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Mathematics for Quantum and Statistical Physics
Special Session A15

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This session is devoted to recent advances in the mathematical understanding of phenomena of interest in Statistical and Quantum Physics, and related developments in modern Probability Theory and Analysis. Topics will include Universality in Statistical Mechanics and Condensed Matter, Symmetry Breaking effects, Localization in Quantum Systems, Non-Equilibrium Phenomena, construction of Quantum Field Theory models and Interaction Effects of Quantum Particles

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Schedule

July 23

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12:00 - 12:25 Chiara Boccato
12:30 - 12:55 Ian Jauslin

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16:00 - 16:25 Davide Gabrielli

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Schedule and Abstracts

July 23, 2024

11:00–11:55 The Nonlinear Schroedinger equation with bounded initial data
Thomas Spencer (IAS, Princeton)

Abstract. We discuss the behavior of the NLS on R or the discrete NLS on Z with bounded initial data. Examples include quasi-periodic and random data. On Z, polynomial bounds in time are proved for all bounded initial data. In the continuum, local existence is established for smooth data. We also discuss the long time behavior of a smoothly driven anharmonic oscillator. This is joint work with B. Dodson and A. Soffer.

12:00–12:25 Ground state properties of interacting bosons
Chiara Boccato (University of Pisa)

Abstract. The interacting Bose gas is a system in quantum statistical mechanics where complex collective behavior emerges from the underlying many-body theory, posing crucial challenges to its rigorous mathematical description.

In recent years, progress has been made in understanding its ground state properties; in certain scaling limits it has been possible to prove the occurrence of the Bose-Einstain condensation phase transition [2,4] and to obtain expressions for the excitation spectrum [1], justifying the linear dispersion relation predicted by Bogoliubov. For the description of disordered materials it is crucial to understand whether similar properties are stable in presence of randomly placed impurities. We show that for a suitably rescaled interactions and in presence of Poisson distributed impurities (Kac–Luttinger model), Bose-Einstein condensation occurs into the minimizer of a Hartree-type functional [3].

References

[1] C. Boccato, C. Brennecke, S. Cenatiempo, B. Schlein. *Bogoliubov Theory in the Gross-Pitaevskii Limit.*
Acta Mathematica 222, 219-335 (2019)

[2] C. Boccato, C. Brennecke, S. Cenatiempo, B. Schlein. *Optimal Rate for Bose-Einstein Condensation in the Gross-Pitaevskii Regime.*
Commun. Math. Phys. 376, 1311–1395 (2020)

[3] C. Boccato, J. Kerner, M. Pechmann. *On Bose-Einstein condensation in interacting Bose gases in the Kac-Luttinger model.* arXiv:2312.14357
Accepted for publication in Journal de Mathématiques Pures et Appliquées (2024)

[4] C. Boccato, R. Seiringer. *The Bose gas in a box with Neumann boundary conditions.*
Ann. Henri Poincaré 24, 1505-1560 (2023)

12:30–12:55 Non-perturbative behavior of interacting Bosons at intermediate densities

Ian Jauslin, (Rutgers University)

Abstract. Much attention has been given to systems of interacting Bosons in the dilute regime, where powerful theoretical tools such as Bogolyubov theory give detailed and accurate predictions. In this talk, I will discuss a different approach to studying the ground state of Boson systems, which Carlen, Lieb and I have recently found to be accurate at all densities. In particular, it allows us to probe the system in the intermediate density regime, which had, until now, only been accessible to costly Monte-Carlo simulations. In this talk, I will first give an overview of this Simplified approach, and will discuss evidence for non-perturbative behavior in the intermediate density regime obtained using this tool.

14:30–14:55 Foundations of kinetic theory: recent progress and open directions.

Sergio Simonella (Sapienza Roma)

Abstract. We consider deterministic, time-reversible dynamics with random initial data, in a low-density scaling. Under suitable assumptions on the initial measure, a strong chaos property is propagated in time, which also encodes the transition to irreversibility. This result is complemented by large deviation estimates and by a theory of small fluctuations, allowing to establish the connection between microscopic and hydrodynamic scales, for perturbations of a global equilibrium. Many of the open problems left require a deeper understanding of the coupling mechanisms between deterministic and stochastic dynamics.

