BOOK OF ABSTRACTS

SS38 - Mathematics Education

2nd Joint Meeting Brazil-Italy in Mathematics

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Organizers:

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SS38

Shared drawings in a mathematical modelling activity: an exploratory study

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The seminar presents the results of a study investigating the role of drawings in supporting students' mathematical modelling (MM) processes, with the aim of identifying ways to support them in carrying out complex MM activities [1]. Existing literature on the use of drawings in MM is limited and mainly focuses on completed drawings and the relationship between drawing accuracy and MM performance, showing that accurate mathematical drawings can positively affect students' MM outcomes (see e.g. [2]). However, these findings cannot easily be transferred to complex MM problems that students work on for days. The study is an exploratory case of 11th-grade students engaged in a realistic rugby scenario. The following research questions were investigated: do drawings help students in going throughout the modelling cycle when dealing with complex MM activities? If so, in which steps and how? To answer such questions, the study networked two theoretical frameworks: the semiotic bundle framework [3] and the MM cycle [4], enriched through the MM sub-competencies proposed by [5]. The semiotic bundle was employed as an analytical tool to study the interplay among words, gestures, and inscriptions, while the MM cycle served as a conceptual tool to characterize students' MM activities. The data consist of a video-recorded classroom discussion, which was transcribed including speech, gestures, and inscriptions, and divided into small analytical units. Synchronic and diachronic analyses were conducted, while qualitative content analysis with deductive categories was employed to code the units according to the steps of the MM cycle. In addition, drawing activities were analyzed by distinguishing between solution-relevant and non-solution-relevant elements. To summarize the interaction between MM activities and drawings, a new representation tool called Modelling and Drawing Activity Diagram was developed. The findings show that drawings accompany much of the MM process and play a central role in the analyzed complex MM activity. In this respect, drawings become effective when they are shared among students as part of the semiotic bundle. More precisely, students support the MM process by connecting drawing elements through different networking strategies, such as making understandable, contrasting, coordinating, and integrating. Integrating existing drawings appears to foster students' understanding, structuring, and mathematization processes, while contrasting and coordinating solution-relevant and non-solution-relevant drawing elements activate important validation processes. The networking strategies previously described appear to play a prominent role in the process of building accurate drawings. Moreover, the analysis shows that non solution-relevant elements are particularly important for developing accurate drawings when these networking strategies are put into play.

References

[1] C. Bassi, D. Brunetto *Shared drawings in a mathematical modelling activity: An exploratory study*, Journal of Mathematical Behavior, 78 (2025), 101234.

[2] J. Rellensmann, S. Schukajlow, J. Blomberg, C. Leopold *Effects of drawing instructions and strategic knowledge on mathematical modelling performance: Mediated by the use of the drawing strategy*, Applied Cognitive Psychology, 36 (2022), 402–417.

[3] F. Arzarello *Semiosis as a Multimodal Process*, Revista Latinoamericana de Investigación en Matemática Educativa, Num Esp (2006), 267–299.

[4] W. Blum, R. Borromeo-Ferri *Mathematical modelling: can it be taught and learned?*, Journal of Mathematical Modelling and Applications, 1 (2009), 45–58.

[5] K. Maas *What are modelling competencies?*, ZDM, 38(2) (2006), 113–142.

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2nd Joint Meeting Brazil-Italy
in Mathematics
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SS38

Education, ideology, and the limits of mathematization

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The problematic phenomena connected to the ideology of certainty have long been discussed by mathematics education research, usually in connection with the use and abuse of mathematics in the public discourse. I will argue that these are not solely local didactic phenomena but may also be framed as tied to the general form in which mathematical knowledge is assumed to be transposed in the schooling system. I will then present proposals of activities which have been designed by taking into account the ideology of certainty and finally discuss a minimal didactic framework for possible improvement.

References

[1] Beccuti, F. (2025). Ideology and mathematics education: A rebound. *For the Learning of Mathematics, 45*(1), 14-19.

[2] Beccuti, F. & Robutti, O. (2022) Teaching mathematics in today's society: didactic paradigms, narratives and citizenship. *For the Learning of Mathematics 42*(2), 29-34.

[3] Borba, M. & Skovsmose, O. (1994) The ideology of certainty in mathematics education. *For the Learning of Mathematics, 17*(3), 17-23.

