



Unione
Matematica
Italiana



SOCIEDADE BRASILEIRA DE MATEMÁTICA



SOCIETÀ ITALIANA DI MATEMATICA
APPLICATA E INDUSTRIALE



Sociedade Brasileira de Matemática
Aplicada e Computacional



Università
degli Studi di
Messina

BOOK OF ABSTRACTS

SS14 - Control, Stability and Delay Effects in Real-Life Mathematical Models

2nd Joint Meeting Brazil-Italy in Mathematics

September 10 and 11, 2026

Messina, Italy

Organizers:

Genni Fragnelli, University of Siena

Elisa Continelli, University of Padova

Juan Bautista Limaco Ferrel, Fluminense Federal University

On the Asymptotic Behavior of Transmission Problems for the Wave Equation

Antonio Guimarães Leite, María Rosario Astudillo Rojas , Higidio Portillo Oquendo
Department of Mathematics , Federal University of Paraná

In this talk, we study the behavior of a string made up of different types of materials, including purely elastic regions (which are conservative) and regions with dissipative effects such as viscoelastic damping and thermoelasticity. Because the physical properties of these materials, such as density and stiffness, may vary from one region to another, the model involves equations with discontinuous coefficients. The analysis of these type of models, known as a transmission problems is fundamental in the study of asymptotic behavior of composite bodies, as real-world structures are rarely homogeneous and instead consist of multiple materials with distinct mechanical and thermal properties. Interfaces between the media introduce nontrivial coupling conditions that significantly affect energy propagation, dissipation and long time dynamics.

We discuss the asymptotic properties of the solutions when one of the dissipative mechanisms is a localized Kelvin–Voigt damping. In particular, we show how the position of the different components plays an important role in the study of stabilization. We analyze how, in certain configurations, the exponential stability of the system, achieved through other types of dissipation, can be destroyed by localized Kelvin–Voigt damping with a discontinuous coefficient at the interface..

References

- [1] M. Alves, J.M. Rivera, M. Sepulveda, O.V. Villagran, *The lack of exponential stability in certain transmission problems with localized Kelvin-Voigt dissipation*, Siam J. Appl. **74** (2014), 345-365.
- [2] J.M. Muñoz Rivera and R. Racke, *Transmission Problems in (Thermo)Viscoelasticity with Kelvin–Voigt Damping: Nonexponential, Strong, and Polynomial Stability*, SIAM J. Math. Anal., **49**(5) (2017), 3741–3765.

Preserving Information: Kolmogorov Entropy of Numerical Conservation Laws

Fabio Ancona

Dip. di Matematica “Tullio Levi Civita”, Univ. di Padova

Alessio Basti, Fabio Camilli

Dip. di Ingegneria e Geologia, Univ. “G. d’Annunzio” di Chieti-Pescara

Motivated by the information-theoretic framework introduced by P. D. Lax, this work investigates the quantitative compactness of numerical solutions to scalar conservation laws with uniformly convex flux. While the qualitative convergence of classical numerical schemes is well established, a rigorous quantification of their information-theoretic resolution has remained an open challenge.

We address this gap by analyzing the Kolmogorov ε -entropy of the set of approximate solutions generated by conservative, monotone finite-difference schemes. We prove that, under suitable grid constraints, schemes satisfying a discrete one-sided Lipschitz condition preserve the optimal $1/\varepsilon$ entropy scaling exhibited by the corresponding set of exact entropy solutions. This upper bound is shown to be sharp: it matches the classical continuous bounds established by De Lellis–Golse and Ancona–Glass–Nguyen, with the lower bound recovered via a uniform approximation argument rooted in a bounded-variation precursor class.

Finally, we abstract the mechanism underlying the lower bound into a general transfer principle. We discuss the immediate consequences of these findings for information recovery through numerical post-processing and outline how this framework establishes a rigorous, information-theoretic validation of the high-resolution character of prototypical first-order schemes.

References

- [1] P. D. Lax, *Accuracy and resolution in the computation of solutions of linear and nonlinear equations*, in Recent Advances in Numerical Analysis (Proc. Sympos., Math. Res. Center, Univ. Wisconsin, Madison, Wis., 1978), Publ. Math. Res. Center Univ. Wisconsin, Academic Press, New York, 1978, pp. 107–117.

