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Arithmetic and Geometric Aspects of Drinfeld Modules, Anderson Motives, and Computational Aspects Special Session A20

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This session of the 2nd Joint Meeting co-organized by the Unione Matematica Italiana (UMI) and the American Mathematical Society (AMS) will be mainly focused on the arithmetic and geometric theory of Drinfeld modules, shtukas and Anderson motives and on related topics, such as L-series, zeta, multizeta values, transcendence, Drinfeld modular varieties, Drinfeld modular forms. The session, scheduled on July 23-24, 2024, will also include computational aspects as different groups have been developing and implementing computational algorithms for Drinfeld modules.

Relevant information will be posted here:
`https://sites.google.com/uniroma1.it/drinfeld-modules/home-page`.

Schedule

July 23, 2024

11:00–11:20 Ernst-Ulrich Gekeler (Universität des Saarlandes, Germany)
Title. The Manin-Drinfeld Property of Drinfeld Modular Varieties.

Abstract. Let A be a Drinfeld coefficient ring (e.g., a polynomial ring $\mathbb{F}[T]$ over a finite field $\mathbb{F}$), and let M be the Drinfeld modular variety attached to either

(1) the linear group $\mathrm{GL}(Y)$ of a projective A-module Y of rank $r \geq 2$, or to
(2) the $\mathfrak{n}$-th congruence subgroup of such a $\mathrm{GL}(Y)$, where $\mathfrak{n}$ is a non-trivial ideal of A.

By work of Kapranov, Pink, Häberli, and the author, there exist natural Satake-like compactifications $\bar{M}$ of such M. The property referred to in the title is:

(MD) The divisors in $\bar{M}\backslash M$ generate a finite subgroup in the Chow group of cycles of codimension 1 of $\bar{M}$.

This had first been observed for the analogous case of classical elliptic modular curves M by Manin and Drinfeld 1973, where it means that the cusps of M generate a finite subgroup in the Jacobian $\mathrm{Jac}(\bar{M})$.

We show that the MD property holds for arbitrary Drinfeld rings A, ranks $r \geq 2$ and levels $\mathfrak{n}$, thereby generalizing earlier work in special cases ($A = \mathbb{F}[T]$ and $r = 2$, Gekeler 1984, $A = \mathbb{F}[T]$ and $r \geq 2$, Kapranov 1988, A arbitrary and $r = 2$, Gekeler 2000). The key ingredients are:

- an explicit formula in terms of partial zeta values for the vanishing orders of certain modular forms (division forms) $d_{\mathbf{u}}$ along boundary divisors of M (Gekeler 2023);
- the action of a finite abelian group $Q(M)$ both on the boundary divisors of M (where it is free and transitive) and on the parameter set $\{\mathbf{u}\}$ of the division forms $d_{\mathbf{u}}$. (In case (a), $Q(M)$ is the class group $\mathrm{Pic}(A)$ of A, while in case (b), it is an extension of $\mathrm{Pic}(A)$ by a group that depends on the rank r and the level $\mathfrak{n}$.)

Thereby, (MD) is reduced to showing the non-vanishing of certain character sums on $Q(M)$. This finally is achieved by relating character sums with values of the zeta function of A which are known to be non-zero.

11:30–11:50 Oğuz Gezmiş (IWR, Ruprecht-Karls-Universität Heidelberg Germany)
Title. Nearly holomorphic Drinfeld modular forms for admissible coefficient rings.

Abstract. Inspired by the work of Shimura, Chen and Gezmiş introduced the notion of nearly holomorphic Drinfeld modular forms for a certain admissible coefficient ring and studied their special values at CM points. In this talk, we discuss nearly holomorphic Drinfeld modular forms for a more general setting, namely for any admissible coefficient rings, as well as their special values at CM points. Moreover, motivated from the classical theory, we describe nearly holomorphic Drinfeld modular forms as global sections of a particular coherent sheaf on the compactification of the Drinfeld moduli space. This is a joint work with Sriram Chinthalagiri Venkata.

12:00–12:20 Sjoerd de Vries (Department of Mathematics, Stockholm University Sweden)
Title. Traces of Hecke operators on Drinfeld modular forms.

