BOOK OF ABSTRACTS

SS18 - Optimal Control, Optimal Transport and Related Fields

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SS18

A new constraint qualification for continuous-time nonlinear programming based on asymptotic KKT conditions

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This work is concerned with the following continuous-time programming problem:

$$\begin{array}{ll} \text{minimize} & P(x) = \displaystyle\int_0^T f(x(t),t)dt \\ \text{subject to} & h(x(t),t) = 0 \text{ a.e. } t \in [0,T], \\ & g(x(t),t) \leq 0 \text{ a.e. } t \in [0,T], \\ & x \in L^\infty([0,T];\mathbb{R}^n), \end{array} \tag{CTP}$$

where $f : \mathbb{R}^n \times [0,T] \to \mathbb{R}$, $h : \mathbb{R}^n \times [0,T] \to \mathbb{R}^p$ and $g : \mathbb{R}^n \times [0,T] \to \mathbb{R}^m$ are given functions. Necessary optimality conditions of the Karush-Kuhn-Tucker (KKT) type can be found, for instance, in Monte and de Oliveira [1]. Asymptotic Karush-Kuhn-Tucker (AKKT) optimality conditions were recently obtained in Monte, Moreira and de Oliveira [2].

AKKT optimality conditions are distinguished from other approaches in the literature by virtue of their capacity to be effectively derived through numerical methods, such as the utilization of an appropriate version of the augmented Lagrangian method. These tools are of a theoretical nature; nevertheless, they possess practical utility in the identification of candidate solutions to continuous-time programming problems. While this type of optimality condition is valid without imposing any constraint qualification, it is not sufficiently robust to generate good candidate solutions. In certain instances, solutions that meet AKKT conditions may not be stationary. In this study, we investigate conditions that effectively refine this set of candidate solutions with the same precision as the classical KKT conditions. This is achieved by introducing a novel constraint qualification, designated AKKT-regularity. It has been demonstrated that, under AKKT-regularity, each local optimal solution satisfies the KKT conditions. Furthermore, it has been shown that this constraint qualification is the weakest possible to ensure such a property. Additionally, sufficient conditions are provided for its applicability.

The complete version of this work is available in Moreira, Monte and de Oliveira [3].

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Optimal control of the viscous wave equation via the Pontryagin maximum principle

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A tracking-type optimal control problem governed by the viscous wave equation with a distributed-source control and L^2-L^1 control costs is investigated.

For this class of PDE-constrained linear-convex problems, a Pontryagin maximum principle (PMP) in the PDE setting is derived, and it is shown that the pointwise maximization condition is also sufficient for optimality.

Based on the PMP, a sequential quadratic Hamiltonian (SQH) method is implemented and well-posedness and convergence of the this method is discussed. Results of numerical experiments are presented that successfully validate the proposed PMP-based optimization framework.

This is joint work with Souvik Roy (University of Texas at Arlington, USA).

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Geometric control of optimal transport maps

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In this talk we consider the problem of transporting one probability measure into another through the flow of a driftless control-affine system. Under suitable regularity conditions, controllability of the system by means of open-loop controls ensures the existence of time-varying feedback controls whose time-one flow realizes the optimal transport map for the quadratic cost. The analysis connects the controllability of finite-dimensional systems with that of the associated continuity equation, providing a macroscopic description of evolving probability measures. These controllability problems arise naturally in the simultaneous control of large-scale multi-agent systems, where agent distributions evolve as controlled densities [3], or in Neural Ordinary Differential Equations, where control inputs correspond to parameters in continuous-depth models [1] [4]. This talk is part of the paper [2].

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An Asymptotic Maximum Principle for Impulsive Optimal Control Problems

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Arising in several practical situations [3, 2, 4, 1], impulsive optimal control problems have been studied very thoroughly over the years. Their most common formulation involves a state dynamic that is affine with respect to the control variables, a fixed initial state, a targeted final state and control variables within a closed convex cone [5]. There, also, may or may not be state constraints present, that is, constraints with respect to state variables other than the dynamic.

However, all results published so far have been *exact* conditions, that is, conditions fulfilled *at the optimal process*. Although of very high importance for theoretical and practical works, such results fail to address the fact that most numerical resolutions of optimization problems work in a different way, where one *builds a sequence of processes close enough to the feasible region* (but which might not be feasible) in the hope that its limit process (or one of them if such sequence has multiple) is, at least, stationary.

Therefore, the goal of this talk is to present an *Asymptotic Maximum Principle* for an impulsive optimal control problem with state constraints — the name *asymptotic* is due to the fact this result is fulfilled at a sequence converging to the optimal solution. The problem considered will be a slight simplification of the (STS) problem presented in [5] done by removing the constraint $\dot{V} = \|u\|$ and swapping $W(\mathcal{K})$ by $]0, \infty[\, \times \mathcal{K}$, so that the convexity hypothesis in [7] is fulfilled. With that, most of the proof follows the work done in [6].

