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BOOK OF ABSTRACTS

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On the Asymptotic Behavior of Transmission Problems for the Wave Equation

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In this talk, we study the behavior of a string made up of different types of materials, including purely elastic regions (which are conservative) and regions with dissipative effects such as viscoelastic damping and thermoelasticity. Because the physical properties of these materials, such as density and stiffness, may vary from one region to another, the model involves equations with discontinuous coefficients. The analysis of these type of models, known as a transmission problems is fundamental in the study of asymptotic behavior of composite bodies, as real-world structures are rarely homogeneous and instead consist of multiple materials with distinct mechanical and thermal properties. Interfaces between the media introduce nontrivial coupling conditions that significantly affect energy propagation, dissipation and long time dynamics.

We discuss the asymptotic properties of the solutions when one of the dissipative mechanisms is a localized Kelvin–Voigt damping. In particular, we show how the position of the different components plays an important role in the study of stabilization. We analyze how, in certain configurations, the exponential stability of the system, achieved through other types of dissipation, can be destroyed by localized Kelvin–Voigt damping with a discontinuous coefficient at the interface..

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Preserving Information: Kolmogorov Entropy of Numerical Conservation Laws

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Motivated by the information-theoretic framework introduced by P. D. Lax, this work investigates the quantitative compactness of numerical solutions to scalar conservation laws with uniformly convex flux. While the qualitative convergence of classical numerical schemes is well established, a rigorous quantification of their information-theoretic resolution has remained an open challenge.

We address this gap by analyzing the Kolmogorov ε -entropy of the set of approximate solutions generated by conservative, monotone finite-difference schemes. We prove that, under suitable grid constraints, schemes satisfying a discrete one-sided Lipschitz condition preserve the optimal $1/\varepsilon$ entropy scaling exhibited by the corresponding set of exact entropy solutions. This upper bound is shown to be sharp: it matches the classical continuous bounds established by De Lellis–Golse and Ancona–Glass–Nguyen, with the lower bound recovered via a uniform approximation argument rooted in a bounded-variation precursor class.

Finally, we abstract the mechanism underlying the lower bound into a general transfer principle. We discuss the immediate consequences of these findings for information recovery through numerical post-processing and outline how this framework establishes a rigorous, information-theoretic validation of the high-resolution character of prototypical first-order schemes.

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Polynomial Stability of the Wave Equation with Pure Boundary Memory: a Frequency-Domain Approach

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We consider the wave equation in a bounded domain with mixed boundary conditions, where the feedback on Γ_1 is of *pure memory type*, with no instantaneous friction term. In contrast with models involving an additional boundary damping u_t , the dissipation generated solely by the memory kernel does not yield, in general, uniform exponential stability. We prove instead that the associated C_0 -semigroup is polynomially stable. The proof relies on a resolvent estimate on the imaginary axis combined with the Borichev–Tomilov characterization of polynomial decay on Hilbert spaces. For the infinite-history (fading memory) case, we prove that the associated C_0 -semigroup is polynomially stable. For the finite-memory case, where the dynamics is no longer time-translation invariant, we establish well-posedness by a Galerkin method and derive polynomial decay of the energy by means of a suitable Lyapunov functional.

Agent-based models and MFG with large discount

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The theory of MFG describes equilibria in a large population of players, where agents aim to optimize some cost that depends on a discount factor. If one assumes that the discount factor is large, these models turn out to behave similarly to agent-based models, where individuals just react to the population distribution according to a given rule. Inspired by the work [1], we will elaborate on this connection, showing in particular the uniqueness of equilibria in MFG with large discount, and their converge as the discount goes to infinity to certain agent-based models, in a quantitative sense.

To make such a connection, the central idea will be to see the MFG as a sort of nonlinear and nonlocal in time evolution, where, instead of time delay effects, agents have the ability to forecast near future events.

