

## Free boundary problems: viscosity and variational approaches Special Session B9

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**This session is scheduled on July 25-26** and it is focused on **Free boundary problems (FBP)**. Free boundary problems are a type of differential equation where the domain in which the equation holds, depends itself on the solution. Examples arise in flame propagation, image reconstructions, jet flows, optimal stopping problems in financial mathematics, tumor growth, and in many other different contexts, providing the opportunity for an interplay between the applied sciences and pure mathematical analysis.

A classical example, which we will use here to illustrate the significance of FBP, is the Bernoulli problem. It arises in two dimensional fluid dynamics and it was first studied systematically by Alt and Caffarelli using a variational approach, in the one-phase setting i.e for non-negative solutions. The two-phase case, in which a solution is allowed to change sign, was then investigated by Alt, Caffarelli, and Friedman who devised a fundamental monotonicity formula. The theory was propelled forward by breakthrough results due to Caffarelli and relying on a viscosity formulation of the problem which allowed to take a geometric approach to answer the essential question of the regularity of the free boundary.

Other analogous problems arise when considering models for which the free boundary can occur on a lower dimensional space, for example the *thin Bernoulli problem*. Related problems also appear in the study of cooperative systems of species, in optimization problems for spectral functions, in optimal partition problems, or in the study of harmonic functions with junctions. Several evolution problems are also connected to the Bernoulli problem, like the Stefan problem or the Hele-Shaw problem used to describe an incompressible flow lying between two nearby horizontal plates. Furthermore, one can consider FBP in non-commutative structures. Other fundamental problems are obstacle-type problems, which also present a vast literature.

Given the interest of the international scientific community for free boundary problems, our special session is dedicated to this topic. In particular, the invited speakers take different approaches to attack important and current questions, whether with variational or viscosity tools.

In the effort of fostering collaborations, especially between senior and junior researchers, we will advertise the session to mathematicians at all levels, at different institutions both in the U.S and in Italy.

This two-day session will feature several lectures of varying length. The special session is supported by the Department of Mathematics of the university of Bologna and INDAM.

The list of the speakers who accepted to deliver a seminar and further information are available at the following working in progress web page:

<https://math.unibo.it/en/events/free-boundary-problems-viscosity-and-variational-approaches>

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**MSC:** 35R35, Free boundary problems for PDEs

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