Abstract. I will talk about recent work related to a trace formula for Hecke operators on Drinfeld modular forms of level 1. The formula uses Böckle-Eichler-Shimura theory [3] and point-counts on the moduli space of rank 2 Drinfeld modules. I will mention several theoretical consequences, such as a Ramanujan bound for Drinfeld modular forms and implications for the theory of oldforms and newforms. The trace formula can also be implemented algorithmically, which leads to interesting data suggesting directions for further research.

12:30–12:50 Shin Hattori (Department of Natural Sciences, Tokyo City University Japan)
Title. $\mathcal{D}$-elliptic sheaves and the Hasse principle.

Abstract. Let p be a rational prime, $q > 1$ a p-power integer and $F = \mathbb{F}_q(t)$. Let $d \geq 2$ be an integer and D a central division algebra over F of dimension d^2 which splits at ∞ and such that for any place x of F at which D ramifies, the invariant of D at x is $1/d$.

A $\mathcal{D}$-elliptic sheaf is a system of locally free sheaves equipped with an action of a sheafified version $\mathcal{D}$ of D. $\mathcal{D}$-elliptic sheaves are parametrized by the Drinfeld–Stuhler variety over F. When $d = 2$, it is also called the Drinfeld–Stuhler curve and it can be considered as a function field analogue of a quaternionic Shimura curve over $\mathbb{Q}$.

For quaternionic Shimura curves, in 1980s Jordan proved a criterion for the non-existence of quadratic points on them and gave an example of a quaternionic Shimura curve curve X and a quadratic field K such that X has no K-rational points but has K_v-rational points for any place v of K. This property of having local points without having global points is often called violation of the Hasse principle.

In this talk, I will explain how to generalize Jordan's criterion to Drinfeld–Stuhler varieties to obtain similar examples of quadratic extensions K/F over which the Drinfeld–Stuhler curve violates the Hasse principle. This is a joint work with Keisuke Arai, Satoshi Kondo and Mihran Papikian.

14:30–14:50 Yujie Xu (Department of Mathematics, Columbia University USA)
Title. Uniformizing the moduli stacks of global G-shtukas.

Abstract. Moduli stacks of global G-Shtukas play an important role in the global Langlands program over function fields. They were used in the proof of the function field global Langlands conjecture by Drinfeld, Lafforgue etc. In this talk, I will speak on my joint work with Urs Hartl, where we show that the moduli spaces of suitably bounded global G-Shtukas with colliding legs satisfy a p-adic uniformization isomorphism by Rapoport-Zink spaces. If time permits, I will mention some applications (e.g. to the Langlands-Rapoport Conjecture).

15:00–15:20 Paola Chilla (Institute for Mathematics, University of Heidelberg Germany)
Title. Towards a Jacquet-Langlands correspondence for function field modular forms.

Abstract. In this talk I will discuss a first step towards a Jacquet-Langlands correspondence for (quaternionic) Drinfeld modular forms. Using étale cohomology computations (facilitated by the theory of function field crystals) and the Eichler-Shimura isomorphism proved by Böckle, I will associate Hecke eigensystems of rank 2 Drinfeld cusp forms to Hecke eigensystems of functions on quaternionic adelic double quotients, for a definite quaternion algebra ramified at exactly one finite place, exploiting the fact that these double quotients describe supersingular Drinfeld modules. The Drinfeld cusp forms obtained are new at the prime where the quaternion algebra is ramified, in the sense of Bandini and Valentino. Under suitable ordinariness assumptions for the special fiber of modular curves, all newforms arise in this way. The talk is based on the speaker's PhD thesis, written under the supervision of Prof. Gebhard Böckle.

15:30–15:50 Sheng-Yang Kevin Ho (Department of Mathematics, The Pennsylvania State University USA)
Title. The rational torsion subgroups of the Drinfeld modular Jacobians for prime-power levels.