15:00–15:45 Decay of Information for the Kac Evolution

Michael Loss (Georgia Institute of Technology)

Abstract. We consider a system of M particles in contact with a heat reservoir of N »M particles. The evolution in the system and the reservoir, together with their interaction, are modeled via the Kac's Master Equation. We chose the initial distribution with total energy N +M and show that if the reservoir is initially in equilibrium then the entropy of the system decays exponentially to a very small value. We base our proof on a similar property for the Information.

16:00–16:25 Current fluctuations for the boundary driven zero-range process: microscopic versus macroscopic approach and a theory of non-reversible resistor-like networks

Davide Gabrielli (University of l'Aquila)

Abstract. We compute the joint large deviation rate functional in the limit of large time for the current flowing through the edges of a finite graph where a boundary driven system of stochastic particles is evolving with zero-range dynamics. This generalizes one dimensional results previously obtained with different approaches [5, 3]. We use new techniques that illuminate various connections and complementary perspectives. In particular, we use a variational approach to derive the rate functional by contraction from a level 2.5 large deviation rate functional. We perform an exact minimization and obtain finally as a rate functional a variational problem involving a superposition of cost functions for each edge. The contributions from different edges are not independend since are related by the values of a potential function on the nodes of the graph. The rate functional on the graph is a microscopic version of the continuous rate functional predicted by the macroscopic fluctuation theory [4] and we show indeed a convergence in the scaling limit. If we split the graph into two connected regions by a cutset and are interested just on the current flowing through the cutset, we show that the result is the same of an effective system composed by just one effective edge. This is what happens at macroscopic level and it is expected also for other model [1]. The characteristics of this effective edge are related to the capacities of the graph and can be obtained by a reduction of the graph by elementary transformations like in electric networks (effective components in parallel, in series and N-star to effective complete N graphs). Our reduction procedure is directly related to the reduction of the so called *trace process* and, since the dynamics is in general not reversible, it is closely related to the theory of non reversible electric networks in [2].

References

[1] Akkermans E., Bodineau T., Derrida B., Shpielberg O. *Universal current fluctuations in the symmetric exclusion process and other diffusive systems* Europhysics Letters, **103**, Number 2 (2013).

[2] Balázs M., Folly Á *An electric network for nonreversible Markov chains.* Amer. Math. Monthly **123** no. 7, 657–682 (2016).

[3] Bertini L., De Sole A., Gabrielli D., Jona-Lasinio G., Landim C.; *Non equilibrium current fluctuations in stochastic lattice gases.* J. Stat. Phys. **123**, 237–276. (2006)

[4] Bertini L., De Sole A., Gabrielli D., Jona-Lasinio G., Landim C. *Macroscopic fluctuation theory.* Rev. Modern Phys. **87** (2015), no. 2, 593–636.

[5] Harris, R. J., Rákos A., Schütz G. M.; *Current fluctuations in the zero-range process with open boundaries,* J. Stat. Mech. Theory Exp. 2005, P08003 (2005).

17:00–17:45 Current fluctuations for the boundary driven zero-range process: microscopic versus macroscopic approach and a theory of non-reversible resistor-like networks

Eric Carlen (Rutgers University)

Abstract. We compute the spectrum for a class of quantum Markov semigroups describing systems of N particles interacting through a binary collision mechanism. These quantum Markov semgroups are associated to a novel kind of quantum random walk on graphs, with the graph structure arising naturally in the quantization of the classical Kac model, and we show that the spectrum of the generator of the quantum Markov semigroup is closely related to the spectrum of the Laplacian on the corresponding graph. For the direct analog of the original classical Kac model, we determine the exact spectral gap for the quantum generator. We also give a new and simple method for studying the spectrum of certain graph Laplacians. We also discuss entropy production for these Master equations.

References

[1] Eric Carlen and Michael Loss *Spectrum for some Quantum Markov semigroups describing N-particle systems evolving under a binary collision mechanism*, arXiv preprint arXiv:2204.07860.

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11:30–11:55 Time-reversal symmetry and response in an external magnetic field
Lamberto Rondoni (Politecnico di Torino)

Abstract. Given a Hamiltonian system, we show that there are infinitely many time reversal symmetries that lead obtain to statistical mechanical results, such as the Onsager Reciprocal Relations, or the fluctuation relations, even in presence of magnetic fields or rotating refefrence frames. Consequently, the Casimir modification of such relations is not necessary. We illustrate both the classical and the non-relativistic quantum mechanical theories. In particular, we prove that the spin-field interaction does not break the time reversal invariance of the dynamics, and that it does not require additional conditions for such a symmetry to hold, compared to the spinless cases. These results are relevant for experiments such as diffusion in solutions, thermoelectricity and spin charge transport.

