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Functional inequalities and PDEs
Special Session A9

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This session deals with the most recent results in PDEs, functional inequalities and their applications to PDEs. We will mainly focus on existence, regularity, qualitative properties of solutions to PDEs, and optimal functional inequalities. In this framework, inequalities play a crucial role in establishing uniform bounds, global existence results and large-time behavior, decay rates, and existence/uniqueness of blow-up solutions to various classes of differential equations. Furthermore, functional inequalities are also intrinsically fundamental in many sub-fields of analysis. The invited speakers are actively involved in this research area with distinguished contributions. They will discuss their latest achievements and future developments. The session aims to bring together experts in different fields to encourage fruitful discussions, develop new ideas, and promote knowledge exchange between participants.

Schedule and Abstracts

July 23, 2024

11:00–12:00 The LGC method: Recent progress on several problems in harmonic analysis

Victor Lie (Purdue University, USA)

*Abstract.*In this talk we touch upon three problems:

- (joint with my postdoc Bingyang Hu) the boundedness of the trilinear Hilbert transform along the moment curve:

$$T_C(f_1,f_2,f_3)(x):=\text{p.v.}\int_{\mathbb{R}} f_1(x-t)f_2(x+t^2)f_3(x+t^3)\frac{dt}{t},\quad x\in\mathbb{R}\,.$$

- (joint with C. Benea and F. Bernicot) the boundedness of the hybrid trilinear Hilbert transform:

$$T_H(f_1,f_2,f_3)(x):=\text{p.v.}\int_{\mathbb{R}} f_1(x-t)f_2(x+t)f_3(x+t^3)\frac{dt}{t},\quad x\in\mathbb{R}\,.$$

- (joint with my graduate student Martin Hsu) the boundedness of the 2D non-resonant Carleson–Radon transform:

$$CR(f)(x,y):=\sup_{a\in\mathbb{R}}\left|\text{p.v.}\int_{\mathbb{R}} f(x-t,y-t^2)\,\frac{e^{a\,i\,t^3}}{t}\,dt\right|,\quad (x,y)\in\mathbb{R}^2\,.$$

The leit-motif in the successful approach of all of the above problems is the appeal to the LGC methodology–introduced by the speaker and further developed in various collaborative works–that involves several key ingredients, including the following interdependent elements:

- a sparse-unform decomposition of the input function(s) adapted to an appropriate time-frequency foliation of the phase-space,
- a structural analysis of suitable maximal “joint Fourier coefficients", and
- a level set analysis with respect to the time-frequency correlation set.

12:00–13:00 The optimal Leray-Trudinger inequality

Giuseppina di Blasio (University of Campania "L. Vanvitelli", Italy)

Abstract. In this talk, we will present some recent results on Leray-Trudinger type inequalities that are closely related to Trudinger-Moser and Hardy inequalities. The plan of the talk is to present the origin and the history of the problem and to present an optimal analogous of the Trudinger's inequality. The presentation is based on joint works with G. di Blasio and G. Psaradakis.

References

[1] G. di Blasio, G. Pisante, G. Psaradakis, *The optimal Leray-Trudinger inequality*, in press on IUMJ.

14:30–15:30 Classification of singular solutions in the half-space

Luigi Montoro (Università della Calabria, ITALY)

Abstract. We provide a classification result for positive solutions to $-\Delta u = 1/u^{\gamma}$ in the half space, under zero Dirichlet boundary condition. We refer to [1].

References

[1] L. Montoro, L. Muglia, B. Sciunzi, Classification of solutions to $-\Delta u = u^{-\gamma}$ in the half-space, Math. Ann., to appear.

15:30–16:00 Mathematical models for chemotaxis system and their applications in medicine

Monica Marras (Universita' degli Studi di Cagliari, Italy)

Abstract. We are interested in qualitative properties as blow-up phenomena, decay in time, boundedness, global existence to solutions of some classes of parabolic systems. In particular we consider a chemotaxis system in a bounded and smooth domain with no-flux boundary condition. In multicellular organisms, the chemotaxis of cell populations plays a crucial role throughout the life cycle:

- in the formation of the embryo and during embryonic development for the cell positioning;
- In the adult, the immune cell migration to sites of inflammation and fibroblasts into wounded regions to initiate healing.

There are same mechanisms during cancer growth, allowing tumour cells to invade the surrounding environment or stimulate new blood vessel growth (angiogenesis).
This talk is connected to a joint research with S. Vernier-Piro and T. Yokota.

