BOOK OF ABSTRACTS

SS31 - Recent Trends in Nonlinear Elliptic and Parabolic PDEs

2nd Joint Meeting Brazil-Italy in Mathematics

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Organizers:

Cristina Tarsi, University of Milan
Joao Marcos Bezerra do Ó, Universidade Federal da Paraíba

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Nonlinearities and singularities in elliptic boundary value problems

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This talk is concerned with the interaction between nonlinearities and the spectrum of linear differential operators in elliptic boundary value problems. The focus is on how solution multiplicity arises from spectral interaction and how singularity theory provides a framework for understanding this phenomenon. Starting with the classic Ambrosetti–Prodi theorem as a paradigm of a global fold, we progress to cusps, swallowtails, and butterfly singularities that appear when convexity assumptions are relaxed and when different types of nonlinearities are considered. The classical results in singularity theory of H. Whitney, R. Thom, V.I. Arnold, and J. Mather are presented, as well as their extension to the infinite-dimensional setting in Banach spaces. Some examples, including cubic nonlinearities and models with local eigenvalue crossing, illustrate the rich geometric structure of solution sets.

Joint works with Carlos Tomei (PUC-Rio) and with Bernhard Ruf (Istituto Lombardo di Scienze e Lettere)

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On the maximum principle for higher order operators and applications to nonlinear PDEs

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It is well known how the Maximum Principle (MP) in general fails to hold for uniformly elliptic operators of order higher than two, even in smooth convex domains. In [1] it was shown in dimension two and three, by establishing a new Harnack type inequality with nonlocal remainder terms, that the validity of the positivity preserving property can be restored when lower order derivatives are taken into account as a perturbation of the higher order differential operator. We will present further advances in this direction, extending the validity of the MP to supersolutions in any dimension and fairly general domains. Moreover, we will discuss how the presence of inertial terms affects the range of the perturbation parameter, providing a balance between the positivity restoring effect of lower order derivatives and the mass energy. Recent applications to semilinear biharmonic equations are presented.

References

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A bifurcation phenomenon for quasilinear p-Laplace equations in the ball

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We focus on positive radial solutions to the critical Dirichlet problem

$$\begin{cases} \Delta_p u(x) + \lambda K(|x|)|x|^\delta u(x)^{q-1} = 0 & |x| \leq 1\,, \\ u(x) = 0\,, & |x| = 1\,, \end{cases}$$

where $x \in \mathbb{R}^n$, $1 < p < n$, $\delta > -p$, and q is the Hardy-Sobolev critical exponent

$$q = p^*(\delta) = p\,\frac{n+\delta}{n-p}.$$

Assuming that K is positive and satisfies a suitable flatness condition at the origin, we prove that its flatness order determines a bifurcation phenomenon: positive radial solutions exist for every $\lambda > 0$ when $\ell < \ell_p^*$, whereas for $\ell \geq \ell_p^*$ a bifurcation value λ_0 separates existence from nonexistence. Under additional assumptions on K, multiplicity phenomena arise in the supercritical case.

The main technical ingredient is the construction of an unstable manifold in a non-smooth setting using the Ważewski principle, combined with dynamical systems techniques, phase-plane analysis, and energy estimates.

We conclude by illustrating how the underlying dynamical framework can extend beyond the present setting to other quasilinear p-Laplace equations, leading to new bifurcation and multiplicity results.

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[2] F. Dalbono, M. Franca, A. Sfecci, *A bifurcation phenomenon for the critical Laplace and p-Laplace equation in the ball: part 2*, Preprint 2026, submitted for publication.

