BOOK OF ABSTRACTS

SS6 - Arithmetic, Geometry and Combinatorics of Projective Varieties

2nd Joint Meeting Brazil-Italy in Mathematics

September 8, 2026

Messina, Italy

Organizers:

Francesco Polizzi, University of Naples "Federico II"
Giuseppe Borrelli, Fluminense Federal University
Matteo Penegini, University of Pavia

2nd Joint Meeting Brazil-Italy
in Mathematics
Messina, 7–11 September 2026

SS6

The classification of surfaces of general type with $q = p_g = 2$, and new progress for the case $K^2 = 7$

Fabrizio Catanese [1]
Department of Mathematics, University of Bayreuth and Accademia dei Lincei, Roma

Abstract of the talk.

I will report on the classification of minimal surfaces of general type S with the lowest possible invariant $\chi := \chi(S) = 1$ (this means $q = p_g$).

Beauville observed that, if $q = p_g$, then $q = p_g \leq 4$, equality holding iff S is a product of 2 curves of genus 2.

The case $q = p_g = 3$ was settled some 25 years ago due to work of several authors, Ciliberto, Mendes-Lopes and myself, then Pirola and Hacon-Pardini gave the final touch: there are two cases, Theta divisors in an Abelian variety, and some surfaces isogenous to a product.

Still open is the case $q = p_g = 2$, which was considered by several authors, and where the number of known families is much larger. I will first briefly survey the known results.

For $q = p_g = 2$ we have $4 \leq K^2 \leq 9$, and the case $K^2 = 4, 5, 6$ was recently settled by Du, Jiang, Zhang, who confirmed that there were no more families than the existing examples.

For higher K^2 , there are many examples with $K^2 = 8$, and the case $K^2 = 9$ is suspected not to occur (but the published proofs of this assertion are badly wrong).

In the case $K^2 = 7$ there are some families where the Albanese map $\alpha : S \to A := Alb(S)$ has degree $d = 2$, respectively one family of surfaces with $d = 3$, which we call the CFPP surfaces, since some surfaces were discovered as product quotient surfaces by Cancian and Frapporti, and later a larger family was described by Pignatelli and Polizzi.

I will report on joint work in progress with Matteo Penegini, in the case of degree $d = 3$.

Theorem 1. *For $K^2 = 7$ and degree of the Albanese map equal to 3, the moduli space is irreducible of dimension* 3*, and consists of the CFPP surfaces.*

Proof. The proof uses the Chen-Hacon-Chen-Jiang decomposition, the eventual paracanonical map, as done by Du, Jiang, Zhang, the Heisenberg symmetry-Mukai transform arguments of Alessandro and Catanese, and a generalization of the Casnati-Ekedahl theory of Gorenstein coverings.

Our surfaces are represented as quotients $S = S'/G$, where S' is a double covering of a Traynard surface W (a smooth quartic surfaces with two different sets of 16 disjoint lines).

The main argument hinges on the unicity of a $(16_6, 16_6)$ configuration of 16 plane cubic curves $C_i \subset W$, and 16 points, and is based on the fact that a Traynard surface is the image of an Abelian surface via the odd sections of a polarization of type $(2, 6)$. This shows the appearance of the main actors, that is, an unramified triple covering of a genus 2 curve C_2 by a curve C_4 of genus 4, explaining the relation with the PP7 construction.

Some work is needed to dispose of the case of a polarization of type $(1, 4)$. □

References

[1] Roberto Pignatelli, Francesco Polizzi: *A family of surfaces with $p_g = q = 2, K^2 = 7$ and Albanese map of degree 3.* Math. Nachr. 290, No. 16, 2684–2695 (2017).

[2] Jiabin Du, Zhi Jiang, and Guoyun Zhang: *Cohomological rank functions and surfaces of general type with $p_g = q = 2$*, Math. Annalen

[3] Fabrizio Catanese, Matteo Penegini, *The Moduli space of surfaces with $p_g = q = 2$, $K^2 = 7$ and Albanese map of degree* 3., preliminary version.

[1] Aknowledgements...
E–mail: Fabrizio.Catanese@uni-bayreuth.de,Fabrizio.Catanese@lincei.it.

2nd Joint Meeting Brazil-Italy
in Mathematics
Messina, 7–11 September 2026

SS6

On the degree of a modular map

Ciro Ciliberto
Department of Mathematics, University of Rome "Tor Vergata"

Let X be a general cubic hypersurface in $\mathbb{P}^4$. If $x \in X$ is a general point, there are exactly six distinct lines in X passing through x, that lie on the rank 3 quadric cone with vertex x of lines that have intersection multiplicity at least 3 with X in x. So there is a natural rational map $X \dashrightarrow \mathcal{M}_2$. In this talk I compute its degree to be 15120. This is joint work with Sandro Verra.

