BOOK OF ABSTRACTS

SS34 - Recent progresses in stochastic fluid dynamics and turbulence

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Turbulence and invariant measures in the Hasegawa–Mima equation

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The Hasegawa–Mima equation is a classical model for drift-wave turbulence in magnetically confined plasmas subject to a strong guide magnetic field. In this talk, we present a series of recent results obtained through complementary analytical and computational approaches toward a fundamental understanding of Hasegawa–Mima turbulence. Using rigorous stochastic analysis, we establish the existence of statistical steady states (invariant measures) for the forced-dissipative dynamics. We then apply the classical Kolmogorov–Kraichnan theory to derive theoretical predictions for the energy and enstrophy cascades in these statistical steady states. These predictions are subsequently investigated through direct numerical simulations. Particular emphasis is placed on turbulent diffusion and its dependence on the strength of the guide magnetic field. This talk is based on a series of works in collaboration with Federico Butori, Paolo Cifani, Franco Flandoli, and Andrea Zanoni. This work received funding from the European Union, ERC, project "NoisyFluid", Grant Agreement No. 101053472.

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The "real" butterfly effect in multiscale models of turbulence

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The butterfly effect is often used as a metaphor for the sensitivity of chaotic systems to small perturbations. In turbulent flows, however, this idea is more subtle: perturbations do not simply grow with a single Lyapunov time, but interact with a hierarchy of spatial and temporal scales [1,2]. Here we focus on the pre-asymptotic stage, in which a perturbation may initially decrease before reaching the exponentially growing regime. This regime is especially relevant for the so-called "real" or "literal" butterfly effect, where one asks whether a small, localized perturbation can survive dissipation and eventually affect larger scales [3].

We will first consider shell models of turbulence, which provide a simplified multiscale framework to study error growth without an explicit spatial structure. In this setting, perturbations initially placed in the dissipative range undergo a strong decay before recovering and participating in the inverse cascade of uncertainty [4,5]. This behavior is consistent with the finite-dimensional chaotic picture: a generic infinitesimal perturbation is not asymptotically absorbed, although it may experience a long transient before aligning with the most unstable direction. When small-scale noise is included, this picture connects naturally with spontaneous stochasticity and the amplification of microscopic fluctuations up to macroscopic scales [6].

We will then turn to Surface Quasi-Geostrophic (SQG) turbulence, a minimal model for mesoscale geophysical flows under strong rotation and stratification [7]. In contrast with shell models, SQG turbulence allows one to address spatially localized perturbations and the role of coherent structures. We find that perturbation growth is preceded by a strongly variable transient regime, similar to that observed in shell models. This transient depends on the initial perturbation scale, the Reynolds number, and the local properties of the flow around the perturbation. Perturbations initialized inside vortices can remain trapped and weakly amplified for long times, whereas perturbations in straining regions rapidly spread and grow [8].

Our findings shed light on the pre-asymptotic regime of predictability in turbulent flows. They show that localized perturbations can eventually affect large-scale geophysical flows, but only after a transient whose duration may be comparable to the Lyapunov time and strongly dependent on the local geometry of the flow. This suggests a connection between coherent structures, local predictability, and temporary windows of opportunity in geophysical forecasting.

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Global Well-Posedness for Small Data in a 3D Temperature-Velocity Model with Dirichlet Boundary Noise

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We study a three-dimensional Boussinesq-type temperature-velocity system on a bounded smooth domain $\mathcal{D} \subset \mathbb{R}^3$. The velocity satisfies the Navier-Stokes equations, while the temperature is driven by Dirichlet boundary noise of intensity $\sqrt{\varepsilon}$. The boundary forcing gives rise to a stochastic convolution which is, in general, only distribution-valued in space; this requires working in negative Sobolev spaces and combining stochastic estimates with maximal regularity arguments for the coupled system.

