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Special Geometries and Physics
Special Session B14

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The list of special geometries includes, for example, Calabi-Yau manifolds, Quaternionic-Kähler manifolds, Hyper-Kähler manifolds, G2-manifolds and Spin(7)-manifolds. These geometries are studied both for their mathematical sake, and because they provide the mathematical formalism for new developments in Physics. In particular, String theory and M-theory are two of the most influential physical theories for these fields: they provide not just motivation and urgency for special geometries and geometries with torsion, but also fundamental insight and conjectures. It is a fair statement that most of recent research on manifolds with special holonomy, such as Calabi-Yau manifolds and G2 manifolds, can be largely attributed to interactions with theoretical Physics. The same is true for certain systems of PDE, such as the Hull-Strominger system.

This special session brings together a diverse group of geometers working with special structures on manifolds and physicists working in String and M-theory, with the aim of discussing new results and research directions in both fields. The list of speakers includes experts in a variety of very active research areas: Calabi-Yau and G2 geometry, mirror symmetry, generalized geometry, geometric flows and gauge theory.

Schedule and Abstracts

July 25, 2024

11:30–12:15 Coupled G_2-instantons
Jason Lotay (Oxford University, UK)

Abstract. Gauge theory in higher dimensions plays a pivotal role in modern mathematics and theoretical physics. One such gauge theory occurs in 7 dimensions and is associated with the exceptional Lie group G_2: here, the special connections are called G_2-instantons and are analogous to the famous anti-self-dual instantons in 4 dimensions. G_2-instantons play a crucial role in the so-called heterotic G_2 system arising from heterotic String Theory in physics: a complex coupled system on a 7-manifold for an ambient G_2-structure and G_2-instantons on it which is very poorly understood.

Motivated by recent developments in theoretical physics and ideas in generalized geometry, in this talk we introduce the notion of *coupled* G_2*-instantons*. We show that coupled G_2-instantons encode solutions to the heterotic G_2 system and are linked to the notion of generalized Ricci-flatness, thus giving us a new perspective on these challenging areas. We will also discuss some non-trivial examples of coupled G_2-instantons, including on the 7-sphere.

12:15–13:00 Mirror of minimal submanifolds and a monotonicity formula
Kotaro Kawai (BIMSA, China)

Abstract. For Hermitian connections on a Hermitian complex line bundle over a Riemannian manifold, we can define the "volume", which can be considered to be the "mirror" of the standard volume for submanifolds. We call the critical points minimal connections. They can be considered as "mirrors" of minimal submanifolds and analogous to Yang-Mills connections.

In this talk, I will introduce some properties of minimal connections and then state a monotonicity formula. As a corollary, we obtain the vanishing theorem for minimal connections in the odd-dimensional case.

14:30–15:15 Deformed Hermitian-Yang-Mills: an example
Gonçalo Oliveira (Instituto Superior Técnico, Portugal)
Abstract. Mirror symmetry is a somewhat mysterious phenomenon that relates the geometry of two distinct Calabi-Yau manifolds. In the realm of trying to understand this relationship an equation for a connection on a line bundle in a Kahler manifold appeared. This is commonly called the deformed Hermitian-Yang-Mills equation and I will explain what it is and some current joint work with Benoit Charbonneau and Rosa Sena-Dias which explicitly solves this equation on a specific setting. This helps in understanding the problem of the existence of solutions and explore (or rule out) possible stability conditions.

15:30–16:15 CR-twistor spaces over manifolds with G_2- and $\mathrm{Spin}(7)$-structures
Hông Vân Lê (Institute of Mathematics, Czech Academy of Sciences)
Abstract. In 1984 LeBrun constructed a CR-twistor space over an arbitrary conformal Riemannian 3-manifold and proved that the CR-structure is formally integrable. This twistor construction has been generalized by Rossi in 1985 for m-dimensional Riemannian manifolds endowed with a $(m-1)$-fold vector cross product (VCP). In 2011 Verbitsky generalized LeBrun's construction of twistor-spaces to 7-manifolds endowed with a G_2-structure. In my talk I shall explain how to unify and generalize LeBrun's, Rossi's and Verbitsky's construction of a CR-twistor space to the case when the underlying Riemannian manifold (M, g) has a VCP structure. Then I shall show that the formal integrability of the CR-structure is expressed in terms of a torsion tensor on the twistor space, which is a Grassmanian bundle over (M, g). If the VCP structure on (M, g) is generated by a G_2- or Spin(7)-structure, the vertical component of the torsion tensor vanishes, if and only if (M, g) has constant curvature, and the horizontal component vanishes, if and only if (M, g) is a torsion-free G_2 or Spin(7)-manifold. Finally I shall discuss related open problems. My talk is based on a joint work with Domenico Fiorenza.

