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New Developments in infinite dimensional Lie algebras, vertex operator algebras and the Monster
Special Session B4

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Vertex operator algebras and Borcherds algebras are rich mathematical structures that play a role in many areas of mathematics. They also appear as symmetries of physical models. For example, the Monster Lie algebra was constructed by Borcherds as a quotient of the physical space of the tensor product $V^\natural \otimes V_{1,1}$ of the Moonshine module vertex operator algebra $V^\natural$ and the vertex algebra $V_{1,1}$ for the even unimodular 2-dimensional Lorentzian lattice $\mathrm{II}_{1,1}$. These algebraic structures were central players in the solution of the Conway-Norton Monstrous Moonshine conjecture. Since then, the study of vertex operator algebras, Borcherds algebras, and related structures have remained a source of discovery in both mathematics and physics. Vertex operators algebras and their representations encode the symmetries of two dimensional conformal field theories. Generalized Moonshine has uncovered many connections between number theory and physics. The Monster Lie algebra has recently been shown to be an algebra of gauge symmetries of a compactification of the Heterotic string. Borcherds superalgebras have been linked to the symmetries of supergravity theories. Furthermore, groups associated to Kac–Moody algebras are conjectured to encode symmetries of supergravity theories and there have been recent constructions of Lie group analogs for Borcherds algebras. In this special session, we explore the mathematical developments and possibilities for physical applications raised by these recent discoveries. The list of speakers comprises both mathematicians and physicists, each who have interest in the collaboration between these disciplines.

Schedule and Abstracts

Thursday July, 25

11.30-11.50 Opening remarks: Infinite dimensional Lie algebras and their symmetries
Lisa Carbone (Rutgers University)

Abstract: The Monster Lie algebra $\mathfrak{m}$ and Carnahan's family of Monstrous Lie algebras $\mathfrak{m}_g$, one for each element g of the Monster finite simple group, appear as symmetries in a model of the compactified Heterotic String by Persson, Paquette and Volpato. A recent construction of a Lie group analog for $\mathfrak{m}$ when $g = 1$ may also represent further symmetries. We give an overview and discuss some conjectures and open questions.

12.00-12.20 Opening remarks on infinite dimensional algebras in physical models
Roberto Volpato (Università degli Studi di Padova, ITALY)

Abstract: Infinite dimensional algebras play a crucial role in the theoretical description of many physical models, ranging from quantum mechanics to conformal field theory, from integrable models to string theory. In turn, physics has inspired and motivated the construction of a rich class of examples of infinite dimensional algebras, including vertex algebras and Borcherds-Kac-Moody algebras. In this overview talk, I will describe some recent instances of this rich interplay between algebra and physics, and provide some background and physical motivation for some of the topics that will be discussed in this special session.

12.30-12.50 Applications of Borcherds's Lie algebra to the uniqueness problem of VOAs of moonshine type

Masahiko Miyamoto (Department of Mathematics, University of Tsukuba)

Abstract:

Theorem 1. *If V is a simple VOA of central charge 24 with a non-singular invariant bilinear form $\langle , \rangle$ and $\sum \dim V_n q^{n-1} = j(\tau) - 744 = q^{-1} + 196884q + ...$, then V is C_2-cofinite. More precisely,*

$$C_2(V) = \sum_{n \geq 5} V_n + L(-1)V.$$

Proof. We use the fact $B(V) \cong B(V^\natural)$, where $B(V)$ denotes a Borcherds's Lie algebra of V. □

We next show its application to the space of 1-point functions of V.

Theorem 2. $\mathcal{F}(V) = \mathcal{F}(V^\natural)$, *where $\mathcal{F}(V)$ denotes the space of 1-point functions associated with V.*

About the monstrous moonshine VOA $V^\natural$, it was shown by C. Dong and G. Mason in 2000 that the space $\mathcal{F}(V^\natural)$ of 1-point functions associated to $V^\natural$ is precisely $\mathbb{C}$-linear space spanned by the (meromorphic) modular forms of level 1 and integer weight $k \geq 0$ satisfy holomorphic on $\mathcal{H}$ and has Fourier expansion $q^{-1} + ...$

Proof. Actually, Dong and Mason has shown that for $\theta = e^{2\lambda} + e^{-2\lambda} \in V^\natural_{12}$ with $\lambda \in \Lambda$ and $\langle \lambda, \lambda \rangle = 6$,

$$0 \neq \mathrm{Tr}_{V^\natural} o(\theta) q^{L(0)-1} = \Delta(\tau) = (2\pi)^{12} \eta(\tau)^{24},$$

where $\eta(\tau)$ is the Dedekind eta function, $\Delta(\tau)$ the modular discriminant, and $o(v) := v_{\mathrm{wt}(v)-1}$.

