BOOK OF ABSTRACTS

SS17 - Category theory and Applications

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Organizers:

Hugo Mariano, University of São Paulo
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2-Categorical Methods in Programming Language Theory

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An important question in computer science is how to assign mathematical meaning to programs and proofs. Since the 1980s, category theory has played an important role in classifying the diverse tapestry of different mathematical domains that can interpret programming languages and type theories. Perhaps the simplest example is how Cartesian closed categories (CCC) can be used to interpret simple type theories and the simply typed λ-calculus, an idealized pure functional programming language.

An important application of such a categorical treatment can be found in the construction of non-starndard models, often called logical relations models, to prove global properties of programming languages. After foundational work by authors such as Jacobs, Hermida and Katsumata, we can identify such models as certain faithful Grothendieck fibrations.

Definition 1. A logical relations fibration for the simply-typed λ-calculus is a faithful Grothendieck fibration that strictly preserves the Cartesian closed structure.

These fibrations encompass useful examples such as predicate fibrations and are stable under pullback along product preserving functors, meaning that they can be repurposed in different applications by judiciously pulling back along appropriate functors. Unfortunately, modern programming languages often require more categorical structure, such as monads, lax monoidal functors and adjunctions, rendering the theory above as inadequate for realistic applications.

In this talk I will present recent work where we show how fibrations internal to 2-categories of models provide a robust and general notion of fibration for logical relations. Our main theorem shows that under mild assumptions fibrations for logical relations are stable under pullback.

A resonable assumption we make is that 2-categories of models are 2-functor categories $[\mathsf{D}, \mathcal{C}]_{\mathsf{lx}}$. In this light, Suppose that $\mathcal{C}$ has a sub-class of *tight* 1-cells which are all fibrations in $\mathcal{C}$ — intuitively, the fibrations that strictly preserve structure—and that the underlying 1-category $\mathcal{C}_0$ is such that (1) the pullback of tight 1-cell along arbitrary 1-cells exists in $\mathcal{C}_0$; and (2) these pullbacks are preserved by every $\mathcal{C}_0(C, -) : \mathcal{C}_0 \to \mathbf{Cat}$.

Theorem 1. *In the situation above, consider a cospan* $(G \xrightarrow{\phi} H \xleftarrow{\kappa} \widehat{H})$ *in* $[\mathsf{D}, \mathcal{C}]_{\mathsf{lx}}$ *such that*

(1) *κ is strict and each 1-cell component κ_d is tight;*

(2) *For each $d \in \mathsf{D}$ the pullback $(\phi_d)^*(\kappa_d)$ of κ_d along ϕ_d exists in $\mathcal{C}$ and is tight.*

Then the 1-cells $(\phi_d)^*(\kappa_d)$ form a strict, componentwise-tight transformation, which is the pullback $\phi^*(\kappa)$ in $[\mathsf{D}, \mathcal{C}]_{\mathsf{lx}}$.

As a concrete example, when choosing D to be the 2-category freely generated by a monad structure — also known as the walking monad — the 2-category $[\mathsf{D}, \mathbf{Cat}]_{\mathsf{lx}}$ is equivalent to the 2-category of monads, monad morphisms and monad functor transformations, as defined by Street.

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Diagrams and Dynamic Programming

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In many branches of mathematics and computer science, it is convenient to organize large, complicated data into smaller pieces. Often this is done by specifying diagrams in some category and viewing the diagram itself as a distributed representation of an object. Examples abound across mathematics, spanning areas from geometric group theory to hybrid dynamical systems and diagrammatic methods in physics.

This perspective appears frequently in graph theory and theoretical computer science as well—often without categorical language—where large graphs or databases are decomposed into simpler pieces, either to support inductive proofs or to enable dynamic programming algorithms.

In this talk, I will emphasize a complementary viewpoint: a diagram can be understood as a system of constraints (or equations), where objects play the role of variables and morphisms encode relationships between them, and where taking limits corresponds to solving the resulting system. This perspective allows us to think not only about data, but also about the solution spaces that sit above it.