[4] Ernest, P. (2016) The problem of certainty in mathematics. *Educational Studies in Mathematics, 92*(3), 379-393.

[5] Lundin, S. (2012) Hating school, loving mathematics: on the ideological function of critique and reform in mathematics education. *Educational Studies in Mathematics, 80*(1-2), 73-85.

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2nd Joint Meeting Brazil-Italy
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Messina, 7–11 September 2026

SS38

Festival of Digital Videos and Mathematics Education

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This presentation is grounded in the experience of the Festival of Digital Videos and Mathematics Education, now in its tenth edition. Over the past eleven years — including the pilot phase — the project has aimed to build curriculum from the bottom up. Students and teachers produce videos for the festival, generating over a thousand productions that now form a robust "video library." The festival features multiple categories, with plural narratives situated in multimodal contexts. Originally developed as a research initiative, the project responded to the limited use of digital technologies in Brazilian distance education ten years ago. Our research group, GPIMEM, designed the festival to accelerate classroom transformation in the face of emerging social and technological shifts. What began as research has evolved into an outreach program that integrates teaching, learning, and ongoing investigations. This outreach program has helped to transform our changing classroom and has generated new research I will present theoretical frameworks such as activity theory, social semiotics and critical mathematics education, alongside the construct of humans-with-media, to discuss how video production becomes a space of co-authoring mathematical knowledge. Our group has supported over more than ten years that knowledge is produced by collective of humans and non-humans. Videos are not merely tools but discursive artifacts with agency, shaping how mathematics is experienced, expressed, and transformed by teacher-student collectives. Among the productions, Mar de Lama (Sea of Mud) exemplifies a critical socioecological discourse, it is an example of critical mathematics education. It resignifies the Brumadinho disaster through mathematical modeling that mobilizes simulations, topographic maps, and volume calculations. Students' authorship transforms the classroom into a space of ethical, mathematical, and environmental critique and engagement. I will also explores the newly created "Indigenous and Traditional Peoples" category, which enhances the dialogue with ethnomathematics and intercultural perspectives. These productions reveal mathematics emerging from territorial, collective, and everyday practices, opening the curriculum to diverse epistemologies.

Through the analysis of selected videos and discussion of methodological approaches, this minicourse invites participants to reflect on digital video production as a pedagogical strategy that integrates culture, criticality, and mathematics education. The goal is to offer both theoretical insights and practical tools for those interested in constructing dialogical and transformative curricula. Anybody interested in doing research on in one of these areas? Anybody interested in developing an international festival of indigenous people?

References

[1] Borba, M.d.C., Souto, D.L.P., Shumway, J. et al. Festival of videos: curricula developed by students and teachers. ZDM Mathematics Education 57, 1315–1328 (2025). https://doi.org/10.1007/s11858-025-01710-2

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2nd Joint Meeting Brazil-Italy
in Mathematics
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SS38

Rethinking Instrumental Genesis in Human-GenAI Interaction: Noise and Drift factors in Mathematical Problem Solving

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Recent advances in Generative Artificial Intelligence (GenAI) have opened new possibilities for mathematics education, while also raising theoretical and methodological challenges for the research community [7]. As the research community experiments with new educational settings involving GenAI tools in teaching-learning processes, the complexity of this field is growing, together with the need to revisit established theoretical and methodological perspectives [4]. In this talk, we focus on a specific class of educational situations: the use of GenAI tools to scaffold mathematical problem solving and self-assessment. Within these scenarios, GenAI may act as a tutor supporting learners' cognitive and metacognitive processes. We address these issues from the perspective of the instrumental approach to mathematics education [5], which offers powerful tools for analysing the relationships between users and artefacts. However, we argue that the specific features of human-GenAI interaction call for a reconceptualisation of some core aspects of instrumental genesis. This argument, which stems from a three-year research programme, is articulated around two main point. The first concerns the unique combination, in GenAI artefacts, of rapidly changing capabilities and persistent epistemic opacity [6]. The second concerns the complex phenomenology emerging from users' interactions with GenAI, in which cognitive, metacognitive and affective factors intertwine and shape both users' activity and their awareness of it [1,3]. Following the perspective developed [2], we introduce the notions of noise and drift as conceptual tools to characterise, metaphorically, theoretical and empirical features of human-GenAI interaction. We discuss how these notions may help interpret and make visible specific dynamics that characterise human-GenAI interactions in mathematical problem solving.