We will reconstruct it by a way which does not depend on the structure of V with the help of Borcherds's Lie algebra and the action of $\mathbb{M}$ on Borcherds's Lie algebra. □

References

[1] M. Miyamoto, *Borcherds's Lie algebra and C_2-cofiniteness of vertex operator algebras of moonshine type*, arXiv:2312.02427 .

13.00-14.30 Lunch Break

14.30-14.50 Vertex operators for imaginary $\mathfrak{gl}_2$ subalgebras in the Monster Lie Algebra

Darlayne Addabbo (Dept. of Mathematics, The University of Arizona)

Abstract: The Monster Lie algebra $\mathfrak{m}$ is a quotient of the physical space of the vertex algebra $V = V^\natural \otimes V_{1,1}$, where $V^\natural$ is the Moonshine module vertex operator algebra of Frenkel, Lepowsky, and Meurman, and $V_{1,1}$ is the vertex algebra corresponding to the rank 2 even unimodular lattice $\mathrm{II}_{1,1}$. We discuss vertex algebra elements which project to bases for subalgebras of m isomorphic to $\mathfrak{gl}_2$ and corresponding to imaginary simple roots of $\mathfrak{m}$. The action of the Monster finite simple group $\mathbb{M}$ on $V^\natural$ induces an action of $\mathbb{M}$ on the set of $\mathfrak{gl}_2$ subalgebras corresponding to a fixed imaginary simple root. We will discuss this action and related open questions. (This talk is based on joint work with Lisa Carbone, Elizabeth Jurisich, Maryam Khaqan, and Scott H. Murray.)

References

[1] D. Addabbo, L. Carbone, E. Jurisich, M. Khaqan, S. H. Murray, *Vertex operators for imaginary $\mathfrak{gl}_2$ subalgebras in the Monster Lie Algebra*, J. Pure Appl. Algebra, 228 (2024), no.7, Paper No. 107651, 25 pp.

15.00-15.20 Prosummabillity in groups for Borcherds algebras

Abid Ali (Dept. of Mathematics and Statistics, University of Saskatchewan)

Abstract: As part of his work on the solution of the Conway-Norton Monstrous Moonshine conjecture, Borcherds introduced a new class of infinite dimensional Lie algebras, known as generalized

Kac-Moody algebras or Borcherds algebras. These Lie algebras have important connections and numerous applications in algebra, number theory, combinatorics and mathematical physics. Therefore, it is important to construct an associated group analogous to the Lie groups in classical Lie theory and Kac-Moody groups in infinite-dimensional Lie theory. However, this is a challenging task because, unlike Kac-Moody algebras, the generators of Borcherds algebras do not act nilpotently or locally nilpotently under their adjoint representation. Carbone, Jurisich and Murray, in their construction of a Lie group analog for the Monster Lie algebra, introduced the notion of prosummability, providing a substitute for local nilpotence and paving the way for group constructions for these infinite-dimensional Lie algebras. We discuss how to extend this notion to all Borcherds algebras and we consider applications to complete adjoint constructions of Kac-Moody groups.

This is a joint work with Lisa Carbone, Elizabeth Jurisich and Scott Murray.

15.30-15.50 A Magnus Group construction for a class of Borcherds algebras
Elizabeth Jurisich (College of Charleston)

Abstract: We present a formal power series construction leading to a group for a subclass of Borcherds Lie algebras. We use a Magnus group construction stemming from the Hopf Algebra. This construction is over the rational numbers, but it can be related to the continuous Lie-type group construction over the complex numbers of Carbone, Jurisich, and Murray [1].

