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PDE Theory for Fluid-Structure Interactions
Special Session B8

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This special session focuses on applied problems in the analysis of partial differential equations (PDEs) related to the interactions between a fluid and a solid. We expect contributions from researchers studying coupled models where the structure can be represented through the theory of elasticity (e.g., plates, beams, bulk solids), while the fluid may be included through the Navier-Stokes Equations or alternative theories (e.g. Stokes, Euler, potential flow). The session is also open to talks relating to analytical questions of fluids or solids independently, as they might pertain to applications in coupled fluid-structure systems. For instance, questions about well-posedness, regularity, asymptotic behavior are welcome. Additionally, techniques and results dealing with relevant nonlinear modeling, approximation methods, or computation of numerical solutions and stability are also among the purposes of the session.

Schedule and Abstracts

July 25, 2024

11:30–11:50 A measure for the stability of structures immersed in a 2D laminar flow
Filippo Gazzola (Politecnico di Milano, ITALY)

Abstract. We introduce a new measure for the stability of structures, such as the cross-section of the deck of a suspension bridge, subject to a 2D fluid force, such as the lift exerted by a laminar wind. We consider a wide class of possible flows, as well as a wide class of structural shapes. Within a suitable topological framework, we prove the existence of an optimal shape maximizing the stability. Applications to engineering problems are also discussed. Based on joint works with Edoardo Bocchi, Politecnico di Milano.

12:00–12:20 Time-periodic incompressible viscous flow around a translating rigid body
Ana Leonor Silvestre (Universidade de Lisboa, PORTUGAL)

Abstract. We consider a time-periodic incompressible three-dimensional Navier-Stokes flow past a translating rigid body. Firstly, we prove existence and uniqueness of strong solutions in the exterior domain Ω, along with pointwise estimates for the velocity and pressure. The fundamental solution of the time-periodic Oseen equations plays a crucial role in the derivation of such estimates. In the second part of the talk, we consider the approximation of the exterior problem in truncated domains $\Omega \cap B_R$ under appropriate artificial boundary conditions at ∂B_R. We prove existence and uniqueness of weak solutions for these bounded domains problems. Finally, the velocity component of the truncation error is estimated in terms of R, showing that, when $R \to \infty$, the velocities of the truncated problems converge, in a suitable norm, to the velocity of the exterior flow. This work generalizes the results of P. Deuring and S. Kračmar for the corresponding steady problem. Based on a joint work with Thomas Eiter, University of Kassel.

12:30–12:50 Long-time behavior of an anistropic rigid body interacting with a Poiseuille flow in an unbounded 2D channel
Clara Patriarca (Université Libre de Bruxelles, BELGIUM)

Abstract. We study the long-time behavior of an elliptic rigid body which is allowed to vertically translate and rotate in a 2D unbounded channel under the action of a Poiseuille flow at large distances. The motion of the fluid is modelled by the incompressible Navier-Stokes equations, while the motion of the ellipse is described through Newton's laws in the presence of additional elastic restoring forces but without any structural damping. Our main contributions are a global-in-time existence result and a proof of return to equilibrium. To our knowledge, this represents the first long-time analysis of fluid-solid interaction problems with a given non-trivial final state. Our results are based on two main ingredients: a precise description of the ellipse motion whenever it comes close to the channel boundaries and a stability analysis to compare solutions to fluid-solid problems. To tackle the description of the ellipse motion close to channel boundaries, we provide a novel extensive analysis of fluid-solid interaction asymptotics allowing all possible motions of the solid and non-symmetric configurations. Our return-to-rest result requires to compare different solutions to time-dependent and stationary problems that involve different solid and fluid domains. For this, we provide the first approach on the stability of the stationary problem in a fully eulerian setting.

July 25, 2024

14:30–14:50 On the long-time behaviour of solutions to unforced evolution Navier-Stokes equations under Navier boundary conditions
Elvise Berchio (Politecnico di Torino, ITALY)

Abstract. We consider the asymptotic behaviour of the solutions to Navier-Stokes unforced equations under Navier boundary conditions in a wide class of merely Lipschitz domains of physical interest. The analysis draws its main motivation from celebrated results by Foias-Saut under Dirichlet conditions; here the choice of the boundary conditions requires carefully considering the geometry of the domain, due to the possible lack of the Poincaré inequality in presence of symmetries. In non-axially symmetric domains we show the validity of the Foias-Saut result about the limit at infinity of the Dirichlet quotient, in axially symmetric domains we provide two invariants of the flow which completely characterize the motion and we prove that the Foias-Saut result holds for initial data belonging to one of the invariants. Based on a joint work with Alessio Falocchi, Politecnico di Milano and Clara Patriarca, Université Libre de Bruxelles.