E–mail: address@server.it.

2nd Joint Meeting Brazil-Italy
in Mathematics
Messina, 7–11 September 2026

SS6

Initial ideal of a general rational or elliptic curve on quadrics

Francesca Cioffi[1]
Department of Matematica e Applicazioni "R. Caccioppoli", University of Naples Federico II

Over an algebraically closed field K of characteristic zero, we investigate the following natural question: *is the generic initial ideal with respect to the degree reverse lexicographic term order of a general non-special curve an almost revlex ideal?*

One motivation for studying this question is its connection with the n-Strong Lefschetz Property, due to the role played by almost revlex ideals in that context.

Another reason is that the generic initial ideal of a general non-special curve is the *double-generic initial ideal* of the irreducible component $\mathcal{Z}^n_{p(t)}$ of the Hilbert scheme $\mathcal{H}\text{ilb}^n_{p(t)}$ containing the general non-special curves with Hilbert polynomial $p(t)$ (see [1]).

We obtain a positive answer to our question for some general arithmetically Cohen–Macaulay non-special curves (see [2, Proposition 2.14]) and for general rational or elliptic curves on quadrics.

Theorem 1. [2, Theorem 5.3] *The initial ideal of a general rational (respectively elliptic) curve $C_d \subset \mathbb{P}^n_K$ of degree d is almost revlex for every d such that $n \leq d < \frac{n^2+3n}{4}$ (respectively $n < d < \frac{n^2+3n+2}{4}$).*

First, we are able to perform a preliminary screening of the strongly stable ideals that could be the initial ideal of a general rational or elliptic curve C_d of a given degree d in $\mathbb{P}^n_K$, based on the general hyperplane section of such a curve (see [2, Propositions 3.1 and 3.2]). This analysis also implies one connection with the Lefschetz Properties.

Corollary 2. [2, Corollary 3.5] *The deficiency module of a general rational or elliptic curve has the maximal rank property.*

Next, focusing on the case of curves on quadrics, we study the pencil of quadrics containing a non-degenerate irreducible curve of degree $d-1$ and prove the existence of a line L that intersects the curve transversely at a unique point. By appropriately choosing two quadrics and applying an interpolation method based on the Möller–Buchberger approach, we show that, given a general rational (respectively, elliptic) curve C_{d-1}, the curve $C_{d-1} \cup L$ has an initial ideal with the expected behavior, at least in the initial degree. Since the curve $C_{d-1} \cup L$ is smoothable in the Hilbert scheme of general rational (respectively, elliptic) curves of degree d, we obtain our main result using both the preliminary screening of possible initial ideals and the notion and properties of the double-generic initial ideal.

The connection with Lefschetz Properties also shows up in a different way.

Corollary 3. [2, Corollary 5.6] *The defining ideal $I(C_d)$ of a general rational (respectively elliptic) curve $C_d \subseteq \mathbb{P}^n_K$ of degree d satisfies the n-Strong Lefschetz Property, for every d such that $n \leq d < \frac{n^2+3n}{4}$ (respectively $n < d < \frac{n^2+3n+2}{4}$).*

This talk is based on a joint work with Davide Franco and Giovanna Ilardi (see [2]).

References

[1] C. Bertone, F. Cioffi, M. Roggero, *Double-generic initial ideal and Hilbert scheme*, Ann. Mat. Pura Appl., 196(1), (2017), 19–41.

[2] F. Cioffi, D. Franco, G. Ilardi, *Initial ideal of a general rational or elliptic curve on quadrics*, Algebras and Representation Theory, vol. 29:1, (2026), https://doi.org/10.1007/s10468-026-10401-9.

[1]The author is a member of GNSAGA (INdAM)
E–mail: cioffifr@unina.it.

2nd Joint Meeting Brazil-Italy
in Mathematics
Messina, 7–11 September 2026

SS6

Complete intersection algebras with binomial Macaulay dual generator and Lefschetz properties

Roberta Di Gennaro
Dipartimento di Matematica e Applicazioni "Renato Caccioppoli",
Università degli Studi di Napoli Federico II

Rosa M. Miró-Roig
Miró-Roig: Department de Mathemàtiques i Informàtica
Universitat de Barcelona

Can we characterize *all* Artinian complete intersections having the Weak/Strong Lefschetz property, in characteristic 0? Over the last several decades, this question has been deeply studied and it is far from being resolved.
In [**?**] we give a positive answer to this question in a specific case. It is known that, via Macaulay-Matlis duality, the study of standard graded Gorenstein algebras A is related to a single homogeneous polynomial in the poynomial ring R, the so-called Macaulay dual generator of A, such that $A = R/Ann_R(F) = \{f \in R \mid f \circ F = 0\}$. We characterize *all* binomials F such that $Ann_R(F)$ is a complete intersection ideal and we prove that all Artinian complete intersection algebras with Macaulay dual generator a binomial satisfy the Strong Lefschetz Property.

