BOOK OF ABSTRACTS

SS19 - Improvements in climate models for attribution, extreme phenomena and climate change impacts

2nd Joint Meeting Brazil-Italy in Mathematics

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Fabio Camilli, University of Chieti-Pescara
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SS19

Data as the Backbone of Climate Science: Novel Data Assimilation Pathways with Transfer Operators and Graph Attention Networks

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High-quality observations and reanalysis products have become a central pillar of climate science: they enable objective validation, support attribution and extreme-event analysis, and increasingly serve as training data for data-driven models. At the same time, they provide the initial conditions required for forecasting systems that feed early-warning and impact-assessment applications. This growing reliance on data places new demands on data assimilation (DA) methods, which must produce physically consistent, high-resolution state estimates in computationally efficient ways, even in nonlinear, non-Gaussian, and high-dimensional regimes typical of the Earth system.

In this contribution, we present two emerging DA approaches designed to address these challenges from complementary perspectives. The first, DATO (Data Assimilation with Transfer Operators), reformulates Bayesian filtering in terms of Koopman and Perron–Frobenius operators learned directly from data, enabling the coherent evolution of observables and probability densities without explicit covariance propagation. The second leverages Graph Attention Networks (GATs) to learn spatiotemporal representations and observation operators for soil-moisture prediction and assimilation, providing a physics-informed, computationally efficient alternative to traditional ensemble-based schemes.

Together, these approaches illustrate how modern DA can evolve to better exploit the richness of reanalysis and observational datasets, improving the consistency of initial conditions, supporting extreme-event monitoring, and opening new pathways for integrating machine learning and physical modelling in climate applications.

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A Multiplicative-Noise Mechanism for Variability Amplification under Radiative Forcing in an Arctic Energy-Balance Model

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We propose and analyse a mechanism by which CO_2-driven radiative forcing can increase Arctic temperature variability in a stochastic Sellers-type energy-balance model. Starting from a fast-slow formulation in which insolation is represented by a rapidly mean-reverting Ornstein-Uhlenbeck process, while temperature evolves on a slower macroweather timescale, a Wong-Zakai reduction leads to a stochastic energy-balance equation with multiplicative noise.

After linearising around the stable equilibrium $T^{*,\lambda}$, we derive an explicit expression for the stationary variance of the temperature anomaly and show that it increases monotonically with the forcing parameter λ whenever $T^{*,\lambda}$ lies in the ice-sensitive regime of the co-albedo. We then extend the analysis to a spatial anomaly model. Under natural stability assumptions and nonnegative noise correlations, we prove a component-wise monotone increase of the stationary covariance matrix with respect to λ. Thus, radiative forcing amplifies not only local temperature variances, but also spatial covariances, indicating an increased similarity of temperature-anomaly fluctuations across the Arctic domain.

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Recent Advances in the Analysis of Energy Balance Climate Models

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In this talk, we investigate the steady-state solutions of Energy Balance Climate Models (EBCMs). We first examine the one-dimensional case characterized by a degenerate diffusion coefficient that vanishes at the poles, before extending our analysis to the general two-dimensional framework. By employing a variational approach, we define the dedicated energy functional to establish critical qualitative properties of these solutions. Ultimately, these results provide a rigorous mathematical foundation for understanding temperature distribution and climate stability in EBCMs. This is joint work with Gianmarco Del Sarto (Technische Universität Darmstadt).

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Null-controllability for degenerate PDEs: A climatological application

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Motivated by the classical Budyko–Seller model, we investigate a class of non-autonomous degenerate parabolic problems. We first establish their well-posedness by means of Kato's Theorem. Then, deriving new Carleman estimates for the associated non-homogeneous adjoint problems, we prove the null controllability of the original systems. We also discuss ongoing research directions and open problems.

