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BOOK OF ABSTRACTS

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Local Regularity Estimates for Degenerate Fully Nonlinear Equations

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In this talk, I will discuss optimal Hölder regularity estimates for the gradients of solutions to a class of degenerate fully nonlinear elliptic equations. The equations under consideration lose ellipticity when the gradient vanishes and involve nonlinear first-order Hamiltonian terms, creating substantial analytical difficulties. To address these challenges, I will present a unified approach combining perturbation techniques with a maximum principle argument. This framework enables us to overcome the effects of degeneracy and nonlinear gradient dependence, leading to optimal interior Hölder regularity estimates for the gradient.

References

- [1] P. Andrade, T. Nascimento. *Optimal regularity for degenerate elliptic equations with Hamiltonian terms*, arXiv:2508.03924

On the Dirichlet problem for a critical Kirchhoff equation in dimension four

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Let $\Omega \subset \mathbb{R}^4$ be a bounded smooth domain. We consider the following Dirichlet problem associated to a nonlinear critical Kirchhoff equation

$$\begin{cases} -\left(a + b \int_{\Omega} |\nabla u|^2 dx\right) \Delta u = u^3 + \lambda u^{p-1}, & u > 0, & \text{in } \Omega, \\ u = 0 & & \text{on } \partial\Omega \end{cases}$$

where $a, b, \lambda > 0$ and $p \in (1, 4)$. A particular feature of this problem is that the critical exponent for the embedding $W_0^{1,2}(\Omega) \hookrightarrow L^m(\Omega)$ and the exponent of the Kirchhoff term in the corresponding energy functional are both equal to 4. In this framework, it is not clear, in some cases, how to prove the boundedness of both Palais-Smale sequences and minimizing sequences. Thus, standard variational methods do not seem directly applicable to establish the existence of solutions. To overcome these difficulties, we present an approach based on an approximation process that, in addition, allows us to obtain precise constraints on the parameters b, λ ensuring the existence of solutions, both in the *concave* case $p \in (1, 2)$ and in the *convex* case $p \in (2, 4)$, improving previous results where such constraints were not made explicit.

References

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Lipschitz regularity of vectorial almost-minimizers for Alt-Caffarelli functionals with non-standard growth

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For a fixed constant $\lambda > 0$ and a bounded Lipschitz domain $\Omega \subset \mathbb{R}^n$ with $n \geq 2$, we establish that almost-minimizers (functions satisfying a sort of variational inequality) of the Alt-Caffarelli type functional

$$\mathcal{J}_G(\mathbf{v}; \Omega) := \int_{\Omega} \left(\sum_{i=1}^m G(|\nabla v_i(x)|) + \lambda \chi_{\{|\mathbf{v}| > 0\}}(x) \right) dx,$$

where $\mathbf{v} = (v_1, \dots, v_m)$ and $m \in \mathbb{N}$, exhibit optimal Lipschitz continuity on compact subsets of Ω , where G is a Young function satisfying specific growth conditions. Furthermore, we obtain universal gradient estimates for non-negative almost-minimizers in the interior of non-coincidence sets.

Our work extends the recent regularity results for weakly coupled vectorial almost-minimizers for the p -Laplacian addressed in Bayrami *et al* [1], and even the scalar case treated in da Silva *et al* [2], Dipierro *et al* [3], and Pellegrino-Teixeira [4], thereby providing new insights and approaches applicable to a variety of non-linear one or two-phase free boundary problems with non-standard growth.

References

- [1] M. Bayrami, M. Fotouhi, and H. Shahgholian, *Lipschitz regularity of a weakly coupled vectorial almost-minimizers for the p -Laplacian*, J. Differential Equations, 412 (2024), 447-473.
- [2] J.V. da Silva, A. Silva and H. Vivas, *Lipschitz regularity of almost minimizers in a Bernoulli problem with non-standard growth*. Discrete Contin. Dyn. Syst. 44 (2024), no. 6, 1555-1586.
- [3] S. Dipierro, F. Ferrari, N. Forcillo, and E. Valdinoci, *Lipschitz regularity of almost minimizers in one-phase problems driven by the p -Laplace operator*. Indiana Univ. Math. J. 73 (2024), no. 3, 813-854.
- [4] D. M. Pellegrino and E.V. Teixeira, *Regularity for quasi-minima of the Alt-Caffarelli functional*. Calc. Var. Partial Differential Equations 63 (2024), no. 6, Paper No. 149, 12 pp.