Polynomial Stability of the Wave Equation with Pure Boundary Memory: a Frequency-Domain Approach

Layanne Camargo, Marcelo M. Cavalcanti, Cintya A. Okawa and Claudete M. Webler
Departamento de Matemática, State University of Maringá, Brazil

We consider the wave equation in a bounded domain with mixed boundary conditions, where the feedback on Γ_1 is of *pure memory type*, with no instantaneous friction term. In contrast with models involving an additional boundary damping u_t , the dissipation generated solely by the memory kernel does not yield, in general, uniform exponential stability. We prove instead that the associated C_0 -semigroup is polynomially stable. The proof relies on a resolvent estimate on the imaginary axis combined with the Borichev–Tomilov characterization of polynomial decay on Hilbert spaces. For the infinite-history (fading memory) case, we prove that the associated C_0 -semigroup is polynomially stable. For the finite-memory case, where the dynamics is no longer time-translation invariant, we establish well-posedness by a Galerkin method and derive polynomial decay of the energy by means of a suitable Lyapunov functional.

Agent-based models and MFG with large discount

Marco Cirant, Elisa Continelli

Dipartimento di Matematica “Tullio Levi-Civita”, Università di Padova

The theory of MFG describes equilibria in a large population of players, where agents aim to optimize some cost that depends on a discount factor. If one assumes that the discount factor is large, these models turn out to behave similarly to agent-based models, where individuals just react to the population distribution according to a given rule. Inspired by the work [1], we will elaborate on this connection, showing in particular the uniqueness of equilibria in MFG with large discount, and their converge as the discount goes to infinity to certain agent-based models, in a quantitative sense.

To make such a connection, the central idea will be to see the MFG as a sort of nonlinear and nonlocal in time evolution, where, instead of time delay effects, agents have the ability to forecast near future events.

References

- [1] M. Bardi, P. Cardaliaguet. *Convergence of some mean field games systems to aggregation and flocking models*. Nonlinear Anal., 204:Paper No. 112199, 24, 2021
- [2] E. Continelli, M. Cirant. *Uniqueness of solutions to MFG systems with large discount*. arXiv:2510.09280, 2025

On Pareto and Nash equilibria for Boussinesq fluids with multiplicative controls

Juan Límaco

Instituto de Matemática e Estatística, Universidade Federal Fluminense, Niterói, RJ, Brasil

Abstract

In this work, we analyze Pareto and Nash equilibrium problems in an optimal control setting for a generalized Boussinesq model describing the evolution of the velocity field, the pressure, and the temperature of a viscous, incompressible fluid subject to thermal effects. The system features multiplicative controls acting simultaneously in the velocity and temperature equations, thereby creating a tightly coupled dynamical setting in which strategic interactions naturally emerge. The main contributions of this work are as follows. First, we develop a framework to address multiple control criteria. By assigning positive weights to each criterion, the resulting multi-criteria problem is reduced to a single, well-posed minimization problem. In this setting, identifying controls that properly balance the different criteria, namely Pareto optimal controls, is equivalent, under suitable conditions, to solving the corresponding weighted optimization problem. Secondly, we show that the competitive multiplayer model associated with the system, equipped with quadratic cost functions and appropriate control sets, admits at least one Nash equilibrium, which can be characterized by means of a coupled optimality system.

Mixed-Order Damping Strategies for Hybrid Flexible Beams

Mohammad Akil

CÉRAMATHS/DEMAV, Université Polytechnique Hauts-de-France, Le Mont Houy,
Valenciennes 59313, France

*Genni Fragnelli*¹

Department of Information Engineering and Mathematics, Università degli Studi di Siena, Via
Roma 56, 53100 Siena, Italy

*Sarah Ismail*²

Dipartimento di Matematica, Università degli Studi di Bari Aldo Moro, Via E. Orabona 4,
70125 Bari, Italy

*Ahmet Özkan Özer*³

Department of Mathematics, Western Kentucky University, Bowling Green, KY 42101, USA

In this talk, we introduce a SCOLE-type hybrid PDE-ODE system consisting of two Euler-Bernoulli beams serially connected at an interior joint, where the joint carries both a point mass and a rotational inertia, and the coupling is imposed through dynamic transmission conditions, while each sub-beam preserves classical Euler-Bernoulli dynamics. For a class of boundary-interface velocity feedback laws combining lower-order and higher-order traces at the joint, we establish a complete classification of the system's asymptotic behavior.

First, we consider the case where boundary dissipation is active. When at least one higher-order interface feedback term is present, the associated C_0 -semigroup is exponentially stable, without arithmetic restrictions on material parameters, feedback gains, or the interface location. In contrast, if higher-order interface feedback is replaced by lower-order damping alone, exponential stability fails and the total energy decays at the sharp polynomial rate t^{-1} .

Then, we consider the case where boundary dissipation is violated. In this scenario, exponential decay is achieved only when both controllers are higher-order. Any other admissible pairing (that is, any combination that does not consist exclusively of higher-order feedback at both the boundary and the interface) yields only polynomial decay of order t^{-1} or t^{-2} .

