Point configurations: energy, designs, and discrepancy
Special Session B3

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Uniformly distributing a large number of points in a domain or on a manifold is a question that arises naturally both in pure mathematics (discrete geometry, probability, analysis) and applications (numerical integration, sampling, frame theory etc), and there are numerous ways to measure the quality of a distribution of points: discrepancy, energy minimization, packing and covering radii, lattices, cubature formulas, designs etc, many of which are closely connected to each other. The methods used to address such problems involve a mixture of a variety of areas of mathematics: discrete geometry (polytopes, equiangular lines), combinatorics (combinatorial discrepancy, combinatorial designs, Latin squares), probability (random point processes, large and small deviation bounds), number theory (lattices, diophantine approximation), approximation theory (cubature formulas, spherical designs, interpolation), applied mathematics (compressed sensing, frames), and others. Moreover, a central role in these topics is played by various branches of analysis, in particular, Fourier, harmonic, and functional analysis, as well as potential theory, orthogonal polynomials, and special functions.

The session will concentrate on numerous problems about distributions of points, with a strong focus on the application of the methods of analysis in this circle of questions.

For more information visit `https://sites.google.com/view/point-configurations/home`.

Schedule and Abstracts

July 25, 2024

11:30–11:50 Irregularities of distribution for bounded sets and half-spaces
Luca Brandolini (Università degli studi di Bergamo, ITALY)

Abstract. Let $\mathcal{P}_N$ be a set of N points in $\mathbb{R}^d$ $(d \geqslant 2)$ and let $E \subseteq \mathbb{R}^d$. We want to estimate the quality of the distribution of these points with respect to a probability measure μ supported in E. We consider a reasonably large family $\mathcal{R}$ of measurable sets and, for $R \in \mathcal{R}$, we introduce the discrepancy

$$\mathcal{D}_N(R) = \operatorname{card}(\mathcal{P}_N \cap R) - N\mu(R) \ .$$

We prove a few theorems which extend several known results.

12:00–12:20 Discrepancy and approximation of absolutely continuous measures with atomic measures
Giancarlo Travaglini (Università di Milano-Bicocca, ITALY)

Abstract. We prove several results concerning the discrepancy, tested on balls in the d-dimensional torus $\mathbb{T}^d$, between absolutely continuous measures and finite atomic measures.

12:30–12:50 Discrete analogues of renowned lower bounds in discrepancy theory
Alessandro Monguzzi (Università degli studi di Bergamo, ITALY)

Abstract. A well-known result due to H. Montgomery states the following. There exists a constant $c > 0$ such that for every given finite sequence $\{p_n\}_{n=1}^N$ in the 2-dimensional torus $\mathbb{T}^2$ the estimate

$$\int_{\mathbb{T}^2} \Big(\Big| \sum_{n=1}^{N} \chi_{x+B_{\frac{1}{2}}}(p_n) - N|B_{\frac{1}{2}}| \Big|^2 + \Big| \sum_{n=1}^{N} \chi_{x+B_{\frac{1}{4}}}(p_n) - N|B_{\frac{1}{4}}| \Big|^2 \Big) \, dx \geqslant cN^{\frac{1}{2}}.$$

holds. In this talk I will present a discrete version of the above estimate for the d-dimensional torus with $d \not\equiv 1 \bmod 4$. Namely, I will show that the lower bound $cN^{(1-\frac{1}{d})}$ holds true if we replace the integration over all the possible translated balls with an average over a finite number of translated balls. Time permitting, I will present two other results in the spirit of the one described above. One concerning the discrepancy on the torus with respect to squares, the other concerning the discrepancy in the unit cube with respect to anchored boxes.

14:30–14:50 Spherical cap $\mathbb{L}_2$-discrepancy for spherical Fibonacci points: recent progress
Johann Brauchart (TU Graz, AUSTRIA)

Abstract. The spherical cap $\mathbb{L}_2$-discrepancy measures the irregularity (i.e., the deviation from uniform distribution) of a point set on the unit sphere in $\mathbb{R}^3$ in the $\mathbb{L}_2$-sense with respect to spherical caps.

