BOOK OF ABSTRACTS

SS24 - Conservation Laws for Fluid and Transport Models: Analysis and Numerics

2nd Joint Meeting Brazil-Italy in Mathematics

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Organizers:

Fabio Ancona, University of Padua
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SS24

A Geometric Vector Field Approach for Multidimensional Conservation Laws: From Scalar Entropy Convergence to Lax Positivity for Systems

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This work presents a rigorous numerical analysis and theoretical foundation for a new class of semi-discrete and fully discrete Lagrangian–Eulerian (LE) schemes for multidimensional systems of hyperbolic conservation laws [1,2]. Built upon the geometric framework of space-time no-flow curves and surfaces, the proposed method bypasses dimensional splitting and field-by-field characteristic decompositions, eliminating the need to solve local Riemann problems or compute eigenvalues of the flux Jacobian matrix. Mathematically, we establish convergence proofs toward the unique Kruzhkov entropy solution for multidimensional scalar problems within the setting of uniqueness and regularity of entropy process solutions in the $L^\infty_t(L^1_{\mathbf{x}})$-norm, linked to measure-valued solutions. Under unstructured triangular grids, optimal a priori error estimates of $\mathcal{O}(h^{\frac{1}{2}})$ are achieved. For multidimensional systems, we prove that the LE formulation satisfies a weak CFL-type stability condition and fulfills the positivity principle proposed by P. Lax and X.-D. Liu [3], alongside a structural weak positivity property. Furthermore, a deep mathematical bridge is established in the functional setting between the geometric notion of no-flow curves and the fundamental results of A. Bressan [4] on the existence and continuous dependence for discontinuous ordinary differential equations. The robustness of this framework is validated through challenging benchmarks on cubic and triangular meshes. Finally, the framework's versatility is demonstrated by its recent extension to multidimensional nonlocal conservation laws [5] and novel numerical insights for the inviscid SQG equation within large Besov-weak-Morrey spaces, underscoring its capacity to tackle complex, emerging mathematical challenges. This is a joint work with J. Agudelo, J. François, W. Lambert, and J. Perez.

References

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[1] E. Abreu acknowledges the financial support of the Brazilian NSF (CNPQ) and São Paulo Research Foundation (FAPESP), both located in Brazil, under grants number 307641/2023-6 and 2025/07662-6, respectively. Pedro Godoi was supported by a scholarship from CAPES/Brazil - Finance Code 001 (PPGMA/IMECC/Unicamp).
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A geometric Lagrangian-Eulerian formulation for the intrinsic shallow water equations on general topography

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We present a geometrically adapted Lagrangian-Eulerian finite volume framework for the intrinsic shallow water equations (ISWE) on surfaces with spatially varying bottom geometry, including localized topographic discontinuities. By introducing a local curvilinear reference system anchored to the bottom surface, we derive an effective first-order and high-resolution space-time discretization of no-flow surfaces. The scheme relies on a two-step evolution-remap procedure: i) a Lagrangian step, solving an initial value problem that governs the evolution of the balance laws; ii) an Eulerian remapping step back onto the original surface. The resulting method preserves monotonicity and captures shock waves, while keeping numerical dissipation under control even in the presence of non-autonomous fluxes. Furthermore, it provides a natural and efficient treatment of localized discontinuities in the bottom topography, requiring no significant modifications to the global system of equations. A representative set of numerical experiments is provided to demonstrate the accuracy and robustness of the proposed formulation on two-dimensional surfaces with general curvature and discontinuous initial conditions.

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Local Asymptotic Patterns for Viscous Approximations of Conservation Laws

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Solutions to hyperbolic conservation laws can be approximated in various ways, including vanishing viscosity, relaxation methods, numerical schemes, nonlocal flux approximations. For some methods, general L^1 convergence results are available.

What is the local asymptotic behavior of approximations near singularities of the real solution? Specifically, different patterns are expected at points along a shock, where two shocks interact, or where a new shock forms. While most questions are still open problems, we describe the case of vanishing viscosity approximations to uniformly convex scalar conservation laws.