The emphasis will be on a *programmatic vision*: I will touch on a range of results from past collaborations as well as some new developments. As usual, this is an invitation to get involved—please feel free to reach out.

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[1]This is based on joint work with a variety of coauthors and students over the years: Anny Beatriz Azevedo, Ernst Althaus, James Fairbanks, Guilherme Gomes, Zoltan Kocis, Laura Larios-Jones, Jade Master, Emilio Minichiello, Matteo Capucci, Daniel Rosiak, Will Turner.

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Relational calculus in algebra and logic

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In this talk we will present connections and interactions between two different frameworks for relational calculus, one coming from algebra and the other one coming from logic and computer science. Both of them were established with the same idea: to develop mathematical theories using general relations between sets rather than functions. While they were developed independently and in different directions, we show how the two theories are deeply connected with each other.

The algebraic framework for relational calculus that we will consider is that of *arcs*, recently introduced by Z. Janelidze and D. Rodelo, in the context of the theory of forms. On the logic side, we will consider the framework of *relational doctrines*, introduced by F. Dagnino and F. Pasquali, in the context of the theory of doctrines.

Like arcs and relational doctrines, also the theory of forms and the theory of doctrines were established with the same objective: to study and advance algebra and logic respectively via a categorical fibrational approach. Both theories brought to fundamental breakthroughs in their fields. But the connections between the two theories remained undiscovered until recently, when the deep similarities have begun to be investigated on multiple fronts.

In this talk, we will explain how we studied and applied the connections between arcs and relational doctrines to establish new notions and results in the two worlds. In this context, we will show that normals and equivalence relations (and dually conormals and coequivalence relations) play the same role in the theory of arcs and relational doctrines respectively. We will also see how relational doctrines seem to be the right framework in logic to study contexts in which not all objects are conormal.

Furthermore, we will show how we generalized the algebraic notions of regularity, exactness and Malt'sev condition from arcs to relational doctrines. And we will explain the fruitful new insights that this generalization has brought on the algebraic side. Finally, we will discuss how to complete arcs and relational doctrines under quotients and comprehensions. We will show that, interestingly, these two completions do not always distribute with each other, but they do under certain assumptions.

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Cartesian and additive opindexed categories

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We give a characterization of cartesian objects in the cartesian 2-category OpICat of opindexed categories. They are given by pseudofunctors $F\colon B \to \mathsf{Cat}$, where B has finite products and the canonical oplax monoidal structure L on F admits a right adjoint R (in a suitable sense), which makes F a lax monoidal pseudofunctor. As a special case, if we restrict our attention to functors $F\colon B \to \mathsf{Set}$, the cartesian ones are just finite-product preserving functors. When moreover B is additive, such F factorizes through the category Ab of abelian groups, and the corestriction is an additive functor. Then we consider opindexed groupoids, i.e. pseudofunctors $F\colon B \to \mathsf{Gpd}$. The cartesian objects here are pseudofunctors preserving finite products up to equivalences. When moreover B is additive, we find that such F factorizes through the 2-category of symmetric 2-groups. In fact, we characterize the latter as 2-additive pseudofunctors (in the sense of Dupont).

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Beyond Mac Lane Strictification

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Mac Lane's coherence theorem [5, 6] says that every monoidal category is monoidally equivalent to a strict one. The standard strict replacement, however, changes the category: its objects are formal finite words of objects of the original category. Thus Mac Lane strictification is a replacement theorem, not an in-place strictification theorem. The question of this talk is whether, and when, the strict monoidal structure can instead be put on the original category itself.

This distinction is familiar from the broader viewpoint of two-dimensional universal algebra. Coherence and strictification questions for pseudoalgebras of 2-monads have a long history, including the work of Kelly, Power, Blackwell–Kelly–Power, Lack, and Shulman [1, 7, 2, 9]. In particular, the existence of strictification adjoints was characterized by biadjoint triangle and lifting theorems introduced and proved in [3, 4]. The monoidal question considered here is a sharper object-level version of this phenomenon: not merely whether a weak structure can be replaced by an equivalent strict one, but whether the strict structure can be placed on the original category itself.