Lower and upper bounds with matching optimal powers $N^{-3/4}$ are known for minimizing N-point configurations. A proof for the conjectured constant of the large N asymptotics is still missing.

In case of a sequence of constructed N-point configurations the best proven upper bound is of order $N^{-1/2}$; cf. [1].

In this talk, we discuss discrepancy bounds and numerical results for spherical Fibonacci points obtained via Lambert equal area transformation applied to the Fibonacci lattice in the unit square.

The talk is based on joint work with Josef Dick (UNSW, Sydney) and Yuan Xu (University of Oregon, USA).

References

[1] C. Aistleitner, J. S. Brauchart, and J. Dick, *Point sets on the sphere* $\mathbb{S}^2$ *with small spherical cap discrepancy*, Discrete Comput. Geom. 48 (2012), no. 4, 990–1024.

15:00–15:20 The L^2-Discrepancy of Planar Convex Bodies averaged over Affine Transformations
Thomas Beretti (SISSA, ITALY)

Abstract. Let $\mathbb{T}^2$ denote the bi-dimensional torus. Consider a convex body $C \subset \mathbb{T}^2$ with piecewise-$\mathcal{C}^1$ boundary, and write $\mathcal{X}_C$ as its characteristic function. For a set $\mathcal{P}_N \subset \mathbb{T}^2$ of N points, consider the quantity

$$D(C, \mathcal{P}_N) = \sum_{p \in \mathcal{P}_N} \mathcal{X}_C(p) - N|C|.$$

In this talk, we study the L^2-discrepancy of C averaged over affine transformations, that is, including translations, dilations, and rotations. Namely, for an angle $\phi \in (0, 2\pi]$, we intend to give estimates on the asymptotic behavior as $N \to \infty$ of

$$\inf_{\#\mathcal{P}_N = N} \int_{-\frac{\phi}{2}}^{\frac{\phi}{2}} \int_{\frac{1}{2}}^{1} \int_{\mathbb{T}^2} |D(\tau + \delta\sigma_\theta C, \mathcal{P}_N)|^2 \, d\tau \, d\delta \, d\theta,$$

where σ_θ is a rotation by an angle θ, and δ is to be understood as a dilation factor.

15:30–15:50 Three problems in discrepancy theory
Christoph Aistleitner (TU Graz, AUSTRIA)

Abstract. I will speak about three of my favourite open problems in discrepancy theory. The first is the problem of the “inverse of the discrepancy”: In a unit cube of dimension d, given $\varepsilon > 0$, what is the minimal cardinality of a point set with discrepancy at most ε? The question is very relevant from the perspective of numerical analysis, since the bound for the integration error in Quasi-Monte Carlo integration is proportional to the discrepancy to the set of sampling points. The second problem is about the discrepancy with respect to general measures. The classical case is that of the uniform measure, but it is natural to study also discrepancies with respect to other measures, and consider this as a problem of approximating such a measure by

a simple atomic measure. A key question then is: Is the uniform measure the one which is most difficult to approximate? The third problem concerns the spherical cap discrepancy on the 2-sphere. While the upper bound from probabilitic existence proofs essentially matches the lower bound (order approx. $N^{-3/4}$), we do not know of a single deterministic point set whose spherical cap discrepancy is smaller than the square-root cancellation which i.i.d. random points exhibit as well. So the problem is to find a deterministic point set whose spherical cap discrepancy is better than that of a random point set.

16:00–16:20 Hyperuniformity and Energy on Projective Spaces
Peter Grabner (TU Graz, AUSTRIA)

Abstract. We study Riesz, Green and logarithmic energy on two-point homogeneous spaces. More precisely we consider the real, the complex, the quaternionic and the Cayley projective spaces. For each of these spaces we provide upper estimates for the mentioned energies using determinantal point processes. Moreover, we determine lower bounds for these energies.