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Inhomogeneous nonlinear Schrödinger equations with competing singular nonlinearities

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We study nonlinear elliptic equations that arise as stationary states of inhomogeneous nonlinear Schrödinger (INLS) equations with competing singular nonlinearities. We consider the problem

$$-\Delta u + \frac{|u|^{q-2}u}{|x|^b} = \lambda \frac{|u|^{p-2}u}{|x|^a} + g(x,u) \qquad \text{in } \mathbb{R}^N,$$

where $0 < a, b < 2$, $2 < p < q$, $\lambda \in \mathbb{R}$, $N \geq 2$, and g is a Carathéodory function defined on $\mathbb{R}^N \times \mathbb{R}$ satisfying suitable growth conditions. The variational formulation involves a weighted Sobolev space defined as the intersection of the homogeneous Sobolev space with a weighted Lebesgue space. Using sharp weighted Sobolev and Caffarelli–Kohn–Nirenberg type inequalities, we establish continuous and compact embeddings of this space into suitable weighted Lebesgue spaces. These embedding results, together with a natural scaling structure of the model, enable us to apply a recent abstract critical point framework of Mercuri and Perera (2026), yielding a sequence of nonlinear eigenvalues for the associated problem via a min–max scheme based on the Fadell–Rabinowitz cohomological index. Within this framework we obtain a broad collection of existence and multiplicity results for equations driven by sums of weighted power nonlinearities, covering both subcritical and critical cases. We also establish a nonexistence result obtained via a Pohozaev-type identity. Finally, we analyze the radial setting, where improved radial CKN inequalities allow us to enlarge some of the admissible embedding ranges. This leads to strengthened radial versions of our main results.

References

[1] E. Gloss, K. Perera, B. Ribeiro, *Inhomogeneous nonlinear Schrödinger equations with competing singular nonlinearities*, preprint, https://arxiv.org/abs/2601.02909.

[2] C. Mercuri and K. Perera, *Variational methods for scaled functionals with applications to the Schrödinger–Poisson–Slater equation*, J. Math. Pures Appl., 212 (2026), paper No. 103885.

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Fractional Schrödinger–Poisson–Slater equations in Coulomb–Sobolev spaces

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We discuss recent developments on fractional Schrödinger–Poisson–Slater equations of the form

$$(-\Delta)^s u + \big(I_\alpha * u^2\big)u = f(|x|, u), \qquad x \in \mathbb{R}^N,$$

where I_α is the Riesz potential.

The classical Schrödinger–Poisson equation was studied by Ruiz. Building upon the fractional Coulomb–Sobolev framework and embedding theory developed by Bellazzini *et al.*, together with the abstract scaling theory recently developed by Mercuri and Perera, we establish the analytical ingredients required to apply this theory to fractional Schrödinger–Poisson–Slater equations, including density, regularity and Pohozaev-type results.

Within this setting, we first investigate radial problems by classifying the nonlinearity according to its scaling behavior into subscaled, scaled and superscaled regimes. This framework introduces nonlinear eigenvalues through the $\mathbb{Z}_2$-cohomological index, yields existence and multiplicity results, and provides a unified treatment of several classes of nonlinearities. We then consider critical equations in the nonradial fractional Coulomb–Sobolev setting with weighted perturbations. By combining concentration–compactness arguments with minimax methods, we establish the existence of multiple solutions for sufficiently large values of the parameter and of a positive solution for every positive value of the parameter.

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Solutions Concentrating on Lower Dimensional Spheres for Singularly Perturbed Elliptic Equations

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We consider a class of singularly perturbed Hamiltonian elliptic systems with weighted linear and nonlinear terms on annular domains in $\mathbb{R}^{2M}$, where $M = 2, 4$, or 8. Exploiting the special geometry associated with the Hopf fibrations, we reduce the original problem to an anisotropic elliptic system in a lower-dimensional space. For the reduced problem, we develop a dual variational framework and establish the existence of least-energy solutions together with their asymptotic concentration behavior as the perturbation parameter $\varepsilon \to 0$.

