BOOK OF ABSTRACTS

SS15 -Logic and Foundations of Mathemtics

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SS15

When are valuations automatically definable?

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Valuations provide a fundamental tool in algebra, number theory, and model theory. Since a field may admit many distinct valuations, a natural question is whether a given valuation can be recovered from the field structure alone, without adding it to the language. In this talk, we approach this question through properties of the value groups and residue fields arising from henselian valuations. We say that an ordered abelian group (respectively, a field) has automatic definability if every henselian valuation having that group as its value group (respectively, that field as its residue field) is definable in the language of rings. We present recent characterizations of such groups and fields in terms of model-theoretic properties — augmentability by infinitesimals for ordered abelian groups and t-henselianity for fields — and explain how these notions admit natural order-topological characterizations.

This is part of a joint work with Blaise Boissonneau (University of Düsseldorf), Franziska Jahnke (University of Münster), and Pierre Touchard (University of Dresden), as well as ongoing work with Lothar Sebastian Krapp (University of Zurich) and Salma Kuhlmann (University of Konstanz).

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The Minimality of First-Order Logic

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We aim to prove a new characterization of first-order logic in terms of minimality, that is, that first-order logic is a minimal abstract logic satisfying some suitable property. In order to do this, we define and study what we have called Lindström's Fundamental Property, denoted by $\mathcal{P}$. This property can be understood as a separation property: $\mathcal{P}(l)$ says that given a compact extension L of l, if the classes of models of two fixed $L(\Sigma)$-sentences φ and ψ can be separated by classes of Σ-structures closed under the partial isomorphism relation $\cong_I$ between reducts to $\Sigma_0 \subseteq_{fin} \Sigma$, then there is a finite set Γ_0 of $l(\Sigma_0)$-sentences such that the classes of models of φ and ψ can be separated by classes of Σ-structures closed under the natural logical equivalence $\equiv_{\Gamma_0}$, induced by Γ_0, between reducts to Σ_0. First-order logic is a minimal logic satisfying this property.

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Reasoning with vague events: Probabilistic Logics Grounded in Łukasiewicz Logic

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MV-algebras stand to the infinite-valued propositional Łukasiewicz logic as Boolean algebras do to classical logic [1], providing a natural algebraic semantics for reasoning with vague events. It has become increasingly evident that MV-algebras also constitute a robust foundation for generalized notions of probability. Early evidence in this direction can be found in [6], where the notion of *average degree of truth* motivated the introduction of *states* on MV-algebras. A state of an MV-algebra A is a normalized map into $[0,1]$ that is additive on disjoint elements, extending de Finetti's framework of subjective probability to non-classical logic. Indeed, the so-called Kroupa–Panti theorem [3, Section 4] establishes a bijective correspondence between states on a MV-algebra A and Borel-regular probability measures defined on the set of maximal ideals of A.

These insights have motivated an extensive line of research on probability in Łukasiewicz logic. A general approach to probabilistic two-layered logics is described in [2] and it allows one to reason explicitly about the probabilities of (non-necessarily boolean) events within a fixed substructural logic. From an algebraic standpoint, alternative approaches have been proposed based on *internal states*, aimed at internalizing probabilistic notions as unary operations, though at the cost of permitting self-application. More recently, a different and fully equational perspective has been introduced through the notion of *equational states* [4]. These are two-sorted structures $(\mathsf{A}_1, \mathsf{A}_2, s)$ in which the elements of each sort form an MV-algebra with customary operations, and the operation s, which we call a *state-operation*, has A_1 as domain and A_2 as codomain. This approach opens the way to studying probabilistic notions with algebraic tools, preventing at the same time the iteration of s. Equational states have been shown to provide the algebraic semantics of the two-layer probabilistic logic $\mathsf{FP}(\text{Ł}, \text{Ł})$ [5], designed to reason about vague events and their probabilities. In this talk, we outline this landscape and present recent results on equational states, including their algebraic structure and logical counterparts. We also discuss a refined two-sorted logical system, EsL, which admits equational states as it equivalent algebraic semantics and allows expressing vague events and probabilistic assertions within the same sentence, offering a coherent and expressive framework for reasoning with vague events under uncertainty.

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Craig interpolation from Horn semantics

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In this work we consider semantics of a logic in a class of first order structures axiomatized by universal Horn sentences, a *Horn class*. We give conditions on such a semantics which ensure that an amalgamation property for the Horn class implies Craig interpolation for the logic. This generalizes the well-known result that for an algebraizable logic the amalgamation property for the associated class of algebras implies Craig interpolation.

Our main technical device is the notion of *κ-filter pair* ([1], [2], [3]): If κ is a regular cardinal, a κ-filter pair is a functor $G\colon \Sigma\text{-Str} \to \kappa\text{-AlgLat}$ together with a natural transformation to the power set functor $i\colon G \to \wp$, which objectwise preserves infima and κ-directed suprema.