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On a critical Kirchhoff type problem in high dimension

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Nonlocal boundary value problems of the type

$$\begin{cases} -\left(a + b \int_{\Omega} |\nabla u|^2 dx\right) \Delta u = f(x, u), & \text{in } \Omega \\ u = 0, & \text{on } \partial\Omega \end{cases}$$

are related to the stationary version of the Kirchhoff equation

$$\frac{\partial^2 u}{\partial t^2} - \left(a + b \int_{\Omega} |\nabla u|^2 dx\right) \Delta u = f(t, x, u),$$

first proposed by Kirchhoff in 1883 to describe the transversal oscillations of a stretched string.

We consider the case when Ω is a bounded domain of $\mathbb{R}^N$ with $N > 4$ and the nonlinearity f is of the type $f(x, u) = |u|^{2^*-2}u + \lambda g(x, u)$, being 2^* the Sobolev critical exponent, g a subcritical perturbation, λ a positive parameter.

In this talk, we will show that the interaction between the Kirchhoff operator and the critical term leads to some variational properties of the energy functional as the sequential weak lower semicontinuity and the Palais–Smale condition, provided a and b satisfy a suitable constraint.

Then, through a careful analysis of the fiber maps associated to the energy functional, we will deduce the existence, non existence and multiplicity of solutions of our problem when the parameters a, b, λ vary in appropriate intervals. When the nonlinearity g is a pure power term, i.e. $g(x, u) = |u|^{p-2}u$ for some $p \in (2, 2^*)$, through a detailed study of the Nehari sets associated to the problem, we will show the existence of two critical hyperbolas on the plane (a, b) that separates the plane into regions where the energy functional exhibits distinct topological properties.

Based on the papers [1]-[3].

References

- [1] F. Faraci, Cs. Farkas, *On a critical Kirchhoff-type problem*, Nonlinear Anal., 192 (2020).
- [2] F. Faraci, K. Silva, *On the Brezis-Nirenberg problem for a Kirchhoff type equation in high dimension*, Calc. Var. Partial Differential Equations, 60 (2021).
- [3] G.N. Cunha, F. Faraci, K. Silva, *On a Perturbed Critical p -Kirchhoff-Type Problem*, submitted.

Mixed effects of fast diffusion and refuge in the logistic equations

Carlos Alberto Santos

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In this talk, let us present some results about the behaviour of the positive solution of a logistic equation with respect to a space dependent diffusion rate. The equation also includes a refuge, a zone where the species grows freely. In contrast to the case of homogeneous diffusion coefficient, where the species dies for large diffusion regardless of the birth rate, we show that the species may die, persist or grow indefinitely, depending on the size of the birth rate, for a large increasing of this diffusion rate in a certain region of the space, even more, this growth causes blow up in this region as well as in the refuge. These results were obtained in collaboration with Vinicius Ramos (Universidade de São Paulo) and Antonio Suárez (Universidad de Sevilla).

Ordered solutions for degenerate Kirchhoff problems

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We study a parametrized Kirchhoff type equation with two degeneracy points. The existence of two $H_0^1(\Omega)$ -norm ordered solutions is established for small value of the parameter via a careful analysis of the fiber maps associated to the energy functional. As a consequence we show existence of multiple or even infinitely many solutions to degenerate Kirchhoff equations. Due to the simplicity of the conditions, many applications are given.

References

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Invariant cones for cooperative systems of fully coupled elliptic equations

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Many natural and artificial multi-component systems are modeled by coupled elliptic partial differential equations (pdes).

In this talk we present some recent results for cooperative systems of elliptic pdes, where any component takes advantage from the other ones. In the case of weakly coupled systems, where no coupling concerns first and second order terms, a maximum principle holds for subsolutions, as in the case of separate equations: this corresponds to the invariance of the non-positive orthant, a cumulative sign propagation property from the boundary to the interior of domains.

If higher order couplings occur, in general this kind of maximum principle fails, but we obtain different invariant cones in the sense of Weinberger, as already seen in the case of gradient coupling. Our approach which allows to extend the results of Kresin and Maz'ya, working for solutions of zero-order free systems, to subsolutions of fully coupled systems.

We also remark that our approach can be also used in unbounded domains with suitable measure-geometric conditions, going back to Cabré and ultimately to Berestycki, Nirenberg and Varadhan.