References

[1] Lisa Carbone, Elizabeth Jurisich, and Scott H. Murray, *Constructing a Lie group analog for the Monster Lie algebra,* Lett. Math. Phys. 112 (2022), no. 3

16.00-16.20 Monstrous Lie algebras as Borcherds algebras
Daniel Tan (Rutgers University)

Abstract: We discuss the construction of Carnahan's non-Fricke monstrous Lie algebras from the point of view of Borcherds algebras.

16.30 - 17.00 Coffee Break

17.00-17.20 Monstrous Moonshine for integral group rings
Scott Carnahan (University of Tsukuba)

Abstract: We propose a conjecture that unifies and generalizes Monstrous Moonshine and Modular Moonshine, and produce some partial results.

For any group G, and any commutative ring R, we may consider the tensor category of RG-modules that are R-free of finite rank. Homomorphisms from the Grothendieck ring of this category (or some other, similar category) to the complex numbers are called "species" by Benson and Parker. Given a $\mathbb{Z}$-graded RG-module whose graded pieces are R-free of finite rank, any species produces a corresponding formal power series with complex coefficients. We conjecture that when R is a subring of $\mathbb{C}$ and G is a subgroup of the monster, for a distinguished R-form of the moonshine module, any such power series is a hauptmodul. That is, the power series obtained by evaluating any species on the moonshine module is the expansion of a modular function that has discrete stabilizer in $SL_2(\mathbb{R})$, and generates the function field of the corresponding upper half-plane quotient.

For the case that $R = \mathbb{C}$, this conjecture reduces to the Monstrous Moonshine conjecture, proposed by Conway and Norton in 1979 and solved by Borcherds for the Frenkel-Lepowsky-Meurman moonshine module in 1992. When R is isomorphic to a ring of p-adic integers, and G is a cyclic group whose order has p-valuation 1, then this reduces to the Modular Moonshine conjecture of Ryba, proved by Borcherds and Ryba for odd p in 1996–1999, and for $p = 2$ by the speaker in 2017. We have found that our conjecture holds for some additional cases, and furthermore we have classified species for some nonabelian groups G.

This is joint work with Satoru Urano, and combines our paper at arXiv:2111.09404 with newer results.

References

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[6] S. Carnahan, *A self-dual integral form of the moonshine module*, SIGMA 15 (2019) 030, ArXiv: 1710:00737

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17.30-17.50 Elliptic Curves and Moonshine of the First Janko Group
Michael Griffin (Vanderbilt University)

Abstract: We identify distinguished virtual graded modules for the first sporadic simple group discovered by Janko, and establish relationships between these modules and the arithmetic of certain families of elliptic curves.

References

[1] J. F. R. Duncan, M. J. Griffin, M. H. Mertens, L. Rolen, *Elliptic Curve Arithmetic and Janko's First Group*, In Preparation.

[2] M. C. N. Cheng, J. F. R. Duncan and M. H. Mertens, *Class numbers, cyclic simple groups, and arithmetic*, J. London Math. Soc. **2** (2023), 1–35.

18.00-18.20 Reflective modular varieties and their cusps
Nils Scheithauer (Technical University of Darmstadt)

Abstract: Automorphic forms for orthogonal groups are natural generalisations of elliptic modular forms. An important class of these functions is given by automorphic products. We show that under some natural assumptions there are exactly 11 reflective automorphic products of singular weight. The corresponding modular varieties have a very rich geometry. Surprisingly their 1-dimensional type-0 cusps are naturally parametrised by Schellekens' list. This gives a new complex-geometric proof of this list.

18.30-18.50 Conformal blocks of vertex operator algebra and colored parenthesized braid operad
Yuto Moriwaki (Riken Institute of Physical and Chemical Research)

Abstract: The following theorem is one of the most important results in the theory of vertex operator algebras, proved by a series of papers by Huang and Lepowsky [1]:

Theorem 3 (Huang-Lepowsky)**.** *The representation category of a regular vertex operator algebra inherits a structure of braided tensor category.*

In the proof of the theorem, a partial operad of tubed Riemann spheres introduced by Huang in his pioneering work plays an important role. However, the proof in [1] is based on a very long formal calculation, since this partial operad is complicated whose product structure is formally defined.