References

[1] Contel, F., Cusi, A. (2025). Investigating the Role of ChatGPT in Supporting Metacognitive Processes During Problem-Solving Activities. Digital Experiences in Mathematics Education, 11, 167–191.

[2] Contel, F., Cusi, A. (in press). Noisy trajectories in human-GenAI interaction: an integrative literature review based on a reconceptualisation of the instrumental genesis framework. In: Engelbrecht, J. & Pepin, B. (Eds.), Generative Artificial Intelligence in the Changing Mathematics Classroom. Springer.

[3] Cusi, A., Contel, F. (2026). A reconceptualisation of the instrumental genesis process in the context of human–LLM interactions: Exploring students' perspective on their use of GPT-4o as a tool to support self-assessment. In E. Geraniou, C. Crisan & M. Mavrikis (Eds.), Digital Technology and Artificial Intelligence in Mathematics Education Assessment (pp. 129–150). Routledge.

[4] Pepin, B., Buchholtz, N., & Salinas-Hernández, U. (2025). A scoping survey of ChatGPT in mathematics education. Digital Experiences in Mathematics Education, 11(1), 9-41.

[5] Rabardel, P. (2002) People and technology: a cognitive approach to contemporary instruments. Paris 8 University.

[6] Van Vaerenbergh, S., & Pérez-Suay, A. (2022). A classification of artificial intelligence systems for mathematics education. In Mathematics education in the age of artificial intelligence: How artificial intelligence can serve mathematical human learning (pp. 89-106). Springer.

[7] Walkington, C. (2025). The implications of generative artificial intelligence for mathematics education. School Science and Mathematics, 1–10.

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2nd Joint Meeting Brazil-Italy
in Mathematics
Messina, 7–11 September 2026

SS38

"Cultures Count": Cultural Contaminations in and for Mathematics Teacher Education

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1. Background and theoretical framework

The phenomenon of multiculturalism in education has been studied for several years, yet few works address subject-specific teaching in non-monocultural contexts, and even fewer concern mathematics. Cultural distance perceived by students unfolds along three main dimensions: epistemology of the discipline, language, and teachers' expectations [1, 2]. Two theoretical frameworks orient our reflection. Ethnomathematics [3] interprets mathematics as a set of practices developed by communities over time, to be integrated into teaching to strengthen students' identities and foster cultural connections. Cultural transposition [4] offers tools for the re-elaboration of knowledge developed in educational contexts other than those of the dominant culture. At the heart of both frameworks lie *decentering*—an epistemic condition allowing teachers to distance themselves from their own practices—and *cultural contamination*, through which new, broader frames of reference emerge.

2. A professional development need and a teacher training course

Data from TALIS 2018 [5] highlight a clear professional development need: only 26.3% of Italian lower secondary teachers report that teaching in multicultural contexts was included in their formal training, against an international average of 38.5%, and only 18.7% consider themselves well prepared, compared to 31.4% internationally. Italy ranks third after Mexico and Brazil—a finding that makes the Italy–Brazil dialogue particularly significant. In response, our research group at the University of Palermo launched *Cultures Count*, involving more than 500 in-service mathematics teachers across Italy in 25 hours of online training, combining theoretical sessions with practical workshop activities grouped by school level [6].

3. Preliminary findings and conclusions

At the outset of the course, teachers were asked: "Is mathematics universal?" Among 114 responses, nearly half held an absolutist view describing mathematics as a universal language transcending culture, and an almost equal share adopted a culturally mediated universalist position, acknowledging that methods and practices are culturally situated [3, 7]. Only around 4% took a fully relativist stance. Although motivated, only a small fraction demonstrated genuine awareness of the cultural complexity of mathematics—a necessary prerequisite for intercultural sensitivity and truly inclusive teaching [4, 8].

Today's multicultural school scenarios demand sustained attention from teachers and researchers. Starting from ethnomathematics [3] and cultural transposition [4]—two frameworks centred on culture as an opportunity for inclusion—our proposal aims to enable teachers to decenter and engage in genuine dialogue with other didactic-cultural systems, toward a polycentric vision of mathematics [9] no longer seen as a mere product of Western civilization. Half of the participating teachers still view mathematics as culture-independent: this confirms the urgent need to encounter "other" practices and move toward classroom action—recognising students' mathematical practices from other contexts, where *recognising* means both identifying something already known and, above all, *acknowledging* its value. Therefore, a relativistic stance free from hierarchical value judgments becomes a necessary condition for genuine inclusion toward an equity-oriented view in which difference is the starting point, not an obstacle.

