BOOK OF ABSTRACTS

SS22 - Singularities in Algebra and Geometry

2nd Joint Meeting Brazil-Italy in Mathematics

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Organizers:

Bruna Oréfice Okamoto, UFSCar
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SS22

Functional inequalities for differential forms on Carnot groups

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In this talk we will discuss functional inequalities in Carnot groups, whose prototypes are the (p, q)-Sobolev and (p, q)-Poincaré inequalities in Euclidean spaces. We recall that, for functions in Euclidean space, Poincaré and Sobolev inequalities in a generic regime (i.e. $p > 1$, $q < \infty$) follow from rather general principles. By contrast, limiting cases ($p = 1$ or $q = \infty$) require extra arguments, or even may fail. Similar inequalities for functions in the setting of Carnot groups have been known for many years. The discussion of Poincaré and Sobolev inequalities for higher-degree differential forms is also possible.

In the last few years, in collaboration with B. Franchi (University of Bologna) and P. Pansu (University of Orsay), we have proved Poincaré and Sobolev inequalities, for Rumin differential forms, in Heisenberg groups. These inequalities can be seen as the analytic counterpart of the well-known topological problem of whether a given closed form is exact. More precisely, one can ask whether a primitive can be upgraded to one that satisfies certain estimates; and geometric applications follow. A key ingredient in the proof is a family of estimates that ultimately relies on the existence of a fundamental solution for a suitable Hodge-type Laplacian defined by M. Rumin in the Heisenberg groups. This is a hypoelliptic homogeneous operator. The estimates that we obtain are sharp.

In particular, for L^p, $p > 1$, the global estimates are a direct consequence of singular integral estimates related to the Folland-Stein theory of homogeneous kernels. When $p = 1$, the singular integral estimates are replaced by div-curl type inequalities, which go back to Bourgain-Brezis and Lanzani-Stein in Euclidean spaces, and to Chanillo-Van Schaftingen and Baldi-Franchi-Pansu in Heisenberg groups. In the first part of the talk I would like to discuss the validity of these types of estimates in the Heisenberg groups. In the second part, I will discuss how this approach can be extended to general Carnot groups. The geometry of arbitrary Carnot groups is radically different from that of Heisenberg groups and one of the difficulties that one encounters is related to the fact that the structure of Rumin differential forms is much more complex. Although the same argument used in the Heisenberg case can still be applied in general Carnot groups, relying on the construction of a zero-order Laplacian, it involves singular integral operators that lack good homogeneity properties. While only non-sharp estimates are generally expected, sharp Sobolev inequalities have been obtained for top-degree forms, and in specific cases, such as the Cartan group (the free Carnot group of step 3 with 2 generators). These results also include recent joint work with F. Tripaldi (University of Leeds) and A. Rosa (University of Bologna).

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On the cohomology of L^2-harmonic forms of an incomplete Riemannian manifold

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In [1] the authors consider the complex given by the de Rham differential acting on the space of harmonic forms on a compact Riemannian manifold with boundary. They compute the cohomology of this complex by showing that it is fully determined by the de Rham cohomology of the underlying manifold. More precisely they prove the following isomorphism:

$$H^p(\mathrm{Harm}^*(M), d) \cong H^p(M, \mathbb{R}) \oplus H^{p-1}(M, \mathbb{R}).$$

Motivated by the above result we extend the notion of cohomology of harmonic forms (of a compact Riemannian manifold with boundary) to the abstract setting of Hilbert complexes. Then, we present some geometric applications of our construction to the L^2-cohomology of incomplete Riemannian manifolds with particular interest to the case of smoothly stratified Thom-Mather spaces.

References

[1] S. Cappell, D. DeTurk, H. Gluck, E. Miller. *Cohomology of harmonic forms on Riemannian manifolds with boundary.* Forum Math., 18 (2006) no. 6, 923–931.

