BOOK OF ABSTRACTS

SS7 - Recent Developments in Nonlinear Partial Differential Equations

2nd Joint Meeting Brazil-Italy in Mathematics

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Organizers:

Giuseppe Riey, University of Calabria
Liliane de Almeida Maia, University of Brasilia
Riccardo Molle, University of Rome Tor Vergata

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A Critical Neumann problem with anisotropic p-Laplacian

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Alânnio Barbosa Nóbrega
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We will discuss the existence of a positive solution of a critical Neumann problem involving the anisotropic p-Laplacian on a C^1 bounded domain inside a convex open cone in $\mathbf{R}^N$. Besides the critical growth, the main challenge to succeed with a variational approach, where the strong convergence of a bounded (PS) subsequence needs to be provided, is to deal with anisotropic norms in the absence of a Tartar's type inequality. The solution obtained is a bounded function of class $C^{1,\alpha}(\Omega)$, $\alpha \in (0,1)$, which is strictly positive within the domain.

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Quasilinear elliptic problems with exponential growth via the Nehari manifold method.

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In this talk, I will discuss some results obtained in the paper [1] concerning the existence of solutions to the quasilinear Schrödinger-type problem

$$-\Delta u - \frac{1}{2}\Delta(a(x)u^2)u + V(x)u = f(u), \quad x \in \mathbb{R}^2,$$

where V is a continuous potential and $f : \mathbb{R} \to \mathbb{R}$ is a superlinear function with either subcritical or critical exponential growth. Our main analytical tool is the Nehari manifold method, which we employ to establish the existence of nonnegative solutions as well as nodal solutions.

These results complement classical contributions on quasilinear Schrödinger equations obtained via the Nehari manifold method, particularly those of Jia-quan Liu, Ya-qi Wang, and Zhi-Qiang Wang in the paper [2], by considering exponential-type nonlinearities in two-dimensional space.

References

[1] G. Figueiredo, S. Moreira, R. Ruviaro *Quasilinear Elliptic Problems with Exponential Growth via the Nehari Manifold Method: Existence of Nonnegative and Nodal Solutions*, Acta. Math. Sin.-English Ser., 41 (2025), 1977–1994.

[2] J. Q. Liu, Y. Q Wang, Z. Q. Wang,*Solutions for quasilinear Schrödinger equations via the Nehari method*, Comm. in PDE, 29 (2004), 879–901.

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Solutions to nonlinear Schrödinger equations involving Coulombic interactions

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I will discuss two separate classes of nonlinear Schrödinger type equations both involving Coulombic interaction terms, and arising from the quantum many body problem. The first kind of results [2, 3], dealing with the multiplicity of stationary solutions, have been recently obtained mainly in collaboration with Kanishka Perera (Florida Institute of Technology) using scaling properties of functionals in an essential way. In the second part of the talk I will instead describe some existence and uniqueness results [1] obtained with collaborators from Eindhoven University of Technology on time dependent Kohn-Sham equations coupled with classical nuclear dynamics.

References

[1] Björn Baumeier; Onur Çaylak; Carlo Mercuri; Mark Peletier; Georg Prokert; Wouter Scharpach. *Local existence and uniqueness of solutions to the time-dependent Kohn-Sham equations coupled with classical nuclear dynamics.* J. Math. Anal. Appl. 541 (2025), no. 2, Paper No. 128688, 33 pp.

[2] Elisandra Gloss, Carlo Mercuri, Kanishka Perera, and Bruno Ribeiro. *Fractional Schrödinger–Poisson–Slater equations in Coulomb–Sobolev spaces.* J. Geom. Anal. 36 (2026), no. 4, Paper No. 148.

[3] Carlo Mercuri and Kanishka Perera. *Variational methods for scaled functionals with applications to the Schrödinger-Poisson-Slater equation.* J. Math. Pures Appl. (9) 212 (2026), Paper No. 103885.

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Second order boundary regularity for degenerate elliptic problems

Luigi Muglia
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We consider positive solutions to $-\Delta_p u = \dfrac{1}{u^\gamma} + f(u)$ under zero Dirichlet condition in the half space. Exploiting a priori estimates and the moving plane technique, we prove that any solution is monotone increasing in the direction orthogonal to the boundary.

Joint work with L. Montoro and B. Sciunzi.

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Fractional Higher Differentiability for Solutions of Stationary Stokes Systems with Orlicz Growth

Antonia Passarelli di Napoli
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The talk presents results on the fractional higher differentiability of solutions of elliptic Stokes systems, whose model case is given by

$$\begin{cases} -\operatorname{div}\left(a(x)\,\varphi'(|Eu|)\,\dfrac{Eu}{|Eu|}\right)+\nabla\pi=f, \\ \operatorname{div} u=0, \end{cases} \qquad \text{in } \Omega\subset\mathbb{R}^n,$$

where Eu denotes the symmetric part of the gradient of $u\in W^{1,\varphi}_{\mathrm{loc}}(\Omega,\mathbb{R}^n)$ and $\pi\in L^{\varphi^*}_{\mathrm{loc}}(\Omega,\mathbb{R})$ denotes the pressure.

