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New trends in self-similarity of groups, trees and fractals
Special Session A19

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A particularly interesting family of groups arises as transformations of discrete spaces called trees. These spaces can be thought of as graphs, i.e. a collection of vertices and edges joining them, with no cycles. If we have such a graph with infinitely many vertices, we may select a concrete vertex and look at all other vertices as descendants of this vertex. This is said to be a *rooted tree*. If each vertex has the same number of descendants we can see that the tree hanging from a specific vertex resembles the full tree. This self-similarity connects rooted trees with fractals.

The main goal of this special session is to develop and bring together experts in topics in all areas of self-similarity of groups and the spaces on which they act, including fractal and branch groups, groups acting on Cantor sets such as Thompson-like groups, tree structures related to random walks.

The study of these groups provided answer to multiple important problems in mathematics. In particular, these groups stem from the famous Burnside problem, which arose in 1902: "Can a finitely generated torsion group, i.e. in which every element has finite order, be itself infinite?". This question went unanswered for over sixty years. Then in 1964, Golod and Shafarevich constructed an infinite finitely generated group where every element has finite order. Further examples were given by Adjan and Novikov, Olshanskii, Grigorchuk, Gupta and Sidki and recently Schlage-Puchta. The constructions given by Olshanskii, Grigorchuk, Gupta and Sidki are geometric in nature, with the groups of Grigorchuk, Gupta and Sidki acting on trees. These latter examples form a more general family of the so-called Grigorchuk-Gupta-Sidki (GGS) groups, which are in turn examples of branch groups.

A *branch group* is an example of a self-similar group acting transitively on a spherically homogeneous rooted tree T and admits a structure of subnormal subgroups similar to the corresponding structure in the full automorphism group $\mathrm{Aut}(T)$ of the tree T. There are many examples of branch groups with remarkable algebraic properties, such as the first example of a finitely generated group with intermediate growth and the first example of an amenable but not elementary amenable group. They also play an important role in the classification of just infinite groups, i.e. infinite groups whose proper quotients are all finite.

Furthermore, self-similar groups have many applications to other areas of mathematics and science, such as to dynamics, probability, and cryptography, and the aim of this workshop is also to see the interdisciplinary aspects of these class of groups.

For more information visit:
https://sites.google.com/unisa.it/conferencetreesfractals/home.

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