators of the tensor, symmetric, and alternating squares of Drinfeld modules of rank 2, and express them in terms of data from the Drinfeld modules themselves.

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The interface between smooth and symplectic 4-manifolds Special Session A21

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Lisa Piccirillo
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Despite spectacular advances, smooth 4-manifold topology remains very mysterious; there are few positive results and no reasonable guesses towards classifications. Symplectic 4-manifolds however are far more rigid, and in simple circumstances can be completely understood. This session will gather researchers from both smooth and symplectic 4-manifold topology to discuss the interface of these fields, and in particular what light symplectic results can shed on smooth phenomena.

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Lens spaces in the complex projective plane

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Which lens spaces embed smoothly in the complex projective plane, and which collections of lens spaces can be disjointly embedded? Work of Manetti and Hacking-Prokhorov showed that each solution to the Markov equation gives rise to a triple of lens spaces which embed disjointly, and Evans-Smith showed this accounts for all symplectic embeddings of the standard rational homology balls bounded by these lens spaces. Further embeddings of lens spaces have since been exhibited, including two families of triples which embed disjointly due to Lisca-Parma.

I will exhibit some new triples of examples, and will show in particular that all lens spaces $L(p^2, pq-1)$ with $\gcd(p,q) = 2$, or with p odd and $\gcd(p,q) = 1$, embed in $\mathbb{CP}^2$.

This is joint work with Marco Golla.

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Deformation inequivalent symplectic structures and Donaldson's four-six question

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Studying symplectic structures up to deformation equivalences is a fundamental question in symplectic geometry. Donaldson asked: given two homeomorphic closed symplectic four-manifolds, are they diffeomorphic if and only if their stabilized symplectic six-manifolds, obtained by taking products with $\mathbb{CP}^1$ with the standard symplectic form, are deformation equivalent? I will discuss joint work with Amanda Hirschi on showing how deformation inequivalent symplectic forms remain deformation inequivalent when stabilized, under certain algebraic conditions. This gives the first counterexamples to one direction of Donaldson's "four-six" question and the related Stabilizing Conjecture by Ruan. In the other direction, I will also discuss more supporting evidence via Gromov-Witten invariants.

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On the universal cork conjecture

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It is well known that every exotic pair of 1-connected, closed 4-manifolds is related by some cork. Akbulut in 2008 asked if the cork W_1 is *universal*, i.e. relates every exotic pair of 1-connected closed 4-manifolds. It was conjectured that every cork is *non*-universal. However, it was not known of *any* example of non-universal cork. We will show that corks in a large class, including the corks W_n constructed by Akbulut and Yasui, are (even sequentially) non-universal. Our counterexample exploits the construction of 1-connected general type surfaces and symplectic manifolds of non-negative signature. Moreover we will show that if we allow for exotic pairs to have boundary, then every cork is non-universal.

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Exotic Dehn twists on 4-manifolds with Seifert-fibered boundary

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Given a 4-manifold X whose boundary is a Seifert fibered 3-manifold, one can use the circle action on the boundary to define a diffeomorphism on X, called the boundary Dehn twist. Such boundary Dehn twist naturally arises as monodromy of Milnor fibrations. By results of Orson–Powell, these Dehn twists are very often topologically isotopic to the identity. In this talk, we will discuss a proof (using monopole Floer homology) that some of these Dehn twists represents exotic elements of infinite order in the smooth mapping class group.

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Non-smoothable homeomorphisms of 4-manifolds with boundary

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By the work of Freedman and Perron-Quinn, any self-homeomorphism of a (smooth), simply-connected, closed 4-manifold X which induces the trivial map on $H_2(X)$ is isotopic to the identity, and hence is isotopic to a diffeomorphism (i.e. is smoothable). By the work of Saeki and Orson-Powell, if we allow the manifold to have boundary, then there exist self-homeomorphisms which induce the trivial map on homology but are not isotopic to the identity. Building on this, we show that there exist infinite families of smooth, simply-connected, compact 4-manifolds which support self-homeomorphisms that induce the trivial map on homology but are not isotopic to any self-diffeomorphism.

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Obstructing smooth sliceness of links in $\mathbb{CP}^2\#\overline{\mathbb{CP}^2}$

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It is well-known that every knot K is smoothly slice in both $S^2 \times S^2$ and $\mathbb{CP}^2\#\overline{\mathbb{CP}^2}$, i.e. K bounds a smooth disc in the punctured manifolds $(S^2 \times S^2) \setminus \mathrm{Int}(B^4)$ and $(\mathbb{CP}^2\#\overline{\mathbb{CP}^2}) \setminus \mathrm{Int}(B^4)$.

In 1997, Miyazaki and Yasuhara showed that there exists a 2-component link L which is not strongly slice in $S^2 \times S^2$, i.e. L is not boundary of disjoint discs in $(S^2 \times S^2) \setminus \mathrm{Int}(B^4)$. Since their proof uses only classical results, it holds in the locally flat category too.

The argument of Miyazaki-Yasuhara is specific to $S^2 \times S^2$, and does not seem to generalise to $\mathbb{CP}^2\#\overline{\mathbb{CP}^2}$. In joint work with McDonald we address the latter case.

Theorem 1 (M.-McDonald)**.** *There exists a 2-component link L which is not* smoothly *strongly slice in* $\mathbb{CP}^2\#\overline{\mathbb{CP}^2}$.

Our strategy is quite different from Miyazaki-Yasuhara, and we can make it work only in the smooth category. The smooth ingredient of the proof is the smooth genus function on $\mathbb{CP}^2\#\overline{\mathbb{CP}^2}$, due to Ruberman and Morgan-Szabó-Taubes.

[1]This project has received funding from the European Union's Horizon 2020 research and innovation programme under the Marie Sklodowska-Curie grant agreement No. 893282.
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Negative-definite fillings of plumbed manifolds

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Motivated by the study of smoothings of rational surface singularities as well as symplectic fillings of plumbed 3-manifolds, we consider an analogue problem in a purely topological setting. The question of when a rational surface singularity admits a unique smoothing is of particular interest and has led to a conjecture of Kollár which has been proved in some cases. We look at smooth, definite fillings of certain plumbed manifolds and consider the question of which intersection forms can be realized by such fillings. We describe various constructions and an obstruction based on Donaldson's diagonalization theorem. Finally, we present a couple of uniqueness results and discuss their relevance for Kollár's conjecture as well as the problem of embedding lens spaces in certain 4-manifolds. While the main motivation lies in problems from singularity theory and symplectic topology, our results are purely topological in nature and the main techniques used are algebro-combinatorial. This is joint work with Duncan McCoy and JungHwan Park.

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Topology of the Dirac equation on spectrally large three-manifolds

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Dirac operators twisted by flat $U(1)$-connections play a key role as a bridge between geometry and topology. For example, for any Riemannian metric on the n-torus, the Atiyah-Singer index theorem implies that some twisted Dirac operator has kernel; this is the key input in Gromov-Lawson's proof that the n-torus does not admit metrics of positive scalar curvature. In this talk, I will explore refinements of this result about the kernel of such operators for Riemannian three-manifolds satisfying natural constraints of spectral nature. While our main theorem will only involve linear operators, its proof relies on the non-linear analysis of the Seiberg-Witten equations and Floer theory.

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Disoriented homology and braided surfaces

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Let F be a properly embedded surface in the 4-ball B^4. In [1] we defined, based on the handle decomposition of F induced by the radial distance function in B^4 and a choice of disorientations of the handles, the disoriented homology, $DH_*(F)$, of the surface F. Recall that a ribbon surface F can be completely described by its ribbon projection into the 3-sphere S^3. Assuming that all the ribbon singularities are formed by 1-handles of the surface passing through the 0-handles, a disorientation of a 1-handle is given by a choice of orientations of the arcs into which ribbon singularities split its core in such a way that any two consecutive arcs have opposite orientations. We also defined a pairing λ on $DH_1(F)$ generalizing the Gordon-Litherland pairing which is defined for surfaces F that are obtained by pushing spanning surfaces of links into B^4. For a general F, any projected 2-handle is split into subdisks by its intersection with the ribbon immersed subsurface and orienting these subdisks incoherently gives a choice of disorientation of the 2-handle of F.

Theorem 1 (Owens-Strle)**.** *The disoriented homology of $F \subset B^4$ is isomorphic to the shifted reduced homology of the double cover $\Sigma_2(B^4, F)$ of the 4-ball branched along F:*

$$DH_*(F) \cong \widetilde{H}_{*+1}(\Sigma_2(B^4, F)).$$

Moreover, the intersection pairing of $\Sigma_2(B^4, F)$ under this identification agrees with λ.

Though the disoriented homology of a surface F is easily computable from the above mentioned description of F, the computation is particularly simple for ribbon surfaces. The main obstacle to algorithmic computation is then the form in which the surface is described. We show that for braided surfaces, defined by Rudolph in [2], the computation is algorithmic based on the band factoriation of the braid that determines the corresponding orientable ribbon surface. We then generalize the computational machinery to nonorientable ribbon surfaces.

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Unknotting nonorientable surfaces

Anthony Conway
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Patrick Orson
California Polytechnic State University

Mark Powell
University of Glasgow

This talk will describe joint work with Mark Powell and Patrick Orson in which we prove that most closed, nonorientable surfaces in S^4 with knot group $\mathbb{Z}/2$ are topologically unknotted.

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Loop spaces and Khovanov–Rozansky homology

Joshua Wang
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I will present a new connection between the colored Khovanov–Rozansky homology of two-stranded torus knots and the cohomology of the free loop space of a complex Grassmannian.

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Stable parabolic bundles over complex curves and singular instanton Floer homology

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Boyu Zhang
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Stable parabolic bundles are objects in algebraic geometry which have been studied by many people. Singular instanton Floer homology is an invariant of links in 3-manifolds introduced by Kronheimer and Mrowka, which has been used to solve many problems in the low dimensional topology. It turns out the two things are closely related: knowledge on the moduli space of stable parabolic bundles can help the calculation of singular instanton Floer homology. In this talk, we will give a precise description of the cohomology ring of the moduli space of rank 2 parabolic bundles over complex curves. Then we will derive all the "universal relations" for singular instanton Floer homology. This is joint work with Boyu Zhang.

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[1] Aknowledgements: We would like to express our most sincere gratitude to Peter Kronheimer and Tom Mrowka for explaining their work to us. We also want to thank Junliang Shen for helpful conversations about the Mumford relations.

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Rational homology ball symplectic fillings of spherical 3-manifolds

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The family of spherical 3-manifolds can be identified with the homeomorphism types of the links of quotient surface singularities. It follows that every spherical 3-manifold, viewed as the oriented link of a normal surface singularity, has a canonical contact structure ξ_{can} (also known as the Milnor fillable contact structure), which is unique up to contactomorphism [1]. The lens spaces are precisely the links of cyclic quotient surface singularities.

The lens space $L(p, q)$ equipped with a contact structure ξ, admits a rational homology ball *symplectic* filling if and only if $\frac{p}{q} = \frac{m^2}{mh-1}$ for some coprime integers $0 < h < m$, and ξ is contactomorphic to ξ_{can}. Independent alternative proofs of this fact recently appeared in [2, 3, 4]. We show that this result extends to all spherical 3-manifolds as follows:

Theorem 1. *A spherical* 3*-manifold* Y *equipped with a contact structure* ξ*, admits a rational homology ball symplectic filling, if and only if* Y *is orientation-preserving diffeomorphic to a lens space* $L(m^2, mh-1)$ *for some coprime integers* $0 < h < m$*, and* ξ *is contactomorphic to* ξ_{can}*.*

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Five tori in the four-dimensional sphere

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Ivanšić proved [1] that there is a link L of five tori in S^4 with hyperbolic complement. We describe L and show that $L \subset S^4$ is in many aspects similar to the Borromean rings in S^3. In particular: (1) any two tori in L are unlinked, (2) the complement $M = S^4 \setminus L$ is hyperbolic, (3) it has many symmetries, (4) the double branched covering over L has geometry $\mathbb{H}^2 \times \mathbb{H}^2$, (5) the fundamental group of M has a nice presentation via commutators, (6) the Alexander ideal is explicit, (7) every generic first cohomology class is represented by a circle-valued perfect Morse function and the Betti numbers of all the infinite cyclic covers can be computed, and (8) by longitudinal Dehn surgery along L we get a closed 4-manifold with fundamental group $\mathbb{Z}^5$.

This leads also to the first descriptions of a cusped hyperbolic 4-manifold as a complement of tori in $\mathbb{RP}^4$ and of some explicit Lagrangian tori in the product of two surfaces.

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Recent Developments on Certain Evolution Partial Differential Equations
Special Session A22

Gigliola Staffilani
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Sergio Polidoro
Università di Modena e Reggio Emilia, ITALY

The goal of this section is to bring together researchers whose interest is in the study of a variety of evolution equations that represent fundamental physical phenomena. The study of these equations involves harmonic and Fourier analysis, geometry and stochastic calculus. We expect our speakers to give colloquium style talks involving the analysis of the Kolmogorov's operator, of many body systems of particles and wave packets, of invariant measures, and of singularity formations. Many of the problems discussed could be collected under the wide umbrella of kinetic theory and wave turbulence theory, two subjects that in recent years have seen an incredible number of fundamental advances.

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On the effective dynamics of Bose-Fermi mixtures

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Joseph Miller
Department of Mathematics, The University of Texas at Austin

Nataša Pavlović
Department of Mathematics, The University of Texas at Austin

Investigating degenerate mixtures of bosons and fermions is an extremely active area of research in experimental physics for constructing and understanding novel quantum bound states such as those in superconductors, superfluids, and supersolids. These ultra-cold Bose-Fermi mixtures are fundamentally different from degenerate gases with only bosons or fermions. They not only show an enriched phase diagram, but also a fundamental instability due to energetic considerations coming from the Pauli exclusion principle. Inspired by this activity in the physics community, recently we started exploring the mathematical theory of Bose-Fermi mixtures. One of the main challenges is understanding the physical scales of the system that allow for suitable analysis. We will describe how we overcame this challenge by identifying a novel scaling regime in which the fermion distribution behaves semi-clasically, but the boson field remains quantum-mechanical. In this regime, the bosons are much lighter and more numerous than the fermions.

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Quantum diffusion and approximate semigroups

Felipe Hernández
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The random Schrödinger equation is a toy model for waves in random media and for the motion of an electron in an imperfect metal. The evolution of smooth observables for this equation are described by a linear Boltzmann equation, which gives a classical picture for the motion of electrons in terms of a Markov process with discontinuous jumps in the momentum. In this talk I will introduce both of these equations and discuss some of the ideas needed to rigorously the classical description from the quantum mechanical system.

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Harnack inequalities for kinetic integral equations

Francesca Anceschi
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Università Politecnica delle Marche

In this talk, based on a joint project with M. Piccinini and G. Palatucci, I will present some recent results for weak solutions to the kinetic integral equation

$$\big[\partial_t + v \cdot \nabla_x\big] f = \mathcal{L}_v f,$$

where the diffusion term $\mathcal{L}_v$ is an integro-differential operator, whose nonnegative kernel is of fractional order $s \in (0,1)$ and has measurable coefficients. In particular, I will focus on the proof of a strong Harnack inequality for nonnegative weak solutions f of the type

$$\sup_{Q^-} f \;\leq\; c \inf_{Q^+} f,$$

where $Q^{\pm}$ are suitable slanted cylinders defined in accordance with the non-Euclidean geometry underlying the operator and no a priori boundedness for the solution is assumed, as it is usually done in literature.

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Self-similar blow up profiles for fluids via physics-informed neural networks

Javier Gómez-Serrano [1]
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In this talk I will explain a new numerical framework, employing physics-informed neural networks, to find a smooth self-similar solution (or asymptotically self-similar solution) for different equations in fluid dynamics, such as Euler or Boussinesq. The new numerical framework is shown to be both robust and readily adaptable to several situations. Joint work with Tristan Buckmaster, Gonzalo Cao-Labora, Ching-Yao Lai and Yongji Wang.