16:00–16:30 Liouville theorems for geometric PDEs

Alberto Roncoroni (Politecnico di Milano, ITALY)

Abstract. In this talk I will present Liouville-type theorems for two critical PDEs of geometric and variational nature. The first one is the critical $p-$Laplace equation in $\mathbb{R}^n$ (and, possibly, in Riemannian manifolds), while the second one is the CR Yamabe equation in the Heisenberg group $\mathbb{H}^n$. It is well-known that both equations have a geometric and variational origin; indeed the critical $p-$Laplace equation is related to the Sobolev inequality and, for $p = 2$, to the Yamabe problem while the CR Yamabe equation is related to the Folland-Stein inequality and to the CR-Yamabe problem.

Liouville theorems regarding the classification of entire and positive solutions to these equations have been obtained in [4, 5] and in [3] under a finite energy assumption and remove this hypothesis is still an open and challenging problem. In the talk I will present two recent Liouville theorems in which we remove the finite energy assumption for the critical $p-$Laplace equation in $\mathbb{R}^n$, with $n = 2$ or $n = 3$ provided $3/2 < p < 2$, and for the CR Yamabe equation in $\mathbb{H}^1$.

The first result has been obtained in collaboration with G. Catino and D.D. Monticelli and is contained in the paper [2], while the second one has been obtained with G. Catino, Y.Y. Li and D.D. Monticelli and is contained in [1].

References

[1] G. Catino, Y.Y. Li, D.D. Monticelli, A. Roncoroni. *A Liouville theorem in the Heisenberg group.* Preprint.

[2] G. Catino, D. D. Monticelli, A. Roncoroni. *On the critical $p-$Laplace equation.* Adv. Math. **433** (2023), 109331.

[3] D. Jerison, J.M. Lee. *Extremals for the Sobolev inequality on the Heisenberg group and the CR Yamabe problem.* J Amer. Math. Soc. **1** (1988), 1–13.

[4] B. Sciunzi. *Classification of positive $\mathcal{D}^{1,p}(\mathbb{R}^n)-$solutions to the critical $p-$Laplace equation in $\mathbb{R}^n$.* Adv. Math. **291** (2016), 12–23.

[5] J. Vétois. *A priori estimates and application to the symmetry of solutions for critical $p-$Laplace equations.* J. Differential Equations **260**, no. 1 (2016), 149–161.

17:00–17:30 Growth of Sobolev norms for completely resonant quantum harmonic oscillator

Maria Teresa Rotolo (SISSA, ITALY)

Abstract. We consider the linear time-dependent quantum harmonic Schrödinger equation in $\mathbb{R}^2$:

$$i\partial_t u = \frac{1}{2}(-\partial_{x_1}^2 - \partial_{x_2}^2 + x_1^2 + x_2^2)u + V(t,x,D)u, \qquad x \in \mathbb{R}^2,$$

where $V(t,x,D)$ is a self-adjoint pseudodifferential operator of degree zero, 2π periodic in time. We prove that under suitable conditions on the symbol of the potential $V(t,x,D)$ there exist solutions of the equation that exhibit unbounded growth in time of their positive Sobolev norms, and we show that the class of symbols satisfying such condition is generic in the Fréchet space of classical time-dependent symbols of order zero.

The strategy to prove growth of positive Sobolev norms mainly consists of two parts: in the first we use a pseudodifferential normal form to extract an effective resonant equation governing the dynamics, and in the second we prove that a dispersive energy estimate holds for the effective equation and use it to find a solution with growing positive norms.
The main difficulty lies in the proof of the energy estimate, that in turn relies on Mourre's theory of positive commutators and on the existence of a conjugate operator for the effective equation. We give generic conditions on the classical Hamiltonian dynamics of the principal symbol of the perturbed operator that turn out to be sufficient to prove a Mourre estimate for the perturbed operator.
The main novelty of the work is the proof of the generic existence of a conjugate operator for an equation with spatial variable in dimension two. To this aim we adapt a geometric construction done by Colin de Verdière in [1] to our context.
In this talk I will try to present the general problem and the key results needed for this proof. This is a joint work with B. Langella and A. Maspero.

References

[1] Y. Colin de Verdière, *Spectral theory of pseudodifferential operators of degree 0 and an application to forced linear waves*, Analysis & PDE, 13 (2020), 1521–1537.

17:30–18:00 A p-Laplacian problem in $\mathbb{R}^N$ with singular, convective, critical reaction

Umberto Guarnotta (Università degli Studi di Catania, ITALY)

Abstract. The talk is devoted to the problem

$$\begin{cases} -\Delta_p u = \lambda w(x) f(u, \nabla u) + u^{p^*-1} & \text{in } \mathbb{R}^N, \\ u > 0 & \text{in } \mathbb{R}^N, \\ u(x) \to 0 & \text{as } |x| \to +\infty, \end{cases}$$

where $N \geq 2$, $1 < p < N$, and $\lambda > 0$. The nonlinear term $f : (0,+\infty) \times \mathbb{R}^N \to (0,+\infty)$ is a continuous function which is singular in the first variable and p-sublinear with respect to the second one. The weight $w : \mathbb{R}^N \to (0,+\infty)$ satisfies suitable summability and decay conditions.