We have given in [2] a more geometric and concise proof of Theorem 3. Huang-Lepowsky's proof is built on the basis of intertwining operators, which is a formal series. However, we use its geometric counterpart, the conformal block. We show that the fundamental groupoid of the E_2 operad (colored parenthesized braid operad) acts naturally on the conformal blocks. This operad is much simpler than Huang's operad.

Theorem 4 (M)**.** *Let V be a VOA (not necessary regular) and V-mod$_{C_1}$ the category of C_1 cofinite V-modules. Then, the colored parenthesized braid operad lax 2-categorically acts on V-mod$_{C_1}$.*

In this work, we will discuss these results and an alternative proof of Theorem 3 from Theorem 4 based on [2].

References

[1] Y.Z. Huang and J. Lepowsky, A theory of tensor products for module categories for a vertex operator algebra, I-IV, etc.

[2] Y. Moriwaki, Vertex operator algebra and colored parenthesized braid operad, arXiv:2209.10443.

Friday July, 26

11.30-11.50 Classification of Self-Dual Vertex Operator Superalgebras of Central Charge at Most 24

Gerald Höhn (Kansas State University)

Abstract: I will discuss recent joint work with Sven Möller on the construction and classification of self-dual vertex operator superalgebras of central charge up to 24. We employ the 2-neighbourhood graph of the self-dual VOAs of central charge 24 and realize these SVOAs as simple-current extensions of a dual pair. This pair includes a VOA derived from the Leech lattice alongside a lattice VOA. We have identified exactly 969 such SVOAs. The remaining open question concerns the uniqueness of the shorter Moonshine module, which was the subject of my 1995 Ph.D. thesis.

References

[1] Gerald Höhn and Sven Möller, *Classification of Self-Dual Vertex Operator Superalgebras of Central Charge at Most* 24*,* arXiv:2303.17190.

12.00-12.20 Genera of Vertex Operator Algebras

Sven Möller (University of Hamburg)

Abstract: We define and investigate several notions of genera of (suitably regular) vertex operator algebras, building on earlier work in [1,2]. In doing so, we shall see that each notion of equivalence for even lattices (e.g., genus, Witt and rational equivalence) has at least two generalisations to vertex operator algebras, one being a more "classical" analogue, and one being a more honest "quantum" analogue.

We study the relations between these various notions. For example, we give a proof that two vertex operator algebras in the same hyperbolic genus (as defined in [2]) are in the same bulk genus (as defined in [1]).

We also study mass formulae, p-neighbourhood and Hecke operators for hyperbolic genera of vertex operator algebras.

This is work in progress.

References

[1] Gerald Höhn, *Genera of vertex operator algebras and three-dimensional topological quantum field theories*. In Vertex Operator Algebras in Mathematics and Physics, volume 39 of Fields Inst. Commun., pages 89–107. Amer. Math. Soc., 2003.

[2] Yuto Moriwaki, *Genus of vertex algebras and mass formula*, Math. Z., 299 (3–4), 1473– 1505, 2021.

12.30-12.50 Replicable functions arising from code lattice VOAs fixed by automorphisms

Lea Beneish (University of North Texas)

Abstract: We ascertain properties of the algebraic structures in towers of codes, lattices, and vertex operator algebras (VOAs) by studying the associated subobjects fixed by lifts of code automorphisms. In the case of sublattices fixed by subgroups of code automorphisms, we identify replicable functions that occur as quotients of the associated theta functions by suitable eta products. We show that these lattice theta quotients can produce replicable functions not associated to any individual automorphisms. Moreover, we show that the structure of the fixed subcode can induce certain replicable lattice theta quotients and we provide a general code theoretic characterization of order doubling for lifts of code automorphisms to the lattice-VOA. Finally, we prove results on the decompositions of characters of fixed subVOAs. This talk is based on joint work with Jennifer Berg, Eva Goedhart, Hussain M. Kadhem, Allechar Serrano López, and Stephanie Treneer.

References

[1] L. Beneish, J. Berg, H. Kadhem, A. Serrano López, S. Treneer *Replicable functions arising from code lattice VOAs fixed by automorphisms*, J. Algebra (2024),Volume 642, Pages 159-202.