The proofs rely on sharp frequency-domain resolvent estimates within the Borichev-Tomilov framework, which is shown to be essential due to the failure of classical spectral, multiplier, and Riesz basis methods for this class of interface-inertial generators. These findings refine the SCOLE paradigm and provide new insights into the distributed control of large-scale networks of flexible structures.

References

- [1] Akil, M., Fragnelli, G., Ismail, S. et al. *Extending the SCOLE Paradigm to an Interconnected Two-Beam System with Mixed-Order Boundary-Interface Control Strategy*. Appl. Math. Optim. 94, 3 (2026). <https://doi.org/10.1007/s00245-026-10452-w>

¹G. Fragnelli and S. Ismail are members of Gruppo Nazionale per l'Analisi Matematica, la Probabilità e le loro Applicazioni (GNAMPA) of the Istituto Nazionale di Alta Matematica (INdAM) and UMI "Climath". They are partially supported by INdAM GNAMPA Project 2026 Partial differential equations methods for control and inverse problems (CUP E53C25002010001).

²S. Ismail acknowledges the Erasmus+ program for funding her traineeship mobility at Université Polytechnique Hauts-de-France, for the academic year 2024-2025, under financial agreement No. 2024-1-IT02-KA131-HED-000205629.

³A. Ö. Özer gratefully acknowledges the support of Université Polytechnique Hauts-de-France (UPHF) for hosting him, and of the 20242025 Fulbright Research Scholar Program, jointly supported by the Franco-American Fulbright Commission, which funded his stay in France.

E-mail: sarah.ismail@uniba.it.

Recent Advances on the Scattering Conjecture for Nonlinear Schrödinger Equations

Carlos Guzman Jimenez¹

Department of Mathematics, Federal Fluminense University

In this talk, we discuss recent developments in scattering theory for critical nonlinear dispersive equations. A fundamental example is the nonlinear Schrödinger equation

$$i\partial_t u + \Delta u + |u|^\alpha u = 0,$$

where $\alpha = \frac{4}{N-2}$. This equation arises in several models of mathematical physics, including quantum mechanics and nonlinear optics.

We review some of the main advances in the study of scattering for critical equations, a central problem concerning the long-time behavior of global solutions. Over the last decades, this topic has attracted considerable attention, leading to important contributions by Kenig and Merle and, later, by Killip, Visan, Dodson, and others.

Finally, we present recent scattering results for critical inhomogeneous nonlinear Schrödinger equations and discuss some related open problems..

References

- [1] T. Cazenave, *Semilinear Schrödinger Equations*, Courant Lecture Notes in Mathematics, Vol. 10, American Mathematical Society, Providence, 2003.
- [2] C.E. Kenig, F. Merle, *Global Well-Posedness, Scattering and Blowup for the Energy-Critical Focusing Nonlinear Schrödinger Equation*, Invent. Math., 166 (2006), 645–675.
- [3] C.M. Guzmán, J. Murphy, *Scattering for the Non-Radial Energy-Critical Inhomogeneous NLS*, J. Differential Equations, 295 (2021), 187–210.

¹Aknowledgements Bolsa PQ2 Cnpq and Bolsa Jovem Cientista do estado de RJ - Faperj
E-mail: carlosguzman@idd.uff.br.

Z-Control for Exponential Consensus in High-Order Multi-Agent Systems

Angela Monti , Fasma Diele

Istituto per le Applicazioni del Calcolo "M. Picone", CNR, Bari, Italy

Consensus in multi-agent systems is a central problem in control and collective dynamics, with applications ranging from robotics to social systems and epidemiology [1,2]. We study consensus control for high-order multi-agent systems by prescribing the consensus objective as a hierarchy of exponentially stable error dynamics within a Z-control framework [3]. The approach includes two constructions: a direct state-feedback law, acting on the highest-order state, and an indirect design in which the control input is applied to lower-order dynamics. In the direct case, for weight-balanced interactions, the method preserves the average and ensures exponential convergence to the desired consensus value. In contrast, the indirect construction leads to a state-dependent algebraic problem whose solvability and conditioning must be assessed along the system trajectory, resulting in a centralized model-based design. Numerical experiments on opinion dynamics and Cucker–Smale-type [4] models illustrate convergence properties, control effort, and solver behaviour, highlighting both the effectiveness of Z-control in the error-dynamics formulation and the limitations of indirect actuation in ill-conditioned regimes.

