BOOK OF ABSTRACTS

SS4 - Algebraic Geometry towards Commutative Algebra, Combinatorics and Applications

2nd Joint Meeting Brazil-Italy in Mathematics

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Organizers:

Gioia Failla, University of Reggio Calabria
Elena Guardo, University of Catania
Zaqueu Alves Ramos, Federal University of Sergipe

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SS4

The Waldschmidt Constant for a Special Partial Intersection

Susan Cooper
Department of Mathematics, University of Manitoba

Roberta Di Gennaro
Dipartimento di Matematica e Applicazioni "Renato Caccioppoli",
Università degli Studi di Napoli Federico II

Łucja Farnik
Department of Mathematics, University of the National Education Commission

Elena Guardo
Dipartimento di Matematica e Informatica, Università di Catania, Catania, Italy

Yuriko Pitones
Departamento de Matemáticas, Universidad Autónoma Metropolitana

Elisa Postinghel
Dipartimento di Matematica, Università di Trento

In [2] we make some advances on the Waldschmidt constant

$$\widehat{\alpha}(I_{\mathbb{X}}) = \lim_{t\to\infty} \frac{\alpha({I_{\mathbb{X}}}^{(t)})}{t}$$

of configurations of points $\mathbb{X}$ in $\mathbb{P}^2$ called partial intersections. They are configurations supported on lines but that are not standard k-configurations. Our investigation is carried out via two approaches.

The first approach is algebraic in nature by considering fixed components and dimensions, according to the method described in [3, Section 3] where the authors study the Waldschmidt constant of the ideal of k-configurations and standard k-configurations.

The second approach utilizes the underlying geometry of the points to repeatedly reduce fat point schemes by lines and smooth conics. Precisely we use a generalization of a reduction process introduced in [1] that results in lower and upper bounds to the Hilbert function of the ideal of a fat point scheme.

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E–mail: digennar@unina.it.

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SS4

When Is the General Projection of a Finite Set of Points a Complete Intersection?

Giuseppe Favacchio
Dipartimento di Ingegneria, Università degli Studi di Palermo

A finite set $Z \subset \mathbb{P}^3_K$ is called an (a,b)-*geproci set* if, for a general linear projection $\pi\colon \mathbb{P}^3_K \dashrightarrow \mathbb{P}^2_K$, the image $\pi(Z)$ is a complete intersection of two plane curves of degrees a and b.

The aim of this talk is to discuss geproci sets supported on lines, focusing on two cases: skew lines and concurrent lines. In both cases, the geproci property imposes strong restrictions on the distribution of the points along the supporting lines.

In the skew lines case, the combinatorics of the supporting line configurations in $\mathbb{P}^3$ [1] is encoded by a natural groupoid of projective morphisms between the lines, together with an associated group attached to the configuration. By applying a matrix model for skew lines, these maps are realized in [2] as explicit Möbius transformations, making the connection with finite subgroups of $\mathrm{PGL}_2(K)$ explicit.

The talk will conclude with recent developments and open questions.

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E–mail: giuseppe.favacchio@unipa.it.

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SS4

On nodal deformations of ordinary isolated singularities of surfaces in $\mathbb{P}^3$

Concettina Galati
Department of Mathematics and Computer Science, University of Calabria - Italy

We will mainly report about nodal defomations of surfaces in $\mathbb{P}^3$ with ordinary isolated singularities. The results presented are part of the joint work [C. Ciliberto, C. Galati, *On nodal deformations of singular surfaces in* $\mathbb{P}^3$, arXiv:2602.09177].

E–mail: concettina.galati@unical.it.

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Betti diagrams of edge ideals with a special shape

Sara Asensio , Philippe Gimenez [1]
Mathematics Research Institute (IMUVa), University of Valladolid, Spain
Ignacio García-Marco
Instituto de Matemáticas y Aplicaciones (IMAULL), University of La Laguna, Spain

Edge ideals of graphs were introduced by Villarreal in 1990 [4] and have been extensively studied since then. The first important result relating algebraic properties of edge ideals to the combinatorics of the associated graph is Fröberg's characterization of edge ideals with regularity 2 in [3]. This result was later generalized to regularity 3 bipartite graphs in [2]. A key ingredient in the proofs of these results is the particular shape of the Betti diagrams of the edge ideal of the graph obtained after removing a hamiltonian cycle from a complete graph and a complete bipartite graph.

