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New trends in self-similarity of groups, trees and fractals
Special Session B11

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A particularly interesting family of groups arises as transformations of discrete spaces called trees. These spaces can be thought of as graphs, i.e. a collection of vertices and edges joining them, with no cycles. If we have such a graph with infinitely many vertices, we may select a concrete vertex and look at all other vertices as descendants of this vertex. This is said to be a *rooted tree*. If each vertex has the same number of descendants we can see that the tree hanging from a specific vertex resembles the full tree. This self-similarity connects rooted trees with fractals.

The main goal of this special session is to develop and bring together experts in topics in all areas of self-similarity of groups and the spaces on which they act, including fractal and branch groups, groups acting on Cantor sets such as Thompson-like groups, tree structures related to random walks.

The study of these groups provided answer to multiple important problems in mathematics. In particular, these groups stem from the famous Burnside problem, which arose in 1902: "Can a finitely generated torsion group, i.e. in which every element has finite order, be itself infinite?". This question went unanswered for over sixty years. Then in 1964, Golod and Shafarevich constructed an infinite finitely generated group where every element has finite order. Further examples were given by Adjan and Novikov, Olshanskii, Grigorchuk, Gupta and Sidki and recently Schlage-Puchta. The constructions given by Olshanskii, Grigorchuk, Gupta and Sidki are geometric in nature, with the groups of Grigorchuk, Gupta and Sidki acting on trees. These latter examples form a more general family of the so-called Grigorchuk-Gupta-Sidki (GGS) groups, which are in turn examples of branch groups.

A *branch group* is an example of a self-similar group acting transitively on a spherically homogeneous rooted tree T and admits a structure of subnormal subgroups similar to the corresponding structure in the full automorphism group $\mathrm{Aut}(T)$ of the tree T. There are many examples of branch groups with remarkable algebraic properties, such as the first example of a finitely generated group with intermediate growth and the first example of an amenable but not elementary amenable group. They also play an important role in the classification of just infinite groups, i.e. infinite groups whose proper quotients are all finite.

Furthermore, self-similar groups have many applications to other areas of mathematics and science, such as to dynamics, probability, and cryptography, and the aim of this workshop is also to see the interdisciplinary aspects of these class of groups.

For more information visit:
`https://sites.google.com/view/marialauranoce/umi-ams-conference`.

Schedule and Abstracts

July 23, 2024

11:00–12:00 Some algebraic and combinatorial properties of tree automaton semigroups

Daniele D'Angeli (Università Cusano, Italy)

Abstract. In this talk I will introduce a class of automaton groups defined starting from graphs. This class contains important classical examples of automaton groups. I will study some algebraic, combinatorial and dynamic properties associated with such (semi-)groups and their action on rooted trees in relation to the structure of the initial graph, giving particular emphasis to the case of trees. In particular, I will focus on freeness, growth, spectral properties and some topological indices defined on the corresponding Schreier graphs.

12:00–13:00 On free groups and semigroups defined by automata

Emanuele Rodaro (Politecnico Milano, Italy)

Abstract. In this talk, I will provide an overview of recent findings concerning the freeness question for automaton (semi)group, that is the problem of checking whether an automaton (semi)group generates a free (semi)group or not. It will be shown that algorithmically, this problem for automaton monoids is undecidable, solving an open problem initially posed by Grigorchuk, Nekrashevych, and Sushchanski. If time allows, I will also explore the problem of identifying sufficient conditions for an automaton group to be not free.

14:30–15:30 Path groups

Gustavo A. Fernández-Alcober (University of the Basque Country, Spain)

Abstract. In this talk we introduce path groups, a generalisation of the so-called multi-EGS groups that allows for directed automorphisms along arbitrary paths of a p-adic tree. We develop the basic theory of these groups, which requires the introduction of variants of the concepts of self-similarity, fractality, or regular branchness that apply to a whole family of groups rather than to a single group. Then we calculate the Hausdorff dimension of path groups with respect to the filtration provided by the level stabilisers.

15:30–16:00 The Hausdorff dimension of the generalized Brunner-Sidki-Vieira Groups

Mikel E. Garciarena (Università di Salerno and University of the Basque Country)

Abstract. Groups of automorphisms of regular rooted trees are a rich source of examples with interesting properties in group theory, and they have been used to solve very important problems, such as the General Burnside Problem and the Milnor Problem. Branch profinite groups are especially interesting because they are one of the two types of just infinite profinite groups. Abercrombie, Barnea and Shalev started the study of the Hausdorff dimension on profinite groups. The Hausdorff dimension of self-similar profinite groups is still the object of several important open problems. We present our result regarding the Hausdorff dimension of the closure of the generalized Brunner-Sidki-Vieira groups (BSV-groups for short) acting on the m-adic tree for $m \geq 2$. Which is the first examples of self-similar topologically finitely generated closed subgroups of transcendental Hausdorff dimension in the group of m-adic automorphisms. Let us briefly describe the construction of the BSV-groups. Let $\mathcal{T}$ be the m-adic tree (regular rooted tree with m descendants at every vertex), where $m \geq 2$, and note that the subtree of all descendants at an arbitrary vertex of $\mathcal{T}$ is isomorphic to $\mathcal{T}$. The automorphisms of $\mathcal{T}$ as a graph form a profinite group Aut $\mathcal{T}$ under composition. Let a and b be the two recursively defined automorphisms of $\mathcal{T}$ that rigidly permute the m subtrees hanging from the root according to the permutation $\sigma = (1\ 2\ \cdots\ m)$ and induce the following automorphisms of $\mathcal{T}$ on the m subtrees hanging from the vertices of the first level respectively:

$$(1, \ldots, 1, a) \qquad \text{and} \qquad (1, \ldots, 1, b^{-1}).$$

Then $H = \langle a, b \rangle$ is said to be the BSV-group.

Theorem 1. *Let H be the generalized Brunner-Sidki-Vieira group acting on the m-adic tree. Then the Hausdorff dimension of its closure in Γ_m is*

$$\mathrm{hdim}_{\Gamma_m}(\overline{H}) = \frac{m - \tau(m-1)\log_m 2}{m+1},$$

where the parameter τ is defined as

$$\tau := \begin{cases} 0 & \text{if } m \text{ is odd,} \\ 1 & \text{if } m \text{ is even.} \end{cases}$$

This is joint work with Jorge Fariña Asategui.

16:00–16:30 Generalized word problem for stabilizers of bounded automata groups
Davide Perego (University of Seville, Spain)

Abstract. The membership subgroup problem asks whether an element in a group belongs to a fixed subgroup. This gives rise to a formal language and, in the case of stabilizers, to a graph-theoretic interpretation. After defining groups described by automata, we will study the membership languages for stabilizers in this class. In particular, we will make use of Lindenmayer's systems, which were originally introduced in biology, but recently have successfully been exploited in combinatorial group theory.

17:00–18:00 Finitely generated Hausdorff spectra in branch groups
Jone Uria (University of the Basque Country, Spain)

Abstract. Since the pioneering work of Abercrombie, and Barnea and Shalev, Hausdorff dimension and thereof, Hausdorff spectra (i.e. the set of numbers obtained as the Hausdorff dimension of closed subgroups) of countably based profinite groups have provided fruitful results. In the particular case of profinite branch groups, Benjamin Klopsch proved that the Hausdorff spectra with respect to the level stabilizers is always the full interval $[0, 1]$.

It is common to study subsets of Hausdorff spectra of profinite groups by restricting the Hausdorff dimension values to specific subgroups of the given group. For instance, the normal Hausdorff spectrum or the finitely generated Hausdorff spectrum. The first one has been extendedly studied, but very little is known about the second one. In this talk we present results related to the study of finitely generated Hausdorff spectra of (weakly) branch groups.

July 24, 2024

11:30–12:30 Twin Towers of Hanoi: On the diameters of the components of the Schreier graphs
Zoran Sunic (Hofstra University, U.S.)

Abstract. The Hanoi Towers group H is a self-similar group acting on the rooted ternary tree. We consider the diagonal action H on pairs of vertices on the same level. It is known from the work of D'Angeli and Alfredo Donno that this action is "as transitive as it can be", given that the tree structure (and thus the length of common prefixes) is preserved.

Proposition 2. *Let Y_n be the set of pairs of vertices at level n. This set is decomposed into $n+1$ orbits $Y_{n,k}$, for $k = 0, \dots, n$, by the diagonal action of the Hanoi Towers group, where the orbit $Y_{n,k}$ consists of all pairs (u, v) such that the longest common prefix of u and v has length k.*

The proposition above follows directly from the following two facts:
- H is a regular branch group over its commutator H' and
- H' acts level-transitively on the ternary tree.

A graph structure is induced on the set Y_n of pairs of vertices at level n by the diagonal action as follows. Two pairs of vertices are neighbors if one can be obtained from the other by the diagonal action of one of the standard generators of the Hanoi Towers group H. The obtained graph Y_n is called the level n Schreier graph of the diagonal action. The orbits $Y_{n,k}$, for $k = 0, \dots, n$, of the diagonal action on level n are precisely the connected components of the Schreier graph Y_n. Note that $Y_{n,n}$ is isomorphic to the usual Schreier graph of the action of H

on level n of the ternary tree. Denote by $D_{n,k}$ the diameter of the orbit $Y_{n,k}$. We provide the exact diameters $D_{n,k}$ for the smallest four orbits $Y_{n,k}$, with $k = n, n-1, n-2, n-3$, at any level.

