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Messina

BOOK OF ABSTRACTS

SS21 - Probability and Statistical Mechanics

2nd Joint Meeting Brazil-Italy
in Mathematics

September 8, 2026

Messina, Italy

Organizers:

Alessandra Bianchini, University of Padova

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Network multiple-node immunization via random rooted forests

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Consider a viral agent spreading over a network. The multiple-node immunization problem consists of identifying a set of k nodes whose removal minimises the spread of the infection, and is a fundamental optimisation problem arising in immunization strategies. Motivated by the analysis of classical compartmental epidemic models on graphs, such as the SIS (contact process) dynamics, a well-established approach is to select the k nodes whose removal minimises the largest eigenvalue of the graph adjacency matrix, see e.g. [5,7]. This optimisation problem is computationally intractable in general, and several heuristic algorithms have been proposed to address it. Among them, the NetShield algorithm is a widely used and computationally method [6]. We introduce a novel algorithm for the multiple-node immunization problem that improves upon the performance of NetShield while retaining the same computational complexity. Our method is based on random rooted spanning forests, a natural generalisation of the uniform spanning tree measure, whose probabilistic properties and related sampling algorithms have been investigated in recent years (see e.g. [2,3,4]). As I will discuss, the proposed framework is flexible and can be adapted to other optimisation problems involving the identification of k central nodes in a graph. This is based on joint work [1] with Alexandre Gaudilli, Irina Gurewitsch, Adorndriamandroso and Alessio Troiani.

References

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Percolation on hierarchical lattices

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In this talk we examine families of hierarchical lattices, with focus on its fine percolative properties. Thanks to the self-similarity of the graphs in these sequences, several properties of the model can be examined, including: critical exponents, scaling relations, and noise sensitivity. Furthermore, for these models, we obtain explicit expressions for several critical exponents of percolation. The results are very sharp and proofs employ a combination of tools from probability and dynamical systems.

Imry-Ma phenomenon for the hard-core model on $\mathbb{Z}^2$

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The *Imry-Ma phenomenon* refers to the dramatic effect that disorder can have on first-order phase transitions for two-dimensional spin systems. The most famous example is the absence of a phase transition for the two-dimensional random-field Ising model. This paper establishes that a similar phenomena takes place for the hard-core model, a discrete model of crystallization: arbitrarily weak disorder prevents the formation of a crystal. Our proof of this behaviour is an adaptation of the Aizenman-Wehr argument for the Imry-Ma phenomenon, with the use of internal (spin space) symmetries for spin systems being replaced by the use spatial symmetries.

References

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Quantitative Wright–Fisher limits for the noisy voter model on Erdős–Rényi graphs

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The voter model on a finite graph is an interacting particle system in which each vertex represents a voter holding one of two possible opinions, 0 or 1. Each voter updates their opinion by adopting the opinion of one of their neighbours at a rate proportional to the number of neighbours holding that opinion. It is known that, under suitable conditions relating the mixing and meeting times of the underlying random walk, the average density of opinion 1 converges—after an appropriate rescaling on a growing sequence of graphs—to the Wright–Fisher diffusion, a classical process from population genetics. In this talk we study the voter model perturbed by noise on a sequence of Erdős–Rényi graphs. In this noisy version, the opinion at each vertex may also flip independently of its neighbours. We show that, when the strength of the noise is chosen appropriately, the density of opinion 1 still converges to a version of the Wright–Fisher diffusion. Our main contribution is a quantitative analysis of the convergence, providing explicit bounds on the rate at which the finite system approaches its limit. We also compare our rates with those recently obtained for the same model on the complete graph by Aljovin, Jara, and Xiang. This is joint work with Guilherme Reis.

References

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Equilibrium Fluctuations of the SSEP on Random Graphs with Conductances

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We study the symmetric simple exclusion process on a random weighted undirected graph whose vertices are given by a point process and whose edge weights (conductances) determine the jump rates. Under mild assumptions on the environment (stationarity, ergodicity, and suitable moment conditions), the empirical density converges, for almost every realization of the environment, to the solution of a heat equation with homogenized diffusivity. Here we analyze equilibrium fluctuations. For $d \geq 3$, under the same assumptions as for the hydrodynamic limit, the density fluctuation field converges, in the sense of finite-dimensional distributions and for almost every realization of the environment, to a generalized Ornstein–Uhlenbeck process. For $d = 2$, the same result holds under additional regularity assumptions ensuring local Hölder estimates for the associated parabolic equations. We also discuss applications to several classical random graph models.

Degree-preserving conservative processes and a unified approach for their hydrodynamics

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In this talk I will consider a broad class of interacting systems characterized by a single conservation law and satisfying the "degree-preserving property", namely, the Markov generator of the processes preserves the degree of polynomials of the state variables up to two. This property, together with a few general assumptions, makes it possible to study many different models within the same framework. In particular, it allows us to rigorously derive their hydrodynamic behavior without having to analyze each model separately. Joint work with: Patrícia Gonçalves, Kohei Hayashi, Makiko Sasada.