In this work, we consider generalized Andrásfai graphs, a family of graphs that extends complete and complete bipartite graphs. We show that some results known for complete and complete bipartite graphs naturally extend to this family. More precisely, when removing a suitable hamiltonian cycle, the resulting edge ideal has regularity and projective dimension determined by the size of the graph, and a Betti diagram with a very special shape. This talk is based on the recent preprint [1].

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E–mail: pgimenez@uva.es.

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SS4

Incidence toric ideals and three-point functions

Barbara Betti
Otto von Guericke University Magdeburg

Sean Grate
Iowa State University

Thiago Holleben
Dalhousie University

Flavio Salizzoni
MPI für mathematik in den naturwissenschaften

We study the ideal of the algebraic relations among 3-point functions from a combinatorial and topological perspective. We place this problem in the broader setting of incidence toric ideals associated with incidence matrices of t-subsets contained in k-subsets of n elements. These ideals generalize the well-known class of toric edge ideals of graphs. Generators of these ideals admit combinatorial interpretations as null t-designs and topological interpretations as balanced orientable normal d-pseudomanifolds without boundary. Generators arising from octahedra play a fundamental role in the structure of these ideals.

References

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E–mail: hollebenthiago@dal.ca.

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SS4

Milnor Algebras, Higher Jacobians and Singular Hyperplane Sections

Giovanna Ilardi
Università degli Studi di Napoli "Federico II"

The Milnor algebra of a projective hypersurface is a classical object which reflects the geometry of the hypersurface through its Jacobian ideal. If $X = V(f) \subset \mathbb{P}^n$ is smooth, then $J(f)$ is a complete intersection and $M(f) = S/J(f)$ is an Artinian Gorenstein algebra. For such algebras, the Weak and Strong Lefschetz Properties ask whether multiplication by a general linear form, or by one of its powers, has maximal rank. The Strong Lefschetz Property is known for Fermat-type hypersurfaces and hence for a generic hypersurface, but it remains open for arbitrary smooth hypersurfaces; the Weak Lefschetz Property is better understood in several ranges, although no complete answer is known for Jacobian complete intersections.

I will first recall joint work with A. Dimca and R. Gondim on higher order Jacobian ideals, higher Milnor algebras, mixed Hessians and higher polar maps. These constructions extend the usual Jacobian ideal and the classical Hessian, and provide a geometric framework in which Lefschetz-type questions are related to polar maps, Hessian-type matrices and higher order derivatives of the defining equation.

The main focus of the talk will be joint work with A. Dimca on singular hyperplane sections. Let $V : f = 0$ be a hypersurface with isolated singularities, and let $H : \ell = 0$ be a hyperplane avoiding the singularities of V, but such that the section $V \cap H$, defined by $g = 0$, is singular. Then the injectivity of the multiplication map

$$\times \ell : M(f)_k \longrightarrow M(f)_{k+1}$$

can be controlled by two invariants of the section: the initial degree $r(g)$ of the module of Jacobian syzygies of g, and the initial degree $s(g)$ of the saturation of its Jacobian ideal. These invariants give an effective range in which multiplication by ℓ is injective. Thus the singularities of the section, together with their syzygies and saturated Jacobian ideal, become concrete tools for detecting Lefschetz-type behavior in $M(f)$.

A parallel statement holds for the Jacobian module $N(f) = I(f)/J(f)$, which is a finite-dimensional substitute for the Milnor algebra in the singular case. Using the duality of $N(f)$, injectivity results obtained from the section also yield corresponding surjectivity statements. The method applies to surfaces with nodal, free or nearly free plane sections, and to higher-dimensional hypersurfaces with special nodal sections, such as Kummer and Chebyshev examples. The guiding idea is that singular hyperplane sections are not merely degenerations: they translate geometric information into explicit Lefschetz-type results for Milnor algebras and Jacobian modules.

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E–mail: giovanna.ilardi@unina.it.

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Characterization of Gorenstein Special Fiber Rings of Ladder Determinantal Modules

Maral Mostafazadehfard
Federal University of Rio de janeiro

We characterize the Gorenstein special fiber rings of ladder determinantal modules, which are defined as arbitrary direct sums of ideals of maximal minors of a generic ladder matrix. Specifically, we establish necessary and sufficient conditions for these special fiber rings to be Gorenstein, phrased entirely in terms of the combinatorial structure of the underlying matrix. Our proofs utilize the combinatorial properties of the join-irreducible elements in the lattices of corresponding Hibi rings.

This is joint work with Louiza Fouli, Kuei-Nuan Lin, and Haydee Lindo.

E–mail: maral@im.ufrj.br.