Furthermore, we extend the notion of hyperuniformity to the projective spaces and study the connection between energy and the Wasserstein distance.

References

[1] A. Anderson, M. Dostert, P. J. Grabner, R. W. Matzke, and T. A. Stepaniuk, *Riesz and Green energy on projective spaces*, Trans. Amer. Math. Soc. Series B **10** (2023), 1039–1076.

[2] C. Beltrán, N. Corral, and J.G. Criado del Rey, *Discrete and continuous green energy on compact manifolds*, J. of Approx. Theory **237** (2019), 160–185.

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[4] B. Borda, P. J. Grabner, and R. W. Matzke, *Riesz energy, L^2-discrepancy, and optimal transport of determinantal point processes on the sphere and the flat torus*, Mathematika **70** (2024), e12245, https://arxiv.org/abs/2308.06216.

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[6] J. S. Brauchart and P. J. Grabner, *Hyperuniform point sets on projective spaces*, https://doi.org/10.48550/arXiv.2403.03572, 2024.

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17:00–17:20 Rate of convergence in ergodic transformations
Leonardo Colzani (Università di Milano-Bicocca, ITALY)

Abstract. We discuss the speed of convergence of means of ergodic transformations in the torus.

17:30–17:50 Gaps in Kronecker sequences and optimal spherical codes
Alexey Glazyrin (University of Texas Rio Grande Valley, USA)

Abstract. The celebrated Three-Gap Theorem states that, if one places first N elements of the Kronecker sequence $\{nx\}$, $n = 1, \ldots, N$, on a unit circle, then distances between consecutive points take no more than three distinct values. I will talk about the higher-dimensional version of this problem asking to find the maximal number of gaps in a high-dimensional Kronecker sequence on a flat torus. Recently, Haynes and Marklof solved this problem in two dimensions by showing that the number of gaps in a two-dimensional Kronecker sequence is no greater than 5. I will show how the problem is connected to the general problem of finding optimal spherical codes and explain several new bounds on the number of gaps in all dimensions confirming, in particular, a weak version of the conjecture of Haynes and Marklof in three dimensions.

18:00–18:30 Fourier analysis and signal recovery
Alex Iosevich (University of Rochester, USA)

Abstract. Let $f : \mathbb{Z}_N^d \to \mathbb{C}$ be a signal, and let $\widehat{f}$ denote its Fourier transform. If the Fourier transform of f is sent and the values $\{\widehat{f}(m)\}_{m\in S}$ are missing, we ask under what conditions the original signal f can be recovered. We are going to give an elementary exposition of this problem and describe some recent results. This is joint work with Azita Mayeli.

July 26, 2024

11:30–11:50 Minimal logarithmic energy and Sobolev discrepancy
Jordi Marzo (Universitat de Barcelona, SPAIN)

Abstract. Given a configuration of points on the sphere $x_1, \dots, x_N \in \mathbb{S}^2$ the discrete logarithmic energy is given by

$$\mathcal{E}(x_1, \dots, x_N) = \sum_{i\neq j} \log \frac{1}{|x_i - x_j|}.$$

The asymptotic behavior of the minimum of this energy

$$\mathcal{E}(N) = \min_{x_1,\dots,x_N\in\mathbb{S}^2} \mathcal{E}(x_1, \dots, x_N)$$

has been extensively studied, but despite all the efforts, only a few terms of the asymptotic expansion are known, see for example [1]. In this talk I will explain the connection of some recent developments with the study of a Sobolev discrepancy introduced by T. Wolff in an unpublished work.

References

[1] S. Borodachov, D. Hardin and E. Saff, *Discrete Energy on Rectifiable Sets*, Springer New York, 2019.

