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The interface between smooth and symplectic 4-manifolds Special Session A21

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In spite of spectacular advances, smooth 4-manifold topology remains very mysterious; there are few positive results and no reasonable guesses towards classifications. Symplectic 4-manifolds however are far more rigid, and in simple circumstances can be completely understood. This session will gather researchers from both smooth and symplectic 4-manifold topology to discuss the interface of these fields, and in particular what light symplectic results can shed on smooth phenomena.h".

Schedule and Abstracts

July 23, 2024

11:00–11:20 Lens spaces in the complex projective plane
Brendan Owens (University of Glasgow, UK)

Abstract. Which lens spaces embed smoothly in the complex projective plane, and which collections of lens spaces can be disjointly embedded? Work of Manetti and Hacking-Prokhorov showed that each solution to the Markov equation gives rise to a triple of lens spaces which embed disjointly, and Evans-Smith showed this accounts for all symplectic embeddings of the standard rational homology balls bounded by these lens spaces. Further embeddings of lens spaces have since been exhibited, including two families of triples which embed disjointly due to Lisca-Parma.

I will exhibit some new triples of examples, and will show in particular that all lens spaces $L(p^2, pq-1)$ with $\gcd(p,q)=2$, or with p odd and $\gcd(p,q)=1$, embed in $\mathbb{CP}^2$.

This is joint work with Marco Golla.)

11:30–11:50 Deformation inequivalent symplectic structures and Donaldson's four-six question
Luya Wang (Stanford University, USA)

Abstract. Studying symplectic structures up to deformation equivalences is a fundamental question in symplectic geometry. Donaldson asked: given two homeomorphic closed symplectic four-manifolds, are they diffeomorphic if and only if their stabilized symplectic six-manifolds, obtained by taking products with $\mathbb{CP}^1$ with the standard symplectic form, are deformation equivalent? I will discuss joint work with Amanda Hirschi on showing how deformation inequivalent symplectic forms remain deformation inequivalent when stabilized, under certain algebraic conditions. This gives the first counterexamples to one direction of Donaldson's "four-six" question and the related Stabilizing Conjecture by Ruan. In the other direction, I will also discuss more supporting evidence via Gromov-Witten invariants.

12:00-12:20 On the universal cork conjecture
Roberto Ladu (MPIM Bonn, GERMANY)

Abstract. It is well known that every exotic pair of 1-connected, closed 4-manifolds is related by some cork. Akbulut in 2008 asked if the cork W_1 is *universal*, i.e. relates every exotic pair of 1-connected closed 4-manifolds. It was conjectured that every cork is *non*-universal. However, it was not known of *any* example of non-universal cork. We will show that corks in a large class, including the corks W_n constructed by Akbulut and Yasui, are (even sequentially) non-universal. Our counterexample exploits the construction of 1-connected general type surfaces and symplectic manifolds of non-negative signature. Moreover we will show that if we allow for exotic pairs to have boundary, then every cork is non-universal.

12:30-12:50 Exotic Dehn twists on 4-manifolds with Seifert-fibered boundary
Jianfeng Lin (Tsinghua University, CHINA)

Abstract. Given a 4-manifold X whose boundary is a Seifert fibered 3-manifold, one can use the circle action on the boundary to define a diffeomorphism on X, called the boundary Dehn twist. Such boundary Dehn twist naturally arises as monodromy of Milnor fibrations. By results of Orson–Powell, these Dehn twists are very often topologically isotopic to the identity. In this talk, we will discuss a proof (using monopole Floer homology) that some of these Dehn twists represents exotic elements of infinite order in the smooth mapping class group.

Lunch

14:30-14:50 Non-smoothable homeomorphisms of 4-manifolds with boundary
Daniel Galvin (University of Glasgow, UK)

Abstract. By the work of Freedman and Perron-Quinn, any self-homeomorphism of a (smooth), simply-connected, closed 4-manifold X which induces the trivial map on $H_2(X)$ is isotopic to the identity, and hence is isotopic to a diffeomorphism (i.e. is smoothable). By the work of Saeki and Orson-Powell, if we allow the manifold to have boundary, then there exist self-homeomorphisms which induce the trivial map on homology but are not isotopic to the identity. Building on this, we show that there exist infinite families of smooth, simply-connected, compact 4-manifolds which support self-homeomorphisms that induce the trivial map on homology but are not isotopic to any self-diffeomorphism.