References

[1] S. Bonella, G. Ciccotti, L. Rondoni, *Time reversal symmetry in time-dependent correlation functions for systems in a constant magnetic field*, Europhysics Letters 108 (2015), 60004

[2] A. Coretti, L. Rondoni, S. Bonella, *Fluctuation relations for systems in a constant magnetic field* Physical Review E 102 (2020), 030101

[3] D. Carbone, P. De Gregorio, L. Rondoni, *Time reversal symmetry for classical, non-relativistic quantum and spin systems in presence of magnetic fields*, Annals of Physics 441 (2022) 168853

12:00–12:25 Weakly and strongly coupled maps: a self-consistent transfer operator approach
Roberto Castorrini (Scuola Normale Superiore, Pisa)

Abstract To investigate the statistical properties of dynamical systems which exhibit chaotic behaviour, researchers have developed a well-established method stemming from Ruelle's work in 1970. This method focuses on analyzing the evolution of measures (densities) rather than individual trajectories. The evolution of density is described by a linear operator known as the transfer operator, which offers significant advantages. However, many natural phenomena, particularly those involving interacting dynamical systems coupled by mean-field forces, cannot be adequately described by linear operators alone. In such cases, the evolution in the thermodynamic limit is characterized by a transfer operator that incorporates the coupling effect, known as the self-consistent transfer operator (see e.g. [1]). Building on a joint work with Stefano Galatolo (University of Pisa) and Matteo Tanzi (King's college, London), in this presentation I will discuss recent and new techniques based on self-consistent transfer operators that provide valuable statistical insights, including convergence to equilibrium, especially in scenarios involving weak or moderate coupling.

References

[1] S. Galatolo, *Self-consistent transfer operators: Invariant measures, convergence to equilibrium, linear response and control of the statistical properties*, Communications in Mathematical Physics (2022), 1–58.

12:30–12:55 Synchronization and averaging in a simple dynamical systems with fast/slow variables
Federico Bonetto (Georgia Institute of Technology)

Abstract. We study a family of dynamical systems obtained by coupling a chaotic (Anosov) map on the two-dimensional torus – the chaotic variable – with the identity map on the one-dimensional torus – the neutral variable – through a dissipative interaction. We show that the

two systems synchronize, in the sense that the trajectories evolve toward an attracting invariant manifold, and that the full dynamics is conjugated to its linearization around the invariant manifold. When the interaction is small, the evolution of the neutral variable is very close to the identity and hence the neutral variable appears as a slow variable with respect to the fast chaotic variable: we show that, seen on a suitably long time scale, the slow variable effectively follows the solution of a deterministic differential equation obtained by averaging over the fast variable.

Systems with fast/slow variables of the kind studied here appear in many physical problem and have been proposed as a first (rather small) step toward the construction of a kinetic limit based understanding of Fourier's Law.

14:30–14:55 Disordered systems beyond the permutation invariance
Pierluigi Contucci (University of Bologna)

Abstract. In this talk I will review some results obtained for the mean-field spin-glass models when the disorder is not permutation invariant. Cases include the convex and non-convex multispecies, with emphasis on the deep Boltzmann machines in the Nishimori line, and the multy-bath SK model.

References

[1] *The solution of the deep Boltzmann machine on the Nishimori line*
D.Alberici, F.Camilli, P.C. and E.Mingione
Communications in Mathematical Physics, V. 387, pp 1191–1214 (2021)

[2] *Deep Boltzmann machines: rigorous results at arbitrary depth*
D.Alberici, PC and E.Mingione
Annales Henri Poincaré, V. 22, pp 2619–2642 (2021)

[3] *Annealing and replica-symmetry in Deep Boltzmann Machines*
D.Alberici, A.Barra, PC and E.Mingione
Journal of Statistical Physics, Volume 180, pp 665-677 (2020)

[4] *A Multi-Scale Spin-Glass Mean-Field Model.*
PC and E.Mingione
Communications in Mathematical Physics, V. 368, 1323–1344 (2019)