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On Pareto and Nash equilibria for Boussinesq fluids with multiplicative controls

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Abstract

In this work, we analyze Pareto and Nash equilibrium problems in an optimal control setting for a generalized Boussinesq model describing the evolution of the velocity field, the pressure, and the temperature of a viscous, incompressible fluid subject to thermal effects. The system features multiplicative controls acting simultaneously in the velocity and temperature equations, thereby creating a tightly coupled dynamical setting in which strategic interactions naturally emerge. The main contributions of this work are as follows. First, we develop a framework to address multiple control criteria. By assigning positive weights to each criterion, the resulting multi-criteria problem is reduced to a single, well-posed minimization problem. In this setting, identifying controls that properly balance the different criteria, namely Pareto optimal controls, is equivalent, under suitable conditions, to solving the corresponding weighted optimization problem. Secondly, we show that the competitive multiplayer model associated with the system, equipped with quadratic cost functions and appropriate control sets, admits at least one Nash equilibrium, which can be characterized by means of a coupled optimality system.

Mixed-Order Damping Strategies for Hybrid Flexible Beams

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In this talk, we introduce a SCOLE-type hybrid PDE-ODE system consisting of two Euler-Bernoulli beams serially connected at an interior joint, where the joint carries both a point mass and a rotational inertia, and the coupling is imposed through dynamic transmission conditions, while each sub-beam preserves classical Euler-Bernoulli dynamics. For a class of boundary-interface velocity feedback laws combining lower-order and higher-order traces at the joint, we establish a complete classification of the system's asymptotic behavior.

First, we consider the case where boundary dissipation is active. When at least one higher-order interface feedback term is present, the associated C_0 -semigroup is exponentially stable, without arithmetic restrictions on material parameters, feedback gains, or the interface location. In contrast, if higher-order interface feedback is replaced by lower-order damping alone, exponential stability fails and the total energy decays at the sharp polynomial rate t^{-1} .

Then, we consider the case where boundary dissipation is violated. In this scenario, exponential decay is achieved only when both controllers are higher-order. Any other admissible pairing (that is, any combination that does not consist exclusively of higher-order feedback at both the boundary and the interface) yields only polynomial decay of order t^{-1} or t^{-2} .

The proofs rely on sharp frequency-domain resolvent estimates within the Borichev-Tomilov framework, which is shown to be essential due to the failure of classical spectral, multiplier, and Riesz basis methods for this class of interface-inertial generators. These findings refine the SCOLE paradigm and provide new insights into the distributed control of large-scale networks of flexible structures.

References

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Time-delayed generalized Korteweg–de Vries–Burgers equation: well-posedness and exponential decay

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This work is devoted to the analysis of a generalized Korteweg–de Vries–Burgers equation with delay feedback and damping. More precisely, we consider the system

$$(1) \quad \begin{cases} u_t(t, x) + u_{xxx}(t, x) - u_{xx}(t, x) + h(x)u(t, x) + \lambda(x)u(t - \tau, x) \\ \quad + a(u)u_x(t, x) = 0 \quad \text{in } \mathbb{R} \times (0, \infty), \\ u(x, s) = u_0(x, s) \quad \text{in } \mathbb{R} \times [-\tau, 0], \end{cases}$$

where $\tau > 0$, $h, \lambda \in L^\infty(\mathbb{R})$, and $a \in C^1(\mathbb{R})$ is a positive nonlinear coefficient satisfying suitable growth conditions, namely

$$|a(\mu)| \leq C(1 + |\mu|^r), \quad |a'(\mu)| \leq C(1 + |\mu|^{r-1}), \quad 1 \leq r < 2.$$

Using semigroup arguments, we first establish the existence of a global solution under these growth assumptions. We then investigate the long-time behavior of the associated energy by means of suitable Lyapunov functionals. In particular, we prove exponential stability estimates under appropriate assumptions on the damping and delay coefficients.

A first stability result is obtained when the damping coefficient is uniformly positive, that is, when there exists $\alpha_0 > 0$ such that

$$h(x) \geq \alpha_0 \quad \text{for a.e. } x \in \mathbb{R}.$$

In this case, the delay coefficient is allowed to satisfy

$$|\lambda(x)| \leq \alpha + g(x), \quad g \in L^p(\mathbb{R}), \quad 1 \leq p < \infty,$$

with

$$0 \leq \alpha < \alpha_0, \quad |g|_p < \left(\frac{\alpha_0 - \alpha}{c(p)} \right)^{1 - \frac{1}{2p}},$$

where $c(p) = \left(1 - \frac{1}{2p}\right) \left(\frac{2}{p}\right)^{\frac{1}{2p-1}}$. We also extend the analysis to a more general framework in which both the damping and the delay feedback coefficients may change sign.