17:00–17:45 Triviality of Frobenius Structures Along Generalized Deformations of Nilmanifolds
Yat Sun Poon (University of California, Riverside, USA)
Abstract. It is known that there exists a natural isomorphism between the Gerstenhaber algebra associated to a primary Kodaira manifold and that associated to any complex manifold obtained as a small generalized deformation of the given primary Kodaira manifold. As a result the Frobenius structure at the primary Kodaira manifolds along the degree-2 direction is trivial. We will present an approach to extend this result on some 2-step nilmanifolds with abelian complex structures.

July 26, 2024

11:30–12:15 T-duality beyond torus bundles
Gil Cavalcanti (University of Utrecht, Netherlands)
Abstract. Target-space duality, or T-duality for short, is a duality that comes from physics in the presence of a torus symmetry but also has very concrete mathematical formulations and consequences. In its simplest form, when the target-space is a Riemannian circle, it is marked by inversion of the radius of the circle and swapping of physical quantities (winding and momentum) to yield equivalent physical theories.

Mathematically, T-duality is made of two ingredients:

- *Topological T-duality* relates the global topology of dual target-spaces: the background form on one side influences the topology of the dual space, there are isomorphisms between the twisted cohomologies of T-dual spaces and also of their twisted K-theories.
- *Geometric T-duality* is an isomorphism of Courant algebroids over T-dual spaces that allows one to transport geometric structures between T-dual spaces.

In this talk we will review the basics of T-duality and delve into progresses in the cases when the torus action has fixed points and when one replaces the torus by non-Abelian objects, extending both topological and geometric T-duality.

12:15-13:00 SKT and Kähler-like metrics on complex manifolds
Nicoletta Tardini (University of Parma, Italy)

Abstract. Several special non-Kähler Hermitian metrics can be introduced on complex manifolds. Among them, SKT metrics deserve particular attention. They can be defined on a complex manifold by saying that the torsion of the Bismut connection associated to the metric is closed. These metrics always exist on compact complex surfaces but the situation in higher dimension is very different. We will discuss several properties concerning these metrics also in relation with the Bismut connection having Kähler-like curvature.

14:30–15:15 Generalized Ricci solitons with large symmetry group
Alberto Raffero (University of Torino, Italy)

Abstract. The *generalized Ricci flow* is a geometric flow evolving a family of Riemannian metrics g_t and closed 3-forms H_t as follows

$$\begin{cases} \frac{\partial}{\partial t} g_t = -2\,\mathrm{Ric}_{g_t} + \frac{1}{2}\mathcal{H}_{g_t,H_t}, \\ \frac{\partial}{\partial t} H_t = -\Delta_{g_t} H_t. \end{cases}$$

Here, Δ_g denotes the Hodge Laplacian and $\mathcal{H}_{g,H}(X,Y) = g(\iota_X H, \iota_Y H)$ is a symmetric 2-covariant tensor. In theoretical physics, this flow appears as the renormalization group flow of a nonlinear sigma model arising in string theory. From the mathematical viewpoint, it can be regarded as a generalization of Hamilton's Ricci flow to metric connections with totally skew-symmetric torsion.

Since the RHS of the flow equation is invariant under diffeomorphisms and simultaneous scalings of the pair (g, H), a natural notion of *generalized Ricci solitons* can be introduced. These are pairs (g, H) satisfying the following system of equations

$$\begin{cases} \mathrm{Ric}_g = \lambda g - \frac{1}{2}\mathcal{L}_X g + \frac{1}{4}\mathcal{H}_{g,H}, \\ \Delta_g H = 2\lambda H - \mathcal{L}_X H, \end{cases}$$

for some $\lambda \in \mathbb{R}$ and $X \in \Gamma(TM)$. Solitons give rise to self-similar solutions of the flow and are expected as long time limits and singularity models for it. If $H \equiv 0$, the system above reduces to the classical Ricci soliton equation. On the other hand, if both $\lambda = 0$ and $X = 0$, the soliton reduces to a fixed point of the flow.

In this talk, I will first describe a construction that allows one to obtain infinite families of compact homogeneous spaces admitting invariant fixed points of the generalized Ricci flow. Then, I will discuss the existence of complete rotationally invariant steady ($\lambda = 0$) generalized Ricci solitons on $\mathbb{R}^3$, providing a generalization of the well-known Bryant's construction of the complete SO(3)-invariant steady soliton for the Ricci flow. Time permitting, I will also discuss some related results and open problems in the context of G_2 geometry.

15:30-16:15 New minimal surfaces in multi-Taub-NUT spaces
Lorenzo Foscolo (University of Rome "La Sapienza", Italy)

Abstract. I will describe the construction of new minimal surfaces in hyperkähler 4-manifolds arising from the Gibbons–Hawking Ansatz, i.e. hyperkähler 4-manifolds that admit a triholomorphic circle action. The minimal surfaces we produce are obtained via a gluing construction using well-known surfaces—the Scherk surface in flat space and the holomorphic cigar in the Taub-NUT space—as building blocks. The minimal surfaces we produce are not holomorphic with respect to any complex structure compatible with the metric, they can be parametrised by a harmonic map that satisfies a first-order Fueter-type PDE, and yet are unstable.

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