BOOK OF ABSTRACTS

SS2 - Continuous Optimization and Related Areas

2nd Joint Meeting Brazil-Italy in Mathematics

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Organizers:

Alfredo Noel Iusem, Escola de Matemática Aplicada (EMAp)-Fundação Getúlio Vargas (FGV)
Francisco Facchinei, University of Rome "La Sapienza"

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The Method of Ellipcenters for Strongly Convex Minimization

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This presentation is about ME, the Method of Ellipcenters. ME was recently introduced as a first order accelerated scheme for unconstrained minimization. Its iterates are all centers of ellipses carefully designed to somehow capture ill-conditioning of the underlying optimization problem. In the first article on ME, we were able to prove that it converges with linear rate when the objective function is quadratic and strongly convex, while here we derive convergence for any differentiable strongly convex objective. This investigation was inspired by the great performance of ME in quadratic minimization against steepest descent with exact line search, Nesterov, Barzilai-Borwein and Conjugate Gradient. The experiments we carry out now, make ME even more attractive from the numerical point of view. On top of that, the theory seems promising for quite more general settings.

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A Jacobi-type Newton method for Nash equilibrium problems with descent guarantees

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A common strategy for solving an unconstrained two-player Nash equilibrium problem with continuous variables is applying Newton's method to the system obtained by the corresponding first-order necessary optimality conditions. However, when taking into account the game dynamics, it is not clear what is the goal of each player when considering they are taking their current decision following Newton's iterates. In this paper we provide an interpretation for Newton's iterate as follows: instead of minimizing the quadratic approximation of the objective functions parameterized by the other player current decision (the Jacobi-type strategy), we show that the Newton iterate follows this approach but with the objective function parameterized by a prediction of the other player action. This interpretation allows us to present a new Newtonian algorithm for unconstrained and linear constrained problems where a backtracking procedure is introduced in order to guarantee that the computed Newtonian directions, for each player, are descent directions for the corresponding parameterized functions. Thus, besides favoring global convergence, our algorithm also favors true minimizers instead of maximizers or saddle points, unlike the standard Newton method, which does not consider the minimization structure of the problem in the non-convex case. Thus, our method is more robust compared to other Jacobi-type strategies or the pure Newtonian approach, which is corroborated by our numerical experiments. We also present a proof of the well-definiteness of the algorithm under some standard assumptions, together with a preliminary analysis of its convergence properties taking into account the game dynamics.

References

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[1]This work was supported by FAPESP (São Paulo Research Foundation) grants 2013/07375-0, 2015/02528-8, 2021/05007-0, 2022/05803-3, 2023/08706-1, and 2026/00596-0 CNPq.

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Low-Dimensional Acceleration Strategies for Gradient Methods

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Gradient Methods represent a fundamental class of algorithms in large-scale convex optimization, thanks to their low computational cost and scalability. In this work, we propose an acceleration strategy for gradient-based methods applied to strictly convex Quadratic Programming problems. The approach consists in performing, at selected iterations, minimization steps along alternative descent directions or within low-dimensional affine subspaces, exploiting the structure of the quadratic objective function. Numerical experiments on randomly generated unconstrained and box-constrained QP problems highlight the effectiveness of the proposed strategy, particularly when combined with Barzilai–Borwein-type methods, showing significant improvements in computational performance under different conditioning and initialization settings. We also discuss results toward extending the proposed acceleration framework to general nonlinear optimization problems.

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Ghost penalties and stochastic programmin

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After recalling some recent results on ghost penalties, we discuss how this approach to constrained optimization can be used to develop new convergence results in stochastic programming. We cover the fully general case where both the objective function and the inequality constraints are non convex expected value functions.

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On circumcentered direct methods for monotone variational inequality problems

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The variational inequality problem (VIP) plays a central role in the theory and applications in continuous optimization. In particular, minimization problems and KKT systems can be regard as VIPs. In this work, we present the first methods using circumcenter iterations for solving VIPs. The circumcenteredreflection method (CRM) is a projection-based tool developed with the aim of finding a point in the intersection of finitely many closed convex sets. CRM has gone through enhancements and adaptations over the last few years and was proven to be faster in many settings than competitors such as alternating projections and the Douglas-Rachford methods. One of CRM features is that it is able to deal with approximate projections, which is exactly what we enforced in this article, both theoretically and numerically. Here, we present circumcenter schemes that solve VIPs concerning two types of operators: paramonotone and monotone. For the former we employ exact projections, whereas for the latter we base our approach on approximate ones. We establish convergence results for the methods as well as perform numerical experiments, which indicate good performance, including advantages over the well-know extragradient method.

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A Finitely Convergent Circumcenter Method for the Convex Feasibility Problem

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The convex feasibility problem (CFP) consists of finding a point in the intersection of a finite family of closed and convex sets. This problem arises in several applications, including image reconstruction, signal processing, and optimization.

Projection and projection-reflection methods are classical approaches for solving CFPs. In recent years, circumcenter-based methods have attracted significant attention due to their ability to accelerate the convergence of projection schemes by reducing zig-zagging behavior. However, most existing circumcenter methods either require restrictive assumptions (such as affine sets) or only guarantee asymptotic convergence.

In this work, we propose a new circumcenter-based algorithm for the convex feasibility problem, called the perturbed approximate circumcenter algorithm (PACA). The method replaces exact projections onto the original sets with projections onto suitably constructed separating half-spaces, combined with a circumcenter step. Additionally, exogenous perturbation parameters are incorporated and driven to zero along the iterations.