The problem is also inspired by Schauenburg's formulation of equivalent monoidal structures on a fixed underlying category [8]. In that setting, given a monoidal category $(\mathcal{C}, \otimes, I)$, one asks whether there is a strict monoidal structure $(\mathcal{C}, \boxtimes, I')$ on the same category such that the identity functor $\mathrm{Id}_{\mathcal{C}} \colon (\mathcal{C}, \boxtimes, I') \to (\mathcal{C}, \otimes, I)$ is a strong monoidal isomorphism.

Based on joint work with Paul Blain Levy, I will explain how this leads to an object-level obstruction theory. A central role is played by a labeling construction: from suitable label-bearing categories and a monoidal structure, one obtains a $\otimes$-labeling monoid recording the equations that any in-place strictified tensor product must satisfy. The strictification problem then becomes a separation problem: the object-level equations forced by strict associativity and strict units must not collapse the original objects in a way incompatible with the given monoidal structure. The resulting criterion gives both obstructions and positive theorems; in particular, every monoidal structure on $\mathbf{Set}$, on $\mathbf{Set}^2$, and on $\mathrm{Fam}(\mathbf{Set})$ admits an in-place strictification.

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Topos-like Completions

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We employ the elementary quotient completion introduced in [3] to freely complete suitable Lawvere doctrines, thereby obtaining elementary toposes, Penon's quasi-toposes, and Frey's q-toposes building on [2] in a modular way.

The main examples include realizability toposes, supercoherent localic toposes, and Hyland's category of assemblies in a classical metatheory, making crucial use of the Axiom of Choice, through the results of [1], [5], [6].

Moreover, combining [1] and [4], we show that the category of assemblies, formulated within a suitable constructive metatheory, is a free q-topos if and only if the Zermelo Axiom of Choice holds.

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Propointed categories

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In recent work, G. Janelidze introduced the notion of an *ideally exact category*: a Barr exact category with finite coproducts equipped with a monadic functor to a semi-abelian category. This framework generalises the classical situation of the forgetful functor from the category of unital rings to the category of possibly non-unital rings. Through such a functor, the correspondence between equivalence relations and normal subobjects in semi-abelian categories gives rise to a theory of *ideals* in ideally exact categories, extending the familiar theory of ideals in unital rings.

The aim of this talk is to explore more generally how algebraic properties may be transported back from pointed to non-pointed categories along suitable functors. To this end, we introduce and characterise *propointed categories* as categories equipped with an appropriate "forgetful functor" to a pointed category. We show that, when the target category satisfies additional categorical-algebraic conditions – for instance, being pointed protomodular or homological – the original category indeed inherits corresponding properties, including versions of the Five Lemma and Noether's isomorphism theorems.

This perspective naturally leads us to discuss a fundamental construction already highlighted in Janelidze's work: for a lex category $\mathbf{A}$ with an initial object, pulling back along the unique morphism $0 \to 1$ yields a functor $\mathbf{A} \simeq \mathbf{A}/1 \to \mathbf{A}/0$ (with the slice $\mathbf{A}/0$ being pointed). We show that this construction is universal, in a two-dimensional sense, among ways of associating a pointed category to $\mathbf{A}$.