The problem exhibits several features:

- the perturbation f is singular, i.e., it blows up when the solution vanishes;
- f encompasses also convection terms, that is, depending on the gradient of the solution;
- the 'dominating' reaction term has critical growth;
- the setting is the whole $\mathbb{R}^N$;
- pointwise decay (at infinity) of the solutions is required.

We will present an existence result that combines variational methods, truncation techniques, and concentration compactness arguments, together with set-valued analysis and fixed point theory. In addition, De Giorgi's technique, a priori gradient estimates, and nonlinear regularity theory will be employed to ensure local $C^{1,\alpha}$ regularity of solutions, as well as their pointwise decay at infinity.

The result is new even in the non-singular case, also for the Laplacian.

July 24, 2024

11:30–12:30 Boundedness results for the p-Laplacian on noncompact Riemannian manifolds

Giuseppina Barletta (University of Reggio Calabria, ITALY)

Abstract. We present some boundedness results for the p-Laplace operator in domains with finite volume, on noncompact Riemannian manifolds.
Specifically, we deal with two different eigenvalue problems for the p-Laplacian: the first has Neumann boundary conditions, the second is a Steklov problem.
We consider also the boundedness of the solutions to the Schrödinger equation, under Neumann boundary conditions, with non-necessarily bounded potential.
The assumptions ensuring L^q or L^∞ bounds for the solutions to the equations are offered either in terms of the isoperimetric function or of the isocapacitary function of the domain.
We also present some examples that show the optimality of our assumptions.

References

[1] G.Barletta, *L^∞ boundedness of the solutions the Schrödinger equation on noncompact Riemannian manifolds*, preprint.

[2] G.Barletta, A.Cianchi & V.Maz'ya, *Bounds for eigenfunctions of the Neumann p-Laplacian on noncompact Riemannian manifolds*, Advances in Calculus of Variations, **n. 34 (2)**, (2024), pp. 319–352.

[3] A.Cianchi & V.Maz'ya, *Boundedness of solutions to the Schrödinger equation under Neumann boundary conditions*, J. Math. Pures Appl., **98** (2012), pp 654–688

12:30–13:00 A stability result for the first Robin-Neumann eigenvalue: a double perturbation approach

Gianpaolo Piscitelli (Università di Napoli "Parthenope", ITALY)

Abstract. Let $\Omega = \Omega_0 \setminus \overline{\Theta} \subset \mathbb{R}^n$, $n \geq 2$, where Ω_0 and Θ are two open, bounded and convex sets such that $\overline{\Theta} \subset \Omega_0$ and let $\beta < 0$ be a given parameter. We consider the eigenvalue problem for the Laplace operator associated to Ω, with Robin boundary condition on $\partial\Omega_0$ and Neumann boundary condition on $\partial\Theta$. In [2] it is proved that the spherical shell is the only maximizer for the first Robin-Neumann eigenvalue in the class of domains Ω with fixed outer perimeter and volume.

We establish a quantitative version of the afore-mentioned isoperimetric inequality; the main novelty consists in the introduction of a new type of hybrid asymmetry, that turns out to be the suitable one to treat the different conditions on the outer and internal boundary. Up to our knowledge, in this context, this is the first stability result in which *both* the outer and the inner boundary are perturbed.

This talk is based on a joint work with Gloria Paoli (Università di Napoli Federico II) and Simone Cito (Università del Salento).

References

[1] S. Cito, G. Paoli, G. Piscitelli, *A stability result for the first Robin-Neumann eigenvalue with negative boundary parameter.* ArXiv (2023).

[2] G. Paoli, G. Piscitelli, L. Trani, *Sharp estimates for the first p-Laplacian eigenvalue and for the p-torsional rigidity on convex sets with holes.* ESAIM Control Optim. Calc. Var. **26** (2020), no. 111, 1-15.

14:30–15:30 The Well-posedness of Cylindrical Jets with Surface Tension
Aram Karakhanyan (University of Edinburgh, UK)

Abstract. In 1879 Rayleigh studied the stability of infinite cylindrical jets, inspired by the experiments of Plateau. The principal question that Rayleigh asked is: under what circumstances the jet is stable, for small displacements. In this talk I will discuss the short time stability for the initial condition belonging to some Sobolev space, and the initial jet boundary being uniformly bounded away from the axis of symmetry. This is proved by the method of paradifferential calculus and paralinearization. The salient feature of these results is that no smallness assumption is imposed on the initial condition. The results are taken from a joint paper with Dr Yucong Huang.