References

[1] A. Alam and A. Mohanty, Cultural beliefs and equity in educational institutions, *International Journal of Adolescence and Youth*, **28**(1) (2023), doi:10.1080/02673843.2023.2270662.

[2] I. Kohanová, M. Slavíčková, S. Rosa, B. Di Paola, J. Michal and E. Çakĭroğlu, Exploring teachers' resource utilization practices and beliefs in mathematics education, *International Journal of Science and Mathematics Education*, **23**(8) (2025), 3545–3575, doi:10.1007/s10763-025-10577-4.

[3] U. D'Ambrosio, *Ethnomathematics: Link between traditions and modernity*, Brill, 2006.

[4] M. Mellone, A. Ramploud, B. Di Paola and F. Martignone, Cultural transposition: Italian didactic experiences inspired by Chinese and Russian perspectives on whole number arithmetic, *ZDM Mathematics Education*, **51** (2019), 199–212, doi:10.1007/s11858-018-0992-7.

[5] OECD, *TALIS 2018 Results (Volume I): Teachers and School Leaders as Lifelong Learners*, OECD Publishing, Paris, 2019.

[6] G. Bianco and B. Di Paola, Lesson plan design for teaching mathematics in multicultural classrooms, *Frontiers in Education* (2025), doi:10.3389/feduc.2025.1441957.

[7] H. Xu and R. Ball, Multiple forms of knowing in mathematics: a scoping literature study, *arXiv preprint arXiv:2406.16921*, 2024, doi:10.48550/arxiv.2406.16921.

[8] G. Bianco and B. Di Paola, La diversità culturale e linguistica degli studenti: risorsa od ostacolo nella didattica della matematica?, in *Insegnare matematica in carcere: Una sfida complessa, un'opportunità di crescita*, Edizioni ETS, 2025, pp. 60–68.

[9] A. Bishop, *Mathematical enculturation: A cultural perspective on mathematics education*, Springer Science & Business Media, 1991.

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2nd Joint Meeting Brazil-Italy
in Mathematics
Messina, 7–11 September 2026

SS38

Mathematics education in the teaching of mathematics at university

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The result may be a [pharmaceutic] product free of ties, but built upon knowledge that only existed because those ties existed.[2]

Many teachers and researchers are asking themselves what should be taught to students from now on, in today's data-rich world. Initial studies have indicated that IA generative does not (yet?) substitute creativity and analytical thinking, revealing these as human singularities. We argue that the classical tensions between visualisation and formalisation explored in [1], which are at the core of the research on students' understanding of real analysis ([2], [3]), and the algebraic versus analytical tensions, revealed in the research project on error's analysis of students' responses to calculus exams, open pathways for teaching. Firstly, a case study in [2] on the dialogue between visualisation and formal mathematical knowledge and the potential use of tools to make sense of mathematics are re-analysed from a semiotic-cognitive perspective ([6], [4], [5], [7], [8]). Results reveals that the (eventually visual) resources can be used to produce knowledge (including formal knowledge) through acts of creation rather than comprehension ([8]). Further joint research with Roberto Capone intends to use the artificial intelligence to re-analyse those mathematical resources. Second, the analytical versus algebraic tension materialised in students' responses to a question on continuity in a Calculus exam, indicate that the notion of limits may not have been mobilised by students as an analytical tool to re-signify the geometrical continuum in numerical terms. In this case, the notion of limits is kept as an algebraic object. The historical origins for its use in teaching of definitions such as continuity, introducing the algebraic (rather than formal) version of limits as an analytical tool, may reveal the contextual ties of limits' role in this case. Calculus tools are potentially analytical; and if they are used as such in classroom practices, they may promote analytical thinking.

References

[1] D. Bressoud et al., *Teaching and Learning of Calculus.* Springer, 2016.

[2] M.M.F. Pinto, *Students' understanding of real analysis.* Coventry, UK: University of Warwick, 1998.