Since the hyperbolic solution is discontinuous at these points, a natural approach is to introduce suitable coordinate rescalings. In the singular limit, the rescaled parabolic solutions converge to "eternal solutions"—globally defined both in space and time—of the normalized approximating equation. The set of all such limits depends only on finitely many parameters.

References

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A Lagrangian representation to the classical LWR model and application to the analysis of commuting times in a linear city.

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In this talk, I will describe a mathematical model (in a simplified framework) that captures the everyday phenomenon of thousands of citizens choosing to commute from one place to another, leaving home at a precise time to go to work and estimating their arrival time. In the discrete setting, I will use the well-known Follow-the-Leader model, featuring the original characteristic of vehicles entering and exiting at different times. In order to describe the main features of the model, we adopt a Lagrangian viewpoint, describing the commuting strategies as measures on Lipschitz curves each associated with an entry time on the road and an exit time from the road, which are compatible with the given initial plan among the distribution of lodging and workplace locations, respectively.

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Structure of Optimal Solutions for Traffic Flow

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In recent years, following the seminal papers by Lighthill and Whitham [4] and Richards [5], there has been increasing interest in using conservation laws to model traffic flow in road networks, primarily driven by practical applications. In particular, the problems of reducing traffic congestion, car accidents, and pollution have been addressed using various approaches.

In this talk, we present a method for controlling traffic flow via a control function acting at junctions [1]. More precisely, these control functions are defined as the traces of the flux functions at the junction. We introduce a suitable concept of solution, we show that the solution exists, and that, in some cases, the input-output map is continuous. Furthermore, we consider the minimization of functionals that represent various traffic performance indices [2]. For the specific case of junctions with one incoming and one outgoing road, we describe several properties of optimal solutions [3] and compare them with entropy solutions.

References

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Conservation Laws with Coinciding Smooth Solutions and the Isentropic Approximation in Gas Dynamics

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We consider two one-dimensional hyperbolic systems of conservation laws that share the same eigenvalues and eigenvectors, while possibly differing in their shock curves and conserved quantities. We show that, for the same initial data, their entropic weak solutions differ only at third order with respect to the total variation of the initial data. This estimate can be refined in the presence of a full set of Riemann coordinates, as is the case for all 2×2 systems. This result leads to improved estimates on the distance between solutions of related models, including the isentropic and non-isentropic inviscid Euler equations under general equations of state, as well as DiPerna's approximation of the p-system.

References

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Theoretical Aspects of a Lagrangian-Eulerian Method for Hyperbolic Conservation Laws

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This talk addresses the theoretical aspects of the Lagrangian–Eulerian (LE) approach for one-dimensional scalar hyperbolic conservation laws of the form

$$u_t + [f(u)]_x = 0, \qquad u(x,0) = u_0(x). \tag{1}$$

Built upon the mathematical framework of no-flow curves $x = \sigma(t)$ [1], the proposed approach treats the space-time divergence-free condition, $\nabla_{x,t} \cdot [f(u), u]^T = 0$, as a vector field defined on a space-time manifold. Since this field depends on the unknown solution u, its dynamics are driven along the space-time $(x,t)_j^n$-LE directions. Within this framework, we construct the LE vector field and demonstrate that the impervious boundaries (the no-flow curves $x = \sigma(t)$) of the advancing space-time region are governed by the specific system of ordinary differential equations

$$\frac{d\sigma_j^n(t)}{dt} = \frac{f(u(\sigma_j^n(t),t))}{u(\sigma_j^n(t),t)}, \qquad \sigma_j^n(t^n) = x_j^n. \tag{2}$$

These curves are consistent with shock wave propagation satisfying the Kružkov entropy condition [2]. In the presence of a discontinuity connecting left and right states (u_L and u_R), the curve velocity matches the classic Rankine–Hugoniot shock speed s [3]:

$$(u_R - u_L)\, s = f(u_R) - f(u_L). \tag{3}$$

This theoretical formulation allows the design of numerical schemes that inherently capture discontinuous solutions. Indeed, we prove that for the 1D scalar case, the resulting scheme is monotone, total variation diminishing (TVD), and converges to the unique Kružkov entropy solution under a suitable CFL condition [3]. Monotonicity is also established by casting the scheme as a convex combination of neighboring cell values with nonnegative coefficients, guaranteeing both the entropy inequality and the absence of spurious oscillations in the numerical compuations.