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Minimal-Degree Foliations on Cominuscule Grassmannians

Vladimiro Benedetti
Université Côte d'Azur, France

Crislaine Kuster
Yau Mathematical Sciences Center, China

Alan Muniz
Universidade Federal de Pernambuco, Brasil

Given X a cominuscule Grassmannian (or irreducible Hermitian symmetric space) and an integer p, we compute the minimum $l(p)$ such that $H^0(\Omega^p_X(l(p))) \neq 0$. This allows us to conclude that any codimension-one foliation of degree zero on a cominuscule Grassmannian is a pencil of hyperplanes, improving a result of the first and third authors with D. Faenzi. We also deduce the structure of codimension-one foliations of degree one. Finally, we provide families of examples of high codimensional foliations of minimal degree on classical Grassmannians, Lagrangian Grassmannians, Spinor varieties, and the Cayley plane.

E–mail: alan.nmuniz@ufpe.br.

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Persistent symmetric tensors and their Hessian

Giorgio Ottaviani [1]
Department of Mathematics and Computer Science Ulisse Dini, University of Florence

Persistent tensors were introduced by Gharahi and Lysikov[3] via a recursive construction inspired by quantum information theory. In a nutshell, they constitute a class of tensors that remain well-behaved under the substitution method and, consequently, admit nontrivial lower bounds on tensor rank. In a work joint with Gharahi[4], we investigate the symmetric case—namely, symmetric persistent tensors, or equivalently, persistent polynomials. We establish that a symmetric tensor is persistent if the determinant of its Hessian equals the power of a nonzero linear form. The converse is verified for cubic tensors and in small dimension, by leveraging classical results of C. Mammana[5] and B. Segre[6]. We discuss connections with invariants of some prehomogeneous spaces[1] and homaloidal polynomials[2].

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E–mail: `giorgio.ottaviani@unifi.it`.

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Twenty years of homaloidal and vanishing hessian (cubic) polynomials between Pernambuco and Sicily

Francesco Russo [1]
Department of Mathematics and Computer Science, University of Catania, Catania (ITALY)

After recalling the notions of homogenous polynomial with vanishing hessian, studied firstly by Hesse, Gordan and M. Nöther, and of homaloidal polynomial (studied in Algebraic Geometry and in Theoretical Physics, at least), we shall survey joint work between Recife and Catania in the last twenty years, starting with the seminal work with Ciro Ciliberto and Aron Simis and the Ph.D. Thesis of Rodrigo Gondim and then summarizing subsequent results obtained with Gondim and Staglianò and very recent new contributions with Alzati and Gondim. Due to the lack of time (one minute for each year!) we shall only briefly mention the strong impact of these circles of ideas on the very active area of research of Lefschetz properties of Artinian Gorenstein Algebras and also some applications to the integrability of the equations of Black Holes model found with Marrani and Van Geemen.

[1]Aknowledgements...
E–mail: francesco.russo@unict.it.

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The Rose-Terao/Yuzvinsky theorem revisited and extended

Ricardo Burity
Department of Mathematics, Federal University of Paraíba

Zaqueu Ramos
Department of Mathematics, Federal University of Sergipe

Aron Simis [1]
Department of Mathematics, Federal University of Pernambuco

Stefan Tohaneanu
Department of Mathematics, University of Idaho

Yuzvinsky and Rose-Terao have shown that the homological dimension of the Jacobian ideal of the defining polynomial of a generic hyperplane arrangement is maximum possible. The present work provides yet another proof of this result, which differs from the one given earlier by Burity-Simis-Tohaneanu. Another main drive of the paper concerns a version of the above result in the case of a product of general forms of arbitrary degrees ≥ 2 (in particular, transverse ones). Finally, some cases of non general forms are also contemplated.

[1]To the memory of Gaetana Restuccia
E–mail: aron.simis@ufpe.br .

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Null Hessian Taylor Hypersurfaces

Paola Supino
Dipartimento di Matematica e Fisica, Università Roma Tre

Hessians of homogeneous multivariate polynomials are algebraic objects interesting for their connection to Lefschetz properties of graded Artinian Gorenstein algebras, while hypersurfaces defined by polynomials with null hessian are geometrically interesting objects. Taylor varieties $\mathcal{T}^n_{d,e,m}$ are subvarieties of the projective space of polynomials in n variables with fixed degree m, defined from Taylor expansion of rational functions. Among them, we look for non-defective hypersurfaces. We contribute to the frame presenting the cases where $n = 2$ and $m = d + 2$, which give new examples of hypersurfaces with identically null Hessian.

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E–mail: psupino@uniroma3.it.