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Non-radial implosion for compressible Euler and Navier-Stokes in $\mathbb{T}^3$ and $\mathbb{R}^3$

Gonzalo Cao-Labora, Gigliola Staffilani , Jia Shi [1]
Department of Mathematics, MIT

Javier Gómez-Serrano,
Department of Mathematics, Brown University

We will discuss the smooth, non-radial solutions of the compressible Euler and Navier-Stokes equation that develop an imploding finite time singularity. The construction is motivated by the radial imploding solutions from Merle–Raphaël–Rodnianski–Szeftel (see [1],[2],[3]), and Cao-Labora–Buckmaster–Gómez-Serrano (see [4]) but is flexible enough to handle both periodic and non-radial initial data. This is a joint work with Gonzalo Cao-Labora, Javier Gómez-Serrano, and Gigliola Staffilani [5].

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[1]Aknowledgements: GCL and GS have been supported by NSF under grant DMS-2052651. GCL, JGS and JS have been partially supported by the MICINN (Spain) research grant number PID2021–125021NA–I00. JGS has been partially supported by NSF under Grants DMS-2245017 and DMS-2247537, by the AGAUR project 2021-SGR-0087 (Catalunya). JS has been partially supported by an AMS-Simons Travel Grant. GS has been supported by the Simons Foundation through the Simons Collaboration on Wave Turbulence. JGS is also thankful for the hospitality of the MIT Department of Mathematics, where parts of this paper were done. We thank Tristan Buckmaster for useful conversations.

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On the effect of the Coriolis force on the enstrophy cascade

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Geophysical fluid dynamics refers to the fluid dynamics of naturally occurring flows, such as oceans and planetary atmospheres on Earth and other planets. These flows are primarily characterized by two elements: stratification and rotation. In this article we investigate the effects of rotation on the dynamics, by neglecting stratification, in a 2D model where we incorporate the effects of the planetary rotation by adopting the β-plane approximation, which is a simple device used to represent the latitudinal variation in the vertical component of the Coriolis force.

We consider the well-known 2D β-plane Navier-Stokes equations in the statistically forced case.

Our problem addresses energy-related phenomena associated with the solution of the equations. To maintain the fluid in a turbulent state, we introduce energy into the system through a stochastic force. In the 2D case, a scaling analysis argument indicates a direct cascade of enstrophy and an inverse cascade of energy. We compare the behaviour of the direct enstrophy cascade with the 2D model lacking the Coriolis force, observing that at small scales, the enstrophy flux from larger to smaller scales remains unaffected by the planetary rotation, confirming experimental and numerical observations. In fact, this is the first mathematically rigorous study of the above equations. In particular, we provide sufficient conditions inspired by [1, 2] to prove that at small scales, in the presence of the Coriolis force, the so-called third-order structure function's asymptotics follows the third-order universal law of 2D turbulence without the Coriolis force. We also prove well-posedness and certain regularity properties necessary to obtain the mentioned results.

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Local behaviour of non-local hypoelliptic equations

Amélie Loher [1]
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We discuss local regularity properties of solutions to a class of non-local kinetic equations in divergence form. In particular, we explain why it is not clear if the Strong Harnack inequality holds for solutions of non-local equations, and how we manage to derive it. The key is to obtain a non-local-to-local bound on the tail terms that naturally affect the function values inside any local domain.

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On the De Giorgi-Nash-Moser regularity theory for hypoelliptic operators

Annalaura Rebucci
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We present new results that extend the De Giorgi-Nash-Moser theory to hypoelliptic equations of Kolmogorov (kinetic) type with rough coefficients. A key ingredient to prove these results is a Poincaré inequality, which we derive from the construction of suitable trajectories (see [1]). The trajectories we rely on are quite flexible and allow us to consider equations with an arbitrary number of commutators and whose diffusive part is either local (second-order) or non-local (fractional order). Following [2], we later combine the Poincaré inequality with a L^2-L^∞ estimate, a Log-transformation and a classical covering argument (Ink-Spots Theorem) to deduce Harnack inequalities and Hölder regularity along the line of De Giorgi method.

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Generalised Schrödinger equations

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We consider the Cauchy problem for some local and nonlocal Schrödinger equations and establish some basic properties of their solutions.

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Interpolative boundedness estimates for kinetic integral equations

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We investigate local properties of weak solutions to a wide class of kinetic equations where the diffusion term in velocity is an integro-differential operator, having nonnegative kernel of fractional order $s \in (0,1)$ with merely measurable coefficients. We provide an explicit local boundedness estimate by combining together a suitable gain of integrability and a kinetic Caccioppoli-type inequality, by giving also a precise control of the long-range interactions arising from the nonlocal behaviour of the involved diffusion operator. Moreover, we will show how to deduce a similar result in the case when a p-growth conditions is assumed on the diffusion operator. This is based on a joint project in collaboration with F. Anceschi and G. Palatucci.

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Asymptotic average solutions and a Pizzetti-type theorem for hypoelliptic PDEs

Alessia E. Kogoj
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By using a Pizzetti's 1909 idea for the classical Laplacian, we introduce a notion of asymptotic average solutions. This notion enables the pointwise solvability of every Poisson equation $Lu(x) = f(x)$ with continuous data f, where L belongs to a class of hypoelliptic linear partial differential operators whose classical solutions can be characterized in terms of mean value formulae.

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Lateral boundary conditions for a Kolmogorov-type PDE

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We consider a hypoelliptic Kolmogorov-type PDE motivated by a particle under the influence of a white noise force. We are interested in imposing Dirichlet conditions at a side (lateral) boundary. Specifically, we consider

$$\begin{aligned}\frac{\partial u}{\partial t}(t,x,y) &= \frac{1}{2}\frac{\partial^2 u}{\partial y^2}(t,x,y) + b(y)\frac{\partial u}{\partial x}(t,x,y) \qquad t>0,\ 0<x<\infty,\ \surd\infty<y<\infty \\ u(0,x,y) &= u_\circ(x,y) \qquad 0<x<\infty,\ \surd\infty<y<\infty \\ u(t,0,y) &= u_\partial(t,y). \qquad t>0,\ \surd\infty<y<\infty\end{aligned}$$

where $u_\circ$ is an initial condition and u_∂ is a lateral boundary condition. We assume that this PDE is *hypoelliptic* and that the lateral boundary is accessible. Namely, we consider functions b similar to

$$b(y) = \surd 2 + \tanh(y);$$

these are bounded, negative, and strictly increasing in y at all points.

We construct a simple Gaussian heat kernel K inside the domain, and investigate a boundary-layer kernel K^∂. We show that this boundary layer heat kernel has a novel jump condition (stemming from a careful Laplace-type asymptotic analysis) which captures the structure of the above PDE u near the lateral boundary. We outline a polynomial refinement of the heat kernels, and then construct a Volterra equation stemming from the variation of parameters ansatz

$$\begin{aligned}u(t,x,y) &= \int_{s=0}^{t}\int_{x_\circ>0,\,-\infty<y_\circ<\infty} K_{x_\circ,y_\circ}(t\surd s,x,y)\psi(s,x_\circ,y_\circ)dx_\circ\,dy_\circ\,ds \\ &\quad+\int_{s=0}^{t}\int_{-\infty<y_\circ<\infty} K^\partial_{y_\circ}(t\surd s,x,y)\psi^\partial(s,y_\circ)ds\,dy_\circ \\ &\quad+\int_{x_\circ>0,\,-\infty<y_\circ<\infty} K_{x_\circ,y_\circ}(t,x,y)u_\circ(x_\circ,y_\circ)dx_\circ\,dy_\circ\end{aligned}$$

where ψ and ψ^∂ are to be found. The boundary condition corresponding to ψ^∂ reflects the novel jump condition. We construct a Volterra equation and show (somewhat nonstandard) convergence of the Volterra equation for ψ and ψ^∂.

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Global existence for the generalized derivative NLS

Nicola Visciglia
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In this talk, based on a joint work with M. Hayashi (Kyoto U.) and T. Ozawa (Waseda U.), we shall discuss a blow-up criterion for H^2 solutions to the generalized DNLS

$$\begin{cases} iu_t + u_{xx} + i|u|^{2\sigma}u_x = 0, \sigma > 1, (t,x) \in \mathbf{R} \times \mathbf{T} \\ u(0) = \varphi \in H^2(\mathbf{T}) \end{cases}$$

For $\sigma > 1$ the equation is no more completely integrable and hence the globalization argument is not trivial. The existence and uniquness of a local solution has been constructed in [1]. Our main result is the following blow-up alternative which improves the one described in [1].

Theorem 1. *Let $\varphi \in H^2(\mathbf{T})$. For the solution $u \in C([0, T_{max}), H^2(\mathbf{T}))$ we have the following alternative:*

(1) $T_{\max} = \infty$,
(2) $T_{\max} < \infty$ *implies* $\limsup_{t\ T_{\max}} \|u(t)\|_{H^1(\mathbf{T})} = \infty$.

As a consequence of the result above and the Hamiltonian structure of the equation, we get the following global existence result for small data, which provides a positive answer to a question raised in [1].

Corollary 2. *There exists $\delta > 0$ such that if $\varphi \in H^2(\mathbf{T})$ satisfies $\|\varphi\|_{H^1(\mathbf{T})} < \delta$, then there exists a unique global solution $u \in C(\mathbf{R}, H^2(\mathbf{T}))$.*

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Wave kinetic theory for the forced/dissipated NLS equation

Ricardo Grande
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We will present some recent developments in the justification of kinetic equations in the presence of forcing and dissipation. Such settings are of particular physical relevance as they allow the study of cascades: the transfer of energy from large scales to small scales.

In this talk, we provide the first rigorous justification of such a kinetic equation in the case of a wave system governed by the cubic Schrödinger equation with a stochastic forcing and viscous dissipation. We will describe various regimes depending on the relative strength of the dissipation, the forcing and the nonlinear interactions, which give rise to different kinetic equations. Based on joint work with Zaher Hani.

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Arithmetic and Geometry of Low Dimensional Algebraic Varieties
Special Session A23

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Algebraic Curves and Algebraic Surfaces are classical and fundamental topics in Algebraic Geometry, featuring profound and, at times, astonishing connections to various disciplines such as Differential Geometry, Symplectic Geometry, Complex Geometry, Commutative Algebra, and Number Theory. While many historical questions have been addressed, several open questions and challenging problems still lie before us.

The Special Session *Arithmetic and Geometry of Low Dimensional Algebraic Varieties*, scheduled on July 23-24, aims to convene numerous experts, with a specific focus on young scientists, to foster collaboration and facilitate discussions on the most recent advancements in these vital research domains.

A broad spectrum of topics will be explored, encompassing special families of curves, K3 and Abelian surfaces, surfaces of general type, Hodge structures, surfaces fibred in curves, automorphism groups, moduli spaces, rational points, and computational aspects. Special attention will be directed towards examining the interplay and connections between the arithmetic and geometric perspectives.

This Special Session will promote a robust exchange of ideas among individuals engaged in various aspects of the theory of Low-Dimensional Algebraic Varieties. Additionally, it will provide a valuable opportunity for Ph.D. students and postdocs to acquire insights into the latest results and techniques within these dynamic and rapidly evolving fields.

For more information visit
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Numerically and cohomologically trivial automorphisms of elliptic surfaces

Fabrizio Catanese, joint work with Wenfei Liu and Matthias Schütt [1]
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Let X be a compact connected complex manifold: the automorphism group $\mathrm{Aut}(X)$ is a finite dimensional complex Lie Group, and we have a chain of subgroups

$$\mathrm{Aut}^0(X) \triangleleft \mathrm{Aut}_{\mathbb{Z}}(X) \triangleleft \mathrm{Aut}_{\mathbb{Q}}(X) \triangleleft \mathrm{Aut}(X), \quad \text{where}$$

i) $\mathrm{Aut}^0(X)$ is the connected component of the identity,

ii) $\mathrm{Aut}_{\mathbb{Z}}(X) := \{\sigma \in \mathrm{Aut}(X) \mid \sigma$ induces the trivial action on $H^*(X, \mathbb{Z})\}$ is the group of cohomologically trivial automorphisms, while the group of numerically trivial automorphisms is

iii) $\mathrm{Aut}_{\mathbb{Q}}(X) := \{\sigma \in \mathrm{Aut}(X) \mid \sigma$ induces the trivial action on $H^*(X, \mathbb{Q})\}$.

These subgroups are important for applications to Hodge Theory and to period mappings.

If X is Kähler, $\mathrm{Aut}_{\mathbb{Q}}(X)/\mathrm{Aut}^0(X)$ and $\mathrm{Aut}_{\mathbb{Z}}(X)/\mathrm{Aut}^0(X)$ are finite groups, and with Wenfei Liu [CatLiu21] we started to investigate lower and upper bounds for the sizes of these groups in the case of surfaces S according to Kodaira dimension and in terms of the invariants of S.

There remained then the problem of studying the case of properly elliptic surfaces, i.e., minimal surfaces of Kodaira dimension one, which have a canonical elliptic fibration $f : S \to B$.

When we began our research, the state of the literature seemed to indicate that the only cases requiring a deeper investigation would be the ones with smallest invariants, as:

Theorem 1. *Assume that S is minimal of Kodaira dimension 1), with $\chi(S) = 0$: then S is isogenous to a higher elliptic product (that is, S is a free quotient quotient $(C \times E)/G$ where E is elliptic and C has genus ≥ 2).*

(I) *In the case of the pseudo-elliptic surfaces (the case where $Aut^0(S)$ is infinite, equivalently G acts by translations on E), $Aut_{\mathbb{Z}}(S) = Aut^0(S)$, except in the case*

(I-a) *$G = \mathbb{Z}/2m$, where m is an odd integer,*

(I-b) *$C/G = \mathbb{P}^1$ and $C \to \mathbb{P}^1$ is branched in four points with local monodromies $\{m, m, 2, 2\}$: for these we have $|Aut_{\mathbb{Z}}(X)/Aut^0(X)| = 2$.*

(II) *If $Aut_{\mathbb{Z}}(S)$ is finite and nontrivial, then it is Abelian with $|\mathrm{Aut}_{\mathbb{Z}}(S)| \leq 9$, and the case where $\mathrm{Aut}_{\mathbb{Z}}(S) \cong \mathbb{Z}/6\mathbb{Z}$ does actually occur.*

The investigation of all the possible groups occurring in (II) above requires the Reidemeister Schreier method and heavy computations.

We discovered later that there were counterexamples to existing theorems, and also for higher values of $\chi(S)$ there was no upper bound for the order $|\mathrm{Aut}_{\mathbb{Q}}(S)|$ of the finite group $\mathrm{Aut}_{\mathbb{Q}}(S)$:

Theorem 2. *There are non-isotrivial properly elliptic surfaces S with $p_g > 0$ admitting numerically trivial automorphisms of order any given prime p.*

For $p > 5$, our examples satisfy $\chi(S) \geq (p^2 - 1)/24$.

The analysis of the case of cohomologically trivial automorphisms, and especially of upper bounds according to $\chi(S)$, is still work in progress.

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Irreducibility of Severi varieties on toric surfaces

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Severi varieties parametrize integral curves of fixed geometric genus in a given linear system on a surface. In this talk, I will discuss the classical question of whether Severi varieties are irreducible and its relation to the irreducibility of other moduli spaces of curves. I will indicate how tropical methods can be used to answer such irreducibility questions. The new results are from ongoing joint work with Xiang He and Ilya Tyomkin.

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On mixed surfaces: construction and examples

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In the last two decades there has been a growing interest in those surfaces (and varieties) birational to the quotient of a product of curves of genus at least 2 modulo the action of a finite group, and several new surfaces and varieties have been constructed in this way. These split naturally in two cases: the *unmixed* case, where each element of the group acts diagonally; and the *mixed* case, where there are elements permuting some factors besides those acting diagonally.

In the talk we will focus on surfaces and discuss the mixed case, namely: let C be a Riemann surface of genus at least 2 and $G < \mathrm{Aut}(C \times C)$ be a finite group with a mixed action (i.e., there are elements in G swapping the factors), then the quotient surface $(C \times C)/G$ is a *mixed quotient* and the minimal resolution of its singularities is a *mixed surface*. We will investigate the geometry of mixed quotients and surfaces, discuss how this is encoded in their "*algebraic data*", and present an algorithm to classify the irregular mixed surfaces with given invariants.

This is based on a joint work with M. Alessandro and C. Gleißner.

