BOOK OF ABSTRACTS

SS3 - New challenges on Numerical methods for differential problems

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An Innovative Lagrangian-Eulerian Approach for Conservation Laws: Structure-Preserving Discretization and Emerging Challenges

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This work introduces a novel class of positive, structure-preserving, semi-discrete Lagrangian-Eulerian (SDLE) schemes [1] for the numerical approximation of multidimensional scalar hyperbolic conservation laws and nonlinear systems of the form:

$$\frac{\partial u}{\partial t} + \frac{\partial H(u)}{\partial x} + \frac{\partial G(u)}{\partial y} + \frac{\partial F(u)}{\partial z} = 0, \quad u = u(x, y, z, t) : \mathbb{R}^3 \times \mathbb{R}^+ \longrightarrow \mathbb{R}^m, \tag{1}$$

$m \in \mathbb{N}$, $m \geq 1$ and H, F and G are flux functions from $\mathbb{R}^m$ to $\mathbb{R}^m$. The framework is built upon the unconventional no-flow curve paradigm [1,2], which enables the derivation of weak CFL-type stability conditions and system positivity without recourse to the spectral information (eigenvalues or eigenvectors) of the flux Jacobian matrix. By reinterpreting the hyperbolic flux as a (3+1) space-time vector field, the method leverages intrinsic geometric properties to ensure structural stability—effectively resolving nonlinear interactions while obviating the need for traditional Riemann solvers. A significant advantage of this formulation is its self-contained nature; the scheme bypasses costly field-by-field characteristic decompositions, rendering it highly efficient for parallel high-performance computing (HPC) platforms [1]. Theoretically, we establish the convergence of the scheme to the unique entropy solution for multidimensional scalar problems via weak asymptotic analysis, and provide a formal proof of a weak positivity property for systems [1]; see also [3]. The robustness and high-resolution capabilities of the SDLE scheme are demonstrated through a suite of rigorous 3D benchmarks, including the compressible Euler equations, the Orszag-Tang magnetohydrodynamic vortex, and non-strictly hyperbolic conservation laws in porous media. Furthermore, applications to the Navier–Stokes equations ("*ABC Flow*") via a novel space-time relaxation formalism demonstrate the scheme's versatility with high fidelity. We also showcase structure-preserving, high-fidelity simulations of the Kelvin–Helmholtz instability and the Forward-Facing Step problem (Mach 3 wind tunnel), supported by HPC-MPI strong scaling studies. Finally, recent extensions to multidimensional nonlocal conservation laws [4] highlight the framework's broad applicability and its readiness for emerging numerical challenges. This is a joint work with J. Agudelo, P. Godoi, W. Lambert, and J. Perez.

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An Evolve-Filter-Relax regularization model for the numerical simulation of incompressible MHD flows

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We study the efficiency of a regularization technique, the so called Leray-α model, for the stabilization of numerical simulations of turbulent flows in magnetohydrodynamics.

We focus on finite volume methods and we use the Evolve-Filter-Relax (EFR) algorithm that consists of three steps: in the evolve step the original problem is solved to obtain an intermediate approximation; in the second step a differential filter is used to regularize the intermediate approximation, and in the relax step a more accurate approximation is obtained as a convex combination between the intermediate and the filtered approximation. This algorithm was deeply investigated for hydrodynamics applications where it is well known that a linear filter is not satisfactory because it applies the same amount of artificial viscosity everywhere. We show that the same holds in MHD flows and propose the use of a nonlinear α-model to avoid the over-diffusion.

We implement the algorithm using the OpenFOAM® library. I will present different numerical tests that validate the implementation and illustrate the performance of this regularization strategy for the MHD system. The numerical tests include the classical Orszag-Tang 2D vortex, test problems with a low magnetic Reynolds number (as in metallurgical applications) and three dimensional turbulent flows at high dynamic and magnetic Reynolds numbers (as in plasma physics applications).