12:00–12:20 Riesz Energy with an External Field: When Is the Minimizer a Sphere?
Ryan Matzke (Vanderbilt University, USA)

Abstract. We will discuss the minimization of Riesz energies with external fields

$$I_{s,V}(\mu) = \int_{\mathbb{R}^d}\int_{\mathbb{R}^d} \Big(\frac{1}{s}\|x-y\|^{-s} + V(x) + V(y)\Big) d\mu(x)d\mu(y)$$

focusing on the cases where $V(x) = \gamma\|x\|^\alpha$ for $\alpha, \gamma > 0$. We are particularly interesting in how the choices of s and α, i.e. the strength of repulsion between "electrons" and strength of attraction towards a positively charged source at the origin, respectively, affect the dimension of the support of the minimizing measure, and are able to classify exactly when the support is the uniform measure on a sphere.

12:30–12:50 A constrained logarithmic energy problem on the unit circle
Liudmyla Kryvonos (Vanderbilt University, USA)

Abstract. We study the problem of minimizing the logarithmic energy,

$$\mathcal{E}(\mu) := \int\int \log \frac{1}{|z-\zeta|} d\mu(z)d\mu(\zeta),$$

of probability measures μ supported on the unit circle with an additional constraint imposed on the mass of a fixed subarc. Namely, for given θ, $0 < \theta < 2\pi$, and given q, $0 < q < 1$, we determine the measure μ^*, such that $\mathcal{E}(\mu^*) = \inf\{\mathcal{E}(\mu) : \mu \in \mathcal{P}(\mathbb{S}^1), \mu(A_\theta) = q\}$, where A_θ is the arc from $e^{-i\theta/2}$ to $e^{i\theta/2}$. The result answers a question raised by E. Meckes in connection with the characterization of behavior of eigenvalues of the kernel of the unitary eigenvalue process. In addition, I will talk about analogous constrained problem for more general energy functionals.

14:30–14:50 Dyadic operators, weights, and sparse domination in the non-doubling setting
Jill Pipher (Brown University, USA)

Abstract. In joint work with Conde Alonso and Wagner ([1]), we proved a bilinear form sparse domination, and discovered new appropriate weight classes, for bounding certain dyadic shift

operators with respect to a class of non-doubling measures. The operators are characterized by a suitable notion of *complexity* which generalize Petermichl's shift operator, featured in [5]. Through examples, we demonstrated that the usual sparse forms and standard Muckenhoupt weight classes are not suitable for these dyadic shifts in the non-doubling setting. The project arose as a natural extension of the work of López-Sánchez, Martell, and Parcet ([4]), where necessary and sufficient conditions on non-doubling measures were derived for bounding a classical dyadic shift operator ([5]). In further joint work with Borges, Conde Alonso, and Wagner, we consider the (recently introduced) dyadic Hilbert transform which Domelevo and Petermichl investigated in connection with its close relationship to the continuous Hilbert transform in [2], and which is featured in their paper (joint with Treil and Volberg) establishing optimal 3/2 bounds for matrix weights ([3]). In particular, we investigate properties necessary and sufficient for the boundedness of this operator in L^p, and similar questions for commutator estimates (with martingale BMO functions). Our work was informed by the results in, and several helpful discussions with Treil on, the non-homogeneous martingale theory established in ([6]).

References

[1] J. Conde Alonso, J. Pipher, N. Wagner , *Balanced measures, sparse domination and complexity-dependent weight classes, https://arxiv.org/pdf/2309.13943.*

[2] K. Domelevo, S. Petermichl, *The dyadic and the continuous Hilbert transforms with values in Banach spaces, https://arxiv.org/abs/2212.00090*

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15:00–15:20 The p^{th} frame potentials: an overview

Kasso Okoudjou (Tufts University, USA)

Abstract. Finding optimal configurations of point masses under the action of energy functionals defined on $d-$dimensional Euclidian space appears in several fields such as numerical integration, coding theory, quantum information theory, and chemistry. The problem often involves minimizing a functional of the form $\iint_{S^{d-1}\times S^{d-1}} f(x,y)d\mu(x)d\mu(y)$ over all probability measures μ defined on the unit sphere S^{d-1}, or its discrete version, minimizing $\sum_{k\neq\ell} f(\varphi_k,\varphi_\ell)$ over all sets of N vectors $\Phi=\{\varphi_k\}_{k=1}^N \subset S^{d-1}$ for some function f.