Theorem 3. *The diameters of the 4 smallest orbits $Y_{n,k}$, with $k = n, n-1, n-2, n-3$, are given by*

$$D_{n,n} = 2^n - 1, \qquad \text{for } n \geq 0.$$

$$D_{n,n-1} = \begin{cases} 2, & \text{for } n = 1, \\ \frac{7}{6} \cdot 2^n - \frac{3+(-1)^n}{6}, & \text{for } n \geq 1. \end{cases}$$

$$D_{n,n-2} = \begin{cases} 6, & \text{for } n = 2, \\ \frac{13}{8} \cdot 2^n - 1, & \text{for } n \geq 3. \end{cases}$$

$$D_{n,n-3} = \begin{cases} 16, & \text{for } n = 3, \\ 32, & \text{for } n = 4, \\ \frac{65}{32} \cdot 2^n - 1 & \text{for } n \geq 5. \end{cases}$$

We also provide a method that, in principle, provides the diameter $D_{n,k}$ of any orbit at any level (modulo some finite, brute force, calculation, which depends only on the difference $n-k$, but not on n). We observe an interesting phenomenon – for a fixed level n, the largest diameter does not correspond to the largest orbit (largest connected component of the graph).

12:30–13:00 Divergence in weakly branch groups

Letizia Issini (University of Geneve, Switzerland)

Abstract. The divergence of a group is a quasi-isometry invariant that measures how difficult it is to connect two points avoiding a large ball around the identity. It is easy to see that it is linear for direct products of infinite groups, and from that to deduce that it is linear for branch groups. In this talk, I will discuss divergence for weakly branch groups.

14:30–15:30 Normal subgroups of non-torsion multi-EGS groups

Anitha Thillaisundaram (University of Lund, Sweden)

Abstract. The family of multi-EGS groups form a natural generalisation of the Grigorchuk-Gupta-Sidki groups, which in turn are well-studied groups acting on rooted trees. Groups acting on rooted trees provided the first explicit examples of infinite finitely generated torsion groups, and since then have established themselves as important infinite groups, with numerous applications within group theory and beyond. Among these groups with the most interesting properties are the so-called regular branch groups. In this talk we investigate the normal subgroups in non-torsion regular branch multi-EGS groups, and we show that the congruence completion of these multi-EGS groups have bounded finite central width. Recall that the *central width* of Γ is defined as

$$w_c(\Gamma) := \sup |H : K|,$$

where H and K range over all central sections of Γ.

In particular, we prove that the profinite completion of a Fabrykowski-Gupta group has width 2, where for a pro-p group Γ the *width* of Γ is

$$w(\Gamma) := \sup_n \log_p |\gamma_n(\Gamma) : \gamma_{n+1}(\Gamma)|.$$

15:30–16:30 Iterated Wreath Products in Product Action: In search of new hereditarily just infinite groups

Matteo Vannacci (University of the Basque Country, Spain)

Abstract. A just infinite group is an infinite group without infinite proper quotients. A group is said to be hereditarily just infinite (HJI) if all of its finite index subgroups are just infinite. A classical classification theorem of Grigorchuk-Wilson states that a residually finite just infinite group is either: (a) a branch group or (b) a HJI group. Branch groups have been extensively studied (e.g. Grigorchuk group), but HJI groups remain a very mysterious class. In this talk, I will report on some recent work on the search for new HJI groups. In particular, I will describe

a way of embedding an infinitely iterated wreath product in product action as a subgroup of the automorphism group of a rooted tree via the "tree of partitions". This talk is based on joint work with G. Corob-Cook, G. Fernández-Alcober, M. Noce and S. Smith.

17:00–18:00 Diagonal Actions of Groups Acting on Rooted Trees
Dmytro Savchuk (University of South Florida, U.S.)

Abstract. For the action of the full group $\mathrm{Aut}(T_d)$ of automorphisms of T_d we describe the ergodic decomposition of its action on $(\partial T_d)^n$ for all $n \geq 1$. To achieve it we analyze the orbits of n-tuples of elements of vertices of any fixed finite level of T_d. For a subgroup G of $\mathrm{Aut}(T_d)$ the corresponding orbits may be smaller, but sometimes they coincide with the orbits of the full group of automorphisms for all levels. In this case we say that the action of G on $\mathrm{Aut}(T_d)$ is maximally tree n-transitive. For example, maximal tree 1-transitivity is equivalent to level transitivity of the action of G on T_d. It follows that the Grigorchuk group and Basilica group act maximally tree 2-transitively on ∂T_2. We show that the action of Grigorchuk group on ∂T_2 is, in-fact, at least maximally tree 4-transitive.

18:00–18:30 Rearrangement groups of fractals
Matteo Tarocchi (Università di Milano-Bicocca, Italy)

Abstract. In 2019 J. Belk and B. Forrest introduced the family of rearrangement groups. These are groups of certain "piecewise-canonical" homeomorphisms of fractals that act by permutations of the self-similar pieces that make up the fractal. Despite being countable, many of these groups seem to be dense in the group of all homeomorphisms of the fractal on which they act. The family of rearrangement groups is a generalization of the main trio of Thompson groups F, T and V, each of which has made its appearance in many different topics. Other groups unrelated to fractals, such as certain Thompson-like groups and the Houghton groups, can also be realized in the framework of rearrangement groups. Results about rearrangement groups include the simplicity of commutator subgroups in many examples, a general result about invariable generation, rationality of the fractal spaces on which they act and a method to tackle their conjugacy problem. This talk will introduce this family of groups and highlight some facts about them.

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