For a κ-Horn theory $\mathbb{T}$, we define a complete lattice: For a Σ-structure A let

- $G^{\mathbb{T}}(A) := \{(\theta, S) \mid \theta$ is a Σ-congruence on A and S an interpretation of $\mathfrak{R}$
  . on A/θ s.t. the resulting Σ^+-structure on A/θ is a $\mathbb{T}$-model$\}$
- We define an order on $G^{\mathbb{T}}(A)$ by declaring $(\theta, S) \leq (\theta', S')$ iff
  $\theta \subseteq \theta'$ and the induced quotient map $q_{\theta\theta'} : A/\theta \twoheadrightarrow A/\theta'$ is a homomorphism of Σ^+-structures for the interpretations S, S'.

Theorem Let τ be a set of *atomic* Σ^+-formulas with at most one free variable, such that $|\tau| < \kappa$. The collection of maps $i^\tau = (i^\tau_A)_{A\in\Sigma-Str}$, defined by

$$\begin{array}{rcl} i^\tau_A : G^{\mathbb{T}}(A) & \to & (\mathcal{P}(A), \subseteq) \\ (\theta, S) & \mapsto & \{a \in A \mid \forall\varphi(x) \in \tau :\ A/(\theta, S) \vDash \varphi(a)\} \end{array}$$

then $(G^{\mathbb{T}}, i^\tau)$ is a κ-filter pair.

Using the formalism of filter pairs, we can prove a general Craig Interpolation result:

Theorem Let $(G^{\mathbb{T}}, i^\tau)$ be a Horn semantics for a logic L. Suppose that the filter pair $(G^{\mathbb{T}}, i^\tau)$ has the "theory lifting property". If $\mathbf{K} := \mathrm{Mod}(\mathbb{T})$ has the atomic amalgamation property, then the logic L associated to $(G^{\mathbb{T}}, i^\tau)$ has the Craig entailment property.

The "theory lifting property" is a technical condition, satisfied by every filter pair presenting an equivalent Horn Semantics, but also in other cases.

In the talk we will review the notion of filter pair and explain the above results with examples.

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Left and Right Effective Actions on Baire and Cantor Space

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The symmetric group $\mathrm{Sym}(\omega)$ acts on Baire space ω^ω and Cantor space 2^ω in two canonical ways: on the left, by relabeling values, and on the right, by reindexing positions. Restricting these actions to effective subgroups, yields natural countable Borel equivalence relations. Writing E^c_L and E^c_R for the orbit equivalence relations of the computable left and right actions, respectively, $E^c_R(2^\omega)$ is exactly recursive isomorphism of subsets of ω. By a classical result of Dougherty and Kechris, $E^c_R(\omega^\omega)$ is universal among countable Borel equivalence relations; Marks later showed that the same is true for $E^c_R(k^\omega)$ for every $k \geq 3$, while the case $k = 2$ remains a long-standing open problem [1,3]. By contrast, the left action—although equally canonical from the group-action viewpoint—has, to our knowledge, not been studied systematically.

In this talk, we begin to fill this gap. We compare the orbit equivalence relations induced by the left and right actions of the computable and fully primitive recursive symmetric groups on Baire and Cantor space, under computable, continuous, and Baire class 1 reducibility.

The resulting picture is unexpectedly rich: sharp separations at the computable level coexist with collapses under weaker notions of reducibility. Thus these elementary permutation actions provide a natural test case for recent work on finer effective reducibilities for equivalence relations [2].

This is joint work with Jun Le Goh, Heer Tern Koh, and Keng Meng Ng.

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SS15

A double-categorical approach to coalgebraic modal logic

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Science is an iterative process; both our understanding of a phenomenon and the aspects of it that interest us evolve. This poses a challenge — how can we efficiently adjust our models of reality to address these changes each time they happen? This talk proposes a framework for producing 'updated understandings' via models of a particular kind: those described by predicates within a (generalised) regular logic over a given context.

Our starting point is a regular hyperdoctrine $\mathcal{C}^{\mathrm{op}} \xrightarrow{P} \mathrm{Cat}$ on a cartesian category $\mathcal{C}$: we view $\mathcal{C}$ as consisting of *contexts* and maps thereof, and P as the specification of categories of *predicates* regarding each context. This is a 'model of comprehension' that is comprised of three main ingredients: the environment (understood by the specification of the category of contexts $\mathcal{C}$), the choice of semantics (determined by the functor P), and the logical language (that includes a special connective determined by the monoidal structure on predicates, usually $\wedge$). Provided this data, the proposed 'update protocol' should be flexible enough to adjust all three factors while maintaining satisfaction of the necessary compatibility constraints. An instructive example is the problem of adapting a static specification into a *temporal* one, or changing contexts from *states* to *systems* of some sort (e.g., automata).

This talk will discuss a double-categorical perspective on this problem. Using the tools of 2-dimensional category theory, I'll explain when a regular hyperdoctrine can be lifted to one whose contexts are coalgebras for a comonad and whose predicates are algebras for a monad. If time permits, I will discuss the connection to traditional 'coalgebraic modal logic' and an interesting follow-up question: given a flavour of system, what is the natural logic that describes it? Dually, what sort of system is naturally described by a given logical modality?