15:00–15:20 Well-posedness of a nonlinear shallow water model for oscillating water columns
Edoardo Bocchi (Politecnico di Milano, ITALY)

*Abstract.*We consider a particular wave energy converter, the so-called oscillating water column. Water waves governed by the one-dimensional nonlinear shallow water equations arrive from the offshore, enter a partially-closed chamber and the consequent variation of air volume activates a turbine. We reformulate the problem as a hyperbolic transmission problem related to the wave-structure interaction at the entrance of the chamber. A time-dependent air pressure inside the chamber is taken into account in the model. First, we impose conservation of the total energy to derive a transmission condition that closes the system. Then, we address its local well-posedness, which is obtained by constructing a Kreiss symmetrizer. Based on a joint work with Jiao He and Gastón Vergara-Hermosilla, Université Paris-Saclay.

15:30–15:50 A fluid-poroviscoelastic structure interaction problem with nonlinear coupling
Jeffrey Kuan (University of Maryland College Park, USA)

Abstract. We present a well-posedness result for a moving-boundary fluid-poroelastic structure interaction (FPSI) problem involving the coupled dynamics between an incompressible fluid modeled by the Navier-Stokes equation and a multilayered structure consisting of a thin reticular plate and a thick Biot poroelastic medium. The fluid domain and the physical Biot domain, on which the Navier-Stokes and the Biot equations respectively are posed, are time-dependent and a priori unknown domains, which are determined by the a priori unknown reticular plate displacement.

The Biot equations, originally introduced by Biot to model soil consolidation, model the dynamics of the displacement and pore pressure of a poroelastic medium, which is a medium that is elastic and admits fluid flow through its pores. Such coupled FPSI problems involving incompressible fluids interacting dynamically with poroelastic structures arise in applications to biomedical engineering and geosciences, and are hence of practical importance.

First, we show an existence result for solutions to a regularized form of the FPSI problem, with a regularization parameter $\delta > 0$. This regularization is needed for the analysis, because solutions in the finite energy space for the original problem do not have sufficient regularity for the weak formulation and the moving domains to be well-defined. We give a constructive existence proof for the regularized FPSI problem, by using a splitting scheme which splits the Biot/fluid dynamics and the reticular plate dynamics to define approximate solutions to the FPSI problem. We then pass to the limit in the semidiscrete weak formulations by using compactness arguments, which include compactness arguments of Aubin-Lions type for functions defined on moving domains, to obtain strong convergence of the approximate solutions.

Next, upon showing existence of solutions to the regularized problem, we establish a weak-classical consistency result under the additional assumption that the Biot medium is poroviscoelastic, which shows that the weak solutions to the regularized problem are physically reasonable in the following sense. In particular, if the original (non-regularized) FPSI problem has a classical solution on the time interval $[0, T]$, we show that the weak solutions to the regularized problem with regularization parameter $\delta > 0$ converge to the classical solution to the original problem as $\delta \to 0$. We accomplish this by using a new bootstrap argument in addition to energy-type estimates in order to obtain control of geometric quantities that is uniform in the regularization parameter $\delta > 0$. Based on a joint work with Sunčica Čanić, University of California Berkeley and Boris Muha, University of Zagreb.

July 26, 2024

11:30–11:50 On the motion of several rigid bodies in a viscous fluid
Eduard Feireisl (Czech Academy of Sciences, CZECH REPUBLIC)

Abstract. We show that the collective effect of N rigid bodies $(\mathcal{S}_{n,N})_{n=1}^{N}$ of diameters $(r_{n,N})_{n=1}^{N}$ immersed in an incompressible non–Newtonian fluid is negligible in the asymptotic limit $N \to \infty$ as long as their total packing volume $\sum_{n=1}^{N} r_{n,N}^{d}$, $d = 2, 3$ tends to zero exponentially – $\sum_{n=1}^{N} r_{n,N}^{d} \approx A^{-N}$ – for a certain constant $A > 1$. The result is rather surprising and in a sharp contrast with the associated homogenization problem, where the same number of obstacles can completely stop the fluid motion in the case of shear thickening viscosity. A large class of non–Newtonian fluids is included, for which the viscous stress is a subdifferential of a convex potential.