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Optimal Policies for Environmental Assets under Spatial Heterogeneity and Global Awareness

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We present a new stochastic model for the management of environmental assets in a geographical context where in each place the local authorities take their policy decisions maximizing their own welfare, hence not cooperating each other. A key feature of our model is that the welfare depends not only on the local environmental asset, but also on the global one, making the problem much more interesting but technically much more complex to study, since strategic interaction among players arise. We study the problem first from the N -players game perspective and find open and closed loop Nash equilibria in explicit form. We also study the convergence of the N -players game (when $n \to +\infty$) to a suitable Mean Field Game whose unique equilibrium is exactly the limit of both the open and closed loop Nash equilibria found above, hence supporting their meaning for the game. Then we solve explicitly the problem from the cooperative perspective of the social planner and compare its solution to the equilibria of the N -players game. Moreover we find the Pigouvian tax which aligns the decentralized closed loop equilibrium to the social optimum.

Keywords
Mean Field Games, N-players game, Pigouvian tax, environmental assets

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Reconstruction of wide spectrum forcing in transport-diffusion and Navier-Stokes equations

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Data assimilation is a machine learning technique well-established in weather and climate science, where incomplete observations are combined with dynamical models. The reconstruction of the state of the atmosphere or ocean using data assimilation has been studied intensively. In many applications, however, not only are the observations incomplete but also model components such as external forcings or surface fluxes are not exactly known.

In this talk, we present a rigorous analysis of two data assimilation algorithms for the simultaneous reconstruction of states and additive forcings in two fundamental PDE models: the transport–diffusion equation and the two-dimensional Navier–Stokes equations. We consider two simplified algorithms suitable for mathematical analysis: an iterative scheme which reconstructs time-dependent forcings, and a continuous-time nudging algorithm based on an augmented-state formulation reconstructing stationary forcings. For weather and climate application it is important that as few reruns of the dynamics as possible are used due to the immense cost to run the corresponding simulations in practice.

A central novelty is the introduction of "quasi-finite-rank forcings", whereas in previous approaches the reconstruction of the forcing was restricted to the observed degrees of freedom. This framework allows forcings with infinitely many active Fourier modes acting on degree of freedoms which cannot be restricted or controlled by the observations to suppress error propagation. More importantly it allows to treat parametrized families beyond trigonometric polynomials.

The focus of the talk is on the mathematical analysis. We establish rigorous convergence results for both algorithms for both the linear transport–diffusion equation and the nonlinear two-dimensional Navier–Stokes equations, providing explicit conditions on the observational resolution and algorithmic parameters in terms of the parameters and properties of the system assumed to be known. The content of this talk is published in [**??**].

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Mathematical Representations of Earth System Dynamics: From Phase Space to Interacting Natural Modes

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The mathematical representation of Earth System dynamics plays a fundamental role in how numerical simulations, climate variability, and the mechanisms underlying extreme events are interpreted. While numerical weather and climate models solve the governing equations in physical space, their interpretation has traditionally combined physical reasoning with empirical diagnostics, such as teleconnection patterns and frequency–wavenumber analyses.

This lecture explores a complementary perspective in which the state of the Earth System is represented in terms of interacting natural modes. Within this framework, linear wave propagation, interactions with the background state, nonlinear mode coupling, and coupling with the ocean and biosphere emerge naturally as distinct dynamical processes governing the evolution of modal amplitudes and phases. Teleconnections and multiscale climate variability are interpreted as manifestations of information flow among dynamically coupled natural modes.

Examples involving tropical–extratropical interactions illustrate how this representation provides new mathematical diagnostics for interpreting numerical simulations and investigating the physical mechanisms underlying climate variability. More broadly, this perspective suggests a unified dynamical framework for examining how climate change modifies wave propagation, modal interactions, and the processes leading to attribution, extreme events, and climate impacts.

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A Possible Dynamical Mechanism for Global Warming Impact on Nature Climate Variability and Extreme Climate Events

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According to the well-known Clausius-Clapeyron relation, the higher the temperature the higher the water vapor saturation pressure and, consequently, the higher the capacity of the atmosphere to keep water vapor. On the other hand, the water vapor content of the atmosphere plays a crucial role in the thermodynamic processes associated with the convection activity and the formation of clouds and precipitation. The proper parameterization of the effects of such processes on the large-scale atmospheric dynamics constitutes one of the main challenges of climate modeling, being certainly one of the main uncertainty sources in future climate projections.