15:00-15:20 Obstructing smooth sliceness of links in $\mathbb{CP}^2\#\overline{\mathbb{CP}^2}$
Marco Marengon (Renyi Institute, HUNGARY)

Abstract. It is well-known that every knot K is smoothly slice in both $S^2\times S^2$ and $\mathbb{CP}^2\#\overline{\mathbb{CP}^2}$, i.e. K bounds a smooth disc in the punctured manifolds $(S^2\times S^2)\setminus\mathrm{Int}(B^4)$ and $(\mathbb{CP}^2\#\overline{\mathbb{CP}^2})\setminus\mathrm{Int}(B^4)$.

In 1997, Miyazaki and Yasuhara showed that there exists a 2-component link L which is not strongly slice in $S^2 \times S^2$, i.e. L is not boundary of disjoint discs in $(S^2 \times S^2) \setminus \mathrm{Int}(B^4)$. Since their proof uses only classical results, it holds in the locally flat category too.

The argument of Miyazaki-Yasuhara is specific to $S^2 \times S^2$, and does not seem to generalise to $\mathbb{CP}^2\#\overline{\mathbb{CP}^2}$. In joint work with McDonald we address the latter case.

Theorem 1 (M.-McDonald)**.** *There exists a 2-component link L which is not* smoothly *strongly slice in* $\mathbb{CP}^2\#\overline{\mathbb{CP}^2}$.

Our strategy is quite different from Miyazaki-Yasuhara, and we can make it work only in the smooth category. The smooth ingredient of the proof is the smooth genus function on $\mathbb{CP}^2\#\overline{\mathbb{CP}^2}$, due to Ruberman and Morgan-Szabo-Taubes.

15:30-15:50 Negative-definite fillings of plumbed manifolds
Paolo Aceto (University of Lille, FRANCE)

Abstract. Motivated by the study of smoothings of rational surface singularities as well as symplectic fillings of plumbed 3-manifolds, we consider an analogue problem in a purely topological setting. The question of when a rational surface singularity admits a unique smoothing is of particular interest and has led to a conjecture of Kollár which has been proved in some cases. We look at smooth, definite fillings of certain plumbed manifolds and consider the question of which intersection forms can be realized by such fillings. We describe various constructions and an obstruction based on Donaldson's diagonalization theorem. Finally, we present a couple of uniqueness results and discuss their relevance for Kollár's conjecture as well as the problem of embedding lens spaces in certain 4-manifolds. While the main motivation lies in problems from singularity theory and symplectic topology, our results are purely topological in nature and the main techniques used are algebro-combinatorial. This is joint work with Duncan McCoy and JungHwan Park.

Break

17:00-18:00 Discussion

July 24, 2024

11:30-11:50 Topology of the Dirac equation on spectrally large three-manifolds
Francesco Lin (Columbia University, USA)

Abstract. Dirac operators twisted by flat $U(1)$-connections play a key role as a bridge between geometry and topology. For example, for any Riemannian metric on the n-torus, the Atiyah-Singer index theorem implies that some twisted Dirac operator has kernel; this is the key input in Gromov-Lawson's proof that the n-torus does not admit metrics of positive scalar curvature. In this talk, I will explore refinements of this result about the kernel of such operators for Riemannian three-manifolds satisfying natural constraints of spectral nature. While our main theorem will only involve linear operators, its proof relies on the non-linear analysis of the Seiberg-Witten equations and Floer theory.

12:00-12:20 Disoriented homology and braided surfaces
Sašo Strle (University of Ljubljana, SLOVENIA)

Abstract. Let F be a properly embedded surface in the 4-ball B^4. Previously we defined, based on the handle decomposition of F induced by the radial distance function in B^4 and a choice of disorientations of the handles, the disoriented homology, $DH_*(F)$, of the surface F. Recall that a ribbon surface F can be completely described by its ribbon projection into the 3-sphere S^3. Assuming that all the ribbon singularities are formed by 1-handles of the surface passing through the 0-handles, a disorientation of a 1-handle is given by a choice of orientations of the arcs into which ribbon singularities split its core in such a way that any two consecutive arcs have opposite orientations. We also defined a pairing λ on $DH_1(F)$ generalizing the Gordon-Litherland pairing which is defined for surfaces F that are obtained by pushing spanning surfaces of links into B^4. For a general F, any projected 2-handle is split into subdisks by its intersection with the ribbon immersed subsurface and orienting these subdisks incoherently gives a choice of disorientation of the 2-handle of F.