[3] M. Pinto, D. Tall, Building formal mathematics on visual imagery: A case study and a theory. *For the Learning of Mathematics*, 22(1), 2–10, 2002.

[4] T. Scheiner, M.M.F. Pinto, Cognitive processes underlying mathematical concept construction: The missing process of structural abstraction. In C. Nicol et al. (Eds.). Proc. of PME 38 and PME-NA 36 (5), 105–112. Vancouver, Canada, 2014.

[5] T. Scheiner, M. Pinto, Images of abstraction in mathematics education: contradictions, controversies, and convergences. In C. Csíkos et al. (Eds.). Proc. PME 40, (4) 155–162. Szeged, Hungary, 2016.

[6] T. Scheiner, New light on old horizon: Constructing mathematical concepts, underlying abstraction processes and sense making strategies. *Educational Studies in Mathematics*, 91(2), 165–183, 2017.

[7] T. Scheiner, M. Pinto, Emerging perspectives in mathematical cognition: Contextualizing, complementizing, and complexifying. *Educational Studies in Mathematics*, 101(3), 357–372, 2019.

[8] T. Scheiner, M. Pinto, M. Sense-making in mathematics: an act of comprehension or creation? *For the Learning of Mathematics*, 42(3), 16–20, 2022.

[1]Acknowledgements: TNE project: DeSK.

[2]Fusaro Faioli, Alice (2026) Interview: Psicodélicos, patentes e colonialismo científico. In Minuano, Carlos, Revista digital independente de jornalismo psicodélico. Carta Capital, 25/05/2026.

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in Mathematics
Messina, 7–11 September 2026

SS38

Interpretative Knowledge and Ethical Awareness in Mathematics Teacher Education: Insights from an International Research Collaboration

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Over the last decade, a research collaboration involving scholars from Italy, Brazil, and Norway has led to the development of the construct of *Interpretative Knowledge* (IK) (e.g. Mellone et al. 2020), conceived as the specialised knowledge that enables mathematics teachers to interpret students' errors, ambiguities and non-standard reasoning as opportunities for learning rather than as deficiencies to be corrected. Initially developed through the analysis of teachers' interpretations of students' mathematical productions, IK has progressively been connected to the *Mathematics Teacher's Specialised Knowledge* (MTSK) model (e.g. Climent, Contreras, Montes and Ribeiro, 2024), particularly to domains concerning the structure of mathematics, mathematical practices, and teachers' beliefs.

In this presentation, we reconstruct the evolution of the IK construct and discuss recent developments that connect it to the ethical turn in mathematics education. Building on a communitarian ethics perspective, we argue that *interpretative tasks* (tasks for teacher education) can create learning environments in which prospective teachers simultaneously develop specialised mathematical knowledge and ethical awareness, fostering attitudes of care, inclusion, responsibility and respect toward students' mathematical thinking. We illustrate how interpretative tasks support teachers in moving from evaluative interpretations of students' productions toward forms of interpretation grounded in inquiry and understanding (e.g. Mellone, Jakobsen, Ribeiro and Parlati, 2023). Particular attention will be devoted to examples of mathematical tasks designed to develop specialised knowledge related to numerical representations and students' unconventional strategies. These examples will show how work on mathematical content can be intertwined with reflection on the ethical dimensions of teaching and learning mathematics.

The presentation aims to contribute to current discussions on mathematics teacher education by proposing IK as a productive bridge between specialised teacher knowledge and the ethical challenges of contemporary mathematics education.

References

[1] Climent, N., Contreras, L.C., Montes, M. and Ribeiro, M. (2024). The MTSK model as a tool for designing tasks for teacher education. *ZDM Mathematics Education*, 56, 1123–1135. https://doi.org/10.1007/s11858-024-01605-8

[2] Mellone, M., Jakobsen, A., Ribeiro, M., & Parlati, A. (2023). Ethical dimension in the use of interpretative tasks in mathematics teacher education: fraction division. In *Proceedings of the Thirteenth Congress of the European Society for Research in Mathematics Education (CERME13)*, Budapest, Hungary, 10–14 July 2023.

[3] Mellone, M., Ribeiro, M., Jakobsen, A., Carotenuto, G., Romano, P., & Pacelli, T. (2020). Mathematics teachers' interpretative knowledge of students' errors and non-standard reasoning. *Research in Mathematics Education*, 22(2), 154–167.

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