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A Mean Field Games Perspective on Evolutionary Clustering

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Clustering is a central task in unsupervised learning and is commonly addressed through finite mixture models and Expectation-Maximization (EM) procedures. In many applications, however, the relevant groups are not static: they evolve in time and should be tracked by a model that combines data fidelity, temporal coherence, and mass conservation. We propose a control-theoretic framework for evolutionary clustering based on Mean Field Games (MFG). The data distribution at time t is described by a density on $\mathbb{R}^d$, while each cluster is represented by a subpopulation whose evolution is governed by a Fokker-Planck equation coupled with a Hamilton-Jacobi-Bellman equation. The coupling is driven by variational costs, such as entropy penalization and Kullback-Leibler-type fidelity terms, and the posterior responsibilities play the role of the link between the observed density and the evolving mixture.

The proposed formulation gives a continuous-time and non-parametric interpretation of evolutionary clustering. To connect this point of view with classical statistical learning, we study the case of time-dependent Gaussian mixtures. For suitable choices of the drift, the MFG dynamics preserve the Gaussian structure and recover, at every time, the same parameters selected by the EM maximization of the log-likelihood. More precisely, the cluster weights, means, and covariance matrices coincide with the corresponding responsibility-weighted moments of the observed density. This shows that the classical EM update can be embedded in a conservative population dynamics.

We also introduce time-averaged log-likelihood functionals, obtained by convolving the instantaneous likelihood with asymmetric or symmetric temporal kernels. This provides a natural regularization mechanism for noisy or rapidly varying data and leads to smoothed cluster parameters. Numerical experiments, based on a sequential EM-PDE scheme, illustrate the stability of the approach in the Gaussian case and support its use as a bridge between discrete evolutionary clustering algorithms and continuous mean-field models.

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Feedback Control and Local Convexification of Wasserstein Gradient Flows

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This talk discusses a feedback construction for entropy-regularized Wasserstein gradient flows

$$\partial_t \mu_t = \nabla \cdot \left(\mu_t \nabla \frac{\delta \mathcal{E}}{\delta \mu}(\mu_t) \right) + \sigma \Delta \mu_t, \quad \sigma > 0, \tag{1}$$

near a prescribed stationary state $\bar{\mu}$ defined on the torus. Equation (1) is the gradient flow of $\mathcal{F} := \mathcal{E} + \sigma \,\mathrm{Ent}$ in the 2-Wasserstein space, where Ent is the Boltzmann entropy. The first step for this construction is to realize the Wasserstein Hessian of $\mathcal{F}$ at $\bar{\mu}$ as a self-adjoint operator A on the weighted space $\dot{H}^1_{\bar{\mu}}$, with compact resolvent. Under the Riesz map $\mathcal{I}_{\bar{\mu}} \phi = -\nabla \cdot (\bar{\mu} \nabla \phi)$, the linearization of (1) is related to A by

$$L = -\mathcal{I}_{\bar{\mu}} A \mathcal{I}_{\bar{\mu}}^{-1}.$$

Slow convergence to $\bar{\mu}$ and linear instability are encoded by the small and negative eigenvalues of A, respectively.

Given a threshold $\delta > 0$, only finitely many eigenvalues of A lie below δ. To impose this spectral threshold, we formulate a linear-quadratic control problem for the linearized dynamics, with fixed shape functions α_j chosen as eigenfunctions associated with eigenvalues $\lambda_j \leq \delta$. The optimal feedback is characterized by an algebraic Riccati equation and yields a bounded finite-rank perturbation of the Hessian. Equivalently, the closed-loop equation is still a Wasserstein gradient flow, now for

$$\mathcal{F}_{\mathrm{cl}}(\mu) = \mathcal{F}(\mu) + \frac{1}{2} \sum_{\lambda_j \leq \delta} \pi_j \left(\int e_j \, d\mu \right)^2,$$

where π_j are the diagonal Riccati coefficients. Under local lower-semicontinuity assumptions on the Hessian, this gives local displacement convexity of $\mathcal{F}_{\mathrm{cl}}$ on an admissible class of densities. A Lipschitz estimate for the nonlinear remainder then yields local exponential stability of $\bar{\mu}$. I will end with examples for McKean–Vlasov dynamics, constrained Fokker–Planck equations, and Fokker–Planck equations on the sphere, together with plots of the modified energy landscape.

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Optimal Control and Reinforcemenl Learning

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In this talk, we will discuss the link between the theory of Optimal Control problems with uncertainty and a class of Model based, Probabilistic, Reinforcement Learning (RL) algorithms. In particular, we will discuss a framework to analyze the convergence properties of such Probabilistic RL algorithms and discuss related open questions.

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Vanishing viscosity limit for Hamilton–Jacobi equations in the Wasserstein space

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Hamilton–Jacobi equations on the Wasserstein space of probability measures play a central role in the analysis of deterministic and stochastic systems, including optimal control and mean field interactions. After introducing a unified viscosity solution framework for both first-order equations and semilinear equations with idiosyncratic noise, based on a suitable notion of subdifferential, we discuss its main comparison and stability properties.

We then present a vanishing viscosity result for semilinear Hamilton–Jacobi equations in the Wasserstein space. In particular, we show that solutions converge to those of the corresponding first-order equation as the noise intensity tends to zero and establish an optimal convergence rate. These results provide a PDE-level description of the transition from stochastic to deterministic dynamics and highlight the role of viscosity solutions in the analysis of zero-noise limits.

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