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Correspondences acting on constant cycle curves on K3 surfaces

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Constant cycle curves on K3 surfaces X have been introduced by Huybrechts as curves whose points all define the Beauville-Voisin class in the Chow group of X. In this talk, we introduce correspondences $Z \subseteq X \times X$ over $\mathbb{C}$ acting on the group $\mathrm{ccc}(X)$ of cycles generated by constant cycle curves. We construct for any $n \geq 2$ and any very ample line bundle L a locus $Z_n(L) \subseteq X \times X$ of expected dimension 2, which yields a correspondence acting on the group $\mathrm{ccc}(X)$, when it has the expected dimension. We then discuss examples for low n and answer the problem of non-emptyness for $(X, |C|)$ very general pricipally polarized K3, C smooth genus g curve with general gonality and n less or equal to the general gonality. Part of the results are work in progress with Andreas Knutsen.

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Higher rank lattice cohomology

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Lattice cohomology provides a powerful framework for constructing invariants of the link of a normal surface singularity. When the link is a rational homology sphere, these invariants can be encoded combinatorially by an infinite rooted tree. Recent results have focused on refining these invariants by introducing weights on the vertices of the graded tree. In this talk, I will show how to obtain infinitely many such invariants by considering weighted rooted trees arising from the data of a root lattice of rank at least 2.

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Tropical and logarithmic enumerative geometry of curves

Renzo Cavalieri, Renzo Cavalieri [1]
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I will present some joint work with Hannah Markwig and Dhruv Ranganathan, in which we interpret double Hurwitz numbers as intersection numbers of the double ramification cycle with a logarithmic boundary class on the moduli space of curves. This approach removes the "need" for a branch morphism and therefore allows the generalization to related enumerative problems on moduli spaces of pluricanonical divisors - which have a natural combinatorial structure coming from their tropical interpretation. I will discuss some generalizations springing out from this approach that are currently being pursued in joint work with Hannah Markwig and Johannes Schmitt.

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Examples of non-rigid modular vector bundles on hyperkähler manifolds.

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We exhibit examples of slope-stable and modular vector bundles on a hyperkähler manifold of $K3^{[2]}$-type. These are obtained by performing standard linear algebra constructions on the examples studied by O'Grady of (rigid) modular bundles on the Fano varieties of lines of a general cubic 4-fold and the Debarre-Voisin hyperkähler. Interestingly enough, these constructions are almost never infinitesimally rigid, and more precisely we show how to get (infinitely many) 20 and 40 dimensional families. This is a joint work with Claudio Onorati.

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Beauville Varieties and Groups

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Beauville surfaces are a certain class of rigid regular surfaces of general type that can be purely described by combinatorial methods. These surfaces play an improtant role in algebraic geometry as well as group theory, and have been studied extensively in recent years. In this presentation, I will report on a joint project with Federico Fallucca, focusing on exploring the higher-dimensional counterparts of Beauville surfaces.

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Asymptotic lines on the moduli space of curves

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The aim is to study the second fundamental form associated with the image of the period map of curves started many years ago (see [1]). We present some computational improvements that allow to study asymptotic lines in the tangent of the moduli space $\mathcal{M}_g$ of the curves of genus g. The asymptotic lines are those tangent directions that are annihilated by the second fundamental form induced by the Torelli map. We give examples of asymptotic lines for any $g \geq 4$ and we study their rank. The rank $r(v)$ of a tangent direction v at $[C] \in \mathcal{M}_g$ is defined to be the rank of the cup product map associated to the infinitesimal deformation map, that is the infinitesimal variation of Hodge structure $v : H^{1,0}(C) \to H^{0,1}(C)$, in that direction. We show that if $v \neq 0$ and $r(v) \leq \text{cliff}(C)$ where $\text{cliff}(C)$ is the Clifford index of C, then v is not asymptotic and we study the case when $r(v) = \text{cliff}(C)$. Finally all asymptotic directions of rank 1 are determined and an almost complete description of the rank 2 case is given.
It is a report on the joint work ([2]) with Elisabetta Colombo and Paola Frediani .

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Surfaces with canonical map of odd degree

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Let S be a smooth minimal complex surface of general type. Assume that the canonical map is generically finite of degree d onto a surface Σ. By classical results of Xiao and Beauville, it is known that $d \leq 8$ if the geometric genus $p_g(S)$ is large enough, and for $d = 8$ there are examples with unbounded p_g, showing that the bound is sharp. Families of examples with $d = 2, 4, 6$ and unbounded p_g are also known, while there are only sporadic examples with $d \geq 3$ odd.

I will report on work in progress aiming at explaining this asymmetry. Under the additional assumption that the general canonical curve of S is smooth, we are able to:

(1) give an explicit lower bound on p_g in case $d = 7$;
(2) in the case when the canonical image Σ is ruled by lines, show that $p_g(S) \leq d + 2$ and Σ is a cone in $\mathbb{P}^{p_g(S)-1}$ over the rational normal curve of degree $p_g(S)$ ✓ 2.

For $d = 3$ surfaces as in (2), i.e., with Σ ruled by lines, were completely described by Mendes Lopes and Pardini and, independently, by Starnone: there are two main examples, with $p_g = 5$, $K^2 = 9$ and $K^2 = 8$, $p_g = 4$ respectively, and examples with lower invariants are obtained by degenerating these two types of surfaces.

For $d = 5$ and Σ ruled by lines a similar picture emerges from a theoretical analysis of the situation. However, quite surprisingly, we are able to rule out the existence of most of the possibilities and we have not yet been able to produce examples with $p_g > 3$.

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Anosov Representations and Higher Teichmüller Theory
Special Session A24

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The conference aims to bring together senior researchers and early career mathematicians from the areas of hyperbolic geometry and higher Teichmüller theory to explore the interplay between these two areas and highlight the recent breakthroughs and connections between them.

The fields of hyperbolic geometry and of higher Teichmüller theory are major directions of current and ongoing mathematical research. Starting in the 70's, Thurston's work had a revolutionary impact on low-dimensional geometry and topology, with hyperbolic geometry playing a central role. In particular, the study of Teichmüller space, the space of (marked) complex structures on a surface S, played an important role as the moduli space of convex cocompact hyperbolic structures on 3-manifolds with boundary S. Work of Goldman, Choi and others on Thurston's (G, X)-structures generalized this to study more general geometric structures, including convex projective structures. From a completely different direction, Hitchin used a gauge theoretic approach to study the moduli space $R(S, G) = \mathrm{Hom}(\pi_1(S), G)//G$ of representations of the fundamental group of a surface into higher rank Lie groups G. In particular Hitchin showed that certain components $H_n(S)$ of $R(S, PSL_n(\mathbb{R}))$, now called Hitchin components, are natural generalizations of such examples. Infact, $H_2(S)$ is the Teichmüller space $\mathrm{Teich}(S)$, and $H_3(S)$ is the space of convex projective structures. Work of Fock and Goncharov used methods from cluster algebras and algebraic geometry to describe moduli spaces of positive representations which they named higher Teichmüller spaces and included Hitchin components.

In his seminal work, Labourie used a dynamical approach to define the notion of Anosov representations and prove many of their important properties. Anosov groups incorporate the prior examples of higher Teichmüller spaces, generalize them, and are now a major field of study, combining elements of the theory of Higgs bundles, hyperbolic geometry, the theory of Lie groups, and dynamics. The theory has been further developed in papers by Bochi-Potrie-Sambarino, Danciger-Gueritaud-Kassel, Gueritaud-Guichard-Kassel-Wienhard, Guichard-Wienhard, Guichard-Labourie-Wienhard, Kapovich-Leeb-Porti, and others. In particular this work showed that Anosov groups are the natural analogue in higher rank of convex cocompact representations into rank one Lie groups. Over the past two decades much work has shown that phenomena for cocompact hyperbolic manifolds persist for Anosov subgroups.

A major theme of the conference is to explore this further and consider to what extent phenomena for cocompact hyperbolic manifolds persist for Anosov subgroups.

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Record breakers, slack calculus, and the Benoist limit cone

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We study the boundary of the Benoist limit cone of a positive representation from a surface group into a semi-simple Lie group G, focusing on the cases $G = SL_{n+1}R$ and $G = (PSL_2R)^n$. The limit cone, a cone in the positive Weyl chamber, is obtained by plotting the Jordan projections (essentially, the log-eigenvalues) of the elements of the group and then taking the closure of the cone spanning these points. It turns out to be convex. We investigate the question of which elements of the group (and more generally, which geodesic currents) appear on the boundary.

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Proper affine deformations of positive representations

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We prove that every positive Anosov representation of a free group into $SO(2n, 2n \surd 1)$ admits a family of R^{4n-1}-valued cocycles defining proper affine actions on R^{4n-1}. We construct fundamental domains bounded by generalized crooked planes for these affine actions. This is joint work with Jean-Philippe Burelle.

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Anosov components of triangle reection groups in rank 2

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We seek to describe rank 2 character variety components of triangle reection groups which contain Anosov representations, and to provide a geometric description of any Anosov boundaries. Building on the work of Lee, Lee and Stecker in $SL(3, \mathbb{R})$, we observe fundamentally new phenomena: for example in $SL(3, \mathbb{C})$, the Anosov component that contains the $SL(3, \mathbb{R})$ Hitchin component has a nontrivial boundary. This project is ongoing, and the talk will cover recent progress and computational work.

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Cubulated hyperbolic groups admit Anosov representations

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While it is still not known if there is any Gromov-hyperbolic right-angled Coxeter group that cannot be realized as a convex cocompact group of isometries of some large-dimensional hyperbolic space, it is a result of Danciger–Guéritaud–Kassel that any Gromov-hyperbolic right-angled Coxeter group admits an Anosov representation. In joint work with Balthazar Fléchelles, Theodore Weisman, and Feng Zhu, we extend the latter result to all Gromov-hyperbolic quasi-convex subgroups of right-angled Coxeter groups.

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Manifolds without real projective structures but with Anosov representations

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Surfaces admit many real projective structures. On the other hand, in dimension at least 3 there are manifolds that do not admit any real projective structure at all. Previously known examples of such "non–projective" manifolds are small, in the sense that their fundamental groups are virtually cyclic. In this talk we construct "non–projective" manifolds in every dimension at least 5 whose fundamental groups are non-elementary Gromov hyperbolic groups. While these groups do not arise from real projective geometry, they are known to admit Anosov representations because they are cubulated.

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Foliations in $\mathrm{PSL}_4(\mathbb{R})$ Teichmüller Theory

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In 2008, Guichard and Wienhard carried out a beautiful study of Hitchin representations of surface groups Γ in $\mathrm{PSL}_4(\mathbb{R})$ [1]. They began by singling out a domain of discontinuity Ω^1_ρ in $\mathbb{RP}^3$ for any Hitchin representation $\rho : \Gamma \quad \mathrm{PSL}_4(\mathbb{R})$. The motivating observation of Guichard-Wienhard's work is that Ω^1_ρ has nested $\rho(\Gamma)$-invariant foliations $\mathcal{F}_{\mathrm{pcf}}$ and $\mathcal{G}_{\mathrm{pcf}}$ whose leaves are properly convex subsets of projective planes and lines, respectively. These foliations are curious and remarkable objects.

We discuss a finiteness counterpart to Guichard-Wienhard's work: we enumerate all geometrically similar foliations of Ω^1_ρ. This proves a strong form of rigidity for Guichard-Wienhard's qualitative characterizations of projective structures associated to $\mathrm{PSL}_4(\mathbb{R})$ Hitchin representations in [1]. A bit more specifically, we begin by constructing a new $\rho(\Gamma)$-invariant foliation $\mathcal{G}_{\mathrm{tcf}}$ of Ω^1_ρ whose leaves are properly convex subsets of projective lines. Our main theorem is that these are all such foliations [2]:

Theorem 1 (Exactly Two)**.** *Let $\rho : \Gamma \quad \mathrm{PSL}_4(\mathbb{R})$ be Hitchin. Then $\mathcal{G}_{\mathrm{pcf}}$ and $\mathcal{G}_{\mathrm{tcf}}$ are the only $\rho(\Gamma)$-invariant foliations of Ω^1_ρ by properly embedded projective line segments.*

$\mathcal{F}_{\mathrm{pcf}}$ is the unique foliation of Ω^1_ρ by properly embedded convex domains in projective planes.

Our proof involves developing a detailed general picture for the structure of the domain of discontinuity Ω^1_ρ for ρ that we will explain.

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Finite-sided Dirichlet domains for Anosov representations

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Dirichlet domains provide polyhedral fundamental domains in hyperbolic space for discrete subgroups of the isometry group. For geometrically finite subgroups these domains are finite-sided, and for convex-cocompact subgroups these domains are finite sided in a stronger sense: they also define a compact fundamental domain on some open domain of discontinuity in the compactification of the hyperbolic space.

Selberg [1] introduced a similar construction of a polyhedral fundamental domain for the action of discrete subgroups of the higher rank Lie group $SL(n, \mathbb{R})$ on the projective model of the associated symmetric space. His motivation was to study uniform lattices, for which these domains are finite-sided. However these domains can also be studied for smaller subgroups, and we will consider the following question asked by Kapovich [2]: for which Anosov subgroups are these domains finite-sided ?

Anosov subgroups are hyperbolic discrete subgroups satisfying strong dynamical properties, but are not lattices in higher rank. We show an example of an Anosov subgroup for which the fundamental domain constructed by Selberg has infinitely many sides and provide a sufficient condition on the limit cone of an Anosov subgroup to ensure that the domain is finitely sided in a strong sense. We obtain in particular the following:

Theorem 1. *Let $\Gamma \subset Sp(2n, \mathbb{R})$ be a $\{n\}$-Anosov representation. The Dirichlet-Selberg fundamental domains are all finite sided in a strong sense.*

Our techniques generalize to give a sufficient condition for an Anosov subgroup to admit finite-sided Dirichlet domains for certain Finsler metrics on the associated symmetric space. In particular we obtain the following result:

Theorem 2. *Let $\Gamma \subset SL(n, \mathbb{R})$ be a Borel-Anosov representation. The Dirichlet domains constructed using the Hilbert metric on the symmetric space are all finite-sided in a strong sense.*

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Hausdorff dimension of hyperconvex representations of surface groups

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A discrete and faithful representation of a surface group in PSL(2,C) is said to be quasi-Fuchsian when it preserves a Jordan curve on the Riemann sphere. Classically these objects lie at the intersection of several areas of mathematics and have been studied for example by means of complex dynamics, Teichmüller theory, and 3-dimensional hyperbolic geometry. From a dynamical perspective, an interesting invariant of such representation is the Hausdorff dimension of the invariant Jordan curve which is typically a very fractal object. A celebrated result of Bowen establishes that this number is 1 if and only if the quasi-Fuchsian representation is actually Fuchsian, that is, it is conjugate in PSL(2,R). I will first provide some context and describe this classical picture and then report on recent joint work with James Farre and Beatrice Pozzetti where we prove a generalization of Bowen's result for the much larger class of hyperconvex representations of surface groups in PSL(d,C).

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Divergent extended geometrically finite representations via flows

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The notion of extended geometrically finite representations introduced by Weisman [3] generalizes Anosov representations by studying the convergence dynamics on group boundary extensions. We prove that divergent, extended geometrically finite representations can be interpreted as admitting dominated splitting over certain flow spaces (restricted Anosov representations in the sense of Tholozan–Wang [1]). In particular, the example constructed in [1] provides a divergent geometrically finite representation. The talk is based on the work [2]

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On the boundary of Higher Teichmuller spaces

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Thurston's length compactification of the classical Teichmuller space can be generalized to general character varieties of finitely generated groups, using Weyl-chamber valued length functions. The boundary points can be interpreted using representations over non archimedean real closed fields, acting on real affine buildings. This provides compactifications for the higher Teichmuller spaces. I will present some results on the structure of the boundary based on joint work withMarc Burger, Alessandra Iozzi and Beatrice Pozzetti.

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Hilbert geometry over non-Archimedean ordered fields

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Convex projective geometry is a rich subject and provides an important generalisation of Riemannian geometry. Convex projective surfaces arise as a geometric interpretation of Hitchin representations in $SL(3, \mathbb{R})$. Their Hilbert metric encodes important information about the representation. Understanding degenerations of convex projective structures on a surface naturally leads to the study of the Hilbert geometry of subsets of the projective plane over a non-Archimedean ordered field F. The goal of this talk is to introduce the Hilbert metric (over F) and to describe the metric spaces associated to convex polygons in $\mathbb{FP}^2$ endowed with the Hilbert metric. This is joint work with Anne Parreau.