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An equational axiomatisation for metrically complete ℓ-groups

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The purpose of this talk is to present an equational description of metrically complete unital lattice-ordered groups. Let $(G, 1)$ be a unital Abelian ℓ-group. The order unit 1 induces a (pseudo)norm $\|\cdot\|_\ell$ and hence a (pseudo)metric $d_\ell(g,h) = \|g-h\|_\ell$; the kernel of $\|\cdot\|_\ell$ consists precisely of the infinitesimal elements of G, so d_ℓ is a genuine metric exactly when $(G,1)$ has no nonzero infinitesimals, i.e. when it is Archimedean. We call *metrically complete* any Archimedean unital ℓ-group that is Cauchy complete with respect to this metric (see [1]).

Our starting point is Mundici's equivalence between unital Abelian ℓ-groups and MV-algebras. Under the equivalence, the unit interval $\Gamma(G,1) = \{g \in G \mid 0 \le g \le 1\}$ becomes an MV-algebra A, and the restriction of $\|\cdot\|_\ell$ yields a canonical (pseudo)metric d_{MV} on A. An ℓ-group $(G,1)$ is Archimedean if and only if $\Gamma(G,1)$ is semisimple and $(G,1)$ is metrically complete exactly when $\Gamma(G,1)$ is metrically complete. Consequently, the category $\overline{\mathsf{G}}$ of metrically complete ℓ-groups is equivalent to the category $\overline{\mathsf{MV}}$ of metrically complete MV-algebras.

The core result is an algebraic description of metric completeness. We introduce an infinitary expansion CMV of the variety of MV-algebras by adding a single operation γ of countable arity. For a sequence $\mathbf{x}$, the intended meaning of $\gamma(\mathbf{x})$ is the limit of a sequence canonically associated with $\mathbf{x}$: using a family of MV-terms $\rho_n(x_1, \dots, x_n)$ we transform an arbitrary sequence into a *fast* sequence, i.e. one in which the distance between the n^{th} term and the next is bounded by $2^{-(n+1)}$. A metric space is complete if and only if all fast sequences converge. We prove that every metrically complete MV-algebra carries a unique γ making it a CMV-algebra, and that homomorphisms of MV-algebras between such structures automatically preserve γ. In particular, the forgetful functor $\mathsf{CMV} \to \mathsf{MV}$ is full and faithful and identifies CMV with the full subcategory $\overline{\mathsf{MV}}$. Moreover, although γ is infinitary, we show that the defining axioms of CMV can be chosen to be *finite* in number.

One can then speak of *free* metrically complete ℓ-groups as the ones that correspond to free CMV-algebras, through the above equivalence. In the standard algebra $[0,1]$, the CMV-term operations are exactly the continuous maps $[0,1]^\kappa \to [0,1]$ that decrease denominators ("a-maps"), and the free CMV-algebra on a set I is realized as the algebra of such maps on $[0,1]^I$. Finally, we prove that the category of metrically complete ℓ-groups is not equivalent to any finitary variety and, more strongly, not equivalent to any elementary class of structures in a finitary language. The argument uses an obstruction to finite concreteness for $\mathsf{KH}^{\mathrm{op}}$ (via Gelfand duality) and the duality between metrically complete ℓ-groups and normal a-spaces [2], showing that no faithful functor $\mathsf{CMV} \to \mathsf{Set}$ can preserve directed colimits.

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The cellular-Lindelöf property and cardinal invariants of (and above) the continuum

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A topological space X is said to be *cellular-Lindelöf* (see [2], [4]) if for every family of pairwise disjoint non-empty open subsets of X (henceforth called a *cellular family*) there is a Lindelöf subspace L of X which has non-empty intersection with every member of the family. A space X is called *almost cellular-Lindelöf* [1] if for every cardinal κ and for every cellular family $\mathcal{U}$ in X such that $|\mathcal{U}| = \kappa$ there is a Lindelöf subspace L of X which has non-empty intersection with κ many members of $\mathcal{U}$.

Cellular-Lindelöf spaces are a common generalization of Lindelöf spaces and spaces with the countable chain condition, that was originally motivated by the problem of finding a common extension to Arhangel'skii's Theorem and the Hajnal-Juhász inequality (see [2], [3]).

After giving an introduction to cellular-Lindelöf spaces, we shall present three new examples regarding this class of spaces, the first two of which solve questions of Ofelia Alas, Luis Enrique Gutiérrez-Dominguez and Richard Wilson from [1]. Their construction involves the classical cardinal invariants of the continuum (or their stepped-up versions). Namely, we will construct:

(1) A consistent example (from $\mathfrak{t}_{\omega_1} = \omega_2$) of a normal almost cellular-Lindelöf space which is neither cellular-Lindelöf nor weakly Lindelöf.
(2) A consistent example (from $\mathfrak{b} = \omega_1$) of a cellular-Lindelöf Tychonoff space with uncountable weak Lindelöf degree for closed sets whose Lindelöf degree doesn't exceed ω_1.
(3) A ZFC example of a space whose cellular-Lindelöf property is independent of ZFC (and whose normality also turns out to be independent of ZFC).

Our talk is based on joint work with Rodrigo Hernández-Gutiérrez (UAM, Mexico City) from [5].

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