12:00–12:20 A proof of Vishik's nonuniqueness theorem for the forced 2D Euler equation
Ángel Castro (Instituto de Ciencias Matemáticas, SPAIN)

Abstract. In this talk we will present a new proof of Vishik's theorem on the non uniqueness of solutions for 2D incompressible Euler equations with forcing. Our proof relies in the construction of an unstable radial vortex into two steps: i) the construction of a piece-wise unstable radial vorticity, ii) the regularization of such a vorticity. The rest of the proof follows the Vishik's strategy and the paper "Instability and nonuniqueness for the 2d Euler equations in vorticity form, after M. Vishik" by D. Albritton et al.

12:30–12:50 Homogenization and low Mach number limit for the evolutionary Navier Stokes-Fourier system
Danica Basarić (Politecnico di Milano, ITALY)

*Abstract.*In this talk, we are going to consider the motion of a compressible, viscous and heat-conducting fluid on a domain perforated by tiny holes. First, we are going to identify a class of dissipative solutions to the Oberbeck-Boussinesq approximation as a low Mach number limit of the primitive system. Secondly, by proving the weak–strong uniqueness principle, we obtain strong convergence to the target system on the lifespan of the strong solution. Based on a joint work with Nilasis Chaudhuri, University of Warsaw.

July 26, 2024

14:30–14:50 Estimates of a possible gap related to the energy equality for Newtonian and non-Newtonian fluids

Francesca Crispo (University of Campania "Luigi Vanvitelli", ITALY)

Abstract. We consider the Navier-Stokes and the p-Navier-Stokes equations, and we investigate on the validity of an energy equality.

It is widely acknowledged that a weak solution of the Navier-Stokes equations a priori enjoys an energy inequality. We construct a weak Leray-Hopf solution enjoying a sort of energy equality, which adds a dissipative quantity to the classical energy equality. The additional dissipation, that measures the possible gap with the classical energy equality, is only expressed in terms of energy quantities.

The result is obtained under minimal assumptions on the initial datum for the existence of a weak solution. It is based on a new existence theorem, where our weak solution is the limit of a sequence of solutions to a regularized problem.

Further, we extend our investigation to weak solutions of the chosen model of non-Newtonian fluids within space-periodic domains.

15:00–15:20 Unpredictable behaviour of a partially damped system of PDEs modeling suspension bridges with piers

Emanuele Pastorino (Politecnico di Milano, ITALY)

Abstract. We investigate the long-time behavior of the solutions of a nonlinear nonlocal system of evolution PDEs governing the dynamics of suspension bridges with intermediate piers under the action of the wind. The deck of the bridge is modeled as a degenerate plate composed by a central beam which can move vertically and by a continuum of cross sections which can rotate around their center located on the beam. This gives rise to a system with two degrees of freedom: the vertical displacement of the central beam, governed by a beam-type equation, and the torsional displacement of the cross sections, described by a wave-type equation. Due to the presence of the piers, the solutions fail to be smooth and a suitable notion of weak solution is required. The dissipative effects of the structure and the action of external sources (such as the vortex shedding generated by the wind) are explicitly taken into account by inserting appropriate damping and forcing terms. Particular attention is also devoted to the purely longitudinal motion, arising when no torsional displacement occurs.

15:30–15:50 Well-posedness of Free Boundary Inviscid Flow-Structure Interaction Systems

Amjad Tuffaha (American University of Sharjah, USA)

Abstract. We obtain the local existence and uniqueness of solutions for a system describing interaction of an incompressible inviscid fluid, modeled by the Euler equations, and an elastic plate, represented by the fourth-order hyperbolic PDE. We provide a priori estimates for the existence with the optimal regularity H^r, for $r > 2.5$, on the fluid initial data and construct a unique solution of the system for initial data $u_0 \in H^r$ for $r \geq 3$.

We also address the compressible Euler equations in a domain with a free elastic boundary, evolving according to a weakly damped fourth order hyperbolic equation forced by the fluid pressure. We establish a priori estimates on local-in-time solutions in low regularity Sobolev spaces, namely with velocity and density initial data in H^3. Based on a joint work with Igor Kukavica, University of Southern California and Sarka Necasova, Czech Academy of Sciences.