Mean Field Games and Related Topics
Special Session A1

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Mean field games are a recent and very active area of research leading to new challenging mathematical questions, from analytical, probabilistic, and numerical sides. They originated from the seminal works of Lasry and Lions, and, simultaneously, Huang, Malhamé and Caines, in 2006. In a nutshell, Mean field games represent limit models for symmetric non-cooperative N-player games, as the number of players tends to infinity. On the other hand, mean field control problems, also called optimal control of McKean-Vlasov dynamics, represent limit models for cooperative N-player games. The aim of this special session is to gather together the big community of researchers working on mean field games, mean field control and related topics mainly in Italy and in the United States.

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Schedule and Abstracts

July 23, 2024

11:00–11:45 Deterministic ergodic Mean Field Games with congestion
Martino Bardi (University of Padua, ITALY)

Abstract. I consider deterministic Mean Field Games (MFG) with a cost functional continuous with respect to the distribution of the agents and satisfying a gap condition at infinity, and compare them with the static MFG with such a cost.

Under the coercivity condition on the Hamiltonian

$$H(x,p,m) - H(x,0,m) \geq a_o|p|^\beta, \quad a_o > 0,\, \beta > 1\,,$$

I show how to build a solution of the ergodic MFG system of 1st order PDEs from any solution of the static MFG with cost $F(x,m) := H(x,0,m)$. This leads to new existence results under general assumptions, in particular for non-separable Hamiltonians. The motivating examples for such Hamiltonians are some models of congestion in crowd dynamics with non-local dependence on the crowd distribution, such as

$$H(x,p,m) = \frac{|p|^\beta}{(k * m(x) + \sigma)^\alpha} - F(x,m)\,, \quad \beta > 1, \sigma \geq 0, \alpha > 0.$$

Next I prove that the measure component of any solution to the ergodic MFG must solve the associated static MFG, under the following assumption on the Hamiltonian

$$H(x,p,m) - H(x,0,m) \leq a_1\, p \cdot H_p(x,p,m)\,, \quad a_1 > 0\,.$$

Such necessary condition for the solvability of the ergodic MFG implies new uniqueness results and, in some cases with multiple solutions, the characterization of all of them.

Some of these results were proved earlier for the special case $H(x,p,m) = |p|^2 - F(x,m)$ in the joint work with Hicham Kouhkouh [1], where also the asymptotics for large time of finite horizon MFG were analysed.

References

[1] M. Bardi, H. Kouhkouh, *Long-time behaviour of deterministic Mean Field Games with non-monotone interactions*, SIAM J. Math. Anal. to appear.

12:00–12:20 Convergence analysis of controlled particle systems arising in deep learning: from finite to infinite sample size

Alpar Meszaros (University of Durham, UK)

Abstract. In this talk we will consider a class of neural SDEs and discuss the limiting behavior of the associated sampled optimal control problems as the sample size grows to infinity. The neural SDEs with N samples can be linked to an N-particle system with centralized control. We analyze the Hamilton–Jacobi–Bellman equation corresponding to the N-particle system and establish regularity results which are uniform in N. The uniform regularity estimates are obtained by the stochastic maximum principle and the analysis of a backward stochastic Riccati equation. Using these uniform regularity results, we show the convergence of the minima of objective functionals and optimal parameters of the neural SDEs as the sample size $N \to +\infty$. The limiting objects can be identified with suitable functions defined on the Wasserstein space of Borel probability measures. Furthermore, quantitative algebraic convergence rates will also be discussed. The talk will be based on a joint work with H. Liao, C. Mou and C. Zhou.