The main novelty of this work is twofold. First, we study the well-posedness and stability of a generalized Korteweg–de Vries–Burgers equation with time delay and a general nonlinear term. Second, we improve the existing stability theory by deriving assumptions on the delay feedback coefficient that are independent of the delay size. This represents an improvement over the conditions used, for instance, in Komornik and Pignotti [1], where the stability assumptions involve the delay parameter τ .

References

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Recent Advances on the Scattering Conjecture for Nonlinear Schrödinger Equations

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In this talk, we discuss recent developments in scattering theory for critical nonlinear dispersive equations. A fundamental example is the nonlinear Schrödinger equation

$$i\partial_t u + \Delta u + |u|^\alpha u = 0,$$

where $\alpha = \frac{4}{N-2}$. This equation arises in several models of mathematical physics, including quantum mechanics and nonlinear optics.

We review some of the main advances in the study of scattering for critical equations, a central problem concerning the long-time behavior of global solutions. Over the last decades, this topic has attracted considerable attention, leading to important contributions by Kenig and Merle and, later, by Killip, Visan, Dodson, and others.

Finally, we present recent scattering results for critical inhomogeneous nonlinear Schrödinger equations and discuss some related open problems..

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Z-Control for Exponential Consensus in High-Order Multi-Agent Systems

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Consensus in multi-agent systems is a central problem in control and collective dynamics, with applications ranging from robotics to social systems and epidemiology [1,2]. We study consensus control for high-order multi-agent systems by prescribing the consensus objective as a hierarchy of exponentially stable error dynamics within a Z-control framework [3]. The approach includes two constructions: a direct state-feedback law, acting on the highest-order state, and an indirect design in which the control input is applied to lower-order dynamics. In the direct case, for weight-balanced interactions, the method preserves the average and ensures exponential convergence to the desired consensus value. In contrast, the indirect construction leads to a state-dependent algebraic problem whose solvability and conditioning must be assessed along the system trajectory, resulting in a centralized model-based design. Numerical experiments on opinion dynamics and Cucker–Smale-type [4] models illustrate convergence properties, control effort, and solver behaviour, highlighting both the effectiveness of Z-control in the error-dynamics formulation and the limitations of indirect actuation in ill-conditioned regimes.

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Higher-Order No-Gap Conditions in optimal impulsive control via Compatible Clarke Tangent and QDQ Approximating Cones

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This talk addresses two related problems in optimal control. The first investigation consists of compatibility issues between two classical approaches to deriving necessary conditions for optimal control problems with a final target: the *set-separation approach* and *penalization techniques*. These methods generally lead to non-equivalent conditions, mainly due to their reliance on different notions of tangency at the target. We address this issue by considering *Quasi Differential Quotient (QDQ) approximating cones* (which are fit for the set-separation approach) and identifying conditions under which the Clarke tangent cone (which is a typical tool within penalization techniques) is also a QDQ approximating cone. In particular, we show that this property holds under suitable local invariance assumptions or when the target coincides locally with an *r-prox regular set*. Then, we apply this compatibility result to the study of *infimum-gap phenomena* in optimal control problems with unbounded controls and impulsive extensions. In particular, we establish a connection between the occurrence of infimum gaps for *strict-sense minimizers* and abnormality in a higher-order Maximum Principle involving Lie brackets. While the abnormality-gap correspondence beyond first-order conditions has been already established for impulsive minimizers, the utilization of the above compatibility issues allow us to extend this correspondence to strict-sense minimizers.

Existence of Complementary and Variational Weak Solutions to Obstacle Problems for a Quasilinear Wave Equation

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In this talk, we prove the existence of weak solutions for the one obstacle problem associated with a class of quasilinear wave equations in one space dimension, extending previous results obtained in the linear case, and we also address the two obstacles problem. In contrast with the linear setting, for both strictly quasilinear cases we obtain continuous solutions in a weak complementary sense, which moreover satisfy a weak entropy condition in the free region where the string is not in contact with the obstacles. We further show that, in both the one and two obstacle cases, these solutions are variational solutions in a hyperbolic sense without the viscosity term.

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Stability for wave models with time-dependent memory

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We are concerned with the existence of global solutions as well as with the decay rates of the energy related to the viscoelastic wave equation with time-dependent memory kernels posed on a bounded domain of the 3-D Euclidian space.

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