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High-Order Boundary Treatment in Unfitted Methods for Partial Differential Equations

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Solving partial differential equations (PDEs) in domains with complex geometries is fundamental to many scientific and engineering applications, but remains computationally challenging, particularly due to the accurate and stable enforcement of boundary conditions. This talk presents high-order boundary discretization strategies within the framework of unfitted boundary methods.

We introduce boundary schemes based on the ghost point method, in which the computational grid is extended beyond the physical domain through the addition of ghost points. Rather than determining ghost point values via local or one-sided extrapolation, we employ a coupled formulation that solves ghost and neighboring interior values simultaneously. This results in an augmented linear system that significantly improves both accuracy and numerical stability.

To efficiently solve the resulting systems, we develop a specific multigrid solver designed to handle curved and irregular boundaries. Numerical experiments for elliptic PDEs demonstrate the accuracy and robustness of the proposed approach, while applications to incompressible flow problems, including dynamic scenarios such as oscillating bubbles, illustrate its versatility. Finally, we discuss recent extensions of the method to hyperbolic equations, highlighting the challenges posed by time-dependent, transport-dominated problems.

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Numerical Simulation of Viscoelastic Fluid Flows

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The numerical simulation of viscoelastic fluid flows remains a major challenge in computational rheology due to the strong coupling between momentum and constitutive equations and the nonlinear behavior of stress evolution. Accurate and stable numerical approximations are essential for understanding complex industrial and engineering processes involving non-Newtonian fluids, such as polymer processing, coating flows, and microfluidic applications.

Among the available numerical methodologies, finite difference methods continue to play an important role because of their simplicity, computational efficiency, and flexibility on structured meshes. These methods provide an effective framework for the approximation of classical viscoelastic models, including the Oldroyd-B model and related differential constitutive equations.

This talk presents an overview of finite difference approaches for viscoelastic flow simulations, with emphasis on stabilization strategies designed to overcome numerical instabilities at moderate and high elasticity levels. Particular attention is devoted to the High Weissenberg Number Problem (HWNP), a well-known difficulty characterized by the loss of stability and convergence of standard discretizations as the Weissenberg number increases.

Several stabilization techniques are discussed within the finite difference context. These include log-conformation reformulations, which improve robustness by preserving the positive definiteness of stress-related tensors, as well as high-resolution schemes for the accurate treatment of convective terms. The presentation also examines the Generalized Lie Derivative (GLD) formulation and other consistent discretization strategies aimed at reducing spurious oscillations and improving numerical stability.

Illustrative numerical results are presented for benchmark problems commonly used in computational rheology, highlighting the balance between stability, accuracy, and computational cost achieved by different approaches. Finally, the talk discusses current research challenges and future directions, including adaptive techniques, hybrid numerical methodologies, and the development of more robust solvers for complex three-dimensional and highly elastic flow regimes.

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Matrix-Oriented Solution of a Pitting Corrosion Model

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The increasing complexity of modern industrial systems calls for advanced computational tools capable of supporting sustainable asset management. In particular, accurate prediction of material degradation can improve maintenance planning, extend the service life of critical infrastructures, and reduce unplanned downtime, thereby mitigating disruptions throughout production and supply-chain networks [4]. Mathematical models describing corrosion phenomena are typically formulated as nonlinear reaction-diffusion systems, whose efficient numerical solution remains a significant computational challenge.

In this talk, we consider the simulation of metal pitting corrosion through a phase-field model [5] that accurately captures the evolution of corrosion fronts and complex interface dynamics without the need for explicit interface tracking. Although phase-field models provide a flexible and physically consistent framework, their numerical approximation is hindered by the stiffness of the governing equations, especially in large-scale simulations [2,3].

To address these challenges, we propose a matrix-oriented numerical strategy that exploits the Kronecker sum structure of the discrete diffusion operator obtained from finite difference discretizations. This formulation enables the construction of efficient time integration schemes that avoid the inversion of large matrices while preserving stability and accuracy. Moreover, the proposed methodology can be extended beyond rectangular computational domains, making it suitable for realistic engineering applications [1].

Numerical experiments in two- and three-dimensional settings demonstrate significant reductions in computational cost compared with conventional approaches while maintaining an accurate description of the corrosion dynamics [1].