12:30–12:50 Hamilton-Jacobi equations on infinite dimensional spaces corresponding to linearly controlled gradient flows of an energy functional

Daniela Tonon (University of Padua, ITALY)

Abstract. In this talk, we study Hamilton Jacobi (HJ) equations corresponding to a Mean Field control problem in which one linearly controls the gradient flow of an energy functional defined on a metric space. The main difficulties are given by the fact that the geometrical assumptions we require on the energy functional do not give any control on the growth of its gradient flow nor on its regularity. Therefore this framework is not covered by previous results on HJ equations on infinite dimensional spaces (whose study has been initiated in a series of papers by Crandall and Lions). Our proof of the comparison principle combines some rather classical ingredients, such as Ekeland's perturbed optimization principle, with the use of the Tataru distance and of the regularizing properties of gradient flows in evolutional variational inequality formulation, that we exploit for constructing rigorous upper and lower bounds for the formal Hamiltonian. In the context of Wasserstein gradient flows with underlying energy functional satisfying McCann's condition, we establish that the value function for a linearly controlled gradient flow problem, whose running cost is quadratic in the control variable and just continuous in the state variable, yields a viscosity solution to the HJ equation. One of the main innovations of this work is to introduce a controlled version of EVI, which turns out to be crucial in establishing regularity properties, energy and metric bounds along optimizing sequences in the controlled gradient flow problem that defines the candidate solution.

14:30–15:15 TBC

Ronnie Sircar (Princeton University, USA)

Abstract. TBC

15:30–15:50 Fourier Galerkin approximation of mean field control problems

Mattia Martini (University Côte d'Azur, FRANCE)

Abstract. The goal of this talk is to introduce a finite dimensional approximation of the solution to a mean field optimal control problem set on the d-dimensional torus, without relying on particle-based methods. Our approximation is obtained by means of a Fourier-Galerkin method, the main principle of which is to truncate the Fourier expansion of probability measures on the torus. However, this operation has the main feature not to leave the space of probability measures invariant, which drawback is know as *Gibbs*' phenomenon.

First, we manage to prove that, for initial conditions in the 'interior' of the space of probability measures and for sufficiently large levels of truncation, the Fourier-Galerkin method actually induces a new finite dimensional control problem whose trajectories take values in the space

of probability measures with a finite number of Fourier coefficients. Subsequently, our main result asserts that, whenever the cost functionals are smooth and convex, the optimal control, trajectory, and value function from the approximating problem converge to their counterparts in the original mean field control problem. Noticeably, we show that our method yields a polynomial convergence rate directly proportional to the data's regularity. This convergence rate is faster than the one achieved by the usual particles approach, offering a more efficient alternative. Furthermore, our technique also provide an explicit method for constructing an approximate optimal control along with its corresponding trajectory. This talk is based on a joint work with François Delarue.

16:00–16:20 Deep Backward and Galerkin Methods for the Finite State Master Equation

Asaf Cohen (University of Michigan, USA)

Abstract. This paper proposes and analyzes two neural network methods to solve the master equation for finite-state mean field games (MFGs). Solving MFGs provides approximate Nash equilibria for stochastic, differential games with finite but large populations of agents. The master equation is a partial differential equation (PDE) whose solution characterizes MFG equilibria for any possible initial distribution. The first method we propose relies on backward induction in a time component while the second method directly tackles the PDE without discretizing time. For both approaches, we prove two types of results: there exist neural networks that make the algorithms' loss functions arbitrarily small and conversely, if the losses are small, then the neural networks are good approximations of the master equation's solution. We conclude the paper with numerical experiments on benchmark problems from the literature up to dimension 15, and a comparison with solutions computed by a classical method for fixed initial distributions.

17:00–17:20 The coupling method in (McKean-Vlasov) stochastic control

Giovanni Conforti (University of Padua, ITALY)

Abstract. The coupling method is a popular and far-reaching technique to quantify the speed of convergence to equilibrium of Markov process. The basic principle is that of constructing on a single probability space two realisations of the same Markov dynamics with different initial conditions and in showing that the law of the two processes get exponentially close in time. The aim of this talk is to promote the idea that one can construct contractive couplings between controlled processes as well, which imply exponential turnpike estimates for the optimal controls and processes, as well as estimates on the derivative of the value function that are uniform in time. In particular, a controlled version of coupling by reflection will be discussed in some detail.

17:30–17:50 Pasting of discrete time mean field equilibria and Donsker-type results for mean field games

Ludovic Tangpi (Princeton University, USA)

Abstract. In this talk we discuss mean field games in discrete time on general probability spaces. Using dynamic programming and a forward-backward algorithm, we will construct mean field equilibria of multi period models as concatenation of equilibria of one-step games. We will also present results on convergence of discrete time games to continuous time counterparts akin to Donsker's invariance principle. The talk is based on a joint work with J. Dianetti, M. Nendel and S. Wang.