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Rigidity of circle packing on projective surfaces

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Observing that the notion of disk in CP^1 is invariant under projective transformations, Kojima, Mizushima and Tan proposed the study of circle packings on surfaces equipped with complex projective structure. The main observation is that combinatorially a circle packing is described by a triangulation of the surface, called the nerve of the circle packing. The Andreev-Koebe-Thurston theorem states that for a fixed triangulation T, there exists a unique Fuchsian projective structure carrying a circle packing with nerve T. Motivated by this result Kojima, Mizushima and Tan conjectured that the space of projective structures carrying a circle packing with nerve T is a section of the natural projection map of the space of projective structures to the Teichmüller space.

In the talk I will explain some recent developments around this conjecture obtained in collaboration with Michael Wolf. In particular we prove that the space of projective structures carrying a circle packing with nerve T is an immersed submanifold of dimension 6g-6 of the space of projective structures, and we prove a local projective rigdity: a circle packing cannot infinitesimally deformed within a fixed projective structure.

If time permits, I will also discuss some results around conformal rigidity.

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TBC

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Branched bending in hyperbolic 3-manifolds

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A conjecture of Menasco and Reid states that a hyperbolic knot complement does not contain a closed embedded totally geodesic surface. One heuristic that is used to study the absence of such a submanifold is parabolic cohomology-in particular, if the parabolic cohomology is known to vanish in specific settings, then it serves as an obstruction to bending along such a totally geodesic hypersurface. In this talk, we consider the Borromean rings complement, which is known to not admit any closed embedded totally geodesic surfaces but still has interesting parabolic cohomology. We construct a complex of surfaces that can be used to explain these deformations.

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CMC hypersurfaces in Anti-de Sitter space

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The Anti-de Sitter space is the Lorentzian analogue of the hyperbolic space, namely it is the model for negatively curved manifolds in signature $(n, 1)$. As its Riemannian counterpart, it comes with a conformal asymptotic boundary.

The classical asymptotic Plateau problem in the the Anti-de Sitter space consists in finding *maximal* hypersurfaces (*i.e.* with zero mean curvature) (CMC) with a prescribed boundary at infinity. This problem has first been studied for in the $(2+1)-$dimensional case, because it is linked to (universal) Teichmüller theory, followed by similar results.

In this talk, we show that there exists a unique hypersurface having constant mean curvature $H \in \mathbb{R}$, for any suitable prescribed boundary data:

Theorem 1. *Let Λ be a non-negative $(n-1)-$topological sphere in the boundary of $\mathbb{H}^{n,1}$. For any $H \in \mathbb{R}$, there exists a unique properly embedded spacelike hypersurface Σ with constant mean curvature H and with asymptotic boundary Λ.*

Furthermore, all the hypersurfaces mentioned above are complete: this result extends the Cheng-Yau theorem from the flat case (Minkowski space) to the negative constant sectional curvature case.

If there is time left, we will introduce some preliminary results of an on-going projects, whose goal is to estimate the geometry of a CMC-hypersurface by studying a suitable generalization of the convex hull of its asymptotic boundary.

In the $(2+1)-$dimensional case, CMC-surfaces induce a special class of quasi-conformal maps on the hyperbolic plane, called $\theta-$landslides. The estimates on the geometry of a CMC-surface allows to bound the maximal dilation of the corresponding $\theta-$landslide with respect to the cross-ratio norm of its extension to the asymptotic boundary of $\mathbb{H}^2$.

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High energy estimates and the Labourie conjecture

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Abstract. High energy harmonic maps to symmetric spaces look almost everywhere like harmonic maps into flats. I will first describe how we used a theorem of Mochizuki to that effect in order to find unstable minimal surfaces in locally symmetric spaces of rank at least three, thereby disproving the remaining cases of a conjecture of Labourie. This will then motivate some recent work removing the 'generically regular semisimple' hypothesis from Mochizuki's theorem. This is all joint work with Nathaniel Sagman.

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Proof-Theory and Theoretical Computer Science Special Session A25

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After the unquestionable role played by Logic in the birth of Computer Science at the beginning of last century, a second period of strong interactions between one among the oldest disciplines of human knowledge and one among the most recent ones occurred in the sixties with the discovery of the so-called Curry-Howard correspondence between proofs (of a particular logical system) and programs (of a paradigmatic programming language). Since then a wide interdisciplinary scientific community has grown all over the world.

An example of the fruitfulness of this interaction is type theory: the old idea of Russell to avoid paradoxes of set theory was later used in the framework of Church's Lambda-Calculus to control its computational power. Later on, Martin Löf proposed intuitionistic type theory as a foundation for constructive mathematics and Thierry Coquand introduced the calculus of constructions, at the base of the interactive theorem prover Coq. The vitality of type theory is also witnessed by the recent inception of Homotopy Type Theory (HoTT) in the landscape.

Another striking example is the birth of Linear Logic (at the end of the eighties) that is at the crossroad of traditional Proof Theory and Theoretical Computer Science: on the one hand it reveals a new structure underlying Logic and computational processes in general thus contributing to Proof Theory and Theoretical Computer Science in the most fundamental sense, and on the other hand it brings new concepts and new tools that have been used in various research areas in the last decades (theory of programming languages, implicit computational complexity, concurrency, game semantics, category theory, philosophy, linguistics,...).

In the proposed special session, we aim at bringing together european and american experts in the field. A list of topics of interest of the session include:

- Linear Logic and its applications;
- Type theory, including Homotopy Type Theory;
- Category Theory in Proof Theory and in Programming Languages;
- Extensions of the Curry-Howard Paradigm, Session Types and Concurrency;
- Proof complexity, Implicit computational complexity, Bounded Arithmetics;
- Automated reasoning: Proof compression, decidability and decision procedures, tableaux systems;
- Proof exchange, concept alignment and proof assistant interoperability;
- Realizability, Semantic types, Constructive Semantics, Classical Realizability.

For more information visit `https://sites.google.com/view/ptcs-palermo/home-page`.

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Classical System of Martin-Lof's Inductive Definitions is not Equivalent to Cyclic Proofs

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A cyclic proof system, called CLKID-omega, gives us another way of representing inductive definitions and efficient proof search. The 2005 paper by Brotherston showed that the provability of CLKID-omega includes the provability of LKID, first order classical logic with inductive definitions in Martin-Löf's style, and conjectured the equivalence. The equivalence has been left an open question since 2011. This paper shows that CLKID-omega and LKID are indeed not equivalent. This paper considers a statement called 2-Hydra in these two systems with the first-order language formed by 0, the successor, the natural number predicate, and a binary predicate symbol used to express 2-Hydra. This paper shows that the 2-Hydra statement is provable in CLKID-omega, but the statement is not provable in LKID, by constructing some Henkin model where the statement is false.

All details about this result can be found in paper with the same title published in Logical Method in Computer Science and available at the address: *https://arxiv.org/abs/1712.09603*. In this abstract we only include the statement of the main theorem.

Theorem 1 (Counterexample to the Brotherston-Simpson Conjecture)**.** *Let H be the2-Hydra formula defined in 1. Then H has a proof in the ciclic system* $\texttt{CLKID}(\Sigma_N, \Phi_N)$*, and no proof in the sub-system* $\texttt{LKID}(\Sigma_N, \Phi_N)$*, having inductive definitions +* $(0, succ)$*-axioms.*

Proof. One possible proof of H in the system $\texttt{CLKID}$ is given in 1. The non-provability of H in $\texttt{LKID}(\Sigma_N, \Phi_N) + (0, s)$-axioms is obtained by showing that the structure $\mathcal{M}$ structure defined in 1 verifies all axioms of the system $\texttt{LKID}(\Sigma_N, \Phi_N)$ with inductive definitions, verifies the $(0, succ)$-axioms, but falsifies H. We conclude that there is a theorem in the system $\texttt{CLKID}$ which is not provable in the sub-system $\texttt{LKID}(\Sigma_N, \Phi_N)$+$(0, succ)$-axioms. □

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What it really needs to be conservative

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Kuroda's double negation shift (DNS) is indispensable for the generalisation to first-order logic of Glivenko's theorem for propositional logic: modulo double negation, classical predicate logic is conservative over intuitionistic logic—with DNS added to the latter in the predicate case. Related conservation theorems have turned out to hinge upon variants of DNS. We try to understand this phenomenon in the borderland of classical and constructive reasoning.

Abstracting Glivenko's theorem from double negation and provability to a nucleus over an inductively generated entailment relation requires that all generating rules of the latter remain admissible in the Kleisli extension, i.e. when the nucleus is applied to the succedent of every premiss and the conclusion. This conservation criterion clearly is automatic for every rule which is right invariant, i.e. all premisses and the conclusion of which have in common the very same succedent. Axioms put as premissless rules are trivially right invariant. In the single-succedent calculi for propositional, infinitary and first-order minimal logic, all rules but the right rules for implication, infinite conjunction and the universal quantifier can be expressed as right invariant rules. The conservation criteria for the critical rules have proved to be counterparts of DNS.

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Second order well-behaved

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Second-order logic extends the expressive power of first-order logic, allowing for the representation of properties that involve quantification over all subsets or families of subsets within a given structure, addressing a fundamental need in mathematical discourse. However, while offering significant advantages, full second-order logic faces challenges due to its impredicative nature and the absence of crucial metalogical properties, which complicates the development of proof systems. Our research tackles these challenges by introducing G3-style sequent calculi that incorporate a predicative comprehension schema, facilitating constructive cut-elimination proofs.

Expanding upon these calculi, we delve into the proof theory of mathematical theories, adapting methods from first-order calculi and establishing structural results for both classical and intuitionistic versions of the calculi. Additionally, we define extensional equality and apartness within second-order logic, demonstrating the ability to reduce mathematical concepts to pure logical terms. To illustrate, we present the theories of von Neumann–Gödel–Bernays set theory and predicative second-order arithmetic. Moreover, we establish a variant of Herbrand's theorem tailored for predicative second-order intuitionistic logic, demonstrating the conservativity of predicative second-order Heyting arithmetic over its first-order counterpart. Furthermore, we extend the interpolation theorem and modal embedding of intuitionistic logic to predicative second-order logic.

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Adventures in Subexponential Non-Associative Non-Commutative Linear Logic

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Adding subexponentials to linear logic enhances its power as a logical framework, which has been extensively used in the specification of, e.g., proof systems and programming languages. Initially, subexponentials allowed for classical, linear, affine or relevant behaviors. Recently, this framework was enhanced so to allow for commutativity as well. In [1], we close the cycle by considering associativity. We formulate the resulting intuitionistic, two-sided system and show that it admits the (multi)cut. In the talk we discuss two new undecidability results that strengthen the undecidability results for fragments/variations of the system, given in [1]. If time permits we also discuss a classical, one-sided multi-succedent classical analogue of our intuitionistic system, presented in [2], following the exponential-free calculi of Buszkowski, and of de Groote and Lamarche. As in linear logic, a large fragment of our intuitionistic calculus is shown to embed conservatively into the classical version. It should be noted that such conservativity results are quite unusual, as they do not hold for traditional, richer logics which enjoy more structural rules for arbitrary formulae. This is joint work with Eben Blaisdell, Max Kanovich, Stepan L. Kuznetsov, and Elaine Pimentel.

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Tropical Mathematics and the Lambda-Calculus

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Tropical geometry has evolved in the last decades into a vast and rich research domain. Tropical methods, based on the tropical semiring $[0, +\infty]$ endowed with the min and + operations, provide a combinatorial counterpart to several algebraic geometry concepts, with important connections with optimisation theory and well-developed applications in convex analysis and machine learning (see [4] for a recent survey). Computationally speaking, working with min and + is generally easier than working with standard addition and multiplication; for instance, the fundamental (and generally intractable) problem of finding the roots of a polynomial admits a *linear time* algorithm in the tropical case.

In this presentation we introduce an interpretation of the λ-calculus based on a variant of the *relational model* of linear logic [3] based on the tropical semiring, and we discuss a few potential applications of tropical methods for the study of non-deterministic and probabilistic programs.

The tropical relational model provides a unifying framework for two well-developed quantitative approaches to program semantics, notably *program metrics*, based on the analysis of program sensitivity via Lipschitz conditions, and *resource analysis*, based on linear logic and higher-order program differentiation. Indeed, in this semantics programs are interpreted as *tropical power series* (tps in short) of the form $f(x)_b = \inf_{n\in\mathbb{N}}\{nx + c_n\}$, where $c_n \in [0, \infty]$. Tps yield at the same time a *Lipschitz approximation* of the program, that is, an approximation by means of *more and more sensitive* linear programs, and a tropical analogue of the usual *Taylor expansion* of λ-terms [1], that is, the decomposition of the program via resource-bounded approximants.

Crucially, in many important situations, tps can be shown equivalent to tropical *polynomials*: this means that the *infinitary* inf above collapses onto a min of *finitely* many elements. This collapse leads then to a purely combinatorial interpretation of probabilistic and non-deterministic higher-order programs, and well-developed tools (e.g. tropical roots, Newton polygons) can be used to develop a *best case analysis* of program behavior.

Finally, we show that this correspondence between metric and differential analysis of higher-order programs can be put at a higher level of generality through an abstract correspondence between tropical algebra and Lawvere's theory of *generalized metric spaces* [5,2]. This generalization suggests then ways to investigate other computational concepts (like e.g. differential privacy) via tropical methods.

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Some remarks on the PAM and its extensions

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The Pointer Abstract Machine (PAM) is an abstract machine implementing the linear head reduction of λ-calculus. Unlike other abstract machines such as the Krivine Abstract Machine (KAM) [5], which explicitly construct environments for evaluating λ-terms, the PAM employs a pointer mechanism to connect the substitution steps applied along the reduction. The PAM has been introduced in an unpublished note by Danos and Regnier [3], and it has successively been studied in relation to game semantics by Danos, Herbelin, and Regnier [2]. A detailed account of it, with an analysis of its efficiency has been recently published by Accatoli, Dal Lago, and Vanoni in [1].

Despite its significance, the PAM's description is often perceived as complex. This complexity arises partly because it is seen as an ad-hoc technique for the implementation of λ-calculus β-rule unrelated with the more standard techniques used in writing compilers or interpreters. Roughly speaking, the pointers at the core of the machine are seen as some smart but obscure mechanism peculiar to the PAM and to the particular reduction strategy that it implements, whose deeper meaning can be understood primarily within game semantics frameworks.

This talk aims to demystify the PAM by revealing the connection to a well-established concept in programming language compilation: the so-called *static chain* linking the activation record instances in the execution stack. The only peculiarity of the PAM is that the activation records in its stack do not contain the value of any variable: the association arguments/variables, the so-called execution environment, can be instead recovered by traversing the history of the linear head substitutions memorized in the stack; substitutions that in the PAM play the role of function calls, and whose sequence corresponds then to the so-called *dynamic chain*. Remarkably, the purely syntactical notion of nesting behind the construction of the static chain corresponds to the nesting of boxes in the so-called call-by-name translation of λ-calculus into linear logic [4] based on the $!D \quad D \quad D$ isomorphism.

As already mentioned, in the PAM the environments for evaluating the λ-terms are not explicitly constructed; instead, they are inferred by following the dynamic chain. The talk will explore how this process can be optimized compared to the original method proposed by Danos and Regnier. Furthermore, it will suggest an extension of the PAM to handle a calculus with continuations, in the style of the $\lambda\mu$-calculus. Finally, an extension of the machine that reduces not just to head normal forms but also to normal forms will be presented.

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What is an algorithm?

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What is a model of computation? What is a program? What is an algorithm?
While theoretical computer science has been founded on computability theory, the latter does not answer these questions. Indeed, it is a mathematical theory of computable functions, and does not account for computation itself: it only focusses on what is computed, and not on how it is computed.

A symptomatic consequence is the notion of Turing-completeness. This standard equivalence between models of computation is purely extensional: it does only care about what is computed and not how, blind to complexity aspects and the question of algorithmic completeness. One may view the multiple *barriers* in computational complexity (negative results showing the limitations of current methods) as a consequence of this failure. More importantly, the theory of computation is growing ever more further from actual computer science practice.