References

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NESS for KPZ

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We consider conservative, weakly asymmetric particle systems on a finite interval of size n , connected to weakly asymmetric reservoirs on each boundary of the interval. We take asymmetries of order $\mathcal{O}(1/\sqrt{n})$, in such a way that the particle system belongs to the weak KPZ universality class, and we take the density of the reservoirs as a perturbation of order $\mathcal{O}(1/\sqrt{n})$ of the *triple point density* of the system. We propose a general strategy to show that the density of particles under the invariant measure of the system has a scaling limit given by the gradient of the *non-equilibrium, stationary state* (NESS) of the KPZ equation with Dirichlet boundary conditions, described by [1], [2]. The proof follows as an application of *quantitative hydrodynamics*, as introduced by J. and Menezes.

References

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- [2] *I. Corwin* and *A. Knizel*, Commun. Pure Appl. Math. 77, No. 4, 2183–2267 (2024).

Weakly self-avoiding random walk in a random potential

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We consider a model of a simple random walk influenced by two competing types of interaction: an attractive one towards high values of a random potential, and a self-repellent one measured by its self-intersections (in the spirit of the weakly self-avoiding random walk). We identify the log asymptotics of the partition function of the model and also the typical path behaviour giving the main contribution to the partition function. The latter comes out of a variational formula and shows concentration on a random finite number of points, each occupied for a positive fraction of time. Joint work with Wolfgang König, Nicolas Pétrélis and Willem van Zuijlen.

References

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The critical KPZ scale of the averaging process

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KPZ-type extremal fluctuations have recently been proved for several models of random walks in space-time random environments (RWRE) in $1 + 1$ dimensions. A general moment criterion predicts the spatial scale at which this behavior should occur, but does not by itself guarantee non-trivial fluctuations at that scale.

In this talk, we show that the averaging process provides an instance in which this criterion is not sharp: because of a degeneracy in the update mechanism, the actual KPZ limit appears only beyond the predicted scale. Our proof builds on Dobrushin local-time limit theorems, refined estimates for tilted k -point motions, and the recent moment-based axiomatic characterization of Hopf–Cole solutions to the one-dimensional KPZ equation. We also identify the behavior on the two sides of the critical scale, thereby sharply separating the subcritical, critical, and supercritical regimes of the model.

Gaussian fluctuations for IDLA on cylinders

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Internal Diffusion Limited Aggregation (IDLA) is a random growth model where random walks sequentially occupy vertices of a graph, typically resulting in highly regular clusters. While the limiting shape of IDLA is well-understood in various geometries, characterizing the fluctuations around this deterministic limit remains challenging. In this talk, we investigate the fluctuations of IDLA clusters on cylinder graphs of the form $G \times \mathbb{Z}$, where G is a finite graph. We establish that, for any vertex-transitive base graph G satisfying an eigenvalue convergence condition, the average fluctuations of IDLA on the cylinder $G \times \mathbb{Z}$ are given by a GFF under appropriate scaling. This extends a previous result by Jerison, Levine and Sheffield for $G = \mathbb{Z}_N$ to a wide class of base graphs, thus shedding new light on how underlying geometry and lower-dimensional features influence the asymptotic behavior of internal aggregation models. On the way, we present an improved bound on IDLA clusters' maximal fluctuations, which is of independent interest and which implies a shape theorem for IDLA on $G \times \mathbb{Z}$ for *any* vertex-transitive base graph. Based on joint work with Ahmed Bou-Rabee and Ariel Yadin.

References

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The grass-bushes-trees process on a scale-free network

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The grass-bushes-trees process is a two-type contact process in which one type (the trees), of infection parameter λ_{tree} , can invade the other type (the bushes) of infection parameter λ_{bush} . We look to show which graph parameters lead to the possibility of coexistence versus the necessity of competitive displacement, i.e. metastability of both types or fast extinction of the bushes.

New results on the hydrodynamic limits for long-range exclusion processes

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We consider a class of long-range exclusion processes evolving either on $\mathbb{Z}$ or on a finite lattice with open boundaries. For the power-law, translation-invariant jump kernel

$$\mathbf{p}(x, y) = (p^+ \mathbf{1}_{\{y-x>0\}} + p^- \mathbf{1}_{\{y-x<0\}}) |y-x|^{-1-\gamma}, \quad \gamma \in (0, 1),$$

the hydrodynamic limit was proved in [1]. Under the space-time rescaling $(N^\gamma t, Nx)$, the particle density profile evolves according to a nonlocal convection–diffusion equation.

In this talk, we extend the hydrodynamic limit to general jump kernels $\mathbf{p}(x, y)$ satisfying the homogeneity of degree $-(1 + \gamma)$:

$$\mathbf{p}(\lambda x, \lambda y) = \lambda^{-1-\gamma} \mathbf{p}(x, y).$$

Assuming additionally that the jump kernel satisfies suitable tail bounds and the particle displacement has infinite expectation, we derive in [2] that, under the same superballistic rescaling, the evolution of the density profile is governed by the nonlinear integral operator

$$\mathbb{L}\rho(u) := \int_{\mathbb{R}} [\mathbf{p}(v, u)\rho(v)(1 - \rho(u)) - \mathbf{p}(u, v)\rho(u)(1 - \rho(v))] dv.$$

We also prove the well-posedness of weak BV solution to the corresponding equation, resolving a problem left open in [1].

References

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¹Supported in part by the Italian Ministry of Foreign Affairs and International Cooperation, grant number BR26GR05.

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