More generally, we consider stationary Stokes systems whose growth is not necessarily of power type, but is governed by certain Young functions φ satisfying both a Δ_2 and a ∇_2 condition. A new feature of the presented result is that the right-hand side f does not have to belong to $L^{\varphi^*}_{\mathrm{loc}}(\Omega)$, but only to a larger space.

We obtain fractional differentiability properties for a nonlinear expression of the symmetric gradient of the solution, as well as for the pressure term, in terms of Besov and Orlicz–Besov spaces, respectively. The degree of differentiability is determined depending on the integrability of the right-hand side, the lack of smoothness of the coefficients, and the growth properties of the nonlinearity.

The results are due to joint work with A. Cianchi, F. Giannetti and A. Passarelli di Napoli.

References

[1] A. Cianchi, F. Giannetti, A. Passarelli di Napoli, C. Scheven, *Fractional higher differentiability of solutions to strongly nonlinear Stokes systems*, (2026), submitted.

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The influence of the data on the sign of the solutions to some elliptic and parabolic PDE's

Maria Michaela Porzio
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In this talk we discuss the influence of the data on the sign of the solutions to equations of Leray-Lions type in the elliptic case together with the evolutionary one. Significant cases are the heat equation and the degenerate p-Laplacian equation (slow diffusion).

In the elliptic and in the parabolic cases it is well known that if the data are non negative then the solutions are non-negative too (maximum principle): is this property local in nature? Which is the role of the data on the sign of the solutions? Is it possible to have positive or non-negative solutions even if the data are not assumed to be non negative? In this talk we try to answer to these questions and we give some new results which show which is the impact of the data on the sign of the solutions in all the domain or in a subset with non null measure.

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Advances in Solving Nonlinear Schrödinger Equations with General Potentials

Romildo Lima
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We establish the existence of a positive solution to nonlinear Schrödinger equations

$$-\Delta u + V(x)u = f(u) \text{ in } \mathbb{R}^N, \tag{P_V}$$

for very general potentials V, with positive or zero limit at infinity (allowing convergence from above, below, or oscillations), but imposing no decay rate assumption. Also, the nonlinearities f may satisfy mild hypotheses, including superlinear or asymptotically linear growth at infinity. This is possible due to the application of the classical Monotonicity Trick approach.

KEY WORDS: Schrödinger operator, Variational methods for second-order elliptic equations, Nonlinear elliptic equations, General topics in linear spectral theory for PDE's.

We aim to deal with non-negative definite general Schrödinger operators, hence, for our purposes, we assume the following conditions on the potential $V \in C^1(\mathbb{R}^N)$.

(V_1) $-\infty < V_0 := \min_{x\in\mathbb{R}^N} V(x) < \lim_{|x|\to+\infty} V(x) := V_\infty$, where $V_0, V_\infty \in \mathbb{R}$ and $V_\infty \geq 0$;

(V_2) There exists some $R > 0$ such that $V(x) < V_\infty$ for all $x \in \overline{B}_R$, up to a translation, and

$$\int_{B_R} (V_\infty - V(x))dx + \int_{\{V>V_\infty\}} (V_\infty - V(x))dx > 0.$$

(V_3) $W(x) := \nabla V(x) \cdot x$, for all $x \in \mathbb{R}^N$, verifies $W \in L^{N/2}(\mathbb{R}^N)$ with $\|W\|_{N/2} < 2S$, where S denotes the best constant of the embedding $D^{1,2}(^N\mathbb{R}) \hookrightarrow L^{2^*}(\mathbb{R}^N)$, $2^* = \frac{2N}{N-2}$.

We assume that $f \in C(\mathbb{R}, \mathbb{R})$ is an odd function with $f(0) = 0$, and $f(s) > 0$ for all $s > 0$, and

(f_1) $\lim_{t\to 0} \frac{f(t)}{t} = 0$; or $(f_1)'$ $\lim_{t\to 0} \frac{f(t)}{t^{2^*-1}} = \lim_{t\to+\infty} \frac{f(t)}{t^{2^*-1}} = 0$, and

(f_2) $\lim_{t\to+\infty} \frac{f(t)}{t} = +\infty$ and $\lim_{t\to+\infty} \frac{f(t)}{t^p} = 0$, for some $p \in (1, 2^* - 1)$; or

$(f_2)'$ $\lim_{t\to+\infty} \frac{f(t)}{t} = a > V_\infty \geq 0$.

Theorem 1 (Positive Mass)**.** *Assume $(V_1) - (V_4)$ with $V_\infty > 0$, (f_1), and either (f_2) or $(f_2)'$ hold. Then the problem (P_V) has a positive solution.*

Theorem 2 (Zero Mass)**.** *Assume $(V_1) - (V_3)$ and $(V_4)'$, with $V_\infty = 0$, $(f_1)'$, and either (f_2) or $(f_2)'$ hold. Then the problem (P_V) has a positive solution.*

References

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