For every finite time horizon $T > 0$, under suitable assumptions on the stochastic forcing and for sufficiently small initial data, we prove existence and uniqueness of a mild solution up to a stopping time $\tau^\varepsilon \leq T$. Moreover, the solution belongs to the expected optimal regularity class, with velocity in a maximal-regularity space and temperature continuous in time with values in a negative Sobolev space. Finally, we obtain a high-probability global existence estimate of the form

$$\mathbb{P}(\tau^\varepsilon = T) \geq 1 - C\varepsilon,$$

showing that, in the small-noise regime, the solution exists on the whole interval $[0, T]$ with probability tending to one.

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Stochastic phase separation in complex fluids

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In recent years, liquid-liquid phase separation has attracted considerable attention as a pivotal mechanism underlying structure formation in complex fluids, soft materials, and biological systems. Stochastic modeling provides a natural way to represent thermal fluctuations, molecular interactions, and unresolved microscopic processes that can strongly influence phase separation, particularly near critical regimes or during nucleation. At the continuum level, these effects are introduced through random forcing in diffuse interface models, leading to stochastic Cahn–Hilliard or Allen–Cahn-type equations, typically coupled with those of fluid motion (as, e.g., the Navier–Stokes system). In this talk, I will present well-posedness results and the existence and properties of invariant measures for the stochastic Allen–Cahn–Navier–Stokes system. The talk is based on joint works with Maurizio Grasselli (Politecnico di Milano), Luca Scarpa (Politecnico di Milano) and Margherita Zanella (Politecnico di Milano).

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On local well-posedness of the stochastic incompressible density-dependent Euler equations

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We study the stochastic inhomogeneous incompressible Euler equations in the whole space $\mathbb{R}$. We prove the existence and pathwise uniqueness of local solutions with both additive and multiplicative stochastic noise. Our approach is based on reducing our problem to a random problem and some estimations for type transport equations.

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A spatio-temporal random synthetic turbulent velocity field: Beyond Gaussianity, skewness, intermittency and vorticity alignments

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We write a stochastic model for the velocity field of a fluid in a three-dimensional, stationary, isotropic, homogeneous turbulent flow. We build the velocity field as a random, incompressible vector field that evolves in time. We expand on the Gaussian framework, which evolves Fourier modes as an Ornstein-Uhlenbeck process and acts as an underlying structure for the development of a multifractal velocity field. Such a field is built on the layered structure of embedded processes for the forcing terms, which provides more suitable regularity in time. Our model statistically reproduces fundamental aspects of turulent flows both in space and in time, such as structure functions up to fourth order, the geometric structure of the velocity gradient and shows qualitative improvement in the vorticity structures compared to the Gaussian models. To report on the accuracy of the model, our numerical results are compared to direct numerical simulations of the Navier-Stokes equation.

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Stochastic Transport by Heavy-Tailed Jump Processes: From Classical to Anomalous Diffusion

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In the study of turbulent fluid dynamics, modeling the velocity field as a superposition of divergence-free vector fields with stochastic weights provides crucial insights into the effect of small-scale turbulence onto large-scale dynamics. Indeed, as shown in the seminal work of Galeati [1] and later by Luo and Teng [2], such transport noise can produce dissipation effects on dynamics via a certain scaling limit. While previous results have mainly focused on the diffusive framework (i.e. when the random weights are described by Brownian motions), this talk [3] explores the transport of a passive scalar advected by a turbulent fluid, focusing specifically on regimes where the driving noise exhibits heavy-tailed jump statistics. More precisely, we investigate advection-diffusion equations in which the stochastic turbulence is modelled using symmetric α-stable-like jump processes. Our analysis reveals a fundamental dichotomy that is strictly governed by the tail behaviour of the driving noise. When the velocity field is driven by standard α-stable processes, the system exhibits convection-enhanced super-diffusion. Conversely, when the extremely long jumps are tamed, either through truncation or exponential tempering, the large-scale transport undergoes a sharp transition back to a classical diffusive regime.

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