BOOK OF ABSTRACTS

SS33 - Spectral Graph Theory

2nd Joint Meeting Brazil-Italy in Mathematics

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Organizers:

Francesco Belardo, University of Naples "Federico II"
Vilmar Trevisan, Universidade Federal Rio Grande do Sul

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Graphs whose second-largest Seidel eigenvalue lies in $[-1, 1]$

Maurizio Brunetti
Department of Mathematics and Applications, University Federico II of Naples

The Seidel matrix of a graph G with n vertices is defined as $\mathbb{S}(G) = J_n - I_n - 2A(G)$, where J_n and I_n denote the all-ones matrix and the identity matrix of order n, respectively, and $A(G)$ is the adjacency matrix of G. In this talk, we give a complete characterization of all graphs whose second-smallest Seidel eigenvalue σ_2 lies in $[-1, 1]$. Indeed, we show that $\sigma_2(G) \in [-1, 1]$ if and only if G is Seidel-switching equivalent to a clique blow-up of chain graphs of type 0^s1^t or 0^s101^t. Moreover, we extend our approach to characterize all signed complete graphs whose second-largest adjacency eigenvalue does not exceed 1. This is joint work with Lei Li and Jianfeng Wang.

References

[1] L. Li, M. Brunetti, J.F. Wang, *Graphs whose second-largest Seidel eigenvalue does not exceed 1*, submitted.

E–mail: mbrunett@unina.it.

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Partial transposition and the construction of cospectral graphs, signed graphs, and complex unit gain graphs

Matteo Cavaleri [1]
Department of Engineering, Università degli Studi Niccolò Cusano

This talk concerns the use of partial transposition in the construction of cospectral pairs of graphs, signed graphs, and complex unit gain graphs. The partial transposition of a matrix, given together with a block structure, is simply the simultaneous transposition of all its blocks. We begin by introducing ($\mathbb{T}$-gain) digraphs and a composition operation that produces ($\mathbb{T}$-gain) graphs from them. The structural framework associated with this composition allows us to reformulate and generalize several sufficient conditions under which a graph and its partial transpose are cospectral, and to derive criteria ensuring their non-isomorphism. Several examples, both in the classical and in the signed and gain cases, will be presented. The talk is based on a joint work with F. Belardo, M. Brunetti and A. Donno.

References

[1] F. Belardo, M. Brunetti, M. Cavaleri, A. Donno, *Construction of cospectral graphs, signed graphs and $\mathbb{T}$-gain graphs via partial transpose.* J. Algebraic Combin. 60 (2024), no. 1, 191–224

[2] Dutta, S., Adhikari, B. *Construction of cospectral graphs.* J. Algebraic Combin. 52 (2020), no. 2, 215–235

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E–mail: matteo.cavaleri@unicusano.it.

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On the Determined-by-Adjacency Spectrum problem of signed tadpoles

Maurizio Brunetti, Adriana Ciampella [1]
Department of Mathematics and Applications, University FEDERICO II - Naples

A signed graph Γ is said to be determined by its spectrum if every signed graph with the same spectrum as Γ is switching isomorphic with Γ. The Determined-by-Spectrum (DS) problem ranks among the most prominent areas of spectral graph theory. Interest from the research community grew substantially after van Dam and Haemers published a celebrated survey article, which clarified decades of scattered results on the DS problem, especially for the adjacency, Laplacian, and signless Laplacian matrices of simple graphs, and placed them within a coherent and unified framework.

This presentation focuses on the DS problem for the adjacency spectrum of the unbalanced bipartite tadpoles $L_{g,t}$. A tadpole consists of a cycle C_g joined by a bridge to an endpoint of the path P_t.

The DS problem for unsigned tadpoles has been addressed with respect to the adjacency matrix, the Laplacian matrix and the signless Laplacian matrix. It turns out that unsigned tadpoles are DS with respect to all three matrices mentioned.

The DS problem for signed tadpoles has been investigated with respect to the Laplacian matrix. Indeed, Belardo and Petecki proved that signed tadpoles are all determined by their Laplacian spectrum.

Our main result shows that the context of the adjacency matrix yields drastically different results.

Theorem 1. *(Brunetti, C. 2026+) Let $L^-_{2(l+1),t}$ be an unbalanced bipartite signed tadpole with girth $2(l+1)$ and order $2(l+1)+t$. The signed graph $L^-_{2(l+1),t}$ is determined by its adjacency spectrum if and only if $l=1$ and $t \notin \{7,8,10\}$.*

Proof. The following three claims lead to our result.