Abstract. Fix a non-zero ideal $\mathfrak{n}$ of $\mathbb{F}_q[T]$. Let $\mathcal{T}(\mathfrak{n})$ be the rational torsion subgroup of the Drinfeld modular Jacobian $J_0(\mathfrak{n})$. A generalized Ogg's conjecture states that $\mathcal{T}(\mathfrak{n})$ coincides with the rational cuspidal divisor class group $\mathcal{C}(\mathfrak{n})$ of the Drinfeld modular curve $X_0(\mathfrak{n})$. First, we prove that for any prime-power ideal $\mathfrak{p}^r$ of $\mathbb{F}_q[T]$, the prime-to-$q(q-1)$ part of $\mathcal{T}(\mathfrak{p}^r)$ is equal to that of $\mathcal{C}(\mathfrak{p}^r)$ by studying the Hecke operators and the Eisenstein ideal of level $\mathfrak{p}^r$. Second, by relating the rational cuspidal divisors of degree 0 on $X_0(\mathfrak{p}^r)$ with Δ-quotients, where Δ is the Drinfeld discriminant function, we are able to compute explicitly the structure of $\mathcal{C}(\mathfrak{p}^r)$. As a result, the structure of the prime-to-$q(q-1)$ part of $\mathcal{T}(\mathfrak{p}^r)$ is completely determined.

16:00–16:20 Andreas Maurischat (Department of Mathematics, RWTH Aachen University Germany)
Title. Solving linear difference equations to check abelianess of Anderson t-modules.

Abstract. Let K be perfect field containing the finite field $\mathbb{F}_q$ of q elements. Let t and τ be indeterminates and consider the skew polynomial ring

$$K[t]\{\tau\} = \left\{ \sum_{i,j=0}^{n} \alpha_{ji} t^j \tau^i \,\middle|\, n \geq 0, \alpha_{ji} \in K \right\}$$

with multiplication uniquely given by additivity and the rules

$$\begin{aligned} \tau \cdot \alpha &= \alpha^q \cdot \tau \quad \forall \alpha \in K, \\ t \cdot \alpha &= \alpha \cdot t \quad \forall \alpha \in K, \\ \tau \cdot t &= t \cdot \tau \end{aligned}$$

For an Anderson t-module over K of dimension d, its t-motive M is a left $K[t]\{\tau\}$-module which is free of dimension d as $K\{\tau\}$-module. The t-motive M and the Anderson t-module are called *abelian*, if M is also free finitely generated as $K[t]$-module.

In this talk, we explain how one can check whether a given t-motive is abelian or not by solving the linear difference equations associated to the t-motive. Using a modification of that algorithm, we also show that after scalar extension to $K(t)$, every t-motive becomes finitely generated, i.e. that $K(t) \otimes_{K[t]} M$ is a finite dimensional $K(t)$-vector space.

17:00–17:20 Quentin Gazda (Centre de Mathématiques Laurent Schwartz (CMLS) École Polytechnique, Palaiseau, France)
Title. Wieferich primes and $\mathfrak{p}$-adic L-functions: a conjectural relation for Drinfeld modules.

Abstract. Let p be an odd prime. The prime p is called *Wieferich* if the multiplicative order of 2 modulo p is the same modulo p^2. Very little is known about Wieferich primes; in fact, 1093 and 3511 are the only known ones and the question whether there are infinitely many of them

remains open.

The notion of Wieferich primes can be easily generalized to a Drinfeld module E following the well-established analogy between the multiplicative group scheme and the Carlitz module. Questions related to their density appear to be as difficult as in the number field situation.

The aim of this short talk is to share the surprising observation that the condition for a prime $\mathfrak{p}$ in $\mathbb{F}[t]$ to be *E-Wieferich* is related to the $\mathfrak{p}$-adic valuation of the $\mathfrak{p}$-adic L-function of E. This generalizes the case where E is the Carlitz module, envisioned and established by Thakur [17] in 2015.

All the conjectures were stated jointly with Xavier Caruso and based on our recent work [2] where an algorithm computing L-functions of t-motives is described.

17:30–17:50 Xavier Caruso (IMB; CNRS, Université de Bordeaux, France)
Title. Computation of classical and v-adic L-series of t-motives.

Abstract. I will present an algorithm for computing the L-series attached to a t-motive and report on an implementation of it in SageMath (with a demo if time permits).