13.00-14.30 Lunch Break

14.30-14.50 Holomorphic vertex operator algebras, Teichmüller modular forms and the monster orbifold

Sebastiano Carpi (University of Rome “Tor Vergata”)

Abstract: Almost forty years ago Friedan and Shenker suggested to describe two-dimensional modular invariant conformal field theories in terms of the geometry of the “universal moduli space” of compact complex curves [1]. From this point of view the fundamental problem is the construction of the partition function of the theory on all compact complex curves. This raised many very interesting mathematical problems. In this talk I will review some recent results in this direction in the mathematical framework of vertex operator algebras [2]. If V is a (complex) simple vertex operator algebra of CFT type having an invariant bilinear form we can define the genus g partition function $\chi_{V,g}$ as a formal series in $3g$ variables defined through the correlation functions of V. The partition function χ_V is the sequence $\{\chi_{V,g}\}_{g\in\mathbb{Z}_{\geq 0}}$ of the genus g partition functions. If V is holomorphic (and strongly rational) with central charge c we show that each genus g partition function gives rise in a natural way to a genus g Teichmüller modular form of weight $c/2$, a higher genus generalization of the well-known modularity of the character of V. This gives strong constraints on the partition functions of holomorphic vertex operator algebras. An important example comes from a weak form the Harris-Morrison slope conjecture about the geometry of the moduli spaces of Riemann surfaces [3]. If this conjecture holds true then the partition function of any holomorphic V with $c = 24$ and zero weight-one subspace must coincide with the one of the moonshine $V^\natural$. Other results concerns the constraints on V given by its partition function. For a not necessarily holomorphic V we define the partition function vertex operator subalgebra PV of V. The latter is invariant for $\mathrm{Aut}(V)$ and, when V is unitary, it is a unitary subalgebra of V containing the conformal vector. If V and U are unitary we show that $\chi_V = \chi_U$ if and only if there is a linear isomorphism $\Phi : V \to U$ restricting to a vertex operator algebra isomorphism $\phi : PV \to PU$ and such that $\Phi Y^V(a, z) = Y^U(\phi(a), z)\Phi$ for all $a \in PV$. In particular, if the PV-module V has a unique VOA structure then U and V must be isomorphic. These results open new perspectives on the famous conjecture of Frenkel, Lepowsky

and Meurman on the uniqueness of the moonshine vertex operator algebra and relate it to other important conjectures in different areas of mathematics. Assume for example that $PV^\natural$ coincide with the monster orbifold $V^{\natural\mathbb{M}}$ and that the latter is strongly rational. Then, the uniqueness of $V^\natural$ would follow from the weak Harris-Morrison slope conjecture.

References

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[3] J. Harris, I. Morrison *Slopes of effective divisors on the moduli space of stable curves*, Invent. Math., 99 (1990), 321-355.

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15.00-15.20 Topological defects in K3 sigma models

Stefano Giaccari (Università di Padova & INFN)

Abstract: In the context of physics, given a K3 surface, a supersymmetric non-linear K3 sigma model is the internal superconformal field theory (SCFT) in a six dimensional compactification of type IIA superstring on $\mathbb{R}^{1,5} \times K3$. Studying these models, Eguchi, Ooguri and Tachikawa found Mathieu moonshine phenomena for the elliptic genera of K3 surfaces. Gaberdiel, Hohenegger and Volpato studied symmetries of K3 sigma models, by comparing the isometry group of the Mukai lattice (which is physically interpreted as the D-brane charge lattice) with the Conway group Co_0, the group of automorphisms of the Leech lattice. Recently, symmetries have been reinterpreted in terms of topological operators supported on codimension 1 submanifolds with group-like and invertible fusion rules. This has naturally led to the notion of generalized symmetries as categories of topological defects supported on arbitrary codimension submanifolds with possibly non-invertible fusion rules.

In [3] we apply the same strategy as in [2] to derive a number of general results for the category of the topological defect lines preserving spectral flow, studying their fusion with boundary states. We argue that while for certain K3 models infinitely many simple defects, and even a continuum, can occur, at generic points in the moduli space the category is actually trivial, i.e. it is generated by the identity defect. Furthermore, we show that if a K3 model is at the attractor point for some BPS configuration of D-branes, then all topological defects have integral quantum dimension. We also conjecture that a continuum of topological defects arises if and only if the K3 model is a (possibly generalized) orbifold of a torus model. These general results are confirmed by the analysis of a couple of significant examples.