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Grothendieck prelopologies: towards a closed monoidal sheaf category

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In this paper, we present a generalization of Grothendieck pretopologies, tailored for semicartesian monoidal categories $\mathcal{C}$, which yields a closed monoidal category of sheaves rather than a closed cartesian one. This result is established via a modified sheafification process: a left adjoint to the appropriate inclusion functor that, unlike the classical case, does not preserve all finite limits. When the monoidal structure on $\mathcal{C}$ is given by the categorical product, our construction recovers the usual theory of Grothendieck pretopologies. The motivation for this generalization arises from a particular notion of sheaves on quantales, which does not form a topos. Since quantales are a generalization of locales well suited to model propositional linear logic, they are natural candidates to obtain a linear version of a topos. In [2] we proposed a new definition of sheaves on semicartesian quantales (a non-idempotent generalization of locales, where $a \otimes b \leq a \wedge b$) and showed that the category of sheaves on a quantale $Sh(Q)$ is not necessarily a topos. Moreover, we proved that for a quantale Q, the poset of subobjects of the terminal sheaf in $Sh(Q)$ is quantalic isomorphic to Q. In this paper, we explore that a cover $u = \bigvee_{i \in I} u_i$ in a quantale does not satisfy the axioms of a Grothendieck pretopology and so we expand it to a *Grothendieck prelopology.* The Grothendieck prelopologies are a notion of covering that are better suited for (semicartesian) monoidal categories with equalizers. We introduce the technical device called *pseudo-pullbacks*, which generalizes pullbacks and replaces them to go from pretopologies to prelopologies. If the monoidal structure is given by the categorical product, then Grothendieck prelopologies are precisely the well-known Grothendieck pretopologies. After we discuss the Grothendieck prelopologies, we introduce sheaves for them as a functor satisfying certain gluing conditions that can be translated in the format of an equalizer diagram, as usual, but again we replace pullbacks with *pseudo-pullbacks.* We conclude that the category of sheaves defined by a (strong) Grothendieck prelopology admits the structure of a closed monoidal category: the monoidal structure in the base category $\mathcal{C}$ induces a monoidal structure in $PSh(\mathcal{C})$ given by the Day convolution; then we show that the inclusion functor from such sheaves to presheaves have a left adjoint functor (called sheafification) that induces the closed monoidal structure in the category of sheaves. Since in the quantalic case $Sh(Q)$ is not a Grothendieck topos, our sheafification cannot preserve all finite limits, in general. We show how this generalization of the sheaf concept leads to applications in cohomology, through the quantalic case ([1]). We believe that further developments of this theory will be useful both for the categorical logician interested in a generalization of toposes with a different internal logic and for those interested in sheaf methods in non-commutative geometry, since the monoidal product does not have to be symmetric.

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Stone–Gelfand duality, arithmetic spaces, and lax comma categories

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Stone–Gelfand duality is the umbrella name for a family of classical theorems—due to Gelfand, Kakutani, the Krein brothers, Stone, Yosida, and others—that represent the category KH of compact Hausdorff spaces, dually, through algebras of continuous real-valued functions. In the version obtained by Yosida in 1941, the dual algebraic objects are the metrically complete unital vector lattices: complete, normed vector lattices of continuous functions, with the supremum norm induced by a distinguished unit.

In this talk I discuss recent joint work with M. Abbadini and L. Spada [1] in which Yosida's duality is extended by relinquishing the requirement that the function systems be real vector spaces, keeping only their structure of lattice-ordered Abelian group. One main consequence is that each point of a dual space now has its own *arithmetic character*. Indeed, such a point comes with a metrically complete subgroup of $\mathbb{R}$—either $\frac{1}{n}\mathbb{Z}$, or the whole of $\mathbb{R}$—which arises as the set of values the functions collectively take at that point. These arithmetic characters can therefore be identified with the natural numbers $\mathbb{N}$, where 0 represents $\mathbb{R}$, and n represents $\frac{1}{n}\mathbb{Z}$. Going beyond the local situation at a point, it turns out that the global arithmetic nature of a dual space is neatly encoded by a continuous map from the space to the natural numbers $\mathbb{N}$, ordered by divisibility and topologised by the upper topology. We call the resulting objects *arithmetic spaces*.

Our main result isolates, by an intrinsic separation axiom, the *arithmetically normal* spaces—those satisfying a divisibility-aware version of normality—and establishes that they are dually equivalent to the category of metrically complete lattice-ordered Abelian groups. The crucial ingredient is a substantial generalisation of Urysohn's Lemma in which denominators must be respected along the way. Yosida's classical duality is transparently recovered as the full subcategory of those arithmetic spaces whose arithmetic character is constantly zero.