References

[1] Di Gennaro R., Miró-Roig R.M., *Complete intersection algebras with binomial Macaulay dual generator*, Mediterr. J. Math. 22, 178 (2025). https://doi.org/10.1007/s00009-025-02946-8

E–mail: digennar@unina.it.

2nd Joint Meeting Brazil-Italy
in Mathematics
Messina, 7–11 September 2026

SS6

Singularities of Calabi–Yau Threefolds and Universal Quilts

Elizabeth Gasparim
Research Group of Geometry and AI
Federal University of Technology - Paraná, Brazil

In recent work we have introduced the mathematical concept of the **Universal Quilt** for a toric Calabi–Yau singularity. The construction of such a quilt benefits from methods of machine learning. I will define the basic underlying notions and provide many examples.

This joint work with C. Simpson, B. Suzuki, and R. Talha.

References

[1] E. Gasparim, C. Simpson, B. Suzuki, R. Talha, *Toric Calabi–Yau Resolutions via Machine Learning and their Universal Quilts*, preprint (2026).

[2] R. Talha, `https://github.com/rashidtalha/polytope-triangulations`.

E–mail: `etgasparim@gmail.com`.

2nd Joint Meeting Brazil-Italy
in Mathematics
Messina, 7–11 September 2026

SS6

Constructing Free Plane Curves by Lines and Osculating Conics

Giovanna Ilardi
Università degli Studi di Napoli "Federico II"

Free and nearly free plane curves occupy a central place in the study of logarithmic vector fields, Jacobian ideals and singularities of plane curves. A natural problem is not only to recognize such curves, but also to construct them in a systematic way. In this talk I will present two geometric constructions, developed in joint works, in which freeness or near freeness is produced by adding components with prescribed high contact to a given curve.

The first construction is based on adding lines. Starting from a reduced plane curve, one chooses lines with special incidence or tangency properties: high order tangent lines at smooth points, lines through singularities, or lines contained in tangent cones. These lines create controlled singularities on the union. The point is that the local contribution of these new singularities can be large, while the global Jacobian syzygies remain sufficiently controlled to force the resulting curve to be free or nearly free. This gives families of examples in which the geometry of contact directly governs the algebraic structure of the Jacobian ideal.

The second construction replaces lines by osculating conics. For a smooth cubic, the relevant conics are hyperosculating conics at sextactic points; for singular cubics, one obtains analogous constructions adapted to the singular geometry. Adding one, two or three such conics to a cubic curve produces explicit free and nearly free curves, with a particularly symmetric picture in the case of the Fermat cubic and its 27 hyperosculating conics. Thus the passage from lines to conics enlarges the construction while preserving the same guiding principle: high contact creates singularities of the right type and size.

The algebraic background is provided by the minimal degree of a Jacobian syzygy, an invariant which measures the first non-trivial relation among the partial derivatives of the defining equation. Addition–deletion results for unions of curves explain how this invariant behaves when a new component is added. In the present talk this will be used as a tool rather than as the main theme: it clarifies why the constructions with special lines and osculating conics lead to freeness or near freeness.

The aim is to show that these constructions are not isolated examples, but part of a coherent method. High-contact geometry supplies the singularity contribution, while Jacobian syzygies provide the global algebraic control. Their interaction gives a flexible and explicit way to build new free and nearly free plane curves.

References

[1] A. Dimca, G. Ilardi, P. Pokora and G. Sticlaru, *Construction of free curves by adding lines to a given curve*, Results Math. 79 (2024), Article 11.

[2] A. Dimca, G. Ilardi, G. Malara and P. Pokora, *Construction of free curves by adding osculating conics to a given cubic curve*, Int. Math. Res. Not. IMRN 2025 (2025), no. 1, rnae273.

[3] A. Dimca, G. Ilardi and G. Sticlaru, *Addition-deletion results for the minimal degree of a Jacobian syzygy of a union of two curves*, J. Algebra 615 (2023), 77–102.

E–mail: giovanna.ilardi@unina.it.

2nd Joint Meeting Brazil-Italy
in Mathematics
Messina, 7–11 September 2026

SS6

Kählerness of moduli spaces of sheaves on K-trivial Kähler surfaces

Arvid Perego
Department of Mathematics, University of Genova

If S is a compact Kähler surface with trivial canonical divisor, moduli spaces of stable coherent sheaves on S (with respect to a Kähler class) with fixed topological invariant are conjectured to be Kähler manifolds. This conjecture is known to hold if S is projective (and ω is generic in the Kähler cone), and in some particular cases if the surface is not projective. In this talk we will present recent progress on this conjecture.

E–mail: arvid.perego@unige.it.