July 24, 2024

11:30–11:50 Fu-Tsun Wei (Department of Mathematics, National Tsing Hua University, Taiwan)
Title. Algebraic relations of special v-adic arithmetic gamma values and their period interpretations.

Abstract. Let v be a finite place of $\mathbb{F}_q(\theta)$. We show that algebraic relations of arithmetic v-adic Γ-functions over $\mathbb{F}_q(\theta)$ are explained by the standard functional equations together with Thakur's analogue of the Gross–Koblitz formula. A key step is working out a formula expressing v-adic crystalline periods of Carlitz motives with complex multiplication as products of these special gamma values (a v-adic Chowla–Selberg formula). The algebraic independence of those periods in question results from determining the dimension of the motivic Galois groups, adapting and further developing existing tools.

12:00–12:20 Yoshinori Mishiba (Mathematical Institute, Tohoku University, Japan)
Title. On generators of v-adic multiple zeta values in positive characteristic.

Abstract. Let $A := \mathbb{F}_q[\theta]$ be the polynomial ring in the variable θ over a finite field of q elements, k the fraction field of A, and v a finite place of k. For each index $\mathfrak{s} \in \bigcup_{r\geq 0} \mathbb{Z}_{\geq 1}^r$, let $\zeta_A(\mathfrak{s})$ be the ∞-adic multiple zeta value (MZV) defined by Thakur, and $\zeta_A(\mathfrak{s})_v$ the v-adic MZV defined by Chang and the speaker. These values are function field analogues of the real-valued and p-adic MZV's respectively. Let $\mathcal{Z}_w$ (resp. $\mathcal{Z}_{v,w}$) be the k-vector space spanned by the ∞-adic (resp. v-adic) MZV's of weight w, and set $\mathcal{Z} := \sum_{w\geq 0} \mathcal{Z}_w$ and $\mathcal{Z}_v := \sum_{w\geq 0} \mathcal{Z}_{v,w}$. One of the central problem in this topic is to determine the structures of the k-vector spaces $\mathcal{Z}$ and $\mathcal{Z}_v$.

In the ∞-adic case, Chang showed that the natural surjection $\bigoplus_{w\geq 0} \mathcal{Z}_w \twoheadrightarrow \mathcal{Z}$ is an isomorphism. For each $\mathcal{Z}_w$, Todd and Thakur gave conjectures about its dimension and basis respectively, and Ngo Dac showed that the candidates given by Thakur indeed generate $\mathcal{Z}_w$. The following theorem gives affirmative answers of Todd's dimension conjecture and Thakur's basis conjecture:

Theorem 1 (Chang-Chen-Mishiba [8]**.** *, Im-Kim-Le-Ngo Dac-Pham] Let $\mathcal{I}_w^{\mathrm{T}}$ be the set of indices $\mathfrak{s} = (s_1, \dots, s_r)$ of weight w such that $s_1, \dots, s_{r-1} \leq q$ and $s_r < q$. Then for each $w \geq 0$, the elements $\zeta_A(\mathfrak{s})$ ($\mathfrak{s} \in \mathcal{I}_w^{\mathrm{T}}$) form a k-basis of $\mathcal{Z}_w$. In particular, we have* $\dim_k \mathcal{Z}_w = |\mathcal{I}_w^{\mathrm{T}}|$.

In this talk, we will discuss v-adic analogues of the above results. In the v-adic case, the result corresponding to Theorem 1 is not known. However, the following theorem holds:

Theorem 2 (Chang-Mishiba [9]**.** *, Chang-Chen-Mishiba [7] $\mathcal{Z}_v$ forms a k-algebra and there exists a well-defined k-algebra homomorphism $\phi_v\colon \mathcal{Z} \twoheadrightarrow \mathcal{Z}_v$ such that $\phi_v(\zeta_A(\mathfrak{s})) = \zeta_A(\mathfrak{s})_v$.*

Theorem 2 means that the v-adic MZV's satisfy the same k-algebraic relations that their corresponding ∞-adic MZV's satisfy. Note that Thakur showed that $\mathcal{Z}_w \cdot \mathcal{Z}_{w'} \subset \mathcal{Z}_{w+w'}$. Since $\zeta_A(q-1)_v = 0$ by Goss, we have the following corollary:

Corollary 3. *For each $w \geq 0$, we have $\dim_k \mathcal{Z}_{v,w} \leq |\mathcal{I}_w^{\mathrm{T}}| - |\mathcal{I}_{w-(q-1)}^{\mathrm{T}}|$, where $\mathcal{I}_n^{\mathrm{T}} := \emptyset$ for $n < 0$.*

It is conjectured that the inequality in Corollary 3 is an equality. We will give candidate generators of the k-vector space $\mathcal{Z}_{v,w}$.