I will present a proposal [1] for alternative foundations more faithful to computer science practice and interests. This mathematisation of computer science is grounded within the theory of dynamical systems. I will argue that it generalises computability theory while still allowing to recover standard results. This approach underlies a number of results: shedding new light and unifying several known (and new) lower bounds results in algebraic complexity, conceiving static analysis methods which can be implemented to analyse real programming languages, providing a formal definition of the notion of algorithm, and leading to new models of (linear) logic.

Mainly, I define an *abstract model of computation* as follows.

Definition 1. An *abstract model of computation* (AMC) is a monoid action $\alpha : \mathrm{Mon}(I) \quad \mathbf{X}$:

(1) $\mathbf{X}$ is a space of configurations, together with a notion of morphisms;
(2) I is a set of instructions, generating the monoid action α;

As examples, one can define abstract models of computation accounting for: Turing machines and automata, algebraic models of computation (e.g. Blum Shub Smale models, iterated multiplication, quantum circuits), models based on rewriting (including lambda-calculus), and Instruction Set Architectures (e.g. RISC-V).

Such a monoid action then induces notions of *machines* and *programs*, defined as generalisations of the notion of graphing used in ergodic theory. Intuitively, a program is a graph whose edges represent elements of the monoid. Models of computation can furthermore be endowed with cost models, providing notions of complexity of programs (generalising space and time complexity).

However, in this talk I will focus on one other aspect of this theory. Indeed, as a last step in abstraction, I propose a formal definition of the notion of *algorithm*. An algorithm is defined a labelled graph endowed with an *abstract data structure*: labels then represent specific operations described by the data structure. A program P then implements the algorithm A if P can be defined as a glueing of programs $P_1, \ldots, P_k$ along A, where $P_1, \ldots, P_k$ are programs interpreting the data structure maps corresponding to the labels. An interesting outcome of the appraoch is that it could be used to define notions of distance between algorithms, or between an algorithm and a program. This proposal differs from those of Y. Gurevich (based on abstract state machines) and Y. Moschovakis (based on recursors), and I will comment on the differences.

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Proofs and algorithms: some philosophical considerations

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Since antiquity, the notion of proof has been seen as tightly connected, in mathematical practice, to the notions of construction and algorithm (think for instance of Euclid's way of proving theorems which is based on the possibility of executing certain constructions, and these constructions can be described as algorithmic procedures). Such a view has also been at the core of a certain logic tradition in the 20th century. In particular, according to the Brouwer-Heyting-Kolmogorov (BHK) interpretation of logical connectives, the meaning of a (logical complex) proposition A has to explained in terms of what counts as a proof of it. But, according to such interpretation, a proposition A also expresses the expectation of the solution of a problem, so that the explanation of A is eventually given in terms of what counts as a construction for solving the problem expressed by A. Now, a formal and precise version of the BHK interpretation can be given by appealing to the so-called Curry-Howard correspondance. This is a formal result allowing one to establish a correspondance between proofs (written in some formal deductive system) and programs (written in some abstract programming language), so that the proof of a proposition A corresponds to a (verified) program of type A. Programs thus play the role of the formal correlates of constructions, and additionally they offer a connection with the notion of algorithm: a program can be seen indeed as the implementation of an algorithm.

In this talk, I will try to investigate in some details the connection between proofs and algorithms at the light of the Curry-Howard correspondance. In particular, as the Curry-Howard correspondance provides a formal framework for establishing identity criteria for proofs/programs, I will try to understand whether such a framework can also be used for establishing identity criteria for algorithms. More generally, the aim of the talk is to examine whether it is possible to have a formal definition of algorithm not being *ad hoc* one, but keeping instead a direct connection with the notion of proof. It is in this sense that I will draw a comparison between the positions of A. Kolmogorov & V. Uspensky [3], Y. Gurevich [2] and Y. Moschovakis [4] concerning the possibility of a formal definition of algorithm and the position of J.-Y. Girard, whose project of the *Geometry of Interaction* was not only a continuation of the Curry-Howard correspondance, but also a "general program of mathematisation of algorithmics" ([1, p. 76]).

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On semantic perspectives for linear logic

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In this talk, we will view linear logic through two different semantical lenses: dialogical games and proof-theoretic semantics.

We first look at substructural calculi from a game semantic point of view, guided by certain intuitions about resource conscious and, more specifically, cost conscious reasoning. To this aim, we start with a game, where player I defends a claim corresponding to a (single-conclusion) sequent, while player II tries to refute that claim. Branching rules for additive connectives are modeled by choices of II, while branching for multiplicative connectives leads to splitting the game into parallel subgames, all of which have to be won by player I to succeed. The game comes into full swing by adding cost labels to assumptions, and a corresponding budget. Different proofs of the same end-sequent are interpreted as more or less expensive strategies for I to defend the corresponding claim. This leads to a labelled calculus, which can be seen as a fragment of SELL (subexponential linear logic). Finally, we generalize the concept of costs in proofs by using a semiring structure, illustrate our interpretation by examples and investigate some proof-theoretical properties.

The second look will view substructural behaviors using proof-theoretic semantics (PTS) lenses. PTS aims not only to elucidate the meaning of a logical proof, but also to provide means for its use as a basic concept of semantic analysis. In this talk, we will propose a "inferences-as-resources" approach to intuitionistic multiplicative linear logic (IMLL), and show how this can be smoothly extended to the full intuitionistic linear logic.

This is a joint work with Timo Lang, Carlos Olarte, Christian G. Fermüller and Victor Nascimento.

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Applying Implicit Computational Complexity

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This presentation is interested in reasoning about the recent development of static analyzers and optimizers inspired by Implicit Computational Complexity (ICC), which illustrate the challenges and benefits of applying approaches stemming from proof theory to software development.

ICC aims at characterizing complexity classes without referring to models of computations such as Turing machines. Doing so generally requires to define non-Turing-complete programming languages using tools coming from proof theory, model theory or recursion theory, and then to prove that they are *sound* and *complete* w.r.t. a particular complexity class $\mathbb{C}$ (i.e., that their programs compute functions in $\mathbb{C}$, and that for any problem in $\mathbb{C}$, there is a program that solves it). This approach allows to sidesteps Rice's theorem, but cannot be *intentionally* complete, i.e., it will always exclude programs that yet belong to the targeted complexity class.

ICC slightly wandered off this original program to get applied in three fairly different areas:

- A flow-analysis "extend[ing] and refin[ing] work in the Implicit Complexity research community" [1] was leveraged to develop two optimization passes [2, 3], hence facilitating (automatic) optimization of programs.
- Inspired respectively by the flow-analysis previously mentioned and tiering techniques, pymwp [4, 5] and ComplexityParser [6] provide complexity analysis that can help with the development of software products.
- Combining a noninterference-based type system with a stratification (or tiering) discipline was used to provide two types of correctness, related to resource usage and security [7].

Those research lines illustrates a shift from the design of ICC programming languages ("pre-bounding" all programs) to the design of *criteria* that may or may not be met by programs written in a Turing-complete programming language. Adapting the theory to make it implementable [4], finding reasonable metrics to compare to pre-existing tools [3, Section 5] or obtaining termination certificate and tight bounds [6, Section 3] are some of the challenges offered by this shift.

This new direction also allows to materialize some of the theoretical developments of ICC, to illustrate how this approach can sidestep some of the difficulties met by e.g., abstract interpretation-based static analyzers, and to demonstrate that ICC can be applied to semantic properties not related to complexity. It is, however, our hope that the design *and implementation* of complexity-bounded programming languages can emerge from ICC research, as those would offer the considerable advantage of *not* having to run any static analyzer to obtain strong and diverse semantic properties.

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[1]This abstract benefited from discussions with Neea Rusch, and reflect on work conducted additionally with Thomas Rubiano and Thomas Seiller [3, 4, 5].

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A Kleene algebra with tests for union bound reasoning about probabilistic programs

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Kleene Algebra with Tests (KAT) [1] provides a framework for algebraic equational reasoning about imperative programs. The recent variant Guarded KAT (GKAT) [4] allows to reason on non-probabilistic properties of probabilistic programs. In this talk we introduce an extension of this framework called approximate GKAT (aGKAT) [3], a variant of GKAT enriched with a partially ordered monoid (real numbers) which enables to express satisfaction of (deterministic) properties *except* with a probability up to a certain bound. This allows to represent in equational reasoning 'à la KAT' proofs of probabilistic programs based on the union bound, a technique from basic probability theory. We show how a propositional variant of approximate Hoare Logic (aHL) [2], a program logic for union bound, can be soundly encoded in our system aGKAT.

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Towards an induction principle for nested data types

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A well-known problem in the theory of dependent types is how to handle so-called *nested data types*. These data types are difficult to program and to reason about in total dependently typed languages such as Agda and Coq. In particular, it is not easy to derive a canonical induction principle for such types. Working towards a solution to this problem, we introduce *dependently typed folds* for nested data types. Using the nested data type `Bush` as a guiding example, we show how to derive its dependently typed fold and induction principle. We also discuss the relationship between dependently typed folds and the more traditional higher-order folds.

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Advances in variational methods and applications
Special Session A26

Micol Amar
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Giovanni Leoni
Carnegie Mellon University, USA

Elvira Zappale
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In the last decade, there has been a remarkable surge of interest within the scientific community, spanning physicists, engineers, biomedical researchers, and materials scientists, in the fields of new materials, micro-devices, artificial intelligence, machine learning, and stochastic modeling. These advancements have opened up a plethora of potential applications, driving the need for innovative mathematical techniques, methodologies, and the creation of novel function spaces to establish robust mathematical models. This requires a significant mathematical undertaking.

Our proposed minisymposium is dedicated to the variational formulation of these applications, aiming to provide well-founded and dependable mathematical descriptions. Our primary objective is to bridge the gap between these real-world applications and the realm of mathematics, uniting a community of experts in the field of Calculus of Variations.

This session is scheduled on July 23-24.

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Shape optimization for nonlocal anisotropic energies

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Nonlocal shape optimization problems involving interaction energies with competing repulsive and attractive terms are of interest in a variety of applications and have been extensively studied in the last decades in the mathematical community. In this talk we will consider a family of nonlocal energies in 3d defined on sets with prescribed mass, where the repulsive interaction is an anisotropic variant of the Coulomb kernel and the attractive interaction is quadratic. Under the sole assumption of strict positivity of the Fourier transform of the anisotropic kernel, we will show that there is a critical value of the mass, above which ellipses are the unique minimizers and below which existence of minimizers fails. If instead the Fourier transform is just nonnegative, there is a dichotomy: either there is a critical mass as in the previous case or ellipses are minimizers for every mass. This behavior is related to the shape of minimizers when considering the energy on the larger class of measures with prescribed mass.

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Non-local perimeters from adversarial learning

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Recent work in machine learning has recognized that many standard algorithms for classification are strongly affected by adversarial attacks. Accordingly, a growing body of research has tried to identify ways to mitigate this issue. This talk will discuss a natural non-parametric formulation of this objective, which can be transformed into a standard classification problem that utilizes a non-local perimeter as a regularizer. Connections with optimal transportation, mean curvature flow, and minimal surfaces, and related open problems will also be discussed. This represents joint work with Nicolás García Trillos, Leon Bungert, and Rachel Morris.

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Geometric rigidity in variable domains and applications in dimension reduction

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Konstantinos Zemas
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In this talk we present a quantitative geometric rigidity estimate in dimensions $d = 2, 3$ generalizing a celebrated result by Friesecke, James and Müller to the setting of variable domains. Loosely speaking, we show that for each function $y \in H^1(U; \mathbb{R}^3)$ and for each connected component of an open bounded set $U \subset \mathbb{R}^d$, the L^2-distance of ∇y from a single rotation can be controlled up to a constant by its L^2-distance from the group $SO(d)$, with the constant not depending on the precise shape of U, but only on an integral curvature functional related to ∂U. We further show that for linear strains the estimate can be refined, leading to a uniform control independent of the set U. The estimate can be used to establish compactness in the space of generalized special functions of bounded deformation (GSBD) for sequences of displacements related to deformations with uniformly bounded elastic energy. We show how this estimate can be applied in the context of dimension reduction by calculating the Γ-limit of a model for thin elastic solids containing voids.

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A variational model for 3D features in films/foams

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Daniel Restrepo
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Area minimization among a suitable class of 2D surfaces is the basic variational model describing the interfaces in films/foams. In this talk we will discuss a modification of this paradigm in which surfaces are replaced with regions of small but positive volume. The model captures features of real films/foams, such as Plateau borders, that cannot be described by zero volume objects. We will also discuss the PDE approximation of this problem via the Allen-Cahn equation and its relation to Plateau's laws, which govern the possible singularities.

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Stochastic homogenization in micromagnetics

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Many key properties and applications of magnetic materials are strongly intertwined with the spatial distribution of magnetic moments inside the corresponding specimens. In addition to classical magnetic structures, magnetic skyrmions have raised interest in spintronics as carriers of information for future storage devices. The chirality of magnetic skyrmions is determined by the so-called Dzyaloshinkskii-Moriya (DM) interaction.
In this talk, we present an advance in the mathematical modeling of magnetic skyrmions by analyzing the interplay of stochastic microstructures and chirality. Under the assumptions of stationarity and ergodicity, we characterize the Gamma-limit of a micromagnetic energy functional, including the DM contribution. Eventually, we present an explicit characterization of minimizers of the effective model in the case of magnetic multilayers.
This talk is based on a joint work with E. Davoli and J. Ingmanns.

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Sharp-interface limit for non-isothermal and nonlocal Modica-Mortola functionals.

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In this talk, we analyze a non-isothermal and nonlocal variant of the Modica-Mortola diffuse model for phase transitions fo the form

$$F_\epsilon(u;\Omega) := \frac{1}{4\epsilon}\int_{\Omega\times\Omega} J_\epsilon(x' \surd x)|u(x') \surd u(x)|^2\,dx'dx + \frac{1}{\epsilon}\int_\Omega W(x,u(x))\,dx.$$

Here the classical gradient penalization is replaced by a nonlocal singular perturbation and the double-well potential is space-dependent. Our main result is the identification of the sharp-interface limit as the width ϵ of the transition layers converges to zero. This is joint work with Elisa Davoli.

[1] This work was partially funded by the Austrian Science Fund (FWF)
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Regularity for minimizers of the Griffith fracture energy

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The Griffith criterion says that the energy to crack a brittle elastic material is proportional to the length of the crack. Understanding the regularity of minimizers requires unraveling the complex interplay of bulk (elastic) and surface (crack) energies in the vectorial setting of linearized elasticity. In dimension 2, we prove that the crack of a minimizer is given by a $C^{1,1/2}$ surface outside of a singular set of points with dimension strictly less than 1, analogous to results for the scalar-valued Mumford-Shah functional.

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Regularity and compactness for critical points of degenerate polyconvex energies

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Let $\Omega \subset \mathbb{R}^2$ be a smooth, bounded domain and consider the polyconvex energy

$$\mathbb{E}[u] \equiv \int_\Omega g(\det \mathrm{D}u)\,\mathrm{d}x, \qquad u\colon \Omega \to \mathbb{R}^2,$$

where g is a C^1 strictly convex function. Energies of this type were studied extensively in the literature, for instance in connection with elastic fluids or with the prescribed Jacobian equation.

In [1] we consider critical points of $\mathbb{E}$, i.e. solutions of the Euler–Lagrange system

$$\operatorname{div}(g'(\det \mathrm{D}u)\operatorname{cof}(\mathrm{D}u)) = 0. \tag{1}$$

Any C^1 solution of (1) satisfies $\det \mathrm{D}u = c$, for some $c \in \mathbb{R}$. Our first result is that the same rigidity continues to hold for Lipschitz solutions:

Theorem 1. *Suppose that u is a Lipschitz solution of (1), where $g \in C^1$ is strictly convex. Then $\det \mathrm{D}u$ is constant a.e. in Ω.*

The proof of this theorem combines ideas from the DiPerna–Lions theory for renormalized solutions of the continuity equation, and methods from quasiconformal analysis.