Here, we consider a toy model of the atmospheric circulation to illustrate how the increase of the water vapor content of the atmosphere associated with a warmer planet can modify both the amplitude and the characteristic frequency of natural climate variability modes and, consequently, to the distribution of extreme climate events. This model consists of a highly truncated version of the moist equatorial $\beta-$plane primitive equations, with the diabatic heating being parameterized by the simple wave-CISK formulation in which the heating is proportional to the lower troposphere moisture convergence, with the background moisture field being prescribed to mimic the climatological spatial distribution of the specific humidity and its maximum (equatorial) value being assumed as a free parameter.

In this way, we have shown that increasing the maximum value of the background moisture field from its reference value can trigger resonant triad interactions that would otherwise be absent. We have illustrated a representative example for an interacting wave triad composed of planetary-scale equatorial Rossby and Kelvin modes, both having the first baroclinic mode vertical structure, and a barotropic Rossby mode. These modes are known to be the dominant variability modes of the so-called Madden-Julian Oscillation (MJO), which is the dominant intraseasonal variability mode of the tropical atmospheric circulation and, consequently, an important mode of natural climate variability. As the MJO is one of the main source of extreme climate events, since it constitutes the main modulator of the weather systems, the results point out that understanding the interplay between the nonlinear dynamics of the atmospheric normal modes and the parameterized thermodynamic processes related to moisture is crucial to understand the response of the natural climate variability modes to the anthropogenic forcing.

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Climate change reduces the subseasonal variability of the North Atlantic jet stream

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The response of the extratropical atmospheric circulation to climate change is one of the key uncertainties in climate science. In the North Atlantic sector, circulation trends are emerging that were not predicted, while future projections exhibit significant disagreement, limiting our ability to quantify climate change impacts over Europe. In this work, we identify an ongoing reduction in the subseasonal variability of the North Atlantic eddy-driven jet stream, a key regulator of European weather. By analyzing observation-based reanalyses and an extensive ensemble of state-of-the-art climate model simulations, we demonstrate that this reduction is a robust signature of climate change that is expected to continue unless strong climate change mitigation is achieved.

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Testing for Non-Stationarity in Time-Dependent Spherical Random Fields

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In this talk, we investigate, from a statistical point of view, time-dependent spherical random fields. After introducing the general framework, we focus on three main topics: Q1) Structural breaks exploration for general time dependent spherical random fields, introducing a test procedure under a broad range of alternatives, including deterministic trend, abrupt change, local stationarity, see [1]; Q2) Parameter estimation of particular models, that are so-called stationary SPHARs, for which is possible to study estimation and prediction via either some classical OLS procedure or penalized LASSO procedure, see [2, 3]; Q3) Unit root type of non-stationarity in SPHARs, which is just a random walk type of non-stationarity. In this context, we present a procedure for estimation and inference on attractor and cointegrating spaces, see [4]. Our results are then applied to NCEP data on global temperature: our estimates suggest that Climate Change does not simply affect global average temperatures, but also the nature of spatial fluctuations at different scales.

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Time-fractional degenerate diffusion equations as possible model for climate changes

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For clarifying mechanism of phenomena under consideration, mathematical analysis based on suitable model is still important. For the climate changes, we discuss the time-fractional degenerate diffusion equations as possible model:

$$d_t^\alpha u(x,t) = -Au(x,t) \qquad (*)$$

in a spatial domain Ω and the time $t > 0$. Here $d_t^\alpha u$ is a fractional derivative with order $\alpha \in (0,1) \cup (1,2)$ and $-A$ is a degenerate elliptic operator. The time-fractional derivative can much more flexibly describe various types of diffusion especially in highly heterogeneous media, while the degeneracy of A is a commonly used model.

There are few mathematical results about time-fractional degenerate diffusion equations as possible model. We discuss the well-posedness for the initial boundary value problem for (*), which is the primary theoretical issue. Then, we consider inverse problems of determining governing parameters in (*) by available data.

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