In this talk, we will consider the case where $f(x,y) = |\langle x,y\rangle|^p$ for $p\in[0,\infty]$ which is referred to as the p^{th} frame potentials. We focus the interplay between the continuous and the discrete problems, especially when the dimension d is small.

This talk is based on joint works with R. Ben Av, X. Chen, M. Ehler, A. Goldberger, E. Goodman, V. Gonzales, and S. Kang.

15:30–15:50 Tchakaloff-like compression of QMC integration

Giacomo Elefante (Università di Torino, ITALY)

Abstract. We present a method based on Tchakaloff-Davis-Wilhelmsen theorems to compress quadrature formulae such as quasi-Monte Carlo (QMC) integration. This means that, when the integral

$$\int_\Omega f(x)\mathrm{d}\mu$$

is approximated with QMC method by

$$\frac{|\Omega|}{N}\sum_{i=1}^N f(x_i)$$

for certain points uniformly distributed accordingly to the measure μ, we aim to extract some points $t_i \subset \{x_1, \ldots, x_N\}, i = 1, \ldots, M$ with $M < N$ and positive weights w_i, such that the quadrature formula

$$\sum_{i=1}^{M} w_i f(t_i)$$

is a *good* approximation of QMC integration. In particular, the compressed quadrature formula extracted is going to be exact (with respect to QMC approximation) on polynomials in Ω up to a given fixed degree. Therefore, such formulae preserve the approximation power of QMC up to the best uniform polynomial approximation error of a given degree to the integrand, but using a much lower number of sampling points.

16:00–16:20 Sampling in inverse problems
Giovanni Alberti (Università di Genova, ITALY)

Abstract. Sampling problems appear whenever a continuous, infinite-dimensional object x, has to be reconstructed from discrete, possibly finite, samples. This requires some assumptions on x. These assumptions can be either expressed by linear conditions, as it happens for Shannon-type results for band-limited functions, or by nonlinear conditions, as with sparsity in compressed sensing or with generative neural networks.

In this talk, I will discuss the case when x is not sampled directly: we sample $F(x)$, where F is the so-called forward map, and wish to reconstruct x. This is the framework of inverse problems, in which an unknown quantity x has to be reconstructed from physical, indirect measurements $F(x)$. Sampling appears in this context since, in practice, it is impossible to directly measure the infinite-dimensional quantity $F(x)$, and only samples are available. I will discuss some abstract results, as well as some examples, including deconvolution and the inverse Radon transform.

References

[1] G. S. Alberti and M. Santacesaria, *Infinite-dimensional inverse problems with finite measurements*, Arch. Rational Mech. Anal., 243(1), 1–31, 2022.

[2] G. S. Alberti, A. Felisi, M. Santacesaria and S. I. Trapasso, *Compressed sensing for inverse problems and the sample complexity of the sparse Radon transform*, arXiv:2302.03577, 2023.

17:00–17:20 Exact and approximative t-design curves
Martin Ehler (Universität Wien, AUSTRIA)

Abstract. In analogy to classical spherical t-design points, we introduce the concept of t-design curves on the sphere $\mathbb{S}^d = \{x \in \mathbb{R}^{d+1} : \|x\| = 1\}$. This means that the line integral along a t-design curve γ integrates exactly all algebraic polynomials f in $d+1$ variables of degree t,

$$\frac{1}{\ell(\gamma)} \int_\gamma f = \int_{\mathbb{S}^d} f\,.$$

For low degrees we construct explicit examples.

We also derive lower asymptotic bounds on the lengths of t-design curves.