We then investigate the compactness properties sequences of approximate solutions to (1), i.e. sequences such that

$$\operatorname{div}(g'(\det \mathrm{D}u_j)\operatorname{cof}(\mathrm{D}u_j)) = \operatorname{div}(F_j), \qquad F_j \to 0 \text{ in } L^1(\Omega). \tag{2}$$

This question is closely related to the stability of (1). We show that the Jacobians of approximate solutions are in fact pre-compact:

Theorem 2. *Let (u_j) be a bounded sequence of Lipschitz maps satisfying (2). Then, up to a subsequence, u_j converges weakly-$*$ in $W^{1,\infty}$ to a solution of (1), and $\det \mathrm{D}u_j$ converges to $\det \mathrm{D}u$ strongly in L^p for any $p < \infty$.*

In particular, we obtain the following, which answers positively [2, Question 10]:

Corollary 3. *The differential inclusion associated with (1) is quasiconvex.*

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Discrete-to-continuous crystalline curvature flows

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We consider a fully discrete variant of the implicit variational scheme for mean curvature flow, in a setting where the flow is governed by a crystalline surface tension defined by the limit of pairwise interactions energy on the discrete grid. The algorithm is based on a new discrete distance from the evolving sets, which prevents the occurrence of the spatial drift and pinning phenomena that usually appear in similar discrete frameworks. We provide the first rigorous convergence result holding in any dimension, for any initial set and for a large class of purely crystalline anisotropies, in which the spatial discretization mesh can be of the same order or even coarser than the time step.

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Existence of minimizers for a two-phase free boundary problem with coherent and incoherent interfaces

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A variational model for describing the morphology of two-phase continua by allowing for the interplay between coherent and incoherent interfaces is discussed. Coherent interfaces are characterized by the microscopical arrangement of atoms of the two materials in a homogeneous lattice, with deformation being the solely stress relief mechanism, while at incoherent interfaces delamination between the two materials occurs. The model is designed in the framework of the theory of Stress Driven Rearrangement Instabilities, which are characterized by the competition between elastic and surface effects. The existence of energy minimizers is established in the plane by means of the direct method of the calculus of variations under a constraint on the number of boundary connected components of the underlying phase, whose exterior boundary is prescribed to satisfy a graph assumption, and of the two-phase composite region. Both the wetting and the dewetting regimes are included in the analysis.

[1]Aknowledgements: Projects P 29681 and TAI 293 by the Austrian Science Fund (FWF), Project MA16-005 by the Vienna Science and Technology Fund (WWTF), Projects CUP: E55F22000270001 and E53C22001930001 by GNAMPA-INdAM.
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Existence of solutions past collisions for viscoelastic solids

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In this talk, we will examine the time evolution of viscoelastic solids within a framework that allows for collisions and self-contact. In the static and quasi-static regimes, corresponding existence results have been shown through variational descriptions of the problem. For the fully dynamical case, however, collisions have so far either been ignored or handled using a simplified model (for example, using repulsive terms). In contrast to this, by employing a newly developed variational technique, we are able to prove the existence of solutions for arbitrary times. This entails the study of the measure-valued contact forces that naturally emerge when enforcing non-interpenetration of matter.

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Global existence and uniqueness of a micro-macro model for reactive transport in elastic perforated media

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We consider an effective model of micro-macro type consisting of a macroscopic elasticity-transport problem and associated cell problems, formulated in a unified *Lagrangian* framework. This model has been derived by a formal asymptotic expansion from a microscopic model defined in an elastically deformable perforated medium and formulated in a mixed *Eulerian*/*Lagrangian* framework. The effective model is nonlinearly coupled through reaction terms as well as effective coefficients which take into account the periodic microstructure and, in the case of the transport problem, the deformation of the domain. We prove global existence and uniqueness of a weak solution under a smallness assumption on the data of the elasticity subsystem.

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Rigorous derivation of effective models for perforated second grade viscoelastic materials

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In this talk, we perform the homogenization for nonlinear viscoelastic non-simple perforated materials at large strain in the quasistatic setting. Starting from a microscopic model on a periodically perforated reference domain depending on a scaling parameter ε giving the ratio between the size of the whole domain and the small periodic perforations, for $\varepsilon \to 0$ we rigorously derive a macroscopic model including homogenized coefficients. The mechanical energy in the micro-model depends on both, the gradient and the second gradient of the deformation, and also respects positivity of the determinant of the deformation gradient. Further, we assume dynamic frame indifference for the viscous stresses. For the homogenization we use the two-scale convergence and the unfolding method. The most crucial step is to establish ε-uniform a priori estimates for the microscopic solutions, in particular for the rate of the deformation gradient. For this we construct new extension operators for second order Sobolev spaces and prove a Korn inequality for non-constant coefficients on the perforated domain. Another crucial aspect is to guarantee an ε-independent positive lower bound for the determinant of the deformation gradient.

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Nematic soap films

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Nematic films are thin fluid structures, ideally two dimensional, endowed with an in-plane degenerate nematic order. Some variational models for nematic films have been introduced by Giomi in 2012 and by Napoli and Vergori in 2018. At equilibrium, the shape of the nematic film results from the competition between surface tension, which favors the minimization of the area, and the nematic elasticity, which instead promotes the alignment of the molecules along a common direction. The main difference between the two mentioned approaches is the way to compute the surface derivative of the nematic vector field. In this seminar I will briefly describe the models and I will present some recent analytical results obtained in collaboration with Giulia Bevilacqua, Chiara Lonati and Alfredo Marzocchi.

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Calderòn Problem for Nonlinear Electrical Conductivity

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We treat an inverse electrical conductivity problem, which deals with the reconstruction of nonlinear electrical conductivity starting from boundary measurements in steady currents operations. In this framework, a key role is played by the Monotonicity Principle, which establishes a monotonic relation, connecting the unknown material property to the (measured) Dirichlet-to-Neumann operator (DtN). We prove that the Monotonicity Principle for the Dirichlet energy in nonlinear problems holds under mild assumptions. Then, we show that apart from linear and p-Laplacian cases, transferring this monotonicity result from the Dirichlet energy to the DtN operator is impossible. To overcome this issue, we introduce a new boundary operator, identified as the average DtN operator. Moreover, the nonlinear constitutive relationships, at a given point in the space, present a behavior for large arguments described by monomials of order p and q so that a weighted $p\sqrt{}$Laplacian problem can approximate the nonlinear problem.

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Joint Meeting 2024
Palermo, July 23–26, 2024

Structure of the boundary singular set of area minimizing currents

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Let T be an area minimizing integral m-current in $\mathbb{R}^{m+n}$. It is well known that regularity theory in geometric measure theory is trickier when considering codimensions higher than 1. Almgren wrote his whole treatise [3] to prove that the Hausdorff dimension of the interior singular set of T cannot exceed $m-2$.

The boundary regularity for an area minimizing integral m-current in $\mathbb{R}^{m+n}$ taking the $C^{3,\alpha}$ boundary Γ with multiplicity Q is known to be weaker. Indeed, the following example given in [2] evidenciate it.

Proposition 1. *There exists a* 2*-dimensional area minimizing integral current* T *in* $\mathbb{R}^4$ *taking its boundary* $\Gamma \in C^\infty$ *(which is given by two disjoint closed curves) with multiplicity* 1 *and such that the Hausdorff dimension of the boundary singular set is* 1*.*

In [1], we are able to prove the following theorem.

Theorem 2. *Let* T *be an area minimizing integral* m*-current in* $\mathbb{R}^{m+n}$ *taking the boundary with multiplicity* $Q \in \mathbb{N} \setminus \{0\}$*. Then the boundary regular set of* T *is open and dense.*

Moreover, denoting $\mathcal{H}^{m-3}$ as the $(m-3)$-dimensional Hausdorff measure in $\mathbb{R}^{m+n}$, we can give further information about the structure of the set of one-sided boundary singular points. We call p an one-sided boundary point of T, if the density of T at p equals to half of the multiplicity of the boundary, i.e., $\Theta^m(T,p) = Q/2$.

Theorem 3. *Let* T *be an area minimizing integral* m*-current in* $\mathbb{R}^{m+n}$ *taking the boundary with multiplicity* $Q \in \mathbb{N} \setminus \{0\}$*. Then the set of one-sided singular points is* $\mathcal{H}^{m-3}$*-rectifiable.*

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Asymptotic analysis of thin structures with point-dependent energy growth

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$3D \surd 2D$ dimensional reduction for hyperelastic thin films modelled through energies with point-dependent growth, assuming that the sample is clamped on the lateral boundary, is performed in the framework of Γ-convergence. Integral representation results, with a more regular Lagrangian related to the original energy density, are provided for the lower dimensional limiting energy, in different contexts.

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Dynamic brittle fracture formulated as a an initial value problem

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A nonlocal model for dynamic brittle fracture is introduced consisting of two phases, one elastic and the other inelastic. Evolution from the elastic to the inelastic phase depends on material strength. Existence and uniqueness of the displacement-failure pair follow from the initial value problem. The displacement-failure pair satisfies energy balance. The length of nonlocality ϵ is taken to be small relative to the domain. The evolution provides an energy that interpolates between volume energy corresponding to elastic behavior and surface energy corresponding to failure. In general the deformation energy resulting in material failure over a region is a bounded $d - 1$ dimensional integral. For fixed $\epsilon > 0$, the failure energy is nonzero for $d - 1$ dimensional regions associated with flat crack surfaces. This failure energy is the Griffith fracture energy given by the energy release rate multiplied by area of the crack. The nonlocal field theory is shown to recover a solution of the linear elastic wave equation equation outside a propagating flat traction free crack in the limit of vanishing spatial nonlocality. For curved or more generally countably rectifiable cracks the failure energy is the Griffith fracture energy but only in the $\epsilon \to 0$ limit. Weak convergence methods, slicing variables and methods of geometric measure theory are used to demonstrate claims. This is part of a joint work with D. Bhattacharya.

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[1]This material is based upon work supported by the U.S. Army Research Laboratory and the U. S. Army Research Office under Contract/Grant Number W911NF-19-1-0245.

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Computational Topology: Foundations, Algorithms, and Applications
Special Session A27

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"Computational Topology: Foundations, Algorithms, and Applications" is a special session scheduled for July 23-24 within the 2nd edition of the "AMS-UMI International Joint Meeting".

Computational topology is by now a well-established field at the crossover of topology and computational geometry. It aims to transfer the power of topology for qualitative analysis to the setting of discrete approximations. Because finite data sets may be sampled from continuous underlying objects, computational topology involves not only discrete objects (such as pixelized images or finite meshes) but also an analysis of the transition between continuous and discrete. The development of this field has followed three distinct, but tightly intertwined directions: the investigation of the mathematical foundations of the subject, the quest for creating higher-performance algorithms, and the paths for new applications that those tools have opened.

The goal of this special session is two-fold. On the one hand, we want to valorize those three directions for their own importance. On the other hand, we intend to bring together well-established researchers and early-career mathematicians, enhancing the exchange of ideas and promoting collaborations between different communities.

For more information visit the following webpage:
`https://sites.google.com/view/ams-umi-computationaltopology/home-page`.

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Welcome to the computational topology special session

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This talk is an introduction to computational topology, by now a well-established field at the crossover of topology and computational geometry. We will provide a general overview of the topic and the three intertwined main directions explored during the special session. We will discuss how they complement and motivate each other and connect with different fields.

This presentation is aimed mainly at researchers outside the field to, hopefully, interest curious researchers and promote proficuous exchanges between different communities.

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Topology of random 2-dimensional cubical complexes

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We study a natural model of random 2-dimensional cubical complexes which are subcomplexes of an n-dimensional cube, and where every possible square (2-face) is included independently with probability p. Our main result exhibits a sharp threshold $p = 1/2$ for homology vanishing as the dimension n goes to infinity. This is a 2-dimensional analogue of the Burtin and Erdős-Spencer theorems characterizing the connectivity threshold for random graphs on the 1-skeleton of the n-dimensional cube. Our main result can also be seen as a cubical counterpart to the Linial-Meshulam theorem for random 2-dimensional simplicial complexes. However, the models exhibit strikingly different behaviors. We show that if $p > 1 \sqrt{1/2} \approx 0.2929$, then with high probability the fundamental group is a free group with one generator for every maximal 1-dimensional face. As a corollary, homology vanishing and simple connectivity have the same threshold.

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Variations on a barcode algorithm

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In this talk, I will provide an introduction to the standard barcode algorithm and an overview of some of its variants. I will explain what the pivot pairing is and why it provides the barcode, and how it behaves together with the two classical optimizations called clear and compress. I provide some insight into how the sparseness of the matrix can affect efficiency, and discuss the worst-case and average complexity for some special class of inputs. Finally, I will show how to update the reduced matrix so that it encodes the barcode of the original simplicial complex after the removal of some simplices from the latter.

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Learning and not learning with topology

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We present two recent results based on the use of algebraic topology and TDA in machine learning: one is the introduction of a novel loss for image processing, that leverages on Morse theory and Persistence; the other is the finding of a topological obstruction to the training of MLP.

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Capturing Robust Topology in Data

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A number of topological descriptors such as merge trees, contour trees, Reeb graphs, and persistence barcodes have been widely used in topological data analysis and visualization. However, a key question that deserves more attention is: **how can we capture robust topology in data by constructing topological descriptors that are robust to noise and outliers**?

A Reeb graph is a graphical representation of a scalar function on a topological space that encodes the topology of the level sets. A Reeb space is a generalization of the Reeb graph to a multiparameter function. We propose novel constructions of Reeb graphs and Reeb spaces that incorporate the use of a measure. Specifically, we introduce measure-theoretic Reeb graphs and Reeb spaces when the domain or the range is modeled as a metric measure space (i.e., a metric space equipped with a measure). Our main goal is to enhance the robustness of the Reeb graph and Reeb space in representing the topological features of a scalar field while accounting for the distribution of the measure. We first introduce a Reeb graph with local smoothing and prove its stability with respect to the interleaving distance. We then prove the stability of a Reeb graph of a metric measure space with respect to the measure, defined using the distance to a measure or the kernel distance to a measure, respectively. Our measure-theoretic approach allows Reeb graphs to capture robust topology in data, in line with recent advances in building robust topological descriptors.

[1]Acknowledgements: This project was partially supported by NSF DMS 2301361, NSF IIS 2145499, NSF IIS 1910733, DOE DE SC0021015, and DOE DE SC0023157.
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Apparent pairs in computational topology

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Apparent pairs (also known as evident, shallow, close, steepness, Pareto, or minimal pairs) are a fundamental construction at the interface of persistent homology and discrete Morse theory. I will discuss their role in the context of algorithmic and computational topology, with connections to the efficient computation of persistent homology, to the geometry of Gromov-hyperbolic spaces, and to shape reconstruction from point clouds.

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Topological Machine Learning: Applications to Raman Spectroscopy

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The advent of *machine* and *deep learning* has led to huge advancements in computer vision and data analysis, enabling a shift from handcrafted features to the automatic extraction of meaningful features through *representation learning*.

On the other hand, topological invariants offer shape informative and computable descriptors suitable for differentiating spaces; unfortunately, when applied to real-world data, these descriptors might seem too rigid. Thanks to the theory of *persistent homology* (PH), it is possible to use them to conduct intrinsically multiscale analysis.

Merging PH and machine learning methods allowed us to design and develop a *Topological Machine Learning (TML)* pipeline [1], leveraging the informative content of topological features which are different and complementary to those used in deep learning architectures, such as convolutional neural networks. Such a pipeline that associates persistence diagrams to digital data, via the most appropriate filtration for the type of data considered. Using a grid search approach, representation methods and parameters optimal for the classification task assigned are determined. We assessed the performance of our pipeline, and in parallel, we compared the different representation methods, on popular benchmark datasets, showing promising results.

In real-world problems, the pipeline has been exploited in the medical domain for a challenging task: the classification of Raman spectroscopy (RS).

RS is based on evaluating the inelastic scattering process in which photons incident on a sample transfer energy to or from molecular vibrational modes. Such information is stored in a spectrum. Instead of focusing the analysis on predetermined peaks or windows in the spectra, current research considers the Raman spectrum as a biochemical signature of the sample. In the biomedical domain (e.g., histopathology and oncology), RS represents a fast and efficient diagnostic tool that has found applications to several kinds of biological samples, including cellular tissues, cell lines and fluids, providing practical tools for assessing disease presence and grade. In this context, the TML pipeline achieved convincing results for the grading of chondrosarcoma [2] while in [3,4] it has been used to distinguish the Alzheimer's disease from other neurodegenerative pathologies (Bando Salute 2018 PRAMA project co-funded by the Tuscany Region). The present contribution will include the description and discussion of strengths and limitations of TML methods applied to the analysis and classification of data from RS in the biomedical domain.