- Claim 1. Every unbalanced bipartite tadpole with girth larger than 4 has a cospectral mate.
- Claim 2. $L^-_{4,7}$, $L^-_{4,8}$ and $L^-_{4,10}$ each have a cospectral mate.
- Claim 3. The graphs $L^-_{4,t}$ for $t \notin \{7,8,10\}$ are determined by their spectrum.

□

References

[1] F. Belardo, P. Petecki, *Spectral characterizations of signed lollipop graphs*, Linear Algebra Appl., 480 (2015), 144–167.

[2] M. Brunetti, A. Ciampella, *Which unbalanced bipartite tadpoles are determined by their spectrum?*, Submitted, (2026+).

[3] E.R. van Dam, W.H. Haemers, *Which graphs are determined by their spectrum?* Linear Algebra Appl., 373 (2003), 241–272.

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E–mail: maurizio.brunetti@unina.it, adriana.ciampella@unina.it.

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On totally synchronizing graphs

Daniele D'Angeli [1]
Department of Engineering, University Niccolò Cusano Roma
Emanuele Rodaro
Department of Mathematics, Politecnico of Milano

Motivated by the celebrated Road Coloring Theorem and the Cerny conjecture, we introduce some spectral ideas for the study and construction of infinite families of totally synchronizing digraphs.

References

[1] D. D'Angeli and E. Rodaro, *On totally synchronizing graphs*, submitted.

[1]Partially supported by GNSAGA
E–mail: daniele.dangeli@unicusano.it.

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Maximum Nullity and Zero Forcing in Generalized Petersen Graphs

Renata R. Del-Vecchio [1]
Instituto de Matemática e Estatística , Universidade Federal Fluminense

Johann Bolckau
PPGMAT, Universidade Federal Fluminense

The zero forcing number is a well-known upper bound for the maximum nullity of a graph, and characterizing graphs for which these parameters are equal remains an important problem. In this work, we study this question for generalized Petersen graphs. We construct matrices in $S(G)$ whose nullity equals the zero forcing number, proving the equality for a large family of generalized Petersen graphs. Our results provide new exact values for the maximum nullity of these graphs. In particular, the family $P(n, 2)$ is almost completely characterized. We also identify families of generalized Petersen graphs $P(n, k)$, with $k > 2$, that are extremal with respect to the zero forcing bound, providing further insight into the relationship between zero forcing and maximum nullity in this graph class.

References

[1] AIM Minimum Rank – Special Graphs Work Group, *Zero forcing sets and the minimum rank of graphs*, Linear Algebra and its Applications, 428(7) (2008), 1628–1648.

[2] J. Alameda, E. Curl, A. Grez et al., *Families of graphs with maximum nullity equal to zero forcing number*, Special Matrices, 6(1) (2018), 56–67.

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E–mail: rrdelvecchio@id.uff.br.

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A sequence of integral and distance integral Cayley graphs with few distance eigenvalues

Alfredo Donno [1]
Department of Engineering, University Cusano of Rome

The Pauli group is well known for its remarkable applications in Physics and in Quantum Information Theory. From an algebraic perspective, it is a nonabelian group of order 16, which can be realized as the central product of the dihedral group of 8 elements and the cyclic group of order 4. Iterating this construction yields the sequence of so-called n-qubit Pauli groups.

For every $n \geq 1$, we construct the Cayley graph of the n-th Pauli group with respect to a suitable generating set arising from the central product construction. We carry out a complete spectral analysis of both the adjacency matrix and the distance matrix of this family of graphs, providing explicit descriptions of their eigenvalues and eigenvectors. We show that the Cayley graph of the n-th Pauli group is a $(3n+2)$-regular bipartite graph on 4^{n+1} vertices, with diameter $2n+1$, and that both its adjacency spectrum and its distance spectrum are integral. Remarkably, for every n, the n-th Cayley graph has exactly six distinct distance eigenvalues. This yields an infinite sequence of graphs answering a problem posed by Atik and Panigrai and, at the same time, providing a counterexample to a conjecture of Aouchiche and Hansen.

This is a joint work with Y. Bavuma, D. D'Angeli, F.G. Russo, and S. Spessato.

References

[1] M. Aouchiche, P. Hansen, *Distance spectra of graphs: a survey*, Linear Algebra Appl., 458 (2014), 301–386.

[2] F. Atik, P. Panigrahi, *On the distance spectrum of distance regular graphs*, Linear Algebra Appl., 478 (2015), 256–273.

[3] Y. Bavuma, D. D'Angeli, A. Donno, F.G. Russo, *On an infinite family of integral Cayley graphs of Pauli groups*, J. Algebra, 659 (2024), 148–182.

[4] D. D'Angeli, A. Donno, S. Spessato, *On an infinite family of integral and distance integral Cayley graphs with few distance eigenvalues*, Linear Algebra Appl., 726 (2025), 180–215.