Proposition 1. *Assume that a piecewise smooth, closed curve $\gamma : [0,1] \to \mathbb{S}^d$ satisfies*

$$\frac{1}{\ell(\gamma)} \int_\gamma f = \int_{\mathbb{S}^d} f \qquad \text{for all } f \in \Pi_t\,.$$

Then its length is bounded from below by

$$\ell(\gamma) \geq C_d t^{d-1}$$

with some constant $C_d > 0$ that may depend on the dimension d but is independent of t and γ.

Our main results prove the existence of asymptotically optimal t-design curves in $\mathbb{S}^2$.

Theorem 2. *In $\mathbb{S}^2$ there exists a sequence of t-design curves $(\gamma_t)_{t\in\mathbb{N}}$ with length $\ell(\gamma_t) \asymp t$.*

We also verify the existence of t-design curves in $\mathbb{S}^d$.

Theorem 3. *In* $\mathbb{S}^d$ *for* $d \geq 3$ *there exists a sequence of* t*-design curves* $(\gamma_t)_{t\in\mathbb{N}}$*, such that* $\ell(\gamma_t) \lesssim t^{d(d-1)/2}$.

We additionally derive approximative t-design curves that asymptotically match the lower length bounds for t-design curves.

References

[1] M. Ehler, K. Gröchenig, *t-design curves and mobile sampling on the sphere*, Forum of Mathematics, Sigma, 11, 2023.

17:30–17:50 Distributing points in some Grassmannian manifolds
Carlos Beltrán (Universidad de Cantabria, SPAIN)

Abstract. The projective logarithmic energy of a collection of points $x_1, \dots, x_N \in \mathbb{P}(\mathbb{R}^3)$ is given by the formula

$$E(x_1, \dots, x_N) = \sum_{i \neq j} \log \frac{1}{\sqrt{1 - \langle x_i, x_j \rangle^2}}.$$

What is the minimal value that this energy can attain? The corresponding question in the sphere has been studied by many authors, see for example [1] for the history and a remarkable collection of results. But in the projective space the question has received much less attention. Upper and lower bounds on that value in the real, complex, quaternionic and octonionic projective spaces were computed by [2], giving quite sharp estimates (sharp from the difference between the upper and lower bounds being small) for that minimum value.

In a recent paper [3] we proved new lower bounds, improving those of [2] for all these cases. In another recent paper [4] we focused on $\mathbb{P}(\mathbb{R}^3)$ showing a constructive sequence of points which attains quite good values of the energy:

Theorem 1. *For any* $N = 1, 2, \dots$*, the minimal possible value* m_N *of the projective logarithmic energy in* $\mathbb{P}(\mathbb{R}^3)$ *satisfies:*

$$W_{log}(\mathbb{P}(\mathbb{R}^3))N^2 - \frac{1}{2}N \log N + cN \leq m_N \leq W_{log}(\mathbb{P}(\mathbb{R}^3))N^2 - \frac{1}{2}N \log N + CN,$$

where

$$W_{log}(\mathbb{P}(\mathbb{R}^3)) = 1 - \log 2$$

is the continuous energy and $c = -0.403426\dots, C = -0.395795\dots$. *Moreover, there exists an explicit, contructible set of* N *points in* $\mathbb{P}(\mathbb{R}^3)$*, depending on some random parameters, whose expected logarithmic energy is the upper bound above.*

Another popular quantity of interest is the so–called spherical discrepancy, see for example [5]. Our result in [4] also shows that the spherical sequence corresponding to the real projective points has order of discrepancy $O(1/\sqrt{N})$ which is certainly not optimal but, to our knowledge, it is if we demand constructivity of the sequence. I will present this result as well as some others, related to the problem of finding well distributed points in Grassmannians, proved in [6].

References

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18:00–18:20 Linear statistics of determinantal point processes and norm representations
Joaquim Ortega-Cerdà (Universitat de Barcelona, SPAIN)

Abstract. We study the asymptotic behaviour of the fluctuations of smooth and rough linear statistics for determinantal point processes on the sphere and on the Euclidean space. The main tool is the generalization of some norm representation results for functions in Sobolev spaces and in the space of functions of bounded variation.

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