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Space-time domain decomposition methods for wave-propagation problems

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The numerical simulation of wave propagation remains challenging, especially in the presence of complex geometries, heterogeneous media, and high-frequency phenomena. Achieving high-order accuracy while maintaining computational efficiency is often difficult with traditional time-stepping approaches.

In this talk, we present and compare several space-time domain decomposition strategies for the wave equation, including waveform relaxation methods and both mapped and unmapped tent-pitching schemes. These approaches provide naturally parallelizable frameworks that enable efficient exploitation of modern computing architectures while preserving the accuracy of the underlying discretizations.

We further address the increased computational complexity associated with space-time formulations and introduce an efficient domain decomposition strategy designed to enhance scalability and reduce computational costs. Numerical experiments illustrate the performance of the proposed methods, highlighting their effectiveness across a range of wave propagation scenarios.

Finally, we discuss ongoing developments, including adaptive strategies for the selection of time subdomains and extensions to more general wave propagation and multiphysics problems.

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Numerical analysis of a supra-convergent method applied to a nonlinear pressure-cell density dynamics problem over non uniform meshes

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After a short review over supra-convergent finite difference methods, I focus on a system of partial differential equations in a nonuniform grid that is used to approximate the nonlinear and non-smooth pressure-cell density dynamics in the colon epithelium. This system couples an elliptic equation with a nonlinear parabolic equation with mixed boundary conditions. It is analyzed a supra-convergent method in nonuniform grid that guarantees a second order approximation for the system solution gradient using differential operators that have only a first order truncation error. The challenge arises in building the right discretizations of differential operator and boundary conditions to preserve the method's high order despite the solution non-smoothness.

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Dealing with arbitrarily distorted 8-node bricks

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The reliability of isoparametric 8-node hexahedra in 3D structural modeling depends heavily on mesh regularity; these elements lose validity whenever the Jacobian mapping becomes singular. Due to this sensitivity, engineers often struggle with complex geometries where generating a valid, non-distorted brick mesh is either a labor-intensive manual process or a geometric impossibility.

In this talk, we present the recently developed Virtual Element Method (VEM) for the 8-node brick [1], named *virtual brick*. The virtual brick is face-compatible with standard isoparametric 8-node elements but remains robust even when the brick is degenerate.

The presentation will focus on the mathematical properties of the virtual brick. As a proof of concept, we will provide numerical experiments for a scalar reaction-diffusion problem to demonstrate the method's effectiveness.

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Parallel-in-time numerical methods: challenges and applications

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With the advent of massively parallel high-performance computing systems, parallel-in-time (PinT) numerical methods have attracted increasing interest over the past three decades as an alternative approach for accelerating the numerical simulation of initial value problems described by ordinary and time-dependent partial differential equations [1]. The goal is to obtain an accurate numerical solution while avoiding the serial time-integration of a computationally expensive discretization of the problem.

One of the most popular PinT schemes is Parareal [2], an iterative predictor-corrector algorithm that combines a fine (expensive) and a coarse (less expensive) discretization of the problem. This iterative procedure allows integrating the fine scheme across several time steps simultaneously, converging to the solution provided by the serial integration. If convergence is attained in a few iterations, an accurate solution is obtained in a smaller computing time compared to the serial integration of the fine discretization.

However, while Parareal (and other PinT methods) has been successfully applied to parabolic, diffusive problems, it is well known to present convergence and stability issues in applications involving hyperbolic, advective, and oscillatory phenomena [3,4]. Therefore, improving the performance of the temporal parallelization in these classes of problems is an active area of research.

In this talk, we discuss the main properties and challenges of Parareal, as well as some approaches to improve convergence and stability. This is illustrated through the application of the method to two problems that are particularly challenging to parallelize in time: geophysical fluid dynamics and celestial mechanics. In the former, we consider the shallow water equations on the rotating sphere and show how analytical stability properties and appropriate parametric choices can enhance convergence of the temporal parallelization; in the latter, we solve the gravitational N-body problem, with large N, and show that careful design and implementation of coarse schemes can yield effective gains of computing time.

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