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E–mail: alfredo.donno@unicusano.it.

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Recent advances on the inverse eigenvalue problem on trees

Luiz Emilio Allem, Carlos Hoppen [1]*, Lucas Siviero Sibemberg*
Instituto de Matemática e Estatística , Universidade Federal do Rio Grande do Sul

Thomás Jung Spier
Departamento de Ciência da Computação, Universidade Federal de Minas Gerais

A graph $G = (V, E)$ with vertex set $\{v_1, \ldots, v_n\}$ is said to be the *underlying graph* of a matrix $M = (m_{ij}) \in \mathbb{R}^{n \times n}$ if the following holds:

$$m_{ij} \neq 0 \text{ if and only if } \{v_i, v_j\} \in E \text{ for all } i \neq j.$$

Let $S(G)$ be the set of all real symmetric matrices with underlying graph G. This means that the off-diagonal nonzero entries of a matrix $M \in S(G)$ are precisely the entries ij for which there is an edge $\{i, j\} \in E$, while there is no constraint on entries on the main diagonal. In general, the inverse eigenvalue problem for a graph G aims at describing all possible spectra for matrices in $S(G)$. Our results focus on the parameter $q(G)$, the minimum number of distinct eigenvalues in a matrix in $S(G)$. Formally, if $\mathrm{DSpec}(M)$ denotes the set of distinct eigenvalues of M,

$$q(G) = \min\{|\,\mathrm{DSpec}(M)| \colon M \in S(G)\}.$$

It is well known that for any tree T, $q(T) \geq d(T) + 1$, where $d(T)$ stands for the diameter of T. Moreover, this inequality is not always tight. In fact, the trees for which equality holds are said to be *diminimal*.

In this talk, I shall describe two types of results. On the one hand, any tree of diameter d may be generated by a sequence of branch duplications of a tree in a finite set of trees of diameter d, known as the *seeds* of diameter d. It turns out that, for some seeds, all trees generated by them are diminimal, so that they are called *diminimal seeds*. It has been shown that all seeds of diameter at most 5 are diminimal and that, for every diameter $d \geq 6$, there are at least three diminimal seeds. Let $\alpha(d)$ denote the proportion of seeds that are diminimal.

Theorem 1. *It holds that* $\lim_{d \to \infty} \alpha(d) = 0$.

On the other hand, I will show that are several operations on diminimal trees that preserve diminimality. The following are two results in this direction.

Theorem 2 (Branch deletion)**.** *Let T be a diminimal tree and let $i_0, \ldots, i_{d(T)}$ be the vertices on a longest path in T. Suppose that i_k is the last vertex along this path with degree at least 3. Consider a branch B at i_k that does not intersect the longest path. If $\tilde{T}$ is obtained from T by deleting the branch B, then $\tilde{T}$ is also diminimal.*

Theorem 3 (Duplication via longest path)**.** *Let T be a diminimal tree and assume that one of its longest paths starts at the leaf i. Let T' be a copy of T where i' is the copy of i. Consider the tree $\tilde{T}$ obtained by combining T and T' by either merging the vertices i and i' or connecting them via a path. Then $\tilde{T}$ is also diminimal.*

References

[1] L.E. Allem, R.O. Braga, C. Hoppen, E.R. Oliveira, L.S. Sibemberg, V. Trevisan, *Diminimal families of arbitrary diameter*, Linear Algebra and its Applications 676 (2023), 318–351.

[2] W. Barrett, S. Butler, S.M. Fallat, H. Tracy Hall, L. Hogben, J. Lin, B.L. Shader, M. Young, *The inverse eigenvalue problem of a graph: Multiplicities and minors*, Journal of Combinatorial Theory, Series B, 142 (2020), 276–306.

[3] C.R. Johnson, C.M. Saiago, *Eigenvalues, Multiplicities and Graphs*, Cambridge, 2018.

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E–mail: choppen@ufrgs.br.

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Graphs and signed graphs with integer reciprocal eigenvalue properties

Callum Huntington [1], *Francesco Belardo*
Department of Mathematics and Applications R. Caccioppoli, University of Naples Federico II
Sasmita Barik
Department of Mathematics, IIT Bhubaneswar

As natural companions to the integer reciprocal eigenvalue properties m-(R) and m-(SR) recently introduced by Barik, Pati, and Verma [1], we introduce the integer anti-reciprocal eigenvalue properties m-($-$R) and m-($-$SR) for graphs and signed graphs. We show that these properties are compatible with the Kronecker product for signed graphs [2]; that simple m-coronae of signed graphs always satisfy m-($-$R), with the stronger properties m-(SR) and m-($-$SR) holding whenever the root graph has symmetric spectrum; and that the unsigned hypercube Q_n satisfies integer reciprocal eigenvalue properties for $n \leq 4$ but not for $n \geq 5$, whereas a suitable signing from [3] always yields n-(SR) and n-($-$SR). Finally, we construct explicit infinite families of graphs satisfying m-(R) but not m-(SR), together with non-corona graphs satisfying both m-(SR) and m-($-$SR), in both the classical and signed settings.