References

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[4] R. Angius, S. Giaccari, Sarah M. Harrison and R. Volpato, “Topological defects in $V^{f\natural}$,” to appear.

15.30-15.50 p-adic vertex operator algebras

Geoffrey Mason (University of California, Santa Cruz)

Abstract: We introduce p-adic vertex algebras. Such an object is not quite a VOA in the usual sense, but rather the completion of a VOA with respect to a compatible nonarchimedean norm.

In the language of physics, it is a chiral half of a nonarchimedean 2-dimensional bosonic CFT in which a p-adic Banach space replaces the traditional Hilbert space. Our main motivation for considering these objects is that they facilitate the introduction of p-adic modular forms *a la* Serre into VOA theory. This gives rise to many new phenomena, and we shall review some of these in the setting of the p-adic Heisenberg VOA. This is joint work with Cameron Franc.

16.00-16.20 Tensor hierarchy algebras and generalised diffeomorphisms
Jakob Palmkvist (Örebro University, Sweden)

Abstract: I will review how certain Borcherds–Kac–Moody superalgebras have been used in physics to encode the bosonic field content of supergravity theories. Any such Borcherds–Kac–Moody superalgebra is a $\mathbb{Z}$-graded extension of a simple finite-dimensional Lie algebra $\mathfrak{g}$ at degree 0 (describing a global internal symmetry of the theory) and includes at degree 1 an integrable lowest-weight module over $\mathfrak{g}$ with lowest weight $-\lambda$. I will show how the construction of the Borcherds–Kac–Moody algebra can be modified, and from the same from the initial data $(\mathfrak{g}, \lambda)$ construct a Lie superalgebra which is not contragredient, so that the $\mathfrak{g}$-module at a degree p is not necessarily dual to the one at degree $-p$. Some well known Lie superalgebras appear as special cases, but also new ones that have not been studied before. These infinite-dimensional Lie superalgebras, known as tensor hierarchy algebras [1], can be used to describe generalised diffeomorphisms, providing a local origin of the global symmetries. The construction can be generalised to cases where $\mathfrak{g}$ is an infinite-dimensional Kac–Moody algebra, but the representation structures that appear in these cases remain to be understood.

The talk is based on collaboration mostly with Martin Cederwall [2] but also with Lisa Carbone [3], Guillaume Bossard, Axel Kleinschmidt, Chris Pope and Ergin Sezgin [4].

References

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[2] M. Cederwall and J. Palmkvist, *Cartanification of contragredient Lie superalgebras*, [2309.14423]. (See references therein for earlier work in this collaboration.)

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16.30 - 17.00 Coffee Break

17.00-17.20 Genus g Zhu Recursion for Vertex Operator Algebras
Michael Tuite (University of Galway)

Abstract: We describe Zhu recursion for a vertex operator algebra (VOA) and its modules on a genus g Riemann surface in the Schottky uniformisation. We describe how n-point correlation functions can be naturally expanded in terms $(n-1)$-point functions with universal coefficients given by holomorphic forms and derivatives of the Bers quasiform. We discuss Heisenberg VOA examples where Zhu recursion leads to novel differential equations for the partition function and various classical structures such as the bidifferential of the second kind, holomorphic 1-forms, the prime form and the period matrix.

References

[1] M.P. Tuite, *The Heisenberg generalized vertex operator algebra on a Riemann surface*, Contemp. Math. 768 (2021) 321–342.

[2] M.P. Tuite, M. Welby, *Some properties and applications of Bers quasiforms on Riemann surfaces*, arXiv:2306.08404 (2023).

[3] M.P. Tuite, M. Welby, *Genus g Zhu recursion for vertex operator algebras and their modules*, arXiv:2312.13717 (2023).

17.30-17.50 On Yangian deformations of $\mathcal{S}$-commutative quantum vertex algebras
Lucia Bagnoli (University of Zagreb)

Abstract: We present the construction of a new class of quantum vertex algebras associated with a normalized Yang R-matrix. They are obtained as Yangian deformations of certain $\mathcal{S}$-commutative quantum vertex algebras and their $\mathcal{S}$-locality takes the form of a single RTT-relation. We establish some preliminary results on their representation theory and then we further investigate their braiding map. In particular we show that the fixed points of the braiding map are related to Bethe subalgebras in the Yangian quantization of the Poisson algebra $\mathcal{O}(\mathfrak{gl}_N((z^{-1})))$, introduced by Krylov and Rybnikov in [2].