We establish global convergence of the generated sequence under mild assumptions. Moreover, we prove that the sequence converges linearly. Most importantly, under a Slater-type condition, we show that PACA achieves finite convergence, i.e., it reaches a feasible point in a finite number of iterations. To the best of our knowledge, this is the first circumcenter-type method for the CFP with such a property.

Numerical experiments on feasibility problems involving ellipsoids illustrate the efficiency of the proposed method compared to existing projection-based algorithms.

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Hidden convexity in speed planning problems along a given path

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We address three distinct problems for speed planning along a given discretized path. In the first problem we minimize travel time of the vehicle under acceleration constraints. In the second problem we minimize a weighted sum of two conflicting objectives, energy consumption and travel time, under constraints on the traction force and the power of the vehicle. Finally, in the third problem we minimize travel time under acceleration and jerk constraints. For the first two problems we show that the feasible region is a lattice. We introduce a feasibility-based bound-tightening technique which allows deriving the minimum and maximum element of the lattice or to establish that the feasible region is empty. Moreover, for the first problem we show that the maximum element of the lattice is also the optimal solution of the problem, which can thus be computed by an algorithm with time complexity $O(n)$, where n is the number of points into which the continuous path is discretized. For the second problem, whose mathematical model is non-convex, we prove the exactness of a convex relaxation of the non-convex problem, obtained by replacing all constraints with the lower and upper bounds for the variables corresponding to the minimum and maximum elements of the lattice, respectively. Moreover, we show that a dynamic programming approach is able to return an approximate solution to this exact convex reformulation with time complexity $O(n^2)$. The third problem is more challenging than the previous ones. Its mathematical model is non-convex and its feasible region is not a lattice. However, we present a convex relaxation of the problem, and, after extensive computational experiments, we conjecture exactness of such relaxation.

References

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Inertial Proximal Point Methods with Degenerate Preconditioning

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In this talk, we introduce a proximal point framework for monotone inclusion problems that combines inertial steps, relative error criteria, and degenerate preconditioning. We discuss its main convergence properties and illustrate how it can be used to derive inertial and inexact variants of classical splitting methods.

References

[1] M.P. Machado, M.R. Sicre, *Inertial degenerate preconditioned HPE methods*, Submitted, 2026.

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An Objective Function Free Trust-Region Framework for Decentralized Optimization

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We consider the problem of decentralized optimization, where a network of computational agents collaboratively minimizes a global objective given as the sum of local functions, each privately held by an individual agent. While several first and second-order methods have been proposed for this setting, one of the main challenges is the choice of the stepsize. In particular, classical globalization strategies are not directly applicable in decentralized contexts, and existing methods typically rely on fixed, conservatively small stepsizes, or on specialized adaptive rules tailored to specific algorithms. In this talk a distributed Trust-Region Objective Function free method is presented. It enables adaptive trust-region radii which vary across both nodes and iterations. The framework includes first and second-order models. Under suitable assumptions on the objective functions and the communication network, we establish convergence guarantees for the proposed method. Finally, we present numerical experiments that show the performance of the algorithm and compare the proposed strategy with methods from the literature, in terms of both computation and communication cost.

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Selecting features in nonlinear SVM: MINLP formulation, convex MIQP relaxations and training algorithms

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We address the problem of training nonlinear Support Vector Machines (SVMs) with polynomial kernels, incorporating a hard cardinality constraint which fixes the number of selected features to a user-defined value B. This formulation improves interpretability by enforcing exact sparsity in the input space, but leads to a Mixed-Integer Nonlinear Program (MINLP) whose continuous relaxation is nonconvex, making it intractable for general-purpose solvers. We derive two equivalent Mixed-Integer Convex Quadratic Programming (MIQP) reformulations of the original MINLP. The reformulations introduce auxiliary variables that linearize the multilinear interactions between binary feature-selection variables and continuous dual variables arising from the polynomial kernel expansion. We prove that the continuous relaxations of the resulting formulations are convex and establish formal equivalence with the original problem. Moreover, the convex relaxation is tight in practice, enabling efficient and scalable resolution. The first formulation exploits the symmetry structure of the kernel expansion to eliminate redundant variables, yielding a symmetry-free model. The second is a more compact formulation that further reduces the number of auxiliary variables by a factor of m (the number of training samples) through aggregation over the dual variables, substantially decreasing problem size. Both reformulations extend naturally to polynomial kernels of arbitrary degree. Computational experiments on benchmark datasets show that the proposed MIQP can be solved to certified global optimality in instances where the original nonconvex formulation is computationally prohibitive, confirming the practical effectiveness of the proposed approach.

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The zeroth order proximal algorithm

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In this talk I will focus on a black-box optimization algorithm, based on a zeroth order approximation of the proximal operator. Such algorithm has recently received significant attention, since it combine the use of function evaluations with the theoretical framework of proximal point algorithms. The zeroth order proximal algorithm depends on a parameter, which is typically vanishing in the available convergence results. However, this regime is computationally unsustainable when sampling is used to estimate the zeroth order proximal operator. We therefore propose a comprehensive analysis of ZOPO for any fixed positive temperature, and prove convergence of the zeroth order proximal algorithm under minimal assumptions on the objective function. The talk is based on [1].

References

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