[5] *Multi-species mean-field spin-glasses. Rigorous results*
A.Barra, PC, E.Mingione and D.Tantari
Annales Henri Poincare, Volume 16, Issue 3, pp 691-708 (2014)

15:00–15:45 A Parisi Formula for Quantum Spin Glasses
Simone Warzel (TU, Munich)

Abstract. We establish three equivalent versions of a Parisi formula for the free energy of mean-field spin glasses in a transversal magnetic field. These results are derived from available results for classical vector spin glasses by an approximation method using the functional integral representation of the partition function. In this approach, the order parameter is a non-decreasing function with values in the non-negative, real hermitian Hilbert-Schmidt operators. For the quantum Sherrington-Kirkpatrick model, we also show that under the assumption of self-averaging of the self-overlap, the optimising Parisi order parameter is found within a two-dimensional subspace spanned by the self-overlap and the fully stationary overlap.

16:00–16:25 Response functions of many-body condensed matter systems
Marcello Porta (Sissa, Trieste)

Abstract. In this talk I will discuss rigorous results about charge transport in gapless interacting fermionic lattice models. I will outline a strategy that has been used in the last years to compute the response functions of a wide class of quantum systems, including one-dimensional metals and two and three-dimensional semimetals. The approach is based on: analytic continuation of real-time response functions to imaginary times; renormalization group analysis of imaginary-time correlations and resolution of the scaling limit; lattice conservation laws and

Ward identities to prove universality. I will focus on the analysis of the edge response function of interacting 2d quantum Hall systems, and on the emergence of the multichannel Luttinger liquid at the macroscopic scale. In the last part of the talk, I will report about recent progress for non-translation invariant systems; in particular, I will discuss the edge transport properties of Haldane-like models in the presence of weak quasi-periodic disorder.

References

[1] F. Caragiulo, V. Mastropietro and M. Porta. Edge transport in Haldane-like models with quasi-periodic disorder. *In preparation.*

[2] V. Mastropietro and M. Porta. Multi-Channel Luttinger Liquids at the Edge of Quantum Hall Systems. *Comm. Math. Phys.* **395**, 1097-1173 (2022)

[3] M. Porta, V. Mastropietro, A. Giuliani. Response functions of many-body condensed matter systems. *Encyclopedia of Mathematical Physics*, Elsevier (2024).

17:00–17:45 Approach to equilibrium for an oscillator interacting with a harmonic thermal bath.

Alberto Mario Maiocchi, University of Milano–Bicocca, Italy

Abstract. We want to understand whether a chain of harmonic oscillators can act approximately as a heat bath over a smaller system, put in contact with it, when the number of degrees of freedom of the chain is sufficiently large. As a simplified model, we consider the Hamiltonian dynamics of a single harmonic oscillator (we call it the *probe*) interacting with the chain (we call it the *bath*) through a linear coupling, and we study the dynamics of the global system over long times. The chain is made by identical particles with nearest neighbour interactions and an on–site pinning potential, while the first particle of the chain is linked with the probe by a harmonic spring.

We consider initial data that are canonically distributed, both for the probe and for the bath, but at different temperatures, and show that when the coupling is turned on, the probability distribution for the coordinates of the probe displays an approach to equilibrium. Indeed, we are able to control the decay of the correlations of the position and the momentum of the probe, and to prove that its average energy approaches thermal equilibrium with the bath. These results are valid only up to a finite time, which grows to infinity as the number of particles of the the bath tends to infinity.

We discuss the dependence of the estimates on the parameter tuning the interaction between bath and probe and how this result compares with the previous ones on the dynamics of coupled oscillators.

17:30–18:15 The gapped phases of $O(n)$ quantum spin chains

Bruno Nachtergaele (University of California, Davis)

Abstract. The ground state phase diagram of the $O(n)$ quantum spin chains with nearest neighbor interactions, for $n \geq 3$ or larger, shows two gapped phases separated by a critical point often referred to as the Reshetikhin point. One of the phases contains the $SU(n)$ invariant $-P^{(0)}$ model which has been analyzed using the Temperley-Lieb algebra and, more recently, by a random loop model. These works show the ground state to be dimerized. The other phase contains a special point with exact MPS ground states that generalize the AKLT state (corresponding to the case $n = 3$). For even n, that point too is a phase with breaking of the translation invariance down to period 2. We show that it is not dimerized in the usual sense of the term and uncover other interesting new properties.

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