2nd Joint Meeting Brazil-Italy
in Mathematics
Messina, 7–11 September 2026

SS6

Surfaces with Non-Polyhedral Effective Cone

Roberto Pignatelli, Federico Fallucca
Dipartimento di Matematica, Università di Trento

Francesco Polizzi
Dipartimento di Matematica e Applicazioni, Università Federico II di Napoli

The project started by a question of Keum and Lee [5]: do there exist minimal surfaces of general type with geometric genus zero which are not Mori dream spaces? Although recent work of Cascini, Catanese, Chen and Keum gives a positive answer in the setting of fake $\mathbf{Q}$-homology quadrics [2], it remains interesting to find effective criteria for detecting when an algebraic variety is not a Mori dream space, even if its geometric genus do not vanish.

Let $f\colon S \to B$ be a fibration from a smooth projective surface onto a smooth curve. We denote by $n(f)$ the weighted number of reducible fibres of f: a fibre with k distinct irreducible components contributes $k-1$. We proved

Theorem 1. *If the closed effective cone of S is polyhedral, then*

$$\rho(S) = n(f) + 2,$$

where $\rho(S)$ is the Picard number of S.

Since the closed effective cone of a Mori dream space is rational polyhedral, the theorem yields the following criterion: if $\rho(S) > n(f) + 2$, then $\overline{\mathrm{Eff}}(S)$ is not polyhedral; in particular, S is not a Mori dream space.

Applying this criterion we show that several surfaces of general type are not Mori dream surfaces. The list includes

- the Craighero–Gattazzo surface, a simply connected minimal surface of general type with $p_g = 0$ and $K^2 = 1$ [3];
- some Fermat surfaces;
- certain double covers of Hirzebruch surfaces, including some Horikawa surfaces [4];
- some Kodaira double fibred surfaces;
- some surfaces isogenous to a higher product;
- some product-quotient surfaces [1].

Combining the theorem with the Shioda–Tate formula [6], we also obtain:

Corollary 2. *If an elliptic surface with a section has polyhedral closed effective cone, then its Mordell–Weil group has rank zero.*

References

[1] I. Bauer, R. Pignatelli, *Product-quotient surfaces: new invariants and algorithms*, Groups Geom. Dyn. 10 (2016), no. 1, 319–363.

[2] P. Cascini, F. Catanese, Y. Chen, J. H. Keum, *Mori dream spaces and $\mathbf{Q}$-homology quadrics*, Pacific J. Math. 339 (2025), no. 2, 223–242.

[3] P. Craighero, R. Gattazzo, *Quintic surfaces of $\mathbf{P}^3$ having a non-singular model with $q = p_g = 0$, $P_2 \neq 0$*, Rend. Sem. Mat. Univ. Padova 91 (1994), 185–198.

[4] E. Horikawa, *Algebraic surfaces of general type with small c_1^2. I*, Ann. of Math. 104 (1976), 357–387; *II*, Invent. Math. 37 (1976), 121–155.

[5] J. H. Keum, K.-S. Lee, *Examples of Mori dream surfaces of general type with $p_g = 0$*, Adv. Math. 347 (2019), 708–738.

[6] T. Shioda, *On the Mordell–Weil lattices*, Comment. Math. Univ. St. Pauli 39 (1990), no. 2, 211–240.

E–mail: roberto.pignatelli@unitn.it.

2nd Joint Meeting Brazil-Italy
in Mathematics
Messina, 7–11 September 2026

SS6

On the Geometry of Conic Fibrations over Mori Dream Spaces

Eleonora Romano
Department of Mathematics, University of Genova

Within the framework of the Minimal Model Program, contractions of fiber type with one-dimensional fibers play a central role in the classification of algebraic varieties.

In the first part of this talk, we review previous results concerning the structure of conic bundles over complex smooth Fano varieties. In the second part, we transition to recent developments where the total space X is a Mori Dream Space with a smooth characteristic space. Exploiting the global combinatorial description of Cox rings via Variation of GIT quotients, we analyze the local structure of these contractions. We prove that the total space exhibits at most finite abelian quotient singularities along the singular locus, forcing the base Y to inherit exclusively mild quotient singularities. Finally, we provide a structural classification of the target variety based on the relative Picard number, establishing the conditions under which the base factors as a projective bundle.

References

[1] E. A. Romano, *Positivity of anticanonical divisors from the viewpoint of Fano conic bundles*, Osaka Journal of Mathematics, 56 (2019), 65–74.

[2] E. A. Romano, *A note on flatness of some fiber type contractions*, Proceedings of the Japan Academy, Ser. A, 95(9) (2019), 103–106.

[3] E. A. Romano, *Non-elementary Fano conic bundles*, Collectanea Mathematica, 70(1) (2019), 33–50.

[4] E. A. Romano, *Conic fibrations over Mori Dream Spaces*, In preparation.

E–mail: elenoraanna.romano@unige.it.