References

- [1] R. Olfati-Saber, *Flocking for multi-agent dynamic systems: Algorithms and theory*, IEEE Transactions on Automatic Control 51(3) (2006), 401–420.
- [2] C. Nowzari, V. M. Preciado, G. J. Pappas, *Analysis and control of epidemics: A survey of spreading processes on complex networks*, IEEE Control Systems Magazine 36(1) (2016), 26–46.
- [3] Y. Zhang, Z. Li, *Zhang neural network for online solution of time-varying convex quadratic programs subject to time-varying linear-equality constraints.*, Physics Letters A 373 (2009), 1639–1643.
- [4] F. Cucker, S. Smale, *On the mathematics of emergence*, Japanese Journal of Mathematics 2 (2007), 197–227.

Higher-Order No-Gap Conditions in optimal impulsive control via Compatible Clarke Tangent and QDQ Approximating Cones

Monica Motta

Department of Mathematics Tullio Levi-Civita, University of Padova

Michele Palladino

Department of Engineering, Information Sciences and Mathematics, University of L'Aquila

Franco Rampazzo

Department of Mathematics Tullio Levi-Civita, University of Padova

This talk addresses two related problems in optimal control. The first investigation consists of compatibility issues between two classical approaches to deriving necessary conditions for optimal control problems with a final target: the *set-separation approach* and *penalization techniques*. These methods generally lead to non-equivalent conditions, mainly due to their reliance on different notions of tangency at the target. We address this issue by considering *Quasi Differential Quotient (QDQ) approximating cones* (which are fit for the set-separation approach) and identifying conditions under which the Clarke tangent cone (which is a typical tool within penalization techniques) is also a QDQ approximating cone. In particular, we show that this property holds under suitable local invariance assumptions or when the target coincides locally with an *r-prox regular set*. Then, we apply this compatibility result to the study of *infimum-gap phenomena* in optimal control problems with unbounded controls and impulsive extensions. In particular, we establish a connection between the occurrence of infimum gaps for *strict-sense minimizers* and abnormality in a higher-order Maximum Principle involving Lie brackets. While the abnormality-gap correspondence beyond first-order conditions has been already established for impulsive minimizers, the utilization of the above compatibility issues allow us to extend this correspondence to strict-sense minimizers.

Existence of Complementary and Variational Weak Solutions to Obstacle Problems for a Quasilinear Wave Equation

*Wladimir Neves*¹

Institute of Mathematics, Federal University of Rio de Janeiro

In this talk, we prove the existence of weak solutions for the one obstacle problem associated with a class of quasilinear wave equations in one space dimension, extending previous results obtained in the linear case, and we also address the two obstacles problem. In contrast with the linear setting, for both strictly quasilinear cases we obtain continuous solutions in a weak complementary sense, which moreover satisfy a weak entropy condition in the free region where the string is not in contact with the obstacles. We further show that, in both the one and two obstacle cases, these solutions are variational solutions in a hyperbolic sense without the viscosity term.

¹Aknowledgements: CNPq through the grant 313005/2023-0, 403675/2025-1, and also by FAPERJ (Cientista do Nosso Estado) through the grant E-26/204.171/2024.

E-mail: wladimir@im.ufrj.br.

Stability for wave models with time-dependent memory

Valéria N. Domingos Cavalcanti¹

Department of Mathematics , State University of Maringá - Universidade Estadual de Maringá

We are concerned with the existence of global solutions as well as with the decay rates of the energy related to the viscoelastic wave equation with time-dependent memory kernels posed on a bounded domain of the 3-D Euclidian space.

References

- [1] M. Cavalcanti, V. N. Domingos Cavalcanti, M. A. Jorge Silva and A. Y. Souza Franco, Exponential stability for the wave model with localized memory in a past history framework. *J. Differential Equations* 264 (2018), 6535–6584.
- [2] M. Cavalcanti, V. N. Domingos Cavalcanti, T. D. Marchiori and C. M. Webler, Asymptotic behavior of the energy to the viscoelastic wave equation with localized hereditary memory and supercritical source term. *Discrete Contin. Dyn. Syst. Ser. B* 19 (2014), no. 7, 1987–2012.
- [3] M. M. Cavalcanti and H. P. Oquendo, Frictional versus viscoelastic damping in a semilinear wave equation. *SIAM J. Control Optim.* 42 (2003), no. 4, 1310–1324.
- [4] M. Conti, V. Danese, C. Giorgi and V. Pata, A model of viscoelasticity with time-dependent memory kernels, *Amer. J. Math.* 140(2018), no.2, 349-389.
- [5] M. Fabrizio, C. Giorgi and V. Pata, A new approach to equations with memory, *Arch. Ration. Mech. Anal.*, 198 (2010), no. 1, 189–232.

¹Research of Valéria N. Domingos Cavalcanti partially supported by the CNPq Grant 304895/2003-2.
E-mail: vndcavalcanti@uem.br.