The category-theoretic heart of the picture is that arithmetic spaces may be conceived of as objects of the *lax comma category* $\mathsf{Top}/\!\!/\mathbb{N}$ of spaces over $\mathbb{N}$. This intriguing connection is currently ill-understood at the conceptual level. Our results suggest a number of interesting research directions that may shed light on that connection.

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Beth companions of locally finitely presentable categories

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The problem of establishing whether all epimorphisms in a class of algebras (or related structures) are surjective — and, more generally, of characterising epimorphic images of algebras — has a long tradition. See e.g. [Isb66, HI67, Sto68, SAC68, Rei70] for some earlier work on this topic. For many interesting classes of structures, this is equivalent to asking whether they form a *balanced* category, meaning that every morphism that is both monic and epic is also an isomorphism. The question of characterising the algebraic theories (in the sense of Lawvere) such that the corresponding categories of algebras are balanced, stated in [Law68] and still open to this day, has led to the investigation of so-called *implicit partial operations*.

For example, Hébert has shown that a locally presentable category is balanced precisely when each implicit partial operation in a certain natural class can be extended to an implicit total operation [Héb98]. In algebraic logic, this corresponds to the so-called *Beth definability property*, stating that every implicit concept admits an explicit definition; cf. e.g. [HMT85, Hoo00, BH06].

We will present the notion of a *Beth companion* for an arbitrary locally finitely presentable category, in which the Beth definability property is "forced" to hold. We will discuss when Beth companions exist and how to describe them explicitly. A notion of Beth companion from the standpoint of universal algebra was introduced independently in [CKM26].

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Hyperdoctrines are double functors

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The specification of a logical doctrine can be stratified in three levels: the choice of the underlying type theory of *contexts*, the designation of the *semantics* for predicates over such contexts, and an *interpretation map* connecting these pieces of data (subject to compatibility conditions). Different flavours of logic amount to the specification of different contexts, semantics, or properties of the interpretation map. In this talk, we will argue that double categories give a natural framework in which such parameters can be adjusted to obtain new logics.

We will focus on *regular hyperdoctrines*, which are powerful enough to talk about relations. It is well-known that pseudo functors from bicategories of spans are equivalent to Beck-Chevalley bifibrations, and therefore capture the relationships underlying the adjunctions suitable as semantics for existential quantification. This was further expanded upon in the context of double categories by Dawson, Paré and Pronk [1]. However, these works do not take into account *Frobënius reciprocity*, which can be understood as a compatibility condition between the monoidal structure on predicates and existential quantification. The main result of the talk is that this, too, can be captured in double-categorical terms:

Theorem: *(Generalised) regular hyperdoctrines correspond to lax symmetric monoidal pseudo double functors* $\mathbb{S}\mathrm{pan}(\mathcal{C})^{\mathrm{op}} \to \mathbb{Q}\mathrm{t}(\mathrm{Cat})$ *whose laxators are companion commuter transformations (in the sense of [3]).*

In fact, Cat can be replaced by any symmetric monoidal 2-category $\mathcal{K}$;; this approach allows us to functorially adjust contexts, semantics, and interpretations. Connecting hyperdoctrines to double functors is thus useful for compositionality results for logics (see Special Session 15), and suggests that they might be conceivable as 2-algebraic structures (in the 2-monadic or $\mathcal{F}$-sketch sense). Moreover, there's a well-developed general theory of systems based on double categories [2], and this result allows one to import regular hyperdoctrines as systems themselves. As an application, we can recover a form of graphical regular logic suitable for modelling specifications of systems (e.g., port-plugging systems) that compose operadically [4]. If time permits, I will mention how these ideas suggest a new notion of *regular double hyperdoctrine*, where 2-dimensional contexts and semantic environments are allowed (in the form of double categories).

This talk is based on [5].

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