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Torsion Invariants on Spaces with Isolated Singularities

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The Cheeger-Müller theorem identifies analytic torsion with Reidemeister torsion on smooth manifolds. In recent years, significant progress has been made towards extending this correspondence to singular spaces.

In this talk, I will present two approaches to the Cheeger-Müller theorem for spaces with isolated conical singularities. One is based on Witten deformation and Morse-theoretic techniques, while the other relies on intersection Reidemeister torsion, and spectral analysis on singular spaces. I will describe the main ideas of these constructions and compare their respective strengths and limitations.

References

[1] U. Ludwig, *An extension of a theorem by Cheeger and Müller to spaces with isolated conical singularities*, Duke Math. J. 169 (2020), no. 13, 2501–2568.

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The proof of the metric Arnold corank problem

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In this article, we approach the Arnold corank problem, posed by Arnold in 1975, which asks whether the corank of holomorphic functions is an ambient topological invariant. Here, we obtain a complete positive answer to the metric Arnold corank problem, which asks whether the corank of holomorphic functions is an ambient bi-Lipschitz invariant. Consequently, we show that for complex hypersurfaces, the multiplicity equal to two is an ambient bi-Lipschitz invariant. We also prove that the Arnold corank problem holds true for holomorphic functions of three variables. Other topological invariants are also presented.

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Topological and Algebraic Structures of Singularity Collisions

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Cancellation of critical points originates in classical Morse theory, where consecutive critical points can be eliminated by local perturbations of the flow without altering the topology of the underlying manifold. In singular settings, however, topological singularities exhibit a rigid local structure that prevents their removal by perturbation without destroying essential topological features.

To address this limitation, we consider the notion of homotopical dynamical cancellation. Rather than eliminating singularities, this approach allows them to collide through homotopical deformations of the phase space, producing more degenerate singularities while preserving the homotopy type of the manifold. We focus on homotopical deformations of flow lines and the resulting collisions of singularities, which reduce their total number and lead to a dynamically richer, yet topologically equivalent, phase space.

References

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Lipschitz embeddings of algebraic curves

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We will show that a real algebraic curve has a semialgebraic Lipschitz normal embedding into the real plane if and only if as a graph it does not contain a subdivision of a Kuratowski graph. Moreover, there always exists a Lipschitz normal embedding as a real algebraic curve in R^3. In the complex case we show that while semialgebraic Lipschitz normal embeddings exist, there may not exist Lipschtz normal embeddings as complex algebraic curves.

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Witt pseudomanifolds, the signature operator and Gysin maps in K-homology

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Gysin homomorphisms in K-theory and K-homology and their functoriality properties have played a major role in index theory (on smooth manifolds, Lipschitz manifolds and foliations). Using stable homotopy theory, Markus Banagl has defined Gysin homomorphisms in K-homology for Witt pseudomanifolds and proved that they preserve the Sullivan-Siegel orientation class. Examples of Witt pseudomanifols include complex projective varieties. In this talk I will explain an analytic approach to these results, using KK-theory and the signature operator. This analytic approach will in fact allow for much more general results and, moreover, it is fully compatible with the topological approach. I will also discuss an application of these techniques to the formulation and proof of an Atiyah-Segal-Singer formula for the Goresky-MacPherson G-signature of a Witt pseudomanifold. The first part of the talk is joint work with Pierre Albin and Markus Banagl. The second part, on the G-signature formula, is joint work with Markus Banagl and Eric Leichtnam.

References

[1] P. Albin, M. Banagl, P. Piazza *Smooth atlas stratified spaces, K-Homology Orientations, and Gysin maps.* Preprint May 2025. 81 pp. arXiv:2505.14952

[2] P. Albin, M. Banagl, P. Piazza *Smooth atlas stratified spaces, K-Homology Orientations, and Gysin maps. Part 2.* Preprint May 2026. 40 pp. arxiv:2605.28620

[3] M. Banagl, E. Leichtnam, P. Piazza *The G-signature Theorem on Witt spaces.* Preprint November 2025. 29 pp. arxiv:2511.21586

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