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Random Simple Homotopy Theory

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We implement an algorithm RSHT (Random simple-homotopy) to study the simple-homotopy types of simplicial complexes. The algorithm combines elementary simplicial collapses with pure elementary expansions, and it represents a valide alternative to Discrete Morse Theory, with a particular focus on contractible spaces and on substructures within higher-dimensional complexes. This is joint work with Crystal Lai, Davide Lofano, and the late Frank Hagen Lutz, to whose memory the talk is dedicated.

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Discrete Morse theory for open complexes

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In this talk, we develop a discrete Morse theory for open simplicial complexes $K = X \setminus T$ where X is a simplicial complex and T a subcomplex of X. A discrete Morse function f on K gives rise to a discrete Morse function on the order complex of K, and the topology change determined by f on K can be understood by analyzing the topology change determined by the discrete Morse function on the order complex. This topology change is given by a structure theorem on the level subcomplexes of the order complex. Finally, we show that the Borel-Moore homology of K, a homology theory for locally compact spaces, is isomorphic to the homology induced by a gradient vector field on K and deduce corresponding weak Morse inequalities.

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Topological Data Analysis of Knowledge Networks

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Knowledge networks can organize complex data by constructing graphs where nodes are concepts or ideas and edges represent connections of significance. Understanding the structure of these knowledge networks to uncover how science progresses over time is of interest to researchers studying the "Science of Science." In this project, we are interested in understanding cycles or holes within a network, which can be thought of as gaps in knowledge. We use topological data analysis, and in particular, persistent homology filtered through time where the nodes represent scientific concepts and edges between two nodes are added at the time when they appear together in an abstract of a scientific paper. We study properties of these knowledge gaps in multiple dimensions such as when they form, when they no longer remain, and the concepts and papers that make up the cycles. We observe that papers involved in the knowledge gaps are cited more frequently than papers that are not.

[1]This work is supported by the National Science Foundation (NSF) grants NSF:CDS&E-MSS-1854703 and NSF:BCS-2318171.
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Max-Flow for Circular Mapper Graphs

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We start from the problem of analyzing sets with periodic boundary conditions, which often arise in materials science, and develop a notion of circular max-flow for mapper graphs with a map into the circumference. Roughly speaking, we compute max-flow slicing the graph with the fibers of the map into the circle. We study theoretic and computational properties of such definition, which we then test on simulated data representing glass structures, finding very high correlation with the conductivity of such materials.

[1] This work was supported by funding from the Independent Research Fund Denmark (1026-00037); we also thank C. A. N. Biscio, A. M. Svane, Y. Bokor Bleile and T. Heiss for helpful discussions and comments which sparkled some of the ideas contained in the manuscript.
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Topology meets signal processing and learning

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In recent years, the ever-growing interest in machine learning and in signal processing communities for data processing and analysis has driven the development of advanced models and tools to encode the interdependencies among data entities. Within this context, topological spaces have emerged as the ideal domain for capturing and encoding the complex relationships among data entities. As a result, Graph Signal Processing (GSP) has risen as a powerful branch of signal processing for analyzing and processing signals defined over the nodes of graphs, which are simple topological spaces able to represent pairwise relations between data.
However, in many applications such as social and communication networks, brain and biological networks, the complex interactions among data must be expressed through multiway relations that cannot be represented using simple dyadic relations as with graphs. To overcome this limitation, the more general framework Topological Signal Processing (TSP) has recently emerged as a fascinating area of research that merges concepts from algebraic topology with signal processing to represent and analyse signals defined over higher-order topological spaces, such as simplicial and cell complexes, which are capable of encoding multiway relations among data.
This talk aims to show how cross-fertilization across diverse fields such as topology, signal processing, and machine learning, not only enriches our theoretical understanding but also enhances practical applications, paving the way for breakthroughs in tackling the intricate challenges of data-driven learning.
I will first introduce basic graph signal processing tools and algorithms for inferring the graph topology from data, with applications to brain networks. Then, I will present the topological signal processing framework for the analysis and processing of signals defined over higher order topological structures such as simplicial complexes, cell complexes and a novel class of generalized cell complexes, named planar hollow cell complexes. We will discuss efficient methods for sampling and recovering edge signals, filtering signals, and strategies for topology inference from data. We quantify the advantages of using topological signal processing for various applications such as anomalous traffic detection, recovering of real data traffic flows and image segmentation.

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Tagged barcodes for the topological analysis of gradient-like vector fields

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In topological data analysis, persistent homology studies a continuous function $f\colon M \to \mathbb{R}$ by filtering M by the sublevel sets of f and then applying homology in order to obtain a persistence barcode. Our goal is to develop a persistence theory for gradient-like vector fields on Riemannian manifolds. Such a vector field may not be the gradient of any function, so a difficulty is that there is no canonical filtration of M. Inspired by some recent papers, we are decomposing parametrized chain complexes, but with epimorphic rather than the usual monomorphic internal maps that come from filtrations.

In line with this program, our first contribution is the study of the category of *tame epimorphic parametrized chain complexes*, i.e. functors that take real values to chain complexes, with internal maps always epimorphic and that may fail to be isomorphisms at most at finitely many times. We show that objects in this category can be decomposed into simple direct summands enumerated by a multiset of tagged real intervals, thus yielding a *tagged barcode*. After extending the standard interleaving and bottleneck distances to this setting, we prove an isometry theorem stating that the interleaving distance between two tame epimorphic parametrized chain complexes is equal to the bottleneck distance between their tagged barcodes.

As a second contribution we give a general procedure that, under some genericity conditions, starts from a chain complex with chosen bases and weights on them, and constructs a tame epimorphic parametrized chain complex.

Having this construction at our disposal, we are in the position of associating, in a stable way, a barcode of tagged intervals with any generic enough smooth vector field (precisely, Morse-Smale with pairwise different distances between singular points), without closed orbits (i.e. gradient-like). Given such a vector field v on a Riemannian manifold M, we apply our construction to its Morse complex, viewed as a weighted based chain complex with bases and weights given by the singular points of v and distances between them, respectively. We show that the map taking such a vector field to its tagged barcode is continuous, thus proving stability.

Thanks to the generality of the approach, we can also apply it to combinatorial gradient-like vector fields. Our third contribution is that the tagged barcode of a smooth vector field on a compact Riemannian manifold can be approximated arbitrarily well, in terms of bottleneck distance, by the tagged barcodes of combinatorial vector fields defined on sufficiently refined triangulations of the manifold.

The full version of this work is available at [1].

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Grounded persistent path homology: a stable, topological descriptor for weighted digraphs

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Weighted digraphs are used to model a variety of natural systems and can exhibit interesting structure across a range of scales. In order to understand and compare these systems, we require stable, interpretable, multiscale descriptors. To this end, we propose grounded persistent path homology (GrPPH) – a new, functorial, topological descriptor that describes the structure of an edge-weighted digraph via a persistence barcode. We show there is a choice of circuit basis for the graph which yields geometrically interpretable representatives for the features in the barcode. Using the theory of path homotopy, we show the barcode is stable, in bottleneck distance, to both numerical and structural perturbations. We compute GrPPH on a range of model vascular flow networks and verify that the descriptor can distinguish between different architectures and boundary conditions, with interpretable features explaining the distinctions. Moreover, in order to simulate pathological networks, we empirically investigate the behaviour of the descriptor under random edge removal and rewiring.

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Using Directed Topology to Understand 2-Parameter Persistence

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Unlike single-parameter persistent homology, which is completely described by a persistence diagram, multi-parameter persistence modules contain more information than it is possible to handle, understand, and visualize easily. Even bi-persistence modules can be quite complicated, although there exists a growing number of visualization tools and methods for computing bi-persistence invariants. The overall goal of this talk is to introduce a new constuction relating the parameter space of a 2-parameter persistence module to a directed space, with the purpose of leveraging tools from directed topology to visualize and compute related invariants.

In this talk, I will introduce directed Euclidean cubical complexes and the associated definition of directed homotopy within these spaces. I will give some examples and share some results on *directed collapse*, a way of collapsing cells of a directed Euclidean cubical complex which preserves the relevant spaces of directed paths in the original complex.

I will then introduce a new construction which uses directed Euclidean cubical complexes to model the parameter space of a 2-parameter persistence module. One goal of this construction is to relate the set of 1-d persistence diagrams along non-decreasing paths for a particular 2-parameter persistence module to the space of directed paths in the parameter space of that module. From this construction, we can leverage the tool of directed collapse to simplify the parameter space while still maintaining all necessary information to understand the module.

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On the use of the extended Pareto grid in 2-parameter persistent homology

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In recent years, the concept of *extended Pareto grid* has been introduced as a tool to study biparametric persistence for regular functions defined on smooth closed manifolds. Given a sufficiently regular function $\varphi = (\varphi_1, \varphi_2)$ from a smooth closed manifold M to $\mathbb{R}^2$, let us assume that $\{p_1, \ldots, p_r\}$ and $\{q_1, \ldots, q_s\}$ are the sets of critical points of φ_1 and φ_2, respectively. The extended Pareto grid of φ is the subset $\Gamma(\varphi)$ of $\mathbb{R}^2$ defined as the union $A \cup B \cup C$ with

(1) $A = \{\varphi(p) \in \mathbb{R}^2 : p \in M, \nabla\varphi_1(p) \text{ and } \nabla\varphi_2(p) \text{ are parallel, and } \nabla\varphi_1(p) \cdot \nabla\varphi_2(p) \leq 0\}$;
(2) $B = \bigcup_{i=1}^{r} v_i$ where v_i is the vertical half-line $\{(x, y) \in \mathbb{R}^2 \mid x = \varphi_1(p_i), y \geq \varphi_2(p_i)\}$;
(3) $C = \bigcup_{j=1}^{s} h_j$ where h_j is the horizontal half-line $\{(x, y) \in \mathbb{R}^2 \mid x \geq \varphi_1(q_j), y = \varphi_2(q_j)\}$.

For every $(a, b) \in]0, 1[\times \mathbb{R}$ and $p \in M$, let us set

$$\varphi^*_{(a,b)}(p) = \min\{a, 1-a\} \max\left\{\frac{\varphi_1(p) - b}{a}, \frac{\varphi_2(p) + b}{1-a}\right\}$$

and consider the line $r_{(a,b)} = \{\big(at + b, (1-a)t - b\big) : t \in \mathbb{R}\}$.

Given two functions $\varphi, \psi : M \to \mathbb{R}^2$, the *matching distance* D_{match} [1] is defined by setting

$$D_{\text{match}}(\varphi, \psi) = \sup_{(a,b) \in]0,1[\times \mathbb{R}} d_B\left(\text{Dgm}\left(\varphi^*_{(a,b)}\right), \text{Dgm}\left(\psi^*_{(a,b)}\right)\right)$$

where d_B is the bottleneck distance between persistence diagrams.

The following result and the stability of persistence diagrams allow us to follow the points of $\text{Dgm}\left(\varphi^*_{(a,b)}\right)$ while the pair (a, b) changes.

Theorem 1. *If $(a, b) \in]0, 1[\times \mathbb{R}$, $(u, v) \in \text{Dgm}\left(\varphi^*_{(a,b)}\right)$, $u \neq v$, $w \in \{u, v\}$ and $w \neq \infty$, then a point (x, y) in $r_{(a,b)} \cap \Gamma(\varphi)$ exists such that $w = \frac{\min\{a,1-a\}}{a}(x - b) = \frac{\min\{a,1-a\}}{1-a}(y + b)$.*

In this talk, we will illustrate how Theorem 1 can be used to prove some properties of biparametric persistence [2, 3].

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Computational complexity in multiparameter persistence

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For topological data analysis to be useful, we need efficient algorithms for computation. Our hunt for computational efficiency involves understanding which problems are NP-hard, and which problems might allow polynomial time algorithms. It is known that computing the interleaving distance for merge trees and multiparameter modules is NP-hard, and in recent work, Magnus Botnan and I proved that NP-hardness also holds in these cases for ℓ^p-versions of the interleaving distances. I will discuss these results and related classes of problems for which the computational complexity is still poorly understood. This includes an important meta-question in multiparameter persistence; namely, how much information do we have to sacrifice to turn a hard problem into a tractable one?

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DREiMac: Dimensionality Reduction with Eilenberg-Maclane Coordinates

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Dimensionality reduction is the machine learning problem of taking a data set whose elements are described with potentially many features (e.g., the pixels in an image), and computing representations which are as economical as possible (i.e., with few coordinates). In this talk, I will present a framework to leverage the topological structure of data (measured via persistent cohomology) and construct low dimensional coordinates in classifying spaces consistent with the underlying data topology.

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Palermo, July 23–26, 2024

Koszul complexes and generalized persistence

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Persistence modules are commonly encoded as functors from a poset to the category of finite dimensional vector spaces. In the case of one-parameter persistence, for example, the indexing poset can be restricted, under certain assumptions, to a finite subset of $\mathbb{R}$, and, in the case of multi-parameter persistence, to a finite subset of $\mathbb{R}^k$, for $k > 1$. A lot of research has been conducted to infer invariants for persistence modules. In this talk, we concentrate on the Betti numbers, homological algebra invariants coming from minimal free resolutions of the persistence modules. In particular, we show that, under certain assumptions on the indexing poset, it is possible to make use of Koszul complexes to determine the Betti numbers of the persistence module, without constructing its entire minimal free resolution. Koszul complexes are chain complexes that can be obtained just by evaluating the persistence module on certain elements of the poset. Thus, in a way, they just depend on the local structure of the indexing poset. After having established how to obtain Betti numbers from Koszul complexes and under which hypothesis this can be done, we show how to translate these methods from standard to relative homological algebra. In particular, we show how to use Koszul complexes, defined in standard homological algebra, to compute relative Betti numbers of persistence modules.

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Mapping spaces of persistence diagram into the Hilbert space with controlled distortion

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Stability is one of the most important properties of persistent homology. Similar inputs yield similar persistence diagrams. In this context, the space of persistence diagrams is typically equipped with the bottleneck metric. In order to apply statistical tools or further data analytic techniques to collections of persistence diagrams, we thus need to use a map from the space of persistence diagrams into a Euclidean or Hilbert space. In the past decade dozens of such maps have been proposed, including persistence landscape and persistence images. These maps are typically stable (Lipschitz). However, none of them has explicit lower bounds on distortion and hence they provide no control on the loss of information. In this talk we will present Lipschitz maps from certain spaces of persistence diagrams into Hilbert and Euclidean spaces with explicit distortion functions. The maps are fairly geometric, consisting essentially of bottleneck distances to specific landmark diagrams, and are thus easily implementable. The idea for the construction comes from the quantification of certain classical constructions in dimension theory.

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Several Complex Variables: Theory and Applications
Special Session A28

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Dusty Grundmeier
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Yunus Zeytuncu
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This special session presents talks on several complex variables (SCV) and its applications to other areas of mathematics and beyond. Complex analysis remains a fundamental tool across various fields–from mathematics to engineering. With recent developments in SCV, new applications have emerged. One such example comes from the study of mapping problems in several complex variables; namely, does there exist a proper, holomorphic mapping from the unit ball in $\mathbb{C}^n$ to the unit ball in $\mathbb{C}^N$ where $n, N \geq 2$? The existence of such monomial mappings between balls is closely related to finding the *sparsest* solution to a certain system of linear equations. Thus, one can study compressed sensing problems in the context of several complex variables. For another application, Wiener-Hopf techniques form an important tool for studying diffraction problems in physics, and extending these techniques in several complex variables is proving to be useful for these applications. For a final example, singular integral operators have been a central object of study in harmonic analysis for many years, and they arise naturally in complex analysis through the Bergman and Szegö projection operators, and the Cauchy-Fantappie singular integral operators. The regularity properties of these operators remain of fundamental interest in the field. The properties of these canonical operators also play crucial roles in operator theory. This special session aims to bring together experts from complex analysis and adjacent fields, fostering cross-pollination of ideas, techniques, and problems.

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A lower bound for the essential norm of the Leray transform

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A Berezin-transform-based method for estimating the essential norm of the Leray transform from below will be presented.