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Symmetry breaking for elliptic equations with exponential nonlinearities in higher dimensions

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In this talk, we present a variational approach to semilinear elliptic problems with exponential nonlinearities in (possibly unbounded) annular domains of $\mathbb{R}^N$. These problems present significant analytical difficulties in dimensions $N \geq 3$, where the classical Sobolev framework is not sufficient to handle nonlinear terms with exponential growth. To overcome these difficulties, we combine tools from Orlicz space theory (which provide a natural functional setting to handle non-polynomial nonlinearities) with nonsmooth critical point methods in the spirit of Szulkin. This approach allows us to recover a suitable variational structure despite the lack of standard compactness properties. Under suitable structural assumptions, we establish the existence of positive solutions that are not radially symmetric, thereby exhibiting symmetry-breaking phenomena for this class of elliptic problems.

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About degenerated Kirchhoff logistic problem

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In this talk, we will present new results on the behavior of positive solutions to a Kirchhoff-type logistic problem involving a spatially degenerate diffusion rate. The equation also includes a degenerate potential. In contrast with the case of a homogeneous diffusion coefficient, where blow-up occurs only within the degenerated region, the presence of heterogeneous diffusion may cause the blow-up in other parts of the domain as well. These results were obtained in collaboration with Vinicius Ramos (Universidade de São Paulo), and Willian Cintra (UnB).

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FitzHugh-Nagumo-type systems with sign-changing coefficients

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We establish existence and multiplicity results for steady-state solutions of spatially heterogeneous FitzHugh-Nagumo-type systems in Euclidean space, extending the existing theory from constant to variable coefficients that may change sign. We prove the existence of three distinct weak solutions: one componentwise positive, one componentwise negative, and one sign-changing. The proof combines variational methods with Weth's theorem, whose use in this context is new. A key ingredient is the construction of a suitable Hilbert space, with a carefully chosen scalar product, which ensures that the associated polar cone satisfies the assumptions required for the application of Weth's theorem.

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Fokker–Planck equations and Poincaré inequalities

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We consider some n-dimensional functional inequalities of the type of Poincaré, with weight, for *isotropic* probability densities. Unlike previous studies, for example by S.G. Bobkov, and M. Ledoux (Ann. Probab. **37** (2009)), the derivation of these inequalities is strongly related to their connection with the problem of convergence to equilibrium for a class of Fokker–Planck type equations characterized by a variable coefficient of diffusion. This is a result obtained in collaboration with G. Furioli (Univ. of Bergamo, Italy), A. Pulvirenti and G. Toscani (Univ. of Pavia, Italy).

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A Supercritical Hamiltonian System

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We study the existence of positive radial solutions for the supercritical Hamiltonian system

$$\begin{cases} -\Delta u = v^{p_1-1+r^\alpha} & \text{in } B, \\ -\Delta v = u^{p_2-1} & \text{in } B, \\ u = v = 0 & \text{on } \partial B, \end{cases} \tag{1}$$

where $B \subset \mathbb{R}^N$ is the unit ball, $r = |x|$, $\alpha > 0$, and the exponents p_1, p_2 satisfy

$$\frac{1}{p_1} + \frac{1}{p_2} = \frac{N-2}{N}. \tag{2}$$

In the absence of the perturbation r^α, nonexistence follows from Mitidieri's identity (1996). Although the exponent is supercritical for every $r > 0$, the critical exponent is attained only at the origin. This restores the compactness required for a variational approach.

The proof relies on a reduction to a perturbed critical p-biharmonic equation together with the estimate

$$U_{p,N} = \sup\left\{ \int_B |u|^{p_2^*+r^\alpha}\, dx : u \in E^p_{\mathrm{rad}}(B),\ \|Au\|_p = 1 \right\} < \infty, \tag{3}$$

which yields the continuous embedding

$$E^p_{\mathrm{rad}}(B) \hookrightarrow L^{p(r)}(B), \qquad p(r) = p_2^* + r^\alpha. \tag{4}$$

Moreover, the perturbation restores the attainability of $U_{p,N}$ and provides the compactness needed to obtain positive radial solutions of the Hamiltonian system.

References

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