References

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18.00-18.20 Unitary representations of minimal W-algebras
Paolo Papi (University of Roma, La Sapienza)

Abstract: I will discuss recent advances on a project joint with Pierluigi Möseneder Frajria (Politecnico di Milano) and Victor Kac (MIT). In [1] we classified the levels k for which the simple affine W-algebra $W_k^{\min}(\mathfrak{g})$ attached to a basic finite dimensional Lie superalgebra $\mathfrak{g}$ and to a minimal even nilpotent element is unitary, i.e. it admits a positive definite invariant Hermitian form. We then started the classification of the unitary representations of these algebras in the Neveu-Schwarz sector. The classification is complete in the so called *non-extremal* cases, and we conjecture that our results in the extremal case provide the complete classification. We also obtained character formulas for the unitary representations, yielding rigorous proofs of the results obtained by physicists (Miki, Eguchi-Taormina) in the 80's for the $N = 3$ and $N = 4$ superconformal algebras, respectively.

We recently moved to study unitary representation in the Ramond sector, which involves studying twisted modules.

The study of unitarity of the Ramond twisted irreducible highest weight modules over the vertex algebra $W^k_{\min}(\mathfrak{g})$, where k is in the unitary range, proceeds along similar lines. The main difference is that in the Ramond sector one has to consider separately two cases: when $\frac{1}{2}\theta$ is not a root of $\mathfrak{g}$, and when it is a root, where θ is the highest root of $\mathfrak{g}$. In both cases the necessary conditions of unitarity are similar to the conditions found in the NS case, except that a canonical constant A has to be replaced by another constant A^{tw}, and the notion of an extremal weight needs to be replaced by that of a Ramond extremal weight.

As in [1] we find sufficient conditions of unitarity of Ramond twisted irreducible highest weight modules over $W^k_{\min}(\mathfrak{g})$ by using its free field realization The parameter space for these representations if given by pairs (ℓ_0, ν), where $\ell_0 \in \mathbb{R}$ and ν is a weight of a certain subalgebra of $\mathfrak{g}$. As a result, we prove unitarity for ℓ_0 larger than an explicit constant B in the cases when ν is not Ramond extremal. It turns out that $B = A^{tw}$ in the cases when $\theta/2$ is a root of $\mathfrak{g}$ which completes the proof of unitarity when ν is not Ramond extremal.

However, in the case when $\theta/2$ is not a root of $\mathfrak{g}$, $B = A^{tw}$ only for some very special weights ν. Generically one has that $B > A^{tw}$, and we need use Euler-Poincaré characters, instead of determinants of ϕ-invariant Hermitian forms for twisted $W^k_{\min}(\mathfrak{g})$-modules, as in the non-twisted sector. At this point we need to use a conjecture, which claims that Arakawa's results on properties of the quantum Hamiltonian reduction functor can be extended to the Ramond twisted case.

The necessary conditions for unitarity of Ramond twisted irreducible highest weight modules over $W^k_{\min}(\mathfrak{g})$ are exhibited in all cases. Moreover, as in [2], any unitary irreducible non-twisted or twisted highest weight module over $W^k_{\min}(\mathfrak{g})$ descends to $W_k^{\min}(\mathfrak{g})$.

Using the character formulas for massless representations in the case of level k_0 when $\dim W_{k_0}^{\min}(\mathfrak{g}) = 1$, we find the denominator identities for $W_{\min}^k(\mathfrak{g})$. As a result, we recover the classical identities of Euler, Gauss and Ramanujan, and find some new identities.

In my talk I will give a quick overview of the above results, with emphasis on the Ramond case and applications to identities.

References

[1] V. G. Kac, P. Möseneder Frajria, and P. Papi, *Unitarity of minimal W–algebras and their representations I*, Comm. Math. Phys., 401 (2023), pp. 79–145.

[2] D. Adamović, V. G. Kac, P. Möseneder Frajria, and P. Papi, *Defining relations for minimal unitary quantum affine W-algebras*, Comm. Math. Phys., 405 (2024), Paper N. 33, 25 pp.

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