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Waves, oscillatory double integrals, and multidimensional complex analysis

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In this talk, I will give an overview of recent developments linking wave theory and multidimensional complex analysis. I will explain how a procedure (developed in [1]) of complex deformation of the integration surface of Fourier-like highly oscillatory double integrals can lead to closed-form far-field asymptotics results in wave diffraction theory. Each far-field component will be shown to be connected to a special point on the singularity set of the integrand. The procedure will be illustrated through the three-dimensional problem of plane wave diffraction by a quarter-plane [2, 3, 4] and the two-dimensional problem of plane wave diffraction by a penetrable wedge [5, 6, 7]. We will also show how it can be used to shed some light on wave propagation in periodic structures [8].

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On spectrum of Hankel operators on the polydisk

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We give sufficient conditions for the essential spectrum of the Hermitian square of a class of Hankel operators on the Bergman space of the polydisk to contain intervals. We also compute the spectrum in case the symbol is a monomial. Joint work with Zhenghui Huo and Sonmez Sahutoglu.

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The Commutator of the Bergman Projection

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Consider a bounded, strongly pseudoconvex domain $D \subset \mathbb{C}^n$ with minimal boundary smoothness (namely, the class C^2) and let b be a locally integrable function on D. We characterize boundedness (resp., compactness) in $L^p(D), p > 1$, of the commutator $[b, P]$ of the Bergman projection P in terms of an appropriate bounded (resp. vanishing) mean oscillation requirement on b. We also establish the equivalence of such notion of BMO (resp., VMO) with other BMO and VMO spaces given in the literature. In particular, we relate dyadic BMO spaces to those defined using Kobayashi metric balls (see [1,4]). Our proofs use a dyadic analog of the Berezin transform and holomorphic integral representations going back (for smooth domains) to N. Kerzman & E. M. Stein, and E. Ligocka in [2,3].

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Gromov hyperbolicity methods in holomorphic iteration

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Karlsson proved in 2001 that the classical Denjoy-Wolff theorem on iteration of holomorphic functions in the unit disc can be generalized to the setting of nonexpanding maps of Gromov hyperbolic metric spaces. In this talk we will discuss the interplay between the horofunction compactification and the Gromov compactification. Applying our results to the case of strongly pseudoconvex domains and of convex domains of D'Angelo finite type we will obtain the existence of horospheres, a Julia Lemma, and a Denjoy-Wolff theorem for backward iteration. This is based on joint works with Fiacchi-Gontard-Guerini and Fiacchi-Guerini-Karlsson.

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Matrix–vector form of Picard–Lefschetz theory

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Let be $U(t) = X \setminus \sigma(t)$, where X is a tubular neighborhood of $\mathbb{R}^2$ in the complex domain $\mathbb{C}^2$ of variables $z = (z_1, z_2)$,

$$\sigma(t) = \cup_j \sigma_j(t), \qquad \sigma_j = \{(z_1, z_2) \in X \,:\, g_j(z,t) = 0\},$$

$g_j(z,t)$ are holomorphic functions of variables z and complex parameters $t = (t_1, t_2, \dots)$. We assume that $g_j(z,t)$ take real values when z and t are all real. Let $\hat{U}(t)$ be a universal covering of $U(t)$. We study the ramification of $H_2(\hat{U}(t))$ as t goes along some closed loop λ bypassing about the Landau set of the singularities. It is well-known that this ramification is described by the Picard–Lefschetz theory [1], however, practical application of this theory may be not simple.

A matrix–vector form of Picard–Lefschetz theory is proposed for this case. A relative homology group

$$\hat{H}_2(X, \sigma(t)) = \psi^{-1}(H_2(X, \sigma(t))),$$

is introduced, where $\psi : \hat{U} \to U$ is a canonical projection. The Picard–Lefschetz results are provided by two theorems presented in the talk. The first theorem expresses the ramification of the elements of $\hat{H}_2(X, \sigma(t))$ in a simple matrix-vector form. The second theorem shows that the elements of $H_2(\hat{U}(t))$ can be obtained from a certain subset of the elements of $\hat{H}_2(X, \sigma(t))$ using an "inflation" procedure. Thus, ramification of $H_2(\hat{U}(t))$ becomes expressed in the matrix–vector form.

The method proposed in the talk is aimed for development into a 2D Wiener–Hopf method (which is an ambitious task indeed). The elements of $H_2(\hat{U}(t))$ are possible integration surfaces Γ for 2D Sokhotsky formulae of the form

$$f_s(t_1, t_2) = \frac{1}{(2\pi i)^2} \int_\Gamma \frac{f(z_1, z_2)}{(z_1 - t_1)(z_2 - t_1)} dz_1 \wedge dz_2,$$

$\sigma_j(t)$ are the lines $z_1 = t_1$, $z_2 = t_2$, and the singularities of $f(z)$. A ramification of Γ corresponds to ramification of the integral and thus of $f_s(z)$, which is of a great importance for applications.

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[1]The work is being done in collaboration with Raphael C. Assier and Andrey I. Korolkov from the University of Manchester

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On extrapolation of analytic functions

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The issue of extrapolation of an analytic function is known to be an ill-posed problem and as such it involves a regularisation strategy. We consider some instances of the problem of analytic continuation problem in a basic one-complex-variable setting. We discuss an efficient procedure of reconstruction of an H^2 function from its partial (and potentially non-exact) boundary values and connections to Carleman's formulas. The latter can yield a generalisation to multidimensional settings. If time permits, I will also show some practical situations of extrapolation in certain subspaces of harmonic functions from partial knowledge of a function inside its domain.

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Spherical Sommerfeld Integrals and Monodromy

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In this talk we give an overview on some recent work involving the application of several complex variables to the 'quarter-plane problem'. That is, the physical problem resulting from the interaction of a time-harmonic acoustic wave-field with a 'sound-hard' or 'sound-soft' quarter of a plane in three spatial dimensions; see [1].

After formulating the corresponding boundary value problem (BVP), which models this interaction, we reduce it to an Eigenvalue problem of the Laplace-Beltrami operator on a ramified covering of the 2-sphere $\mathbb{S}^2 = \{\boldsymbol{x} \in \mathbb{R}^3 |\ x_1^2 + x_2^2 + x_3^2 = 1\}$. This reformulation of the BVP enables us to utilise the theory of several complex variables: We complexify the covering space's base-space, the 2-sphere, and study the propagation of singularities (which include the real ramification points) within the complexification $\mathbb{S}^2_{\mathbb{C}} = \{\boldsymbol{z} \in \mathbb{C}^3 |\ z_1^2 + z_2^2 + z_3^2 = 1\}$. This approach was pioneered in the euclidean setting by Assier and Shanin in [2]. Here, we employ a *local* analytical continuation procedure which allows us to represent the *global* physical wave-field (i.e., the non-complexified wave-field) as a contour integral on some 'sphere at infinity' (this notion is made precise by viewing $\mathbb{S}^2_{\mathbb{C}}$ as a projective variety). This generalises the concept of Sommerfeld integrals from the (complexified) circle (which is directly linked to the classical, one-complex-variable, Wiener-Hopf technique) to the (complexified) 2-sphere. Our 'spherical Sommerfeld integral' involves some unknown spectral functions, which are defined on the sphere at infinity and involve only a *single* complex variable. Conversely, these spectral functions can be expressed as a contour integral on the original 2-sphere $\mathbb{S}^2$ which serves as the base-space of our covering. This allows us to derive a monodromy representation which describes the singularity structure of our spectral functions. In turn, this allows us to link the spectral functions to the theory of Fuchsian ODE's (this is Hilbert's 21$^{\text{st}}$ problem). Time permitting, we will discuss further aspects of our theory and potential future directions.

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[1] V. Kunz would like to thank the London Mathematical Society and the University of Cambridge for financially supporting this research with an LMS Early Career Fellowship, and a David Crighton Fellowship, respectively.
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Quasi-finite typeness and 1-regular types on algebraic CR manifolds: global boundedness I

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We first review the well-known notion of regular types on real hypersurfaces or CR manifolds, i.e. the order of contact of non-singular holomorphic curves with such manifolds. We discuss the global boundedness properties of such a type for a manifold and show that such a type is always uniformly bounded on any (not necessarily bounded) real-algebraic manifold containing no complex-analytic curves. Such a result follows from introducing and studying a more general notion of quasi-finite typeness for real-analytic maps and establishing its global boundedness for arbitrary real-algebraic/Nash maps. This is joint work by B. Lamel, N. Mir and G. Rond.

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Quasi-finite typeness and 1-regular types on algebraic CR manifolds: global boundedness II

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Bergman logarithmically flat and obstruction flat CR manifolds

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Let $\Omega \subset \mathbb{C}^n$ be a smoothly bounded, strictly pseudoconvex domain. The boundary $\partial\Omega$ is said to be *Bergman logarithmically flat* if the log singularity in Fefferman's asymptotic expansion of the Bergman kernel vanishes (to infinite order). It is called *obstruction flat* if the log singularity (the obstruction function) of the Cheng–Yau log-potential of the complete Kähler-Einstein metric in Ω vanishes. The Ramadanov Conjecture asserts that if $\partial\Omega$ is Bergman logarithmically flat, then it is spherical. There is a similar conjecture for obstruction flat boundaries. Both conjectures, suitably reformulated, fail for domains in more general complex manifolds in higher dimension ($n \geq 3$), but the situation is still unclear for domains in $\mathbb{C}^n$ (for $n \geq 3$). In this talk, we shall present recent work and open questions concerning these conjectures and the general structure of Bergman logarithmically flat and obstruction flat CR manifolds.

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Palermo, 23–26 July 2024

The $\overline{\partial}$-problem in $Z(q)$-domains

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A smoothly bounded, relatively compact domain Ω in a complex manifold M of dimension n is said to satisfy condition $Z(q)$ for some $1 \leq q \leq n-1$ if the Levi form of the boundary $\partial\Omega$ has at least $n-q$ positive or at least $q+1$ negative eigenvalues. It is well-known that this condition implies the Sobolev-$\frac{1}{2}$ estimates on the $\overline{\partial}$-problem and consequently the L^2-cohomology $H^{p,q}_{L^2}(\Omega)$ is finite dimensional with respect to any Hermitian metric on M. We consider the problem of giving sufficient biholomorphically invariant geometric conditions for the vanishing of this cohomology. Such conditions involve the interaction of the partial convexity of the boundary (given by the Levi form) and the interior convexity of the domain Ω (given by the complex Hessian of a weight function), where the interaction is encoded in a smoothly varying bundle of common positive directions at the boundary. We also discuss the sufficiency of these conditions.

References

[1] The $\overline{\partial}$-problem in $Z(q)$-domains, preprint available at
URL: `https://arxiv.org/abs/2403.04756`

[1]This work is partially supported by grants from the National Science Foundation and the Simons Foundation
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The Julia-Wolff-Carathéodory Theorem in convex domains of finite type

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The classical Julia-Wolff-Carathéodory Theorem says that, if f is a holomorphic self-map of the unit disk $\mathbb{D}$ and $\xi \in \partial\mathbb{D}$ such that

$$\liminf_{z\to\xi} \frac{1-|f(z)|}{1-|z|} < +\infty$$

then the derivative f' has non-tangential limit at ξ and the limit value can be computed in terms of the Poincaré distance. The Theorem has been generalized to several complex variables by Rudin in the unit ball, by Abate in strongly convex domains and by Abate-Tauraso in convex domains of finite type, under some technical assumptions. In this talk we will present a version of the Julia-Wolff-Carathéodory Theorem for a holomorphic map $f : D \to D'$ between convex domains of finite type. In particular, given a point $\xi \in \partial D$ with finite dilation we show that the K-limit of f at ξ exists and is a point $\eta \in \partial D'$, and we obtain asymptotic estimates for all entries of the Jacobian matrix of the differential df_z in terms of the multitypes at ξ and at η. Moreover, we introduce a generalization of Bracci-Patrizio-Trapani's pluricomplex Poisson kernel which, together with the dilation at ξ, gives a formula for the restricted K-limit of the normal component of the normal derivative $\langle df_z(n_\xi), n_\eta\rangle$. This is a joint work with L. Arosio.

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CR functions at CR singularities: approximation, extension, and hulls

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We study three possible definitions of the notion of CR functions at CR singular points, their extension to a fixed-neighborhood of the singular point, and analogues of the Baouendi–Trèves approximation in a fixed neighborhood. In particular, given the existence of a large enough disc hull shrinking to a point, we find the fixed-neighborhood extension and hence approximation properties. We provide examples showing the distinctions between the classes and the various properties studied.

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Kähler-Einstein Bergman metrics on pseudoconvex domains

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A well-known conjecture of Yau asserts that the Bergman metric on a bounded pseudoconvex domain in $\mathbb{C}^n$ is Kähler-Einstein if and only if the domain is homogeneous. A special case of this conjecture was posted earlier by Cheng: if the Bergman metric of a smoothly bounded strongly pseudoconvex domain is Kähler-Einstein, then the domain is biholomorphic to the unit ball. In this talk, we will discuss old and new results concerning the conjectures of Cheng and Yau.

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Potential theory and exceptional sets for the Drury Arveson space

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Michael Hartz
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In this talk we will introduce an energy functional for measures on the boundary of the unit ball of $\mathbb{C}^n$ which is appropriate for the study of function theory in the Drury Arveson space. In particular, the energy functional leads to a notion of capacity for sets on the boundary of the unit ball. Using this notion of capacity we characterize radial or admissible exceptional sets for the Drury Arveson space, to- tally null sets and we give a stronger necessary condition for a function to be cyclic. Hence, in some sense, the proposed notion of capacity plays the same role that linear Lebesgue measure plays in the theory of the Hardy space in the unit disc, or logarithmic capacity plays for the classical Dirichlet space. The talk is based on a joint work with M. Hartz.

References

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Bergman metrics of constant holomorphic sectional curvature

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In 2023, Huang and Li considered complex manifolds admitting a Bergman metric of constant holomorphic sectional curvature. Building on their work, in this talk we show no complex manifold whose Bergman space is base-point free, separates directions and separates points can have a Bergman metric with identically zero holomorphic sectional curvature.

References

[1] X. Huang and S.-Y. Li, *Bergman metrics as pull-backs of the Fubini-Study metric*, https://arxiv.org/abs/2302.13456, (2023).

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On the Hunag-Ji-Yau algebraicity Conjecture

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In our joint work with Jan Gregorovic, we investigate the problem of holomorphic algebraizibility for real hypersurfaces in complex space. We introduce a new invariant of a (real-analytic) Levi-nondegenerate hypersurface called *the jet transcendence degree*. Using this invariant, we solve in the negative the Conjecture of Huang, Ji and Yau on the algabraizability of real hypersurfaces with algebraic syzygies.

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On (some classes of) Levi-nondegenerate homogeneous CR manifolds

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The notion of Levi-nondegenerate of order k for CR manifolds is based on the higher order Levi form introduced by M. Freeman in [2], generalising the classical notion of Levi-nondegenerate CR manifold (which corresponds to $k = 1$).

For homogeneous CR manifolds we analysed Levi-nondegeneracy in terms of CR-algebras introduced in [3] (see also [4]). Generic Levi-nondegenerate CR manifolds have finite dimensional algebras of automorphisms (see, for instance, [5]).

In this talk we will consider the submanifolds M of a complex flag manifold $F = G/P$ of a semisimple Lie groups G that are orbits of a real forms $G_{\mathbb{R}}$ (see [1]). We analyse Levi-nondegenerate submanifolds, showing that they have order $k \leq 3$ (see [6]). In particular, the compact orbits have order at most 2 (see [7]). These estimates are sharp.

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[7] S. Marini, C. Medori, M. Nacinovich, *On finitely nondegenerate closed homogeneous CR manifolds*, Ann. Mat. Pura Appl. (4), 202 (2023), 2715–2747.

[1]This work was partially supported by the PRIN Project "Real and Complex Manifolds: topology, geometry and holomorphic dynamics" and by the Program "FIL-Quota Incentivante" of the University of Parma, co-sponsored by Fondazione Cariparma.

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Hardy Space and Szegő projection on quotient domains

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In this talk, we will briefly introduce the quotient domain setup in the theory of Hardy space and Szegő projection. Then we will study the analytic behavior of the Szegő projection on some non-smooth domains as applications.

References

[1] L. Chen, S. Krantz, Y. Yuan, *L^p regularity of the Bergman projection on domains covered by the polydisc*, J. Funct. Anal., 279(2), 2020.

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