Finally, we formally discuss extending these concepts to multidimensional scalar equations and systems of hyperbolic conservation laws. By utilizing a component-wise approach, we adapt the space-time path tracking to accommodate characteristic wave family interactions, building upon preliminary results found in [2].

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Geometric intrinsic modeling of shallow water flows

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We present an intrinsic shallow water model for geophysical flows over terrains with non-negligible slopes and curvatures, such as mountain landscapes. The model is derived from the Navier-Stokes equations formulated in a local reference frame anchored to the bottom surface, yielding equations with non-autonomous flux functions and source terms that encode geometric information. The proposed model satisfies several desirable theoretical properties: rotational invariance, energy conservation, well-balance, and second-order consistency with the Navier-Stokes equations with respect to a geometry-based order parameter. A first-order upwind Godunov Finite Volume scheme, intrinsically defined on the bottom surface, is proposed for the numerical discretization. Convergence properties of the scheme are analyzed both theoretically and numerically. The model and solver are validated on a range of synthetic test cases, demonstrating their accuracy and robustness.

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Entropy Solutions for Temple-Class Systems with Discontinuous Flux

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This presentation focuses on a one-dimensional 2×2 system of conservation laws of Temple class featuring flux functions with spatial discontinuities, inspired by traffic flow modeling. Abrupt variations in roadway characteristics or driving behavior naturally induce such discontinuities, which act as interfaces separating different dynamical regimes. We analyze the resulting mathematical framework and establish the existence of entropy-admissible solutions under suitable hypotheses on the discontinuous flux.

The analysis relies on a wave-front tracking methodology, offering a constructive procedure to generate approximate solutions and justify convergence to a limiting entropy solution. In this context, we develop a Riemann solver adapted to the presence of flux jumps and study its fundamental features, such as solvability, coherence, and appropriate admissibility conditions for shocks and contact waves.

Complementing the theoretical analysis, we provide numerical experiments that reproduce the main qualitative behaviors predicted by the theory and clearly show how flux discontinuities influence wave interactions and propagation. These simulations highlight the applicability of the model to traffic flow problems and clarify the role played by discontinuities in shaping traffic dynamics.

References

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Stability of dispersive boundary layers for scalar conservation laws in one space dimension

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We consider the following class of Korteweg-De Vries (KdV)-type initial-boundary value problems on the half-line,

$$\begin{cases} \partial_t u^\varepsilon + \partial_x[f(u^\varepsilon)] + \varepsilon^2 \partial_{xxx} u^\varepsilon = 0, & (t,x) \in \mathbb{R}_+ \times \mathbb{R}_+ \\ u^\varepsilon|_{t=0} = u_{in}, \quad u^\varepsilon|_{x=0} = u_b. \end{cases} \tag{1}$$

We study the zero-dispersion limit $\varepsilon \to 0^+$ in the outflow regime $f' < 0$, so that the solution of the limiting scalar conservation law does not attain the boundary condition imposed on the dispersive problem. We construct a boundary layer profile, depending on the fast variable, which is uniquely determined, through the associated stationary third-order boundary layer equation, by the mismatch between the boundary conditions, and by the exponential decay at infinity in the fast variable. Our main result shows that, under suitable regularity and compatibility assumptions on the data, the dispersive solution is well approximated by a WKB expansion given by the sum of the smooth solution of the conservation law and the boundary layer profile. The proof is based on the analysis of a linearized energy functional and does not rely on complete integrability or inverse scattering techniques. It applies to general fluxes and requires no smallness assumption on the amplitude of the boundary layer. To the best of our knowledge, this is the first stability result for boundary layers of KdV-type equations on the half-line.

References

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A differential structure for general scalar conservation laws

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We introduce a general framework to study first-order perturbations of entropy solutions to scalar conservation laws with general flux functions. We first show that such perturbations are represented by measures satisfying a continuity equation with a vector field given by the characteristic speed. Next, by "lifting" the perturbations to measures in the graph space, we establish a shift differentiability result: to first order, near the shocks of the limiting solution, any perturbation essentially behaves as a shift (translation) of the shock in a direction determined by the lift.

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