Theorem 2 (Owens-Strle)**.** *The disoriented homology of $F \subset B^4$ is isomorphic to the shifted reduced homology of the double cover $\Sigma_2(B^4, F)$ of the 4-ball branched along F:*

$$DH_*(F) \cong \widetilde{H}_{*+1}(\Sigma_2(B^4, F)).$$

Moreover, the intersection pairing of $\Sigma_2(B^4, F)$ under this identification agrees with λ.

Though the disoriented homology of a surface F is easily computable from the above mentioned description of F, the computation is particularly simple for ribbon surfaces. The main obstacle to algorithmic computation is then the form in which the surface is described. We show that for braided surfaces, defined by Rudolph, the computation is algorithmic based on the band factoriation of the braid that determines the corresponding orientable ribbon surface. We then generalize the computational machinery to nonorientable ribbon surfaces.

12:30-12:50 Unknotting nonorientable surfaces
Anthony Conway (University of Texas, USA)

Abstract. This talk will describe joint work with Mark Powell and Patrick Orson in which we prove that most closed, nonorientable surfaces in S^4 with knot group $\mathbb{Z}/2$ are topologically unknotted.

Lunch

14:30-14:50 Loop spaces and Khovanov–Rozansky homology
Joshua Wang (MIT, USA)

Abstract. I will present a new connection between the colored Khovanov–Rozansky homology of two-stranded torus knots and the cohomology of the free loop space of a complex Grassmannian.

15:00-15:20 Stable parabolic bundles over complex curves and singular instanton Floer homology

Yi Xie (Pecking University, CHINA)

Abstract. Stable parabolic bundles are objects in algebraic geometry which have been studied by many people. Singular instanton Floer homology is an invariant of links in 3-manifolds introduced by Kronheimer and Mrowka, which has been used to solve many problems in the low dimensional topology. It turns out the two things are closely related: knowledge on the moduli space of stable parabolic bundles can help the calculation of singular instanton Floer homology. In this talk, we will give a precise description of the cohomology ring of the moduli space of rank 2 parabolic bundles over complex curves. Then we will derive all the "universal relations" for singular instanton Floer homology. This is joint work with Boyu Zhang.

15:30-15:50 Rational homology ball symplectic fillings of spherical 3-manifolds

Burak Ozbagci (Koc University, HUNGARY)

Abstract. The family of spherical 3-manifolds can be identified with the homeomorphism types of the links of quotient surface singularities. It follows that every spherical 3-manifold, viewed as the oriented link of a normal surface singularity, has a canonical contact structure ξ_{can} (also known as the Milnor fillable contact structure), which is unique up to contactomorphism [**?**]. The lens spaces are precisely the links of cyclic quotient surface singularities.

The lens space $L(p,q)$ equipped with a contact structure ξ, admits a rational homology ball *symplectic* filling if and only if $\frac{p}{q} = \frac{m^2}{mh-1}$ for some coprime integers $0 < h < m$, and ξ is contactomorphic to ξ_{can}. Independent alternative proofs of this fact recently appeared in [**?**, **?**, **?**]. We show that this result extends to all spherical 3-manifolds as follows:

Theorem 3. *A spherical* 3*-manifold* Y *equipped with a contact structure* ξ*, admits a rational homology ball symplectic filling, if and only if* Y *is orientation-preserving diffeomorphic to a lens space* $L(m^2, mh-1)$ *for some coprime integers* $0 < h < m$*, and* ξ *is contactomorphic to* ξ_{can}*.*

16:00-16:20 Five tori in the four-dimensional sphere

Bruno Martelli (University of Pisa, ITALY)

Abstract. Ivanšić proved [**?**] that there is a link L of five tori in S^4 with hyperbolic complement. We describe L and show that $L \subset S^4$ is in many aspects similar to the Borromean rings in S^3. In particular: (1) any two tori in L are unlinked, (2) the complement $M = S^4 \setminus L$ is hyperbolic, (3) it has many symmetries, (4) the double branched covering over L has geometry $\mathbb{H}^2 \times \mathbb{H}^2$, (5) the fundamental group of M has a nice presentation via commutators, (6) the Alexander ideal is explicit, (7) every generic first cohomology class is represented by a circle-valued perfect Morse function and the Betti numbers of all the infinite cyclic covers can be computed, and (8) by longitudinal Dehn surgery along L we get a closed 4-manifold with fundamental group $\mathbb{Z}^5$.

This leads also to the first descriptions of a cusped hyperbolic 4-manifold as a complement of tori in $\mathbb{RP}^4$ and of some explicit Lagrangian tori in the product of two surfaces.

Break

17:00-18:00 Discussion

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