12:30–12:50 Nathan Green (Department of Mathematics, Louisiana Tech University, USA)
Title. New Families of Multiple Polylogarithm Relations over Function Fields

Abstract. We described a new family of relations between Carlitz multiple polylogarithms obtained using a new noncommutative factorization of the exponential function. We describe these relations both at the finite level, as well as in a motivic setting. Additionally, we give precise formulas for the coefficients of these relations and describe how these coefficients relate to Carlitz multiple zeta values.

14:30–14:50 Changningphaabi Namoijam (Department of Mathematics, Colby College, USA)
Title. Periods of extensions of Drinfeld modules by the Carlitz module

Abstract. Drinfeld introduced 'Elliptic modules' now called Drinfeld modules, which can be regarded as a function field analogue of elliptic curves. By Anderson's work, there are higher dimensional generalizations of Drinfeld module called t-modules. As they are modules, if we consider two t-modules E and F, we can study the properties of $\mathrm{Ext}^1(E, F)$. Papanikolas and Ramachandran showed that when F is of rank 1 and dimension 1 (called the Carlitz module) and E is a Drinfeld module, $\mathrm{Ext}^1(E, F)$ has the structure of a t-module, and a special subgroup $\mathrm{Ext}^1_0(E, F)$ is isomorphic to the dual t-module associated to E. In this talk, we present recent work regarding the periods of the extensions in $\mathrm{Ext}^1_0(E, C)$ where we establish their connection with the periods and logarithms of the dual t-module associated to E. Our result generalizes the work of Chang for rank 2 Drinfeld modules.

15:00–15:20 Yen-Tsung Chen (Department of Mathematics, Texas A&M University, USA)
Title. On extensions of Drinfeld modules by the tensor powers of the Carlitz module

Abstract. Let E be a Drinfeld module of rank $r \geq 2$ and $\mathbf{C}^{\otimes n}$ be the n-th tensor powers of the Carlitz module. By the theory of Anderson, we can associate a dual t-motive $\mathcal{M}_E$ for E and a dual t-motive $\mathcal{M}_{\mathbf{C}^{\otimes n}}$ for $\mathbf{C}^{\otimes n}$ respectively. In this talk, we aim to present some results concerning the $\mathbb{F}_q[t]$-module structure of $\mathrm{Ext}^1_{\mathfrak{F}}(\mathcal{M}_E, \mathcal{M}_{\mathbf{C}^{\otimes n}})$, where we consider the extensions of $\mathcal{M}_E$ by $\mathcal{M}_{\mathbf{C}^{\otimes n}}$ in the category of Frobenius modules. Consequently, we prove that $\mathrm{Ext}^1_{\mathfrak{F}}(\mathcal{M}_E, \mathcal{M}_{\mathbf{C}^{\otimes n}})$ itself defines a t-module. Furthermore, we have the following short exact sequence of t-modules

$$0 \to \mathbf{C}^{\otimes n-1} \otimes \wedge^{r-1} E \to \mathrm{Ext}^1_{\mathfrak{F}}(M_E, M_{\mathbf{C}^{\otimes n}}) \to \partial \mathbf{C}^{\otimes n} \to 0,$$

where $\partial \mathbf{C}^{\otimes n}$ is the n-dimensional t-module whose $\mathbb{F}_q[t]$-module structure is determined by

$$t \cdot \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} := \begin{pmatrix} \theta & 1 & & \\ & \ddots & \ddots & \\ & & \theta & 1 \\ & & & \theta \end{pmatrix} \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}.$$

As an application of our results on $\mathrm{Ext}^1_{\mathfrak{F}}(\mathcal{M}_E, \mathcal{M}_{\mathbf{C}^{\otimes n}})$, we will present an effective construction of the rigid analytic trivialization for extensions of $\mathcal{M}_E$ by $\mathcal{M}_{\mathbf{C}^{\otimes n}}$.