References

[1] S. Barik, S. Pati, P. Verma, *Trees with integer reciprocal eigenvalue property*, Linear and Multilinear Algebra, online (2026), DOI: 10.1080/03081087.2026.2666137

[2] K.A. Germina, S. Hameed K, T. Zaslavsky, *On products and line graphs of signed graphs, their eigenvalues and energy*, Linear Algebra and its Applications, 435 (2011), 2432–2450.

[3] S. Laplante, R. Naserasr, A. Sunny, Z. Wang, *Sensitivity conjecture and signed hypercubes*, ACM Transactions on Computation Theory, 18 (2026), 1–25.

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E–mail: callum.huntington@unina.it.

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The NEPS of gain graphs over arbitrary groups and its spectra

Matteo Cavaleri, Alfredo Donno, Stefano Spessato [1]
Department of Engineering, University Niccolò Cusano

This talk is devoted to the spectral theory of gain graphs and their non-complete extended p-sums (NEPS). After a brief introduction to gain graphs and their spectral theory, based on Fourier transforms at representations of the gain groups, we present two natural constructions of gain functions on NEPS over arbitrary groups. We discuss their main structural properties, with particular attention to switching isomorphism and balance. We provide some formulas expressing the spectra of NEPS in terms of the spectra of the factor graphs, generalizing the classical formula describing the spectrum of a NEPS of ordinary graphs.

References

[1] Cavaleri M., Donno A., Spessato S. *The NEPS of gain graphs over arbitrary groups and its spectra*, Adv. in Group Th. and App. (2025).

[1] Aknowledgements...
E–mail: stefano.spessato@unicusano.it.

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Brouwer's Conjecture: Recent Developments

Vilmar Trevisan
Instituto de Matemática e Estatística
Universidade Federal do Rio Grande do Sul

In Spectral Graph Theory, Brouwer's Conjecture is one of the central open problems relating the Laplacian spectrum of a graph to its combinatorial structure. For a graph (G=(V,E)) with (n=|V|) vertices and (m=|E|) edges, the conjecture states that, for every (k=1,...,n), the sum (S(k)) of the (k) largest Laplacian eigenvalues satisfies

$$S(k) \leq m + \binom{k+1}{2}.$$

Over the past two decades, the conjecture has been verified for numerous classes of graphs and has inspired several related problems, revealing deep connections between Laplacian eigenvalues, degree sequences, threshold graphs, and graph invariants. Recently, a preprint has claimed a complete proof of Brouwer's Conjecture, together with an equivalence between the conjecture and the Grone–Merris–Bai theorem, bringing renewed attention to this longstanding problem. Since these developments are still very recent, they also motivate a careful examination of the underlying arguments and their consequences.

In this talk, we survey the mathematical significance of Brouwer's Conjecture, discuss some of the main partial results and recent advances, present ongoing developments, and highlight several related problems and possible future directions, including weighted and distance-Laplacian analogues.

E–mail: trevisan@mat.ufrgs.br.

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Characterizing graphs for which the second largest distance eigenvalue is less than $-\frac{1}{2}$

Miriam Abdón, Cybele T.M. Vinagre
Instituto de Matemática e Estatística, Universidade Federal Fluminense, Niterói, RJ, Brazil

Lilian Markenzon
Universidade Federal do Rio de Janeiro, Rio de Janeiro, RJ, Brazil

Let G be a connected graph and let $\mathbf{D}(G)$ denote its distance matrix. We write $\lambda_2(G)$ for the second largest eigenvalue of $\mathbf{D}(G)$. Characterizing connected graphs satisfying $\lambda_2(G) < -\frac{1}{2}$ has recently become an active topic in Spectral Graph Theory. A major advance was obtained by Guo and Zhou [1], who proved that every connected graph satisfying this spectral condition is necessarily chordal. Building on this structural insight, we exploit properties of minimal vertex separators in chordal graphs to obtain the complete characterization of the connected graphs satisfying $\lambda_2(G) < -\frac{1}{2}$.

References

[1] H. Guo and B. Zhou, *Graphs for which the second largest distance eigenvalue is less than* $-\frac{1}{2}$, Discrete Math. **347** (2024), no. 9, Paper No. 114082, 11 pp..

E–mail: cybele$_v$*inagre@id.uff.br.*