15:30–16:00 On the approximation of optimal thin layers in thermal insulation
Paolo Acampora (University of Naples Federico II, ITALY)

Abstract. We are interested in the thermal insulation of a body Ω surrounded by a bulk layer Σ of insulating material. We consider a Robin boundary condition on the boundary of Σ that does not touch Ω; this corresponds to a model of heat transfer between the insulated body and the environment determined by convection. We will address the problem of proving the existence of an optimal Σ, and we will study a way to approximate the problem via first-order asymptotic development by Γ-convergence.

References

[1] P. Acampora and E. Cristoforoni *On the asymptotic behavior of a diffraction problem with a thin layer*, Preprint, 2024.

[2] P. Acampora, E. Cristoforoni, C. Nitsch, and C. Trombetti. *On the optimal shape of a thin insulating layer*, To appear on: SIAM Math. Anal., 2024.

[3] E. Acerbi and G. Buttazzo. *Reinforcement problems in the calculus of variations*, Annales de l'Institut Henri Poincaré C, Analyse non linéaire 3.4, 1986.

[4] H. Brézis, L. A. Caffarelli, and A. Friedman. *Reinforcement problems for elliptic equations and variational inequalities.* Annali di matematica pura ed applicata 123.1 (1980).

[5] F. Della Pietra, C. Nitsch, R. Scala, and C. Trombetti. *An optimization problem in thermal insulation with Robin boundary conditions.* Communications in Partial Differential Equations 46.12 (2021).

16:00–16:30 On the singular planar Plateau problem
Marco Caroccia (Politecnico di Milano, ITALY)

Abstract. The classical Plateau problem asks which surface in three-dimensional space spans the least area among all the surfaces with boundary given by an assigned curve S. This problem has many variants and generalizations, along with (partial) answers, and has inspired numerous new ideas and techniques. In this talk, we will briefly introduce the problem in both its classical and modern contexts, and then we will focus on a specific vectorial (planar) type of the Plateau problem. Given a curve S in the plane, we can ask which diffeomorphism T of the disk D maps the boundary of D to S and spans the least area, computed as the integral of the Jacobian of T, among competitors with the same boundary condition. For simply connected curves, the answer is provided by the Riemann map, and the minimal area achieved is the Lebesgue measure of the region enclosed by S. For more complex curves, possibly self-intersecting, new analysis is required. I will present a recent result in this sense, obtained in collaboration with Prof. Riccardo Scala from the University of Siena, where the value of the minimum area is computed with an explicit formula that depends on the topology of S.

References

[1] Harrison, Jenny, and Harrison Pugh, *Plateau's problem*, Open problems in mathematics (2016): 273-302.

[2] Caroccia, M., Scala, R *On the singular planar Plateau problem*, arXiv preprint arXiv:2402.13050 (2024).

17:00–17:30 On the Shape of Small Liquid Drops Minimizing Nonlocal Energies
Konstantinos Bessas (University of Pavia, ITALY)

Abstract. We study the equilibrium shape of liquid drops minimizing the fractional perimeter under the action of a potential energy. We prove, with a quantitative estimate, that the small volume minimizers are convex and uniformly close to a ball. This is a joint work with Matteo Novaga (Pisa) and Fumihiko Onoue (Munchën).

17:30–18:00 Stability for the logarithmic Sobolev inequality
Giovanni Brigati (IST, Austria)

Abstract. This talk is devoted to stability results for the Gaussian logarithmic Sobolev inequality, with explicit stability constants.

18:00–18:30 Higher-order singular perturbation models for phase transitions
Giuseppe Cosma Brusca (SISSA, ITALY)

Abstract. Variational models of phase transitions take into account double-well energies singularly perturbed by gradient terms, such as the Cahn-Hilliard free energy. The derivation by Γ-convergence of a sharp-interface limit for such energy is a classical result by Modica and Mortola. We consider a singular perturbation of a double-well energy by derivatives of order k, and show that we still can describe the limit as in the case $k = 1$ with a suitable interfacial energy density, in accord with the case $k = 1$ and with the case $k = 2$ previously analyzed by Fonseca and Mantegazza. The main issue is the derivation of an optimal-profile problem on the real line describing the interfacial energy density, which must be conveniently approximated by minimum problems on finite intervals with homogeneous condition on the derivatives at the endpoints up to order $k - 1$. To that end a careful study must be carried on of sets where sequences of functions with equibounded energy are "close to the wells" and have "small derivatives", in terms of interpolation inequalities and energy estimates.

References

[1] I. Fonseca, C. Mantegazza, *Second Order Singular Perturbation Models for Phase Transitions*, SIAM J. Math. Anal., 2000.

[2] L. Modica, S. Mortola, *Un esempio di* Γ*-convergenza*, Boll. Un. Mat. It. B, 1977.

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