15:30–15:50 Giacomo Hermes Ferraro (Department of Mathematics Guido Castelnuovo, University Sapienza of Rome, Italy)
Title. A duality result about special functions in Drinfeld modules of arbitrary rank.

Abstract. Given a Drinfeld module ϕ over a ring A with an $\mathbb{F}_q$-rational point at ∞, Poonen proved in [15] that the kernel of the adjoint exponential $\exp^*_\phi$ is isomorphic to $\mathrm{Hom}_{\mathbb{F}_q}(\Lambda_\phi, \mathbb{F}_q)$. We make explicit the inverse of this bijection: $g \in \mathrm{Hom}_{\mathbb{F}_q}(\Lambda_\phi, \mathbb{F}_q)$ is sent to $-\sum_{\lambda \in \Lambda_\phi \setminus \{0\}} \frac{g(\lambda)}{\lambda}$.

We use this result to prove that the series $\zeta_\phi := -\sum_{\lambda \in \Lambda_\phi \setminus \{0\}} \lambda^{-1} \otimes \lambda \in \mathbb{C}_\infty \hat{\otimes} \Lambda_\phi$ is a *dual Drinfeld eigenvector*, in the sense that $(\phi^*_a \otimes 1)\zeta_\phi = (1 \otimes a)\zeta_\phi$ for all $a \in A$.

Under a functorial point of view, it turns out that ζ_ϕ is the universal object with this property; similarly, we find the universal *Drinfeld eigenvector* $\omega_\phi \in \mathbb{C}_\infty \hat{\otimes} \mathrm{Hom}_A(\Lambda_\phi, \Omega_{A/\mathbb{F}_q})$, a substitute of the special functions studied by Anglès, Ngo Dac and Tavares Ribeiro in [1]. Finally, we state that, under the $\mathbb{C}_\infty \hat{\otimes} A$-linear pairing $\_ \cdot \_$, the element $\zeta_\phi \cdot \omega_\phi \in \mathbb{C}_\infty \hat{\otimes} \Omega_{A/\mathbb{F}_q}$ is a rational differential form, partly generalizing previous rationality results by Pellarin [14], Green and Papanikolas [12], and the speaker [11].

16:00–16:20 Bo-Hae Im (Korea Advanced Institute of Science and Technology, Korea)
Title. Zagier-Hoffman's conjectures in positive characteristic

Abstract. Zagier-Hoffman's conjectures in the classical setting on multiple zeta values over $\mathbb{Q}$ of Euler and Euler sums are still open. As analogues of the classical case, multiple zeta values and alternating multiple zeta values in positive characteristic were introduced by Thakur and Harada. In this talk, we determine the dimension and a basis of the span of all alternating multiple zeta values over the rational function field by finding all linear relations among them. As a consequence, we completely establish Zagier-Hoffman's conjectures in positive characteristic formulated by Todd and Thakur which predict the dimension and an explicit basis of the span of multiple zeta values of Thakur of fixed weight. In fact, our results have completed the conjectures as a corollary of the vastly generalized results which proves the conjectures for alternating MZV's [4] and cyclotomic MZV's [5].

This is a joint work with Hojin Kim, Khac Nhuan Le, Tuan Ngo Dac, and Lan Huong Pham.

17:00–17:20 María Inés de Frutos-Fernández (Department of Mathematics, Universidad Autónoma de Madrid and Instituto de Ciencias Matemáticas, Spain)
Title. The refined class number formula for Drinfeld modules.

Abstract. Taelman proved a formula for a special value of the Goss L-function attached to a Drinfeld module [16], which can be interpreted as a function field analogue of the analytic class number formula. In the same article, he stated that 'it should be possible to formulate and prove an equivariant version' of this formula.

Given a finite field $\mathbb{F}_q$, a finite Galois extension K/k of function fields over $\mathbb{F}_q$ and a Drinfeld $\mathbb{F}_q[t]$-module E defined over the ring of integers of k, in [10] we formulate and prove an equivariant refinement of Taelman's formula for $(E, K/k)$. We also derive explicit consequences for the Galois module structure of the Taelman class group of E over K.

17:30–17:50 Florian Breuer (University of Newcastle, Australia)
Title. Coefficients of Drinfeld modular polynomials

Abstract. For any monic polynomial $N \in A = \mathbb{F}_q[t]$ we can define the Drinfeld modular polynomial $\Phi_N(X,Y) \in A[X,Y]$, which vanishes at pairs of j-invariants of rank 2 Drinfeld modules linked by a cyclic N-isogeny. The height of this polynomial is the largest t-degree of its coefficients, and grows rapidly with $\deg N$.

The precise asymptotic growth of this height was determined by Hsia in 1998 [13].

In this talk, we give fully explicit bounds on this height, similar to recent work in the elliptic curve case [6].

18:00–18:20 Maxim Mornev (Institute of Mathematics, EPFL, Switzerland)
Title. Local Kummer theory for Drinfeld modules

Abstract. Let φ be a Drinfeld A-module of finite residual characteristic $\bar{\mathfrak{p}}$ over a local field K. We study the action of the inertia group of K on a modified adelic Tate module $T^\circ_{\mathrm{ad}}(\varphi)$ which differs from the usual adelic Tate module only at the $\bar{\mathfrak{p}}$-primary component. After replacing K by a finite extension we can assume that φ is the analytic quotient of a Drinfeld module ψ of good reduction by a lattice $M \subset K$. The image of inertia acting on $T^\circ_{\mathrm{ad}}(\varphi)$ is then naturally a subgroup of $\mathrm{Hom}_A(M, T^\circ_{\mathrm{ad}}(\psi))$.

This subgroup is described by a canonical local Kummer pairing that is the central subject of our study. In particular we give an effective formula for the image of inertia up to finite index, and obtain a necessary and sufficient condition for this image to be open. We also determine the image of the ramification filtration.

18:30–18:50 Chien-Hua Chen (National Center for Theoretical Sciences, Taiwan)
Title. On natural density of rank-2 Drinfeld modules with big Galois image

Abstract. In the groundbreaking work of Jones, he investigated the natural density of elliptic curves over $\mathbb{Q}$ with adelic Galois image that are index-2 subgroups of $\mathrm{GL}_2(\hat{\mathbb{Z}})$. Zywina later extended this result to elliptic curves over arbitrary number fields. It turns out that the natural density of elliptic curves over a number field with maximal adelic Galois image is equal to one.

As a function field analogy, Pink and Rütsche proved the open image theorem for Drinfeld modules of arbitrary rank without complex multiplication. And Zywina proved the existence of rank-2 Drinfeld modules with surjective adelic Galois representation. Hence it makes sense to study the natural density of Drinfeld modules of rank 2 with surjective adelic Galois representation. Due to Van der Heiden's result on the determinant of adelic Galois representation for Drinfeld modules, the natural density estimation can be split into two cases:

Case 1: Natural density of rank-1 Drinfeld modules with surjective adelic Galois representation.

Case 2: Natural density of rank-2 Drinfeld modules whose adelic Galois image containing $\mathrm{SL}_2(\widehat{\mathbb{F}_q[T]})$.

In this talk, we compute the natural density of rank-1 Drinfeld module over $\mathbb{F}_q[T]$ with surjective adelic Galois representation; and the natural density of rank-2 Drinfeld modules over $\mathbb{F}_q[T]$ whose $\mathfrak{l}$-adic Galois image containing the special linear subgroup for finitely many prime ideal $\mathfrak{l}$.

19:00–19:20 Wei-Cheng Huang (Department of Mathematics, University of Rochester)
Title. Regulators of Drinfeld modules or t-modules appear in the class module formulas of Demeslay, Fang, and Taelman. In this talk, we will investigate regulators of the tensor, symmetric, and alternating squares of Drinfeld modules of rank 2, and express them in terms of data from the Drinfeld modules themselves.

Abstract.

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