July 24, 2024

11:30–12:15 Long Range Games

Paolo Dai Pra (University of Verona, ITALY)

Abstract. I will introduce a general extension of Mean Field Games that includes Kac-Potential interactions and Graphon interaction. In the context of finite state space we show that the basic properties of mean-field games continue to hold: existence of Nash equilibria, uniqueness under monotonicity, approximation of the N-player game by the limit independent games.

12:30–12:50 Self-organizing equilibria and their local stability in a Kuramoto mean field game

Annalisa Cesaroni (University of Padua, ITALY)

Abstract. Recently a Mean Field Game version of the classical Kuramoto model has been proposed in [1], describing synchronization phenomena in a large population of rational interacting oscillators. In this talk, I will discuss existence and uniqueness (up to phase transition) of the incoherent equilibrium and the self-organizing equilibrium, given that the interaction parameter is sufficiently large. Furthermore, I will also present some local stability properties of the self-organizing equilibrium with respect to dynamic equilibria in a long time regime.

References

[1] R. Carmona, Q. Cormier, H. M. Soner, *Synchronization in a Kuramoto mean field game*, Comm. Partial Differential Equations 48 (2023), no 9, 1214–1244.

14:30–15:15 Stationary Mean-field Games of Singular Control

Giorgio Ferrari (University of Bielefeld, GERMANY)

Abstract. In this talk I will present recent and ongoing results on existence, uniqueness, and characterization of equilibria for mean-field games with singular controls. This class of problems finds natural applications in Economics and Finance, such as in investment problems in oligopolies. In those games, the representative agent employs a bounded-variation control in order to maximize an ergodic profit functional depending on a long-time average of the controlled state-process. Several variants of the considered games will be presented, which will differ with respect to the dimension of the state-process and the optimality criterion employed.

15:30–15:50 Asymptotic behavior of mean field games: coercive and non coercive case

Cristian Mendico (University of Rome Tor Vergata, ITALY)

Abstract. In this talk we will revisit the recent results on the description of the asymptotic behavior of some deterministic mean field game models, namely: the classical system, the system arising from the control of acceleration and the case nonholonomic dynamics. We will discuss about the results and about the questions that are still open. Finally, we will see how the ergodic mean field game system associated with a calculus of variation problem captures the behavior of Nash equilibria.

16:00–16:20 Mean field games on homogeneous Lie groups

Claudio Marchi (University of Padua, ITALY)

Abstract. We study short-time existence of classical solutions to mean field games systems defined on homogeneous Lie groups.

More precisely, we consider an homogeneous Lie group, endowed with a family of dilations, which can be identified with $\mathbb{R}^d$. Let $\{X_1, \ldots, X_m\}$ (with $m < d$) be a family of vector fields which satisfies the Hörmander condition; in particular, together with their commutators, these vector fields generate the Lie group. We consider second-order mean field games systems where the differential operators are given in terms of these vector fields and where the couplings are strongly regularizing. In our model, each agent can move only along the directions generated by $X_1, \ldots, X_m$ but it can still reach every point due to the Hörmander condition.

In order to obtain existence of solution to these mean field games, we first study existence and uniqueness of the subelliptic Fokker-Planck equation and separately of the Hamilton-Jacobi equation.

17:00–17:20 Second order PDEs on the Wasserstein space

Ibrahim Ekren (University of Michigan, USA)

Abstract. We prove a comparison result for viscosity solutions of second order parabolic partial differential equations in the Wasserstein space. The comparison is valid for semisolutions that are Lipschitz continuous in the measure in a Fourier-Wasserstein metric and uniformly continuous in time. The class of equations we consider is motivated by McKean-Vlasov control problems with common noise and filtering problems. We also mention applications for prediction problems with expert advice. The proof of comparison relies on a novel version of Ishii's lemma, which is tailor-made for the class of equations we consider.

17:30–18:15 TBC

